電子回路論第11回
Electric Circuits for Physicists

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- 6.2.1 Noise figure
- 6.2.2 Noise impedance matching

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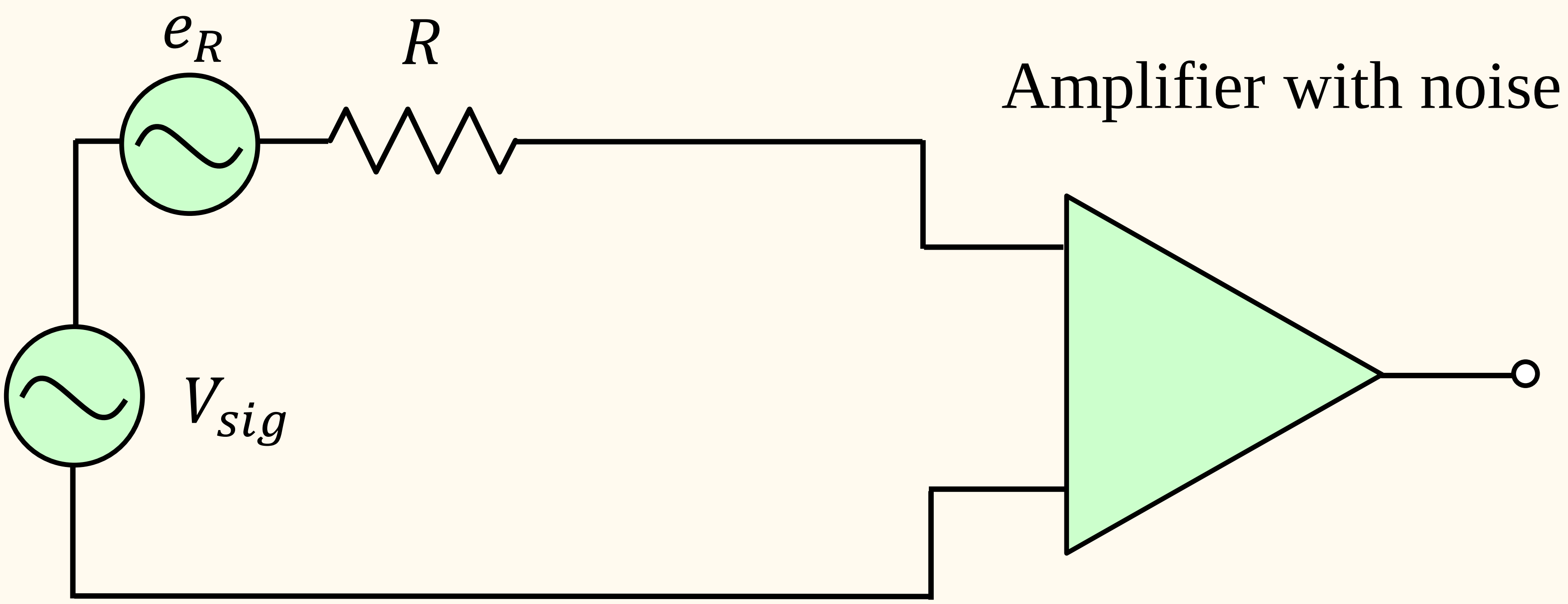
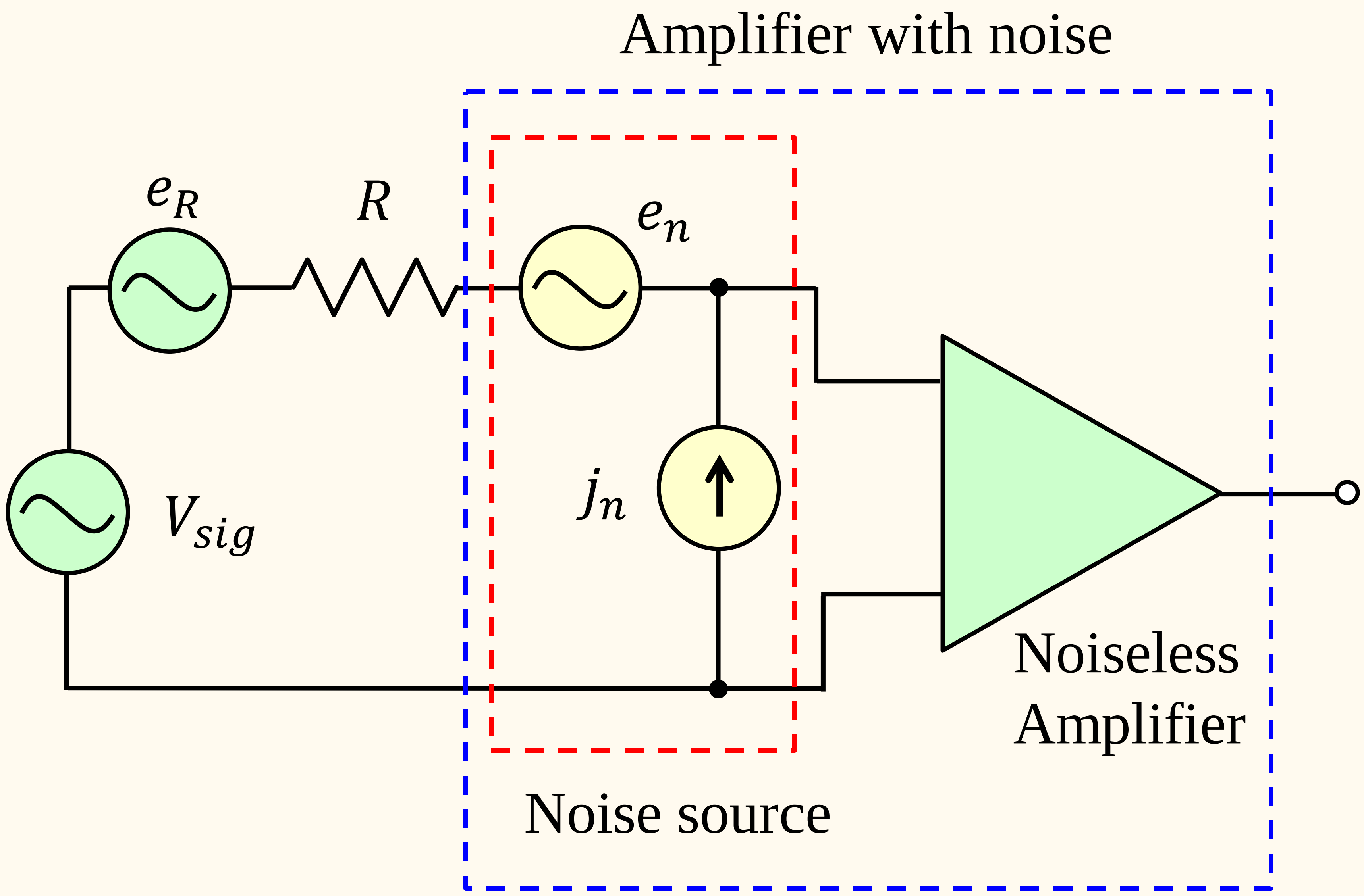
- 6.3.1 Modulation/demodulation
- 6.3.2 Amplitude modulation
- 6.3.3 Angle modulation
- 6.3.4 Demodulation of frequency modulated signal
- 6.3.5 Modulation and noise

FM broadcast test

6.2 Noise from Amplifiers



6.2 Noise from Amplifiers



6.2 Noise from Amplifiers

Amplifiers: the elements have characteristic noises,
power sources work as noise sources

→ Noiseless amplifier + Noise source = Amplifier with noise

Power gain G_p $$e^2_{\mathrm{intotal}} = j_n^2 R^2 + e_{\mathrm{R}}^2 + e_n^2 = e^2_{\mathrm{out}}/G_{\mathrm{p}}$$

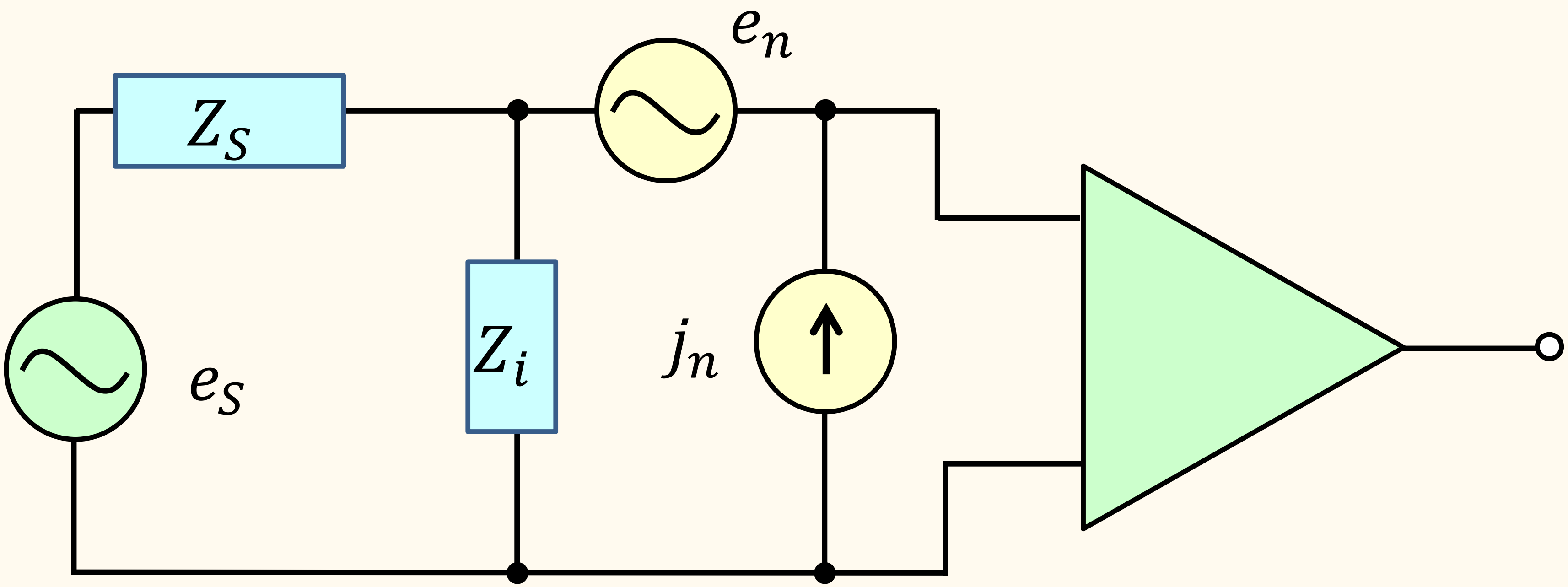
Signal to noise ratio: S/N ratio

Noise Figure: $$\mathrm{NF} = 10\log_{10}\frac{(S/N)_{\mathrm{in}}}{(S/N)_{\mathrm{out}}} = 10\log_{10}\frac{S_{\mathrm{in}}N_{\mathrm{out}}}{S_{\mathrm{out}}N_{\mathrm{in}}}$$

$$N_{\mathrm{out}} = G_{\mathrm{p}}\overline{e_{\mathrm{N}}^2}$$

$$\mathrm{NF} = 10\log_{10}\frac{S_{\mathrm{in}}G_{\mathrm{p}}\overline{e_{\mathrm{N}}^2}}{S_{\mathrm{in}}G_{\mathrm{p}}\overline{e_{\mathrm{R}}^2}} = 10\log_{10}\frac{\overline{e_{\mathrm{N}}^2}}{\overline{e_{\mathrm{R}}^2}} = 10\log_{10}\frac{\overline{e_n^2} + \overline{e_{\mathrm{R}}^2} + \overline{j_n^2}R^2}{\overline{e_{\mathrm{R}}^2}}$$

6.2.2 Noise impedance matching



Optimization of S/N ratio including the noise-source in the amplifier
(a care should be taken to the effect of noise to the object)

Noises from the signal source, amplifiers: repel as much as possible
Signals from the source: absorb …

Noise temperature method: not almighty

6.2.2 Noise impedance matching

Nyquist theorem:

$$\sqrt{\overline{J^2}\ \overline{V^2}} = 2k_\mathrm{B} T \varDelta f$$

Noise temperature definition $(J(f), V(f))$

Noise temperature and matched source impedance

$$T_\mathrm{a} = \frac{\sqrt{\overline{e_n^2}\ \overline{j_n^2}}}{2k_\mathrm{B}}, \quad R_\mathrm{bs} = \sqrt{\frac{\overline{e_n^2}}{\overline{j_n^2}}}$$

Output noise temperature:

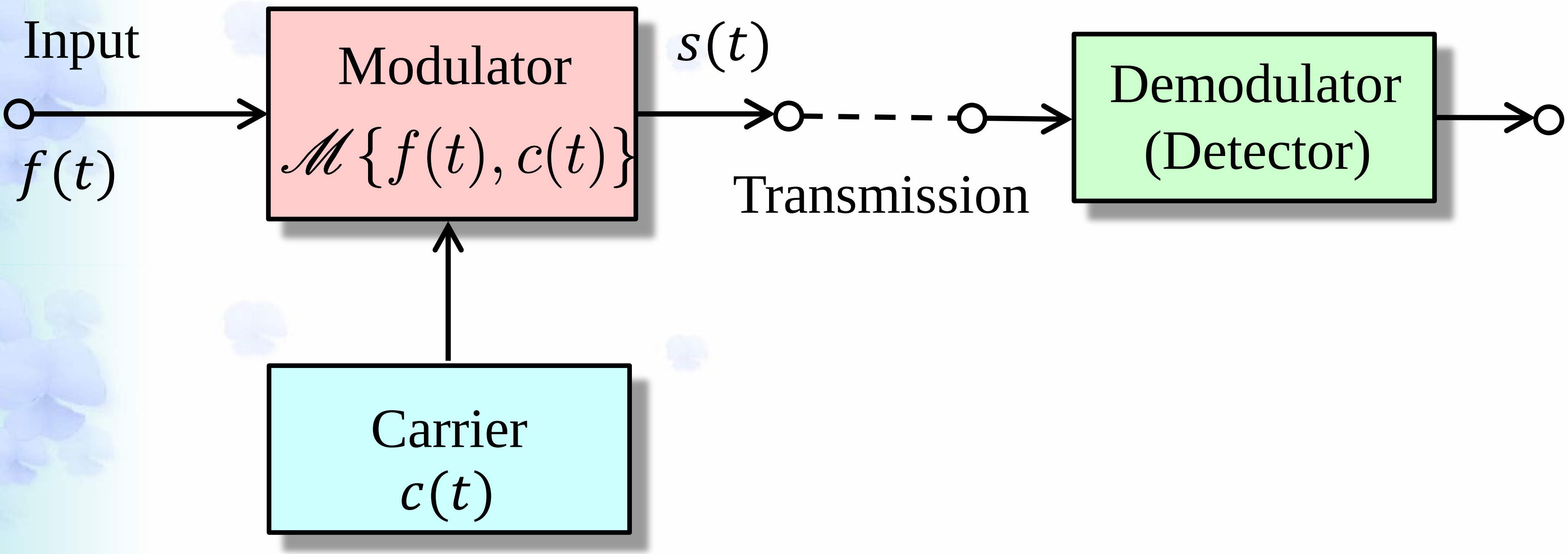
$$T_\mathrm{n} = \left(1 + \frac{\mathrm{Re}(1/Z_\mathrm{i})}{\mathrm{Re}(1/Z_\mathrm{s})}\right) \frac{T_\mathrm{a}}{2\mathrm{Re}Z} \left(\frac{|Z|^2}{R_\mathrm{bs}} + R_\mathrm{bs}\right), \quad \frac{1}{Z} \equiv \frac{1}{Z_\mathrm{i}} + \frac{1}{Z_\mathrm{s}}$$

Minimize T_n: $Z_\mathrm{i} = \dfrac{1}{R_\mathrm{bs}^{-1} - Z_\mathrm{s}^{-1}}$ Noise matching condition

$$T_\mathrm{n} = \left(1 + \frac{\mathrm{Re}(1/Z_\mathrm{i})}{\mathrm{Re}(1/Z_\mathrm{s})}\right) T_\mathrm{a}$$

6.3 Signal transmission

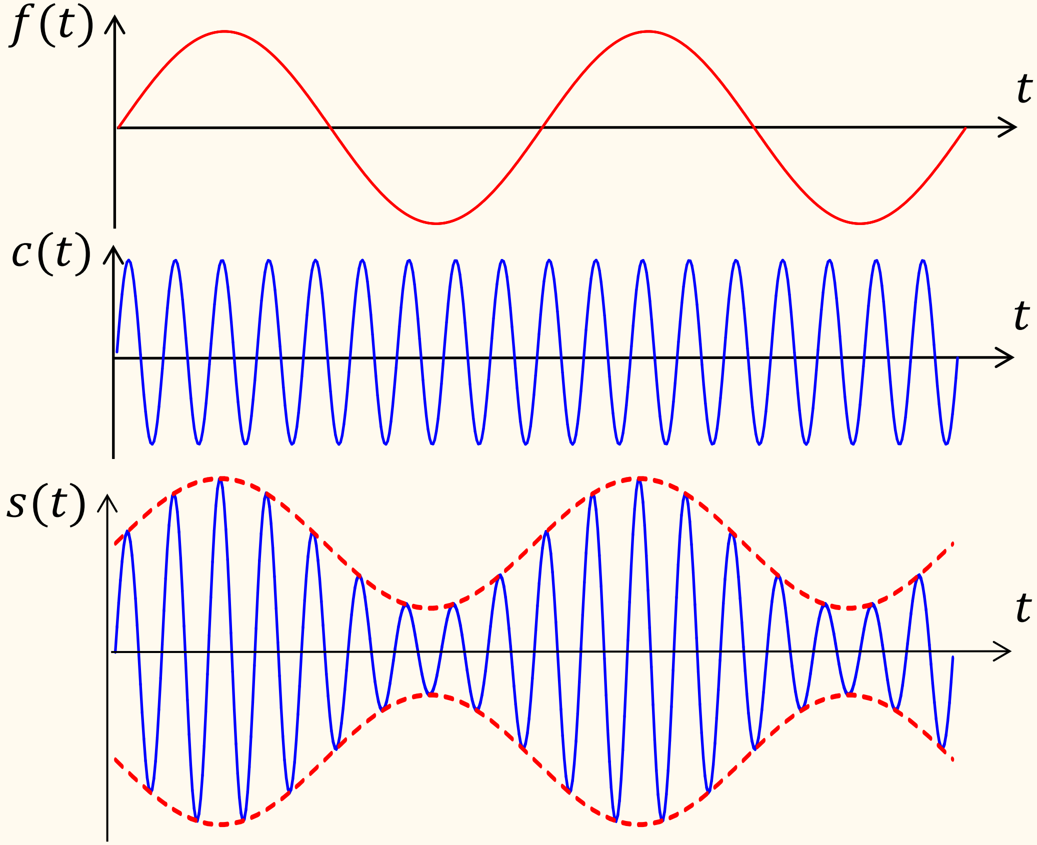
Electric communication
- Baseband communication
- Carrier communication



Modulation
- Amplitude modulation
- Frequency (Phase) modulation

→ Analog / Pulse

6.3.2 Amplitude modulation



$$c(t) = A\cos\omega_c t$$

$$s(t) = A[1 + mf(t)]\cos\omega_c t$$

m: Modulation index

$$0 < m \leq 1$$

$$S(i\omega) = \int_{-\infty}^{\infty} s(t)e^{i\omega t}dt = \int_{-\infty}^{\infty} A[1 + mf(t)]\cos(\omega_c t)e^{i\omega t}dt$$

$$= A\left\{\pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] + \frac{m}{2}[F(i(\omega - \omega_c)) + F(i(\omega + \omega_c))]\right\}$$

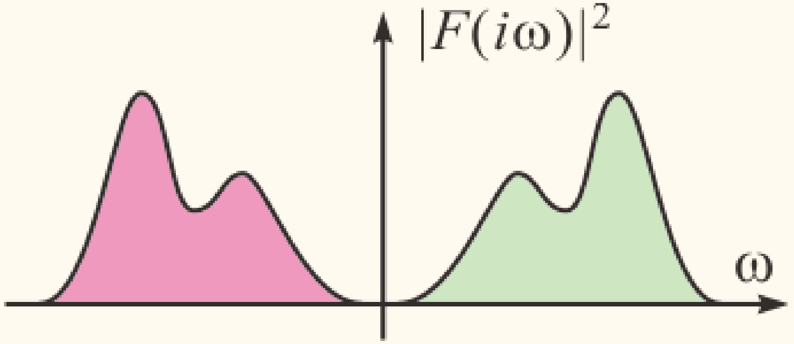
6.3.2 Amplitude modulation

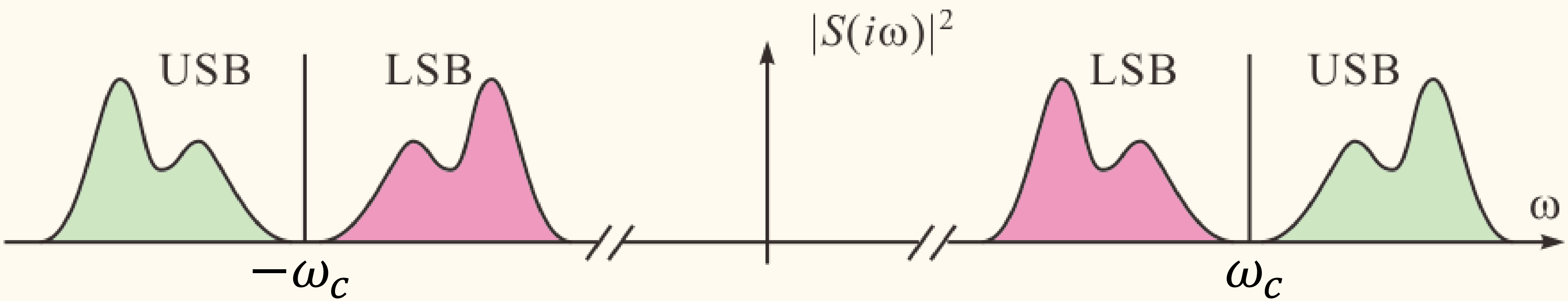
$$S(i\omega) = \int_{-\infty}^{\infty} s(t) e^{i\omega t} dt = \int_{-\infty}^{\infty} A[i + mf(t)] \cos(\omega_c t) e^{i\omega t} dt$$

$$= A \left\{ \pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] + \frac{m}{2}[F(i(\omega - \omega_c)) + F(i(\omega + \omega_c))] \right\}$$

$$F(i\omega) = \mathscr{F}\{f(t)\}$$

$f(t)$: Real $\quad F(i\omega) = F^*(-i\omega)$



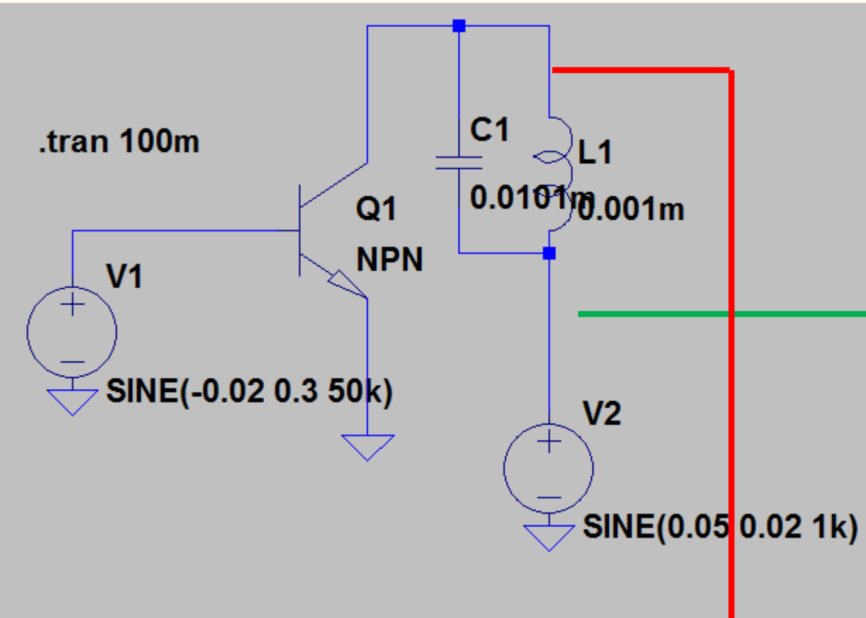


Upper side band (USB), Lower side band (LSB)

6.3.2 Amplitude modulation (circuit example)

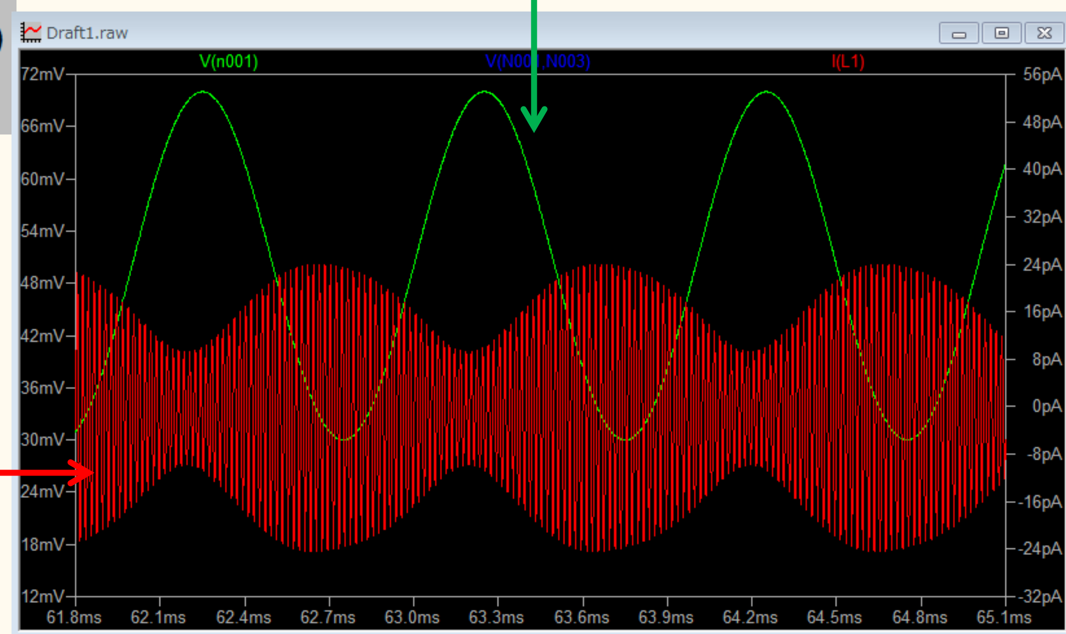
Collector modulation circuit

C-class amplification (non-linear) region



Modulation voltage

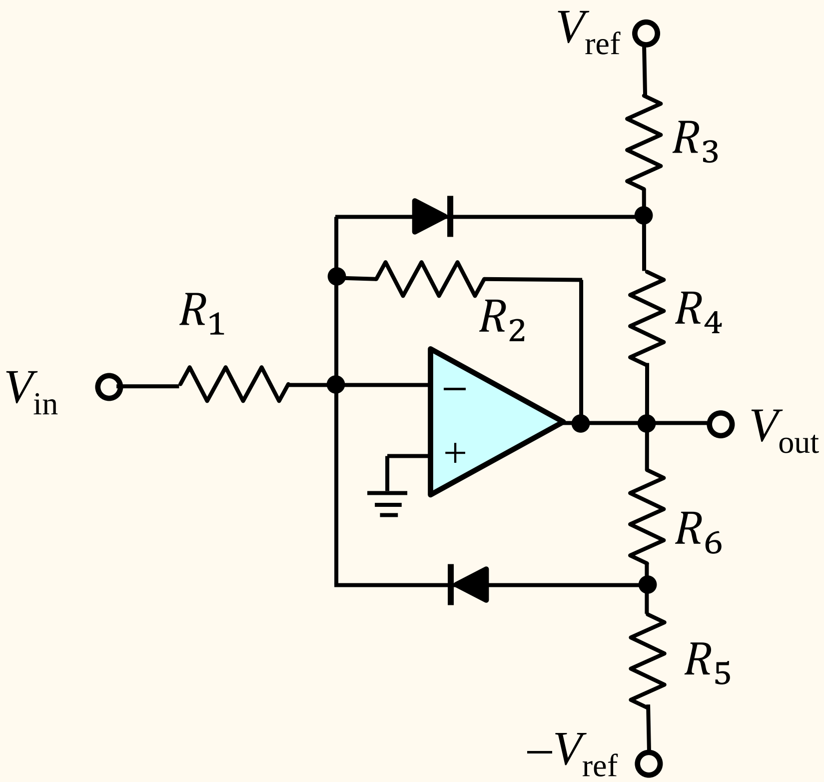
Current through the inductor

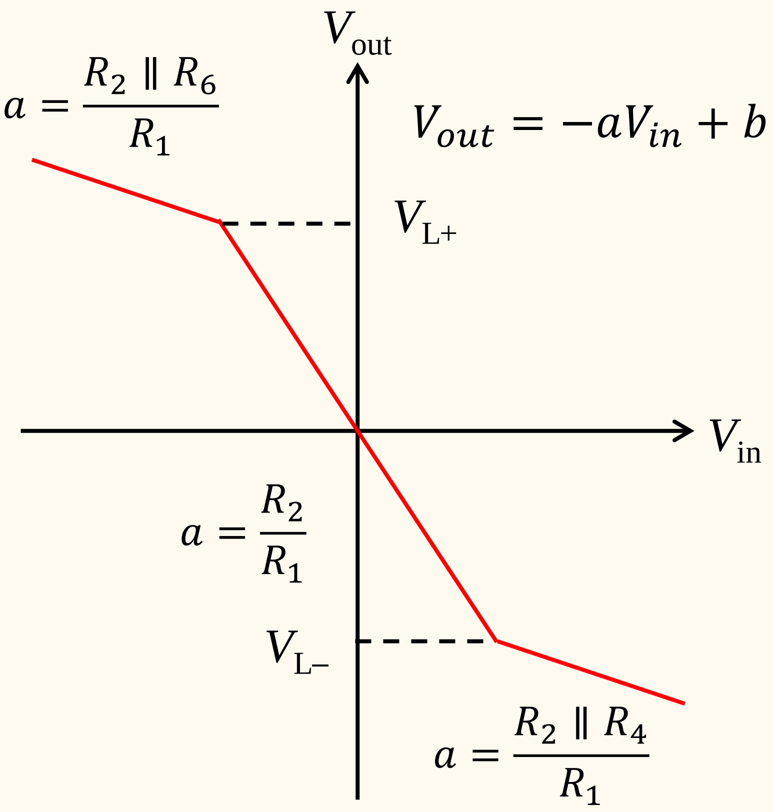


6.3.2 Amplitude modulation (circuit example2)

Modulation of oscillator circuit

Soft limiter circuit

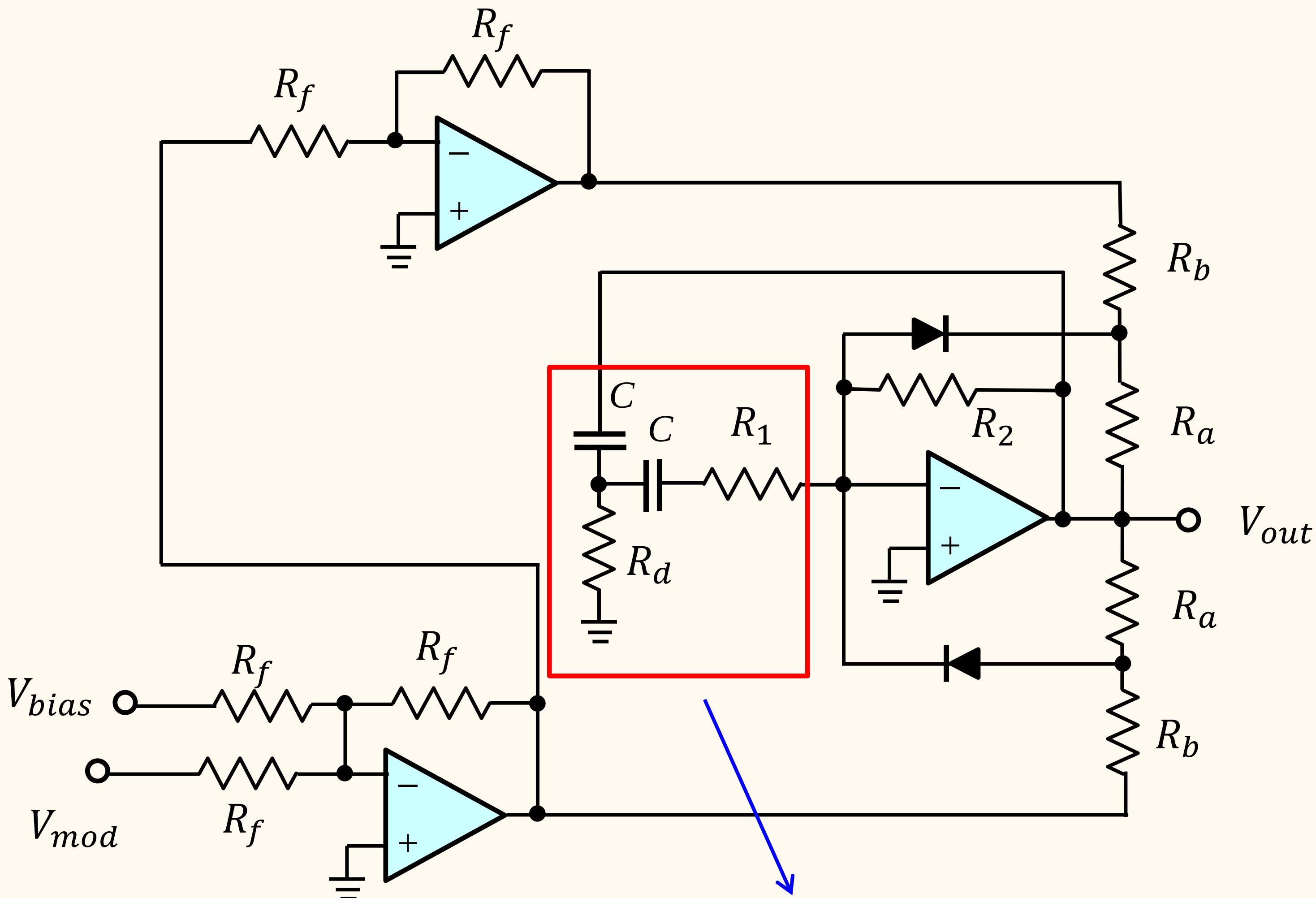




$$V_{L-} = -\frac{R_4}{R_3} V_{ref} - \left(1 + \frac{R_4}{R_3}\right) V_{th}$$

: controllable with V_{ref}

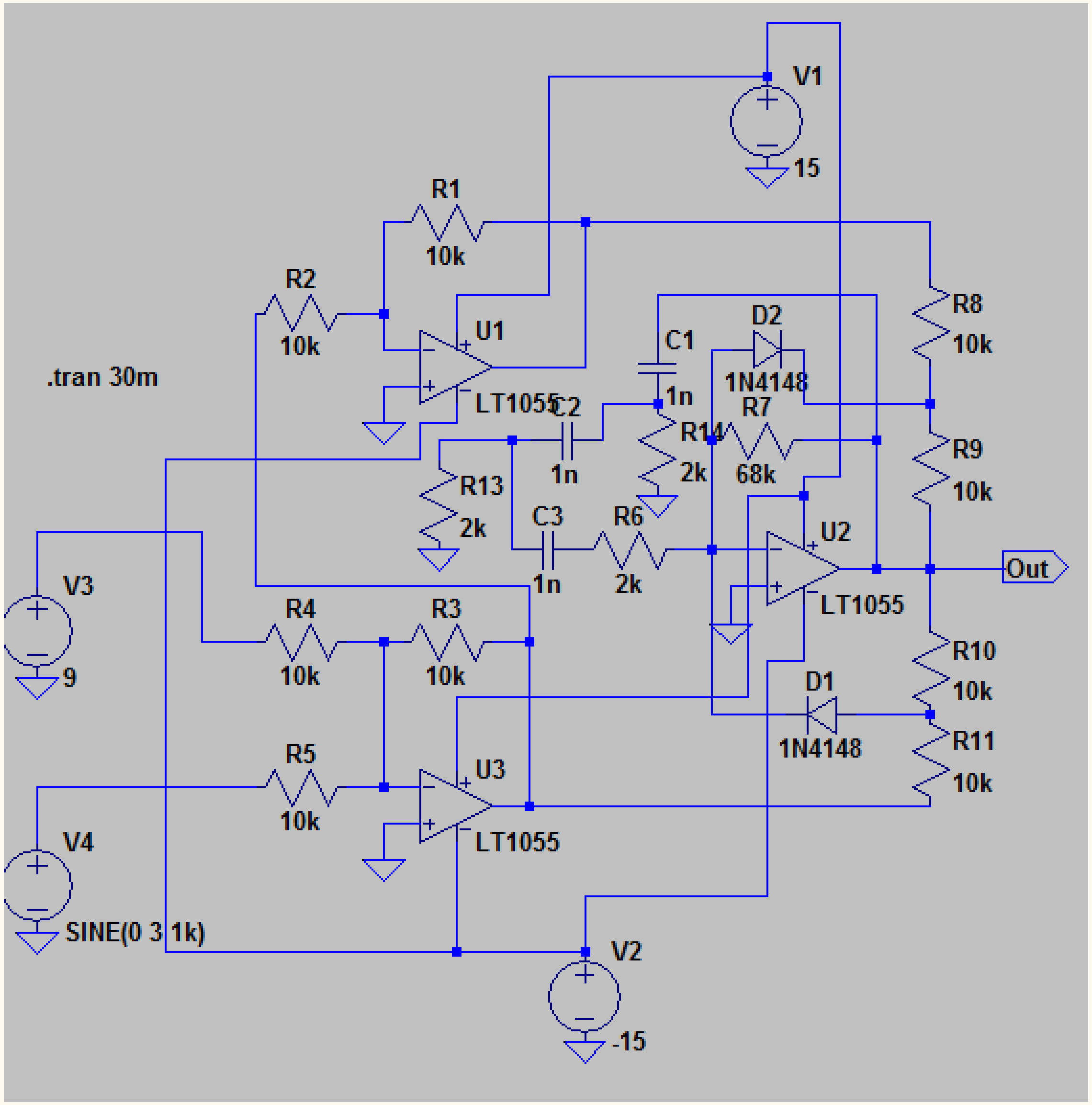
6.3.2 Amplitude modulation (circuit example2)



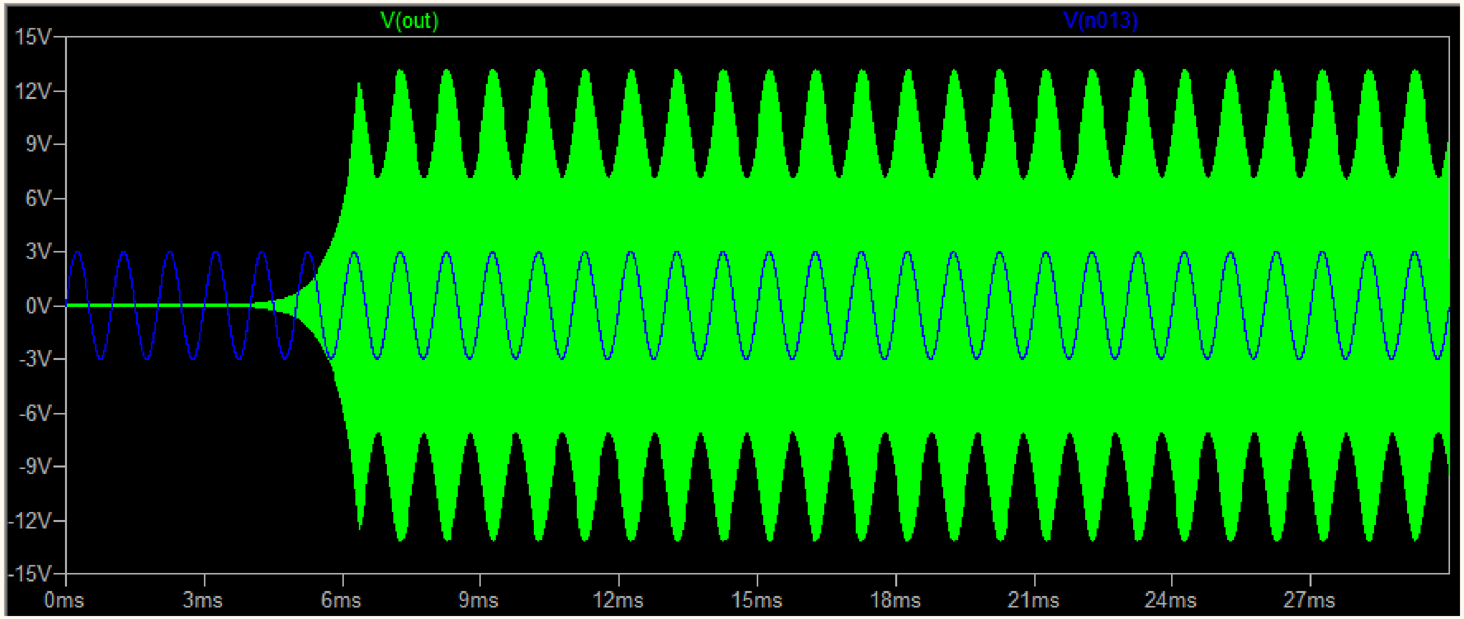
Delayed feedback → Oscillation

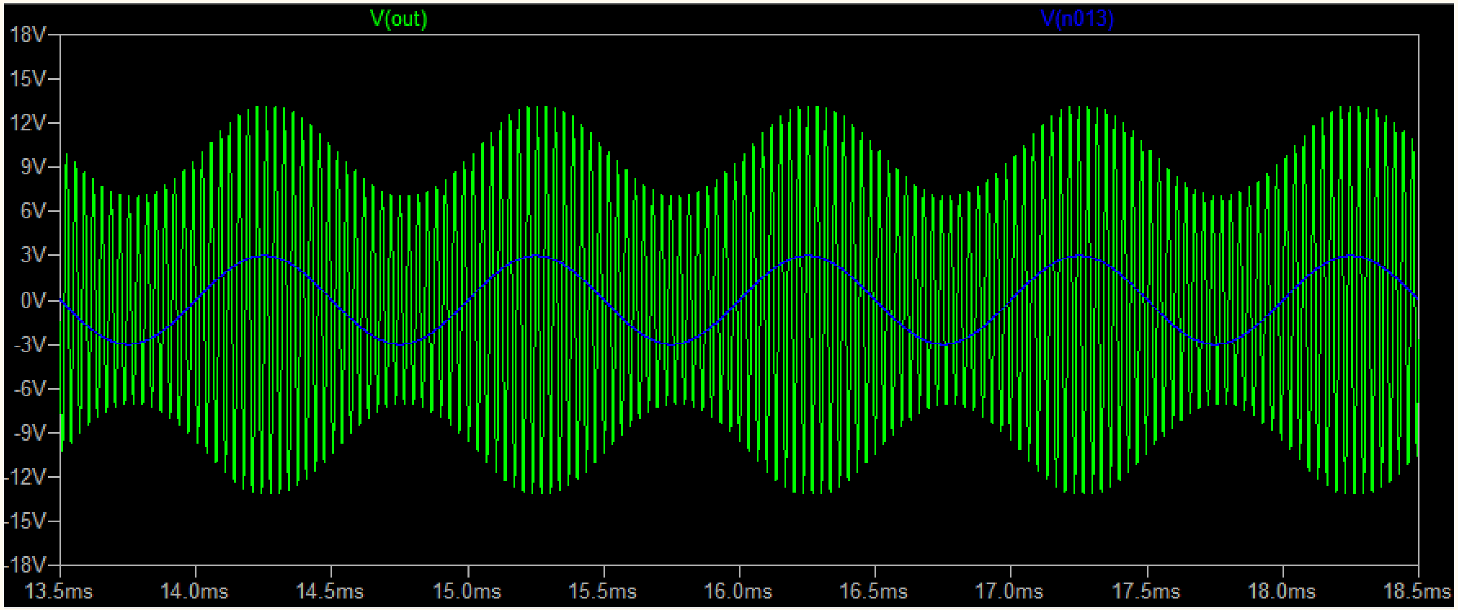
The amplitude is softly limited with the modulation voltage.

6.3.2 Amplitude modulation (circuit example2)

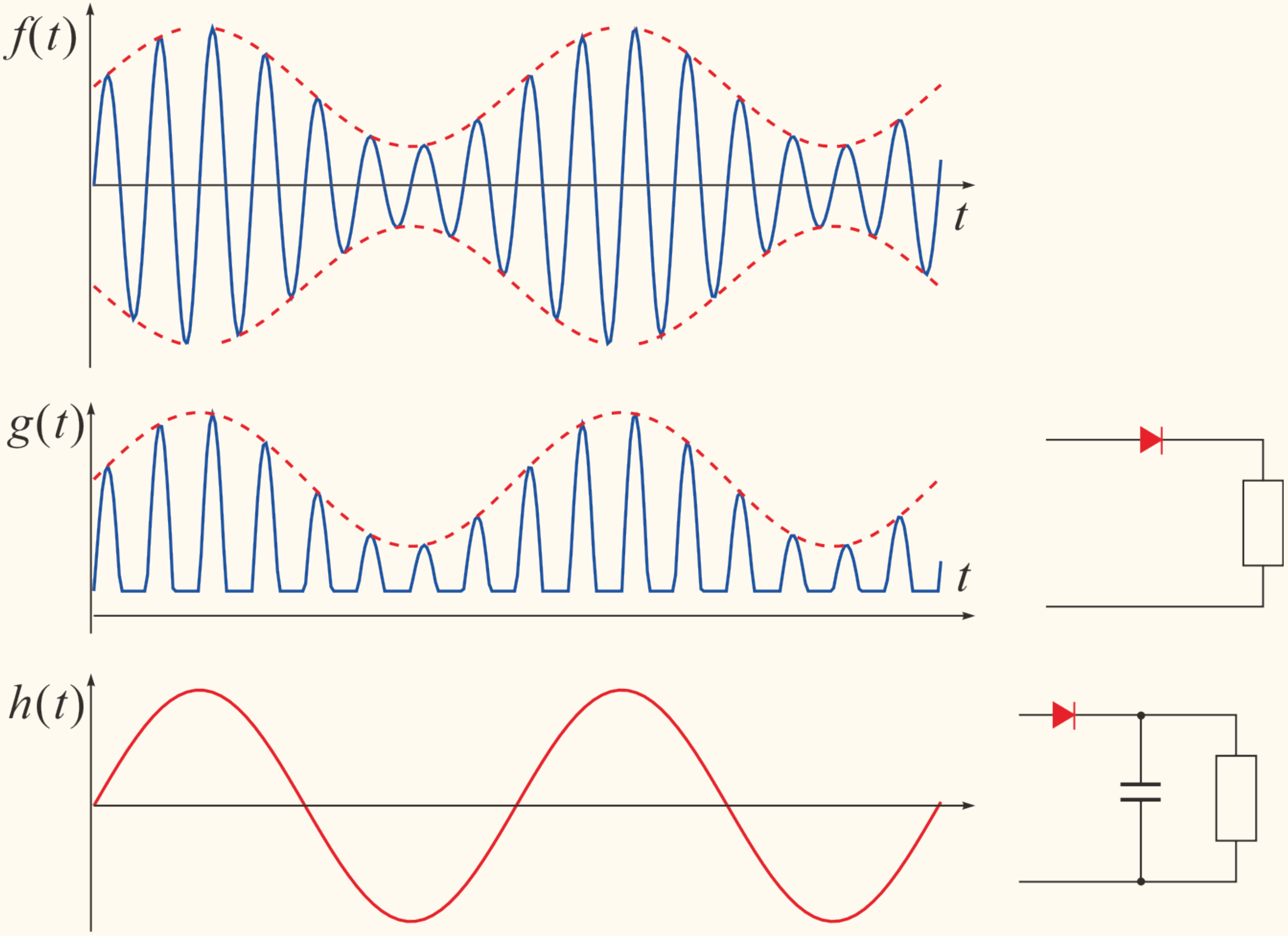


6.3.2 Amplitude modulation (circuit example2)

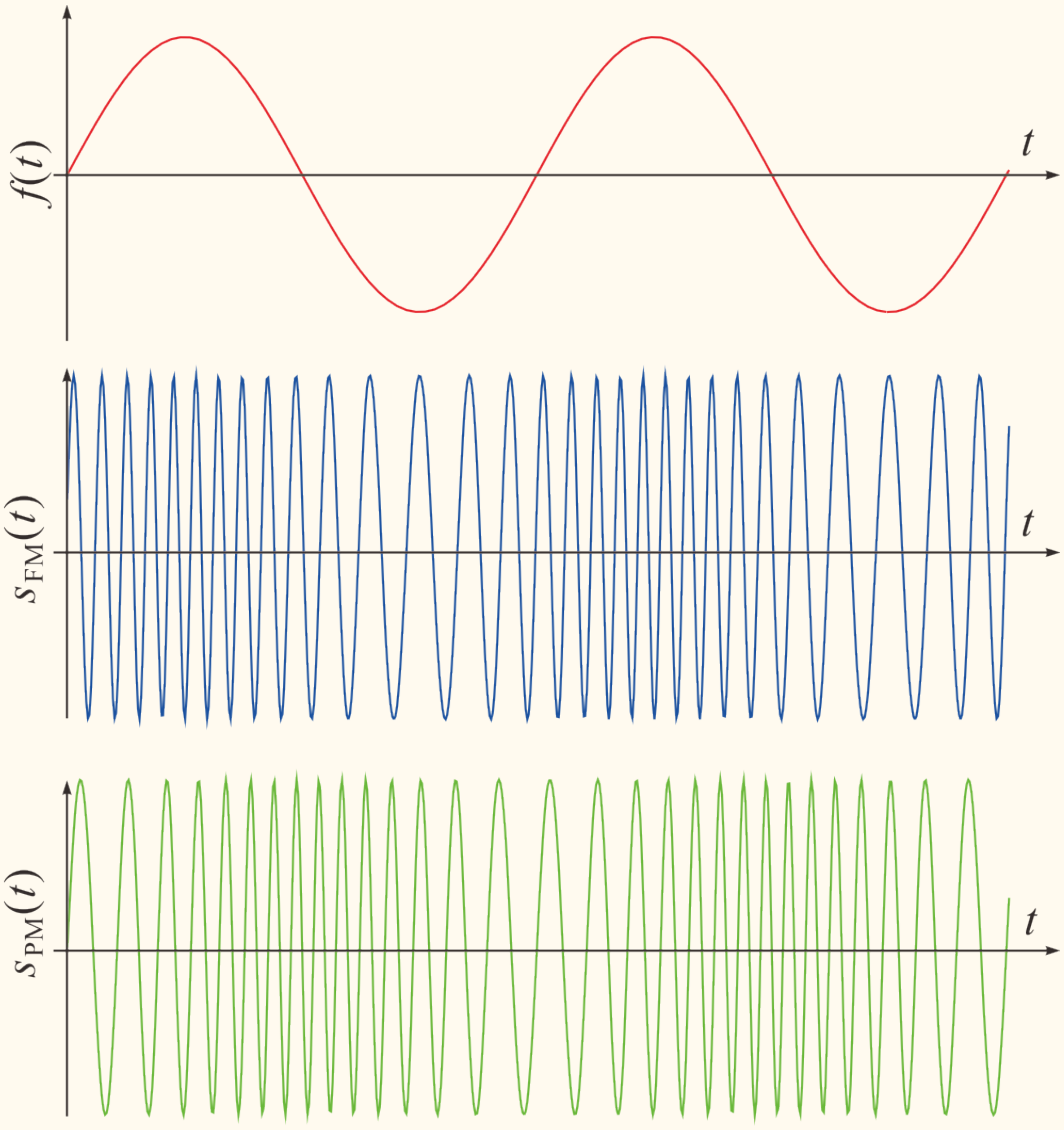




6.3.2 Amplitude modulation (demodulation)



6.3.3 Angle modulation



6.3.3 Angle modulation

$$s(t) = A\cos\theta_{\mathrm{i}}(t), \quad \theta_{\mathrm{i}}(t) = \omega_{\mathrm{c}}t + \phi[t, f(t)]$$

Differential angular frequency $\omega_{\mathrm{i}}(t) = \dfrac{d\theta_{\mathrm{i}}(t)}{dt} = \omega_{\mathrm{c}} + \dfrac{d\phi[t, f(t)]}{dt}$

$$\frac{d\phi[t, f(t)]}{dt} = k_f f(t) \quad (\mathrm{Frequency Modulation, FM}),$$

$$\phi[t, f(t)] = k_p f(t) \quad (\mathrm{Phase Modulation, PM})$$

$$s_{\mathrm{FM}}(t) = A\cos\left[\omega_{\mathrm{c}}t + k_f \int^t f(\tau)d\tau\right],$$

$$s_{\mathrm{PM}}(t) = A\cos[\omega_{\mathrm{c}}t + k_f f(t)]$$

Frequency ω component: only phase shift $\pi/2$:
No difference in signal outlook.

6.3.3 Angle modulation (Frequency modulation)

$$f(t) = A_p \cos \omega_p t$$

$$s_{\mathrm{FM}} = A\cos(\omega_c t + \beta \sin \omega_p t) = A\mathrm{Re}\left[\exp(i\omega_c t)\exp(i\beta \sin \omega_p t)\right]$$

$$\left(\beta \equiv \frac{k_f A_p}{\omega_p} = \frac{\Delta f}{f_p}\right)$$

$\sin \omega_p t$: Periodic function with $T = 2\pi/\omega_p$

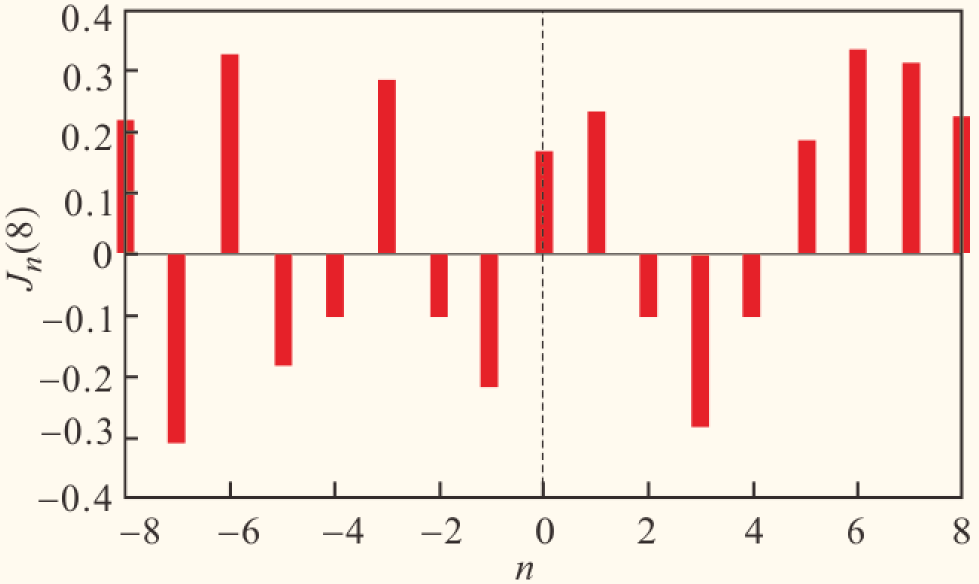
$$\exp(i\beta \sin \omega_p t) = \sum_{n=-\infty}^{\infty} c_n \exp(i\omega_p t),$$ Fourier series expansion

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} \exp(i\beta \sin \omega_p t)\exp(-in\omega_p t)dt$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp[i(\beta \sin \theta - n\theta]d\theta = J_n(\beta)$$

First kind Bessel function

6.3.3 Angle modulation (Frequency modulation)



$$s_{\mathrm{FM}}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[(\omega_{\mathrm{c}} + n\omega_p)t]$$

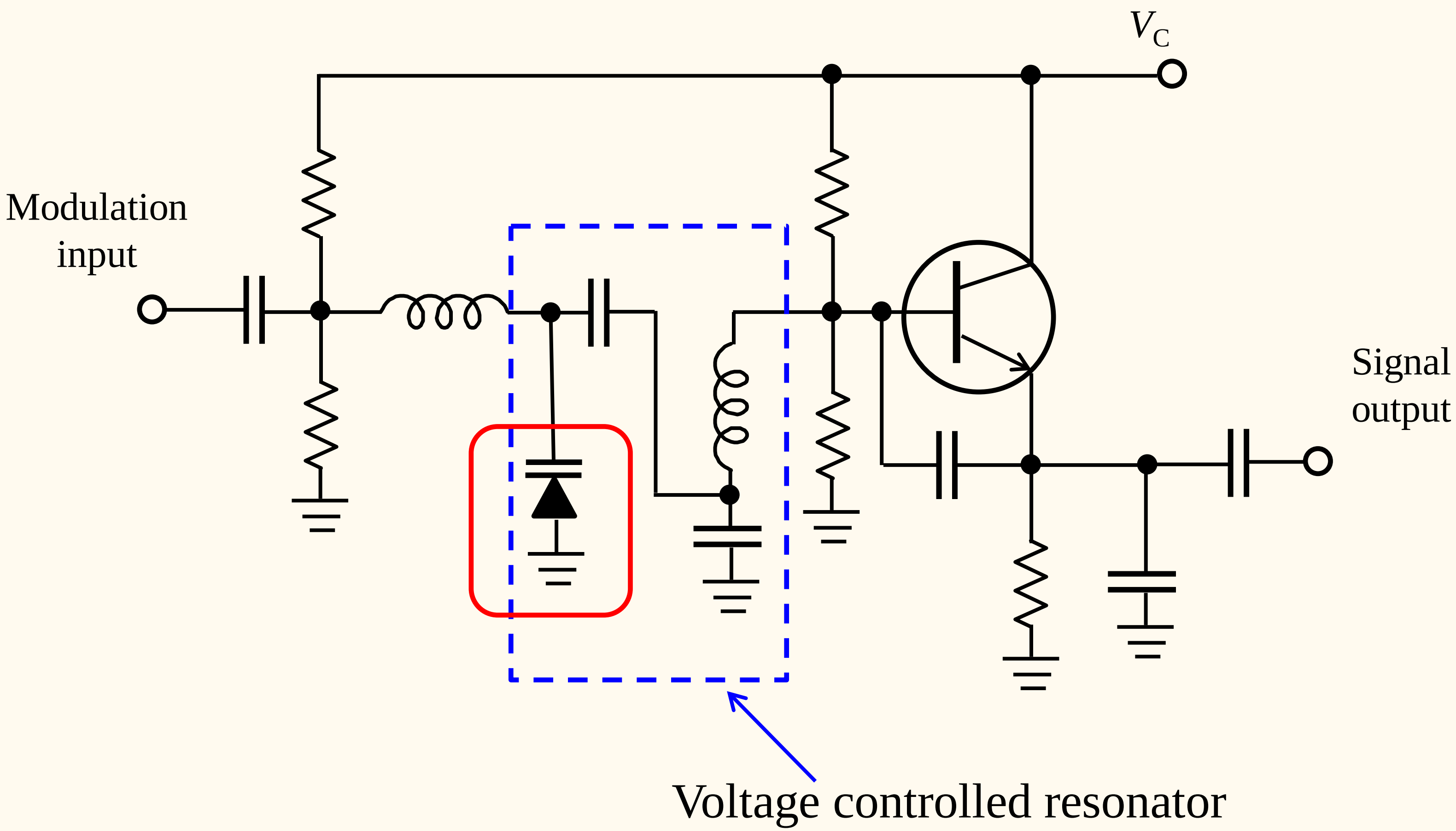
$$S_{\mathrm{FM}}(i\omega) = \pi A \sum_{n=-\infty}^{\infty} J_n(\beta) \{\delta[\omega - (\omega_{\mathrm{c}} + n\omega_p)] + \delta[\omega + (\omega_{\mathrm{c}} + n\omega_p)]\}$$

Actual band width:

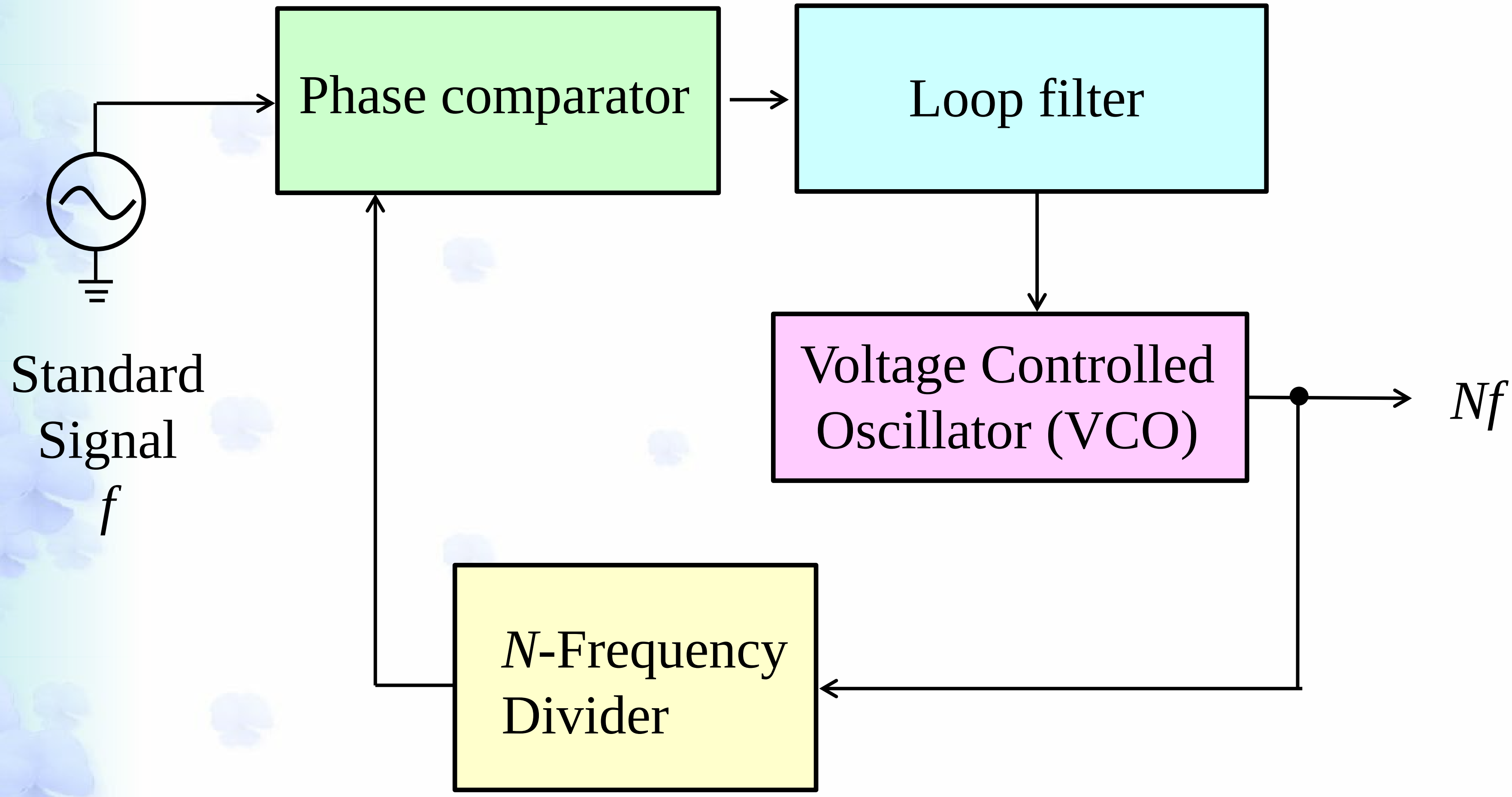
$$\omega_{\mathrm{bw}} = 2(\omega_{\mathrm{f}} + \xi\omega_{\mathrm{w}}) \qquad 1 \leq \xi \leq 2$$

6.3.3 Angle modulation (circuit example)

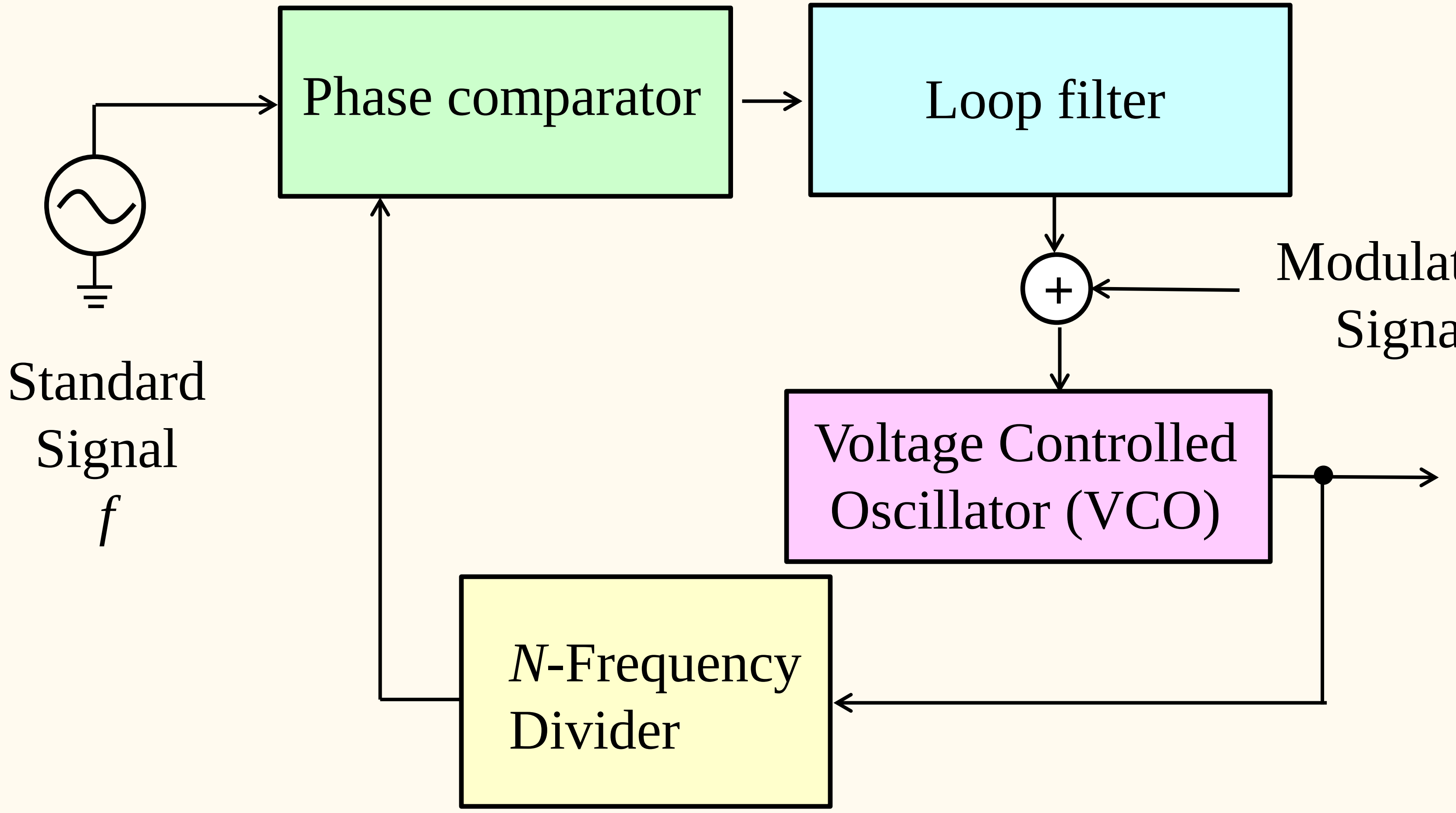
Voltage Controlled Oscillator (VCO)



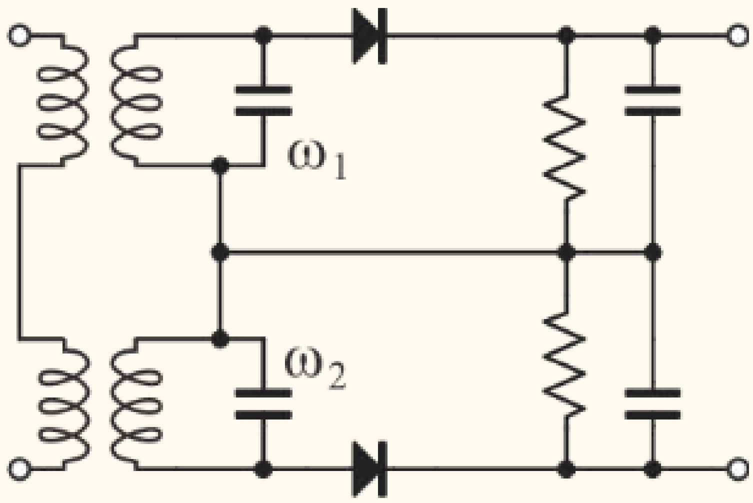
Phase Lock Loop (PLL)



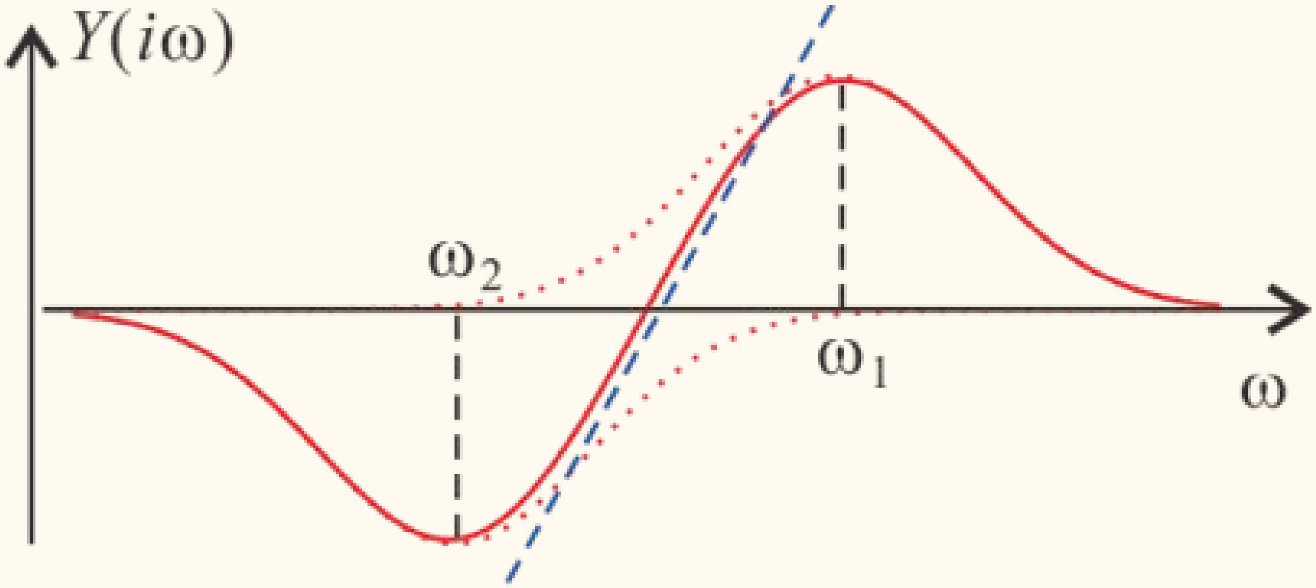
6.3.3 Angle modulation (circuit example)



6.3.4 Angle modulation (Frequency demodulation)

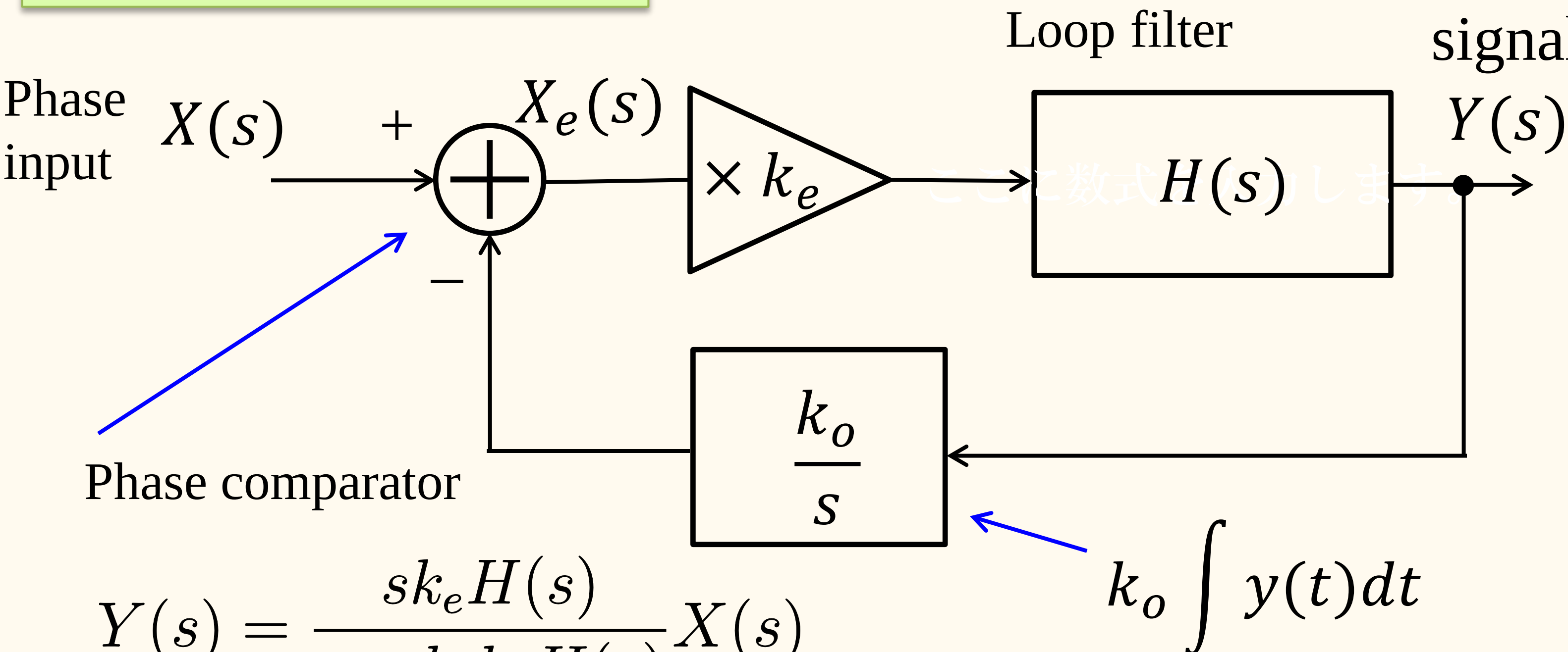


Double tuned circuit



6.3.4 Angle modulation (frequency demodulation)

PLL FM demodulation



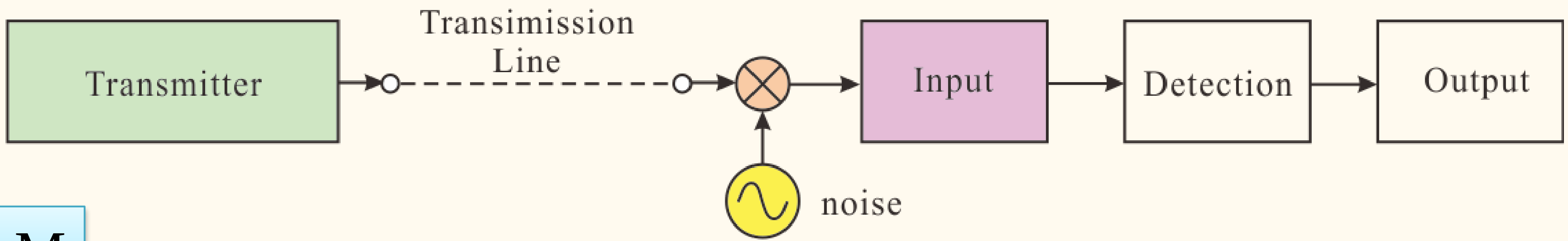
$$Y(s) = \frac{s k_e H(s)}{s + k_e k_o H(s)} X(s)$$

$g(t)$: Frequency modulation signal (original)

$$\phi(t) = k_f \int_{-\infty}^{t} g(\tau) d\tau, \quad sX(s) = k_f G(s)$$

$$\therefore \; Y(s) = \frac{k_f k_e H(s)}{s + k_e k_o H(s)} G(s) \approx \frac{k_f}{k_o} G(s)$$

6.3.5 Modulation and noise



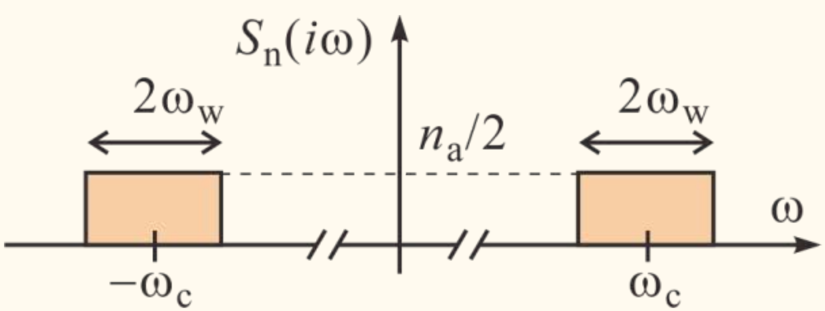
AM

Received signal $$r(t) = A_r[1 + mf(t)]\cos\omega_c t + n_i(t)$$

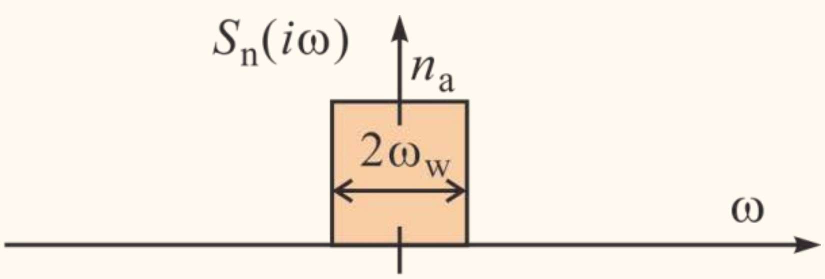
Demodulated output $$g(t) = A_r m f(t) + n_o(t)$$

Averaged signal power

$$\text{received: } S_{pr} = \frac{A_r^2}{2} + \frac{(A_r m)^2}{2}\langle f^2\rangle, \quad \text{output: } S_{po} = A_r^2 m^2 \langle f^2 \rangle$$



Received



Demodulated

6.3.5 Modulation and noise

ω_W : Noise bandwidth (assumption: white)

Noise power: $2 \times \frac{n_a}{2 \times 2\pi} \times 2\omega_w = \frac{n_a \omega_w}{\pi}$, $\frac{n_a \times 2\omega_w}{2\pi} = \frac{n_a \omega_w}{\pi}$

Received — Demodulated

$$\left.\frac{S}{N}\right|_{\text{in}} = \frac{\pi[A_r^2 + (A_r m)^2 \langle f^2 \rangle]}{2 n_a \omega_w}, \quad \left.\frac{S}{N}\right|_{\text{out}} = \frac{\pi A_r^2 m^2 \langle f^2 \rangle}{n_a \omega_w} = 2\eta \left.\frac{S}{N}\right|_{\text{in}}$$

$$\eta = \frac{m^2 \langle f^2 \rangle}{1 + m^2 \langle f^2 \rangle}$$:Power transmission efficiency

$$0 < m \leq 1 \rightarrow \eta < \frac{1}{2}$$

Input sinusoidal: $\langle f^2 \rangle = \frac{1}{2} \rightarrow \eta < \frac{1}{3}$

6.3.5 Modulation and noise

FM, PM

In phase — Out of phase

$$r(t) = \underbrace{A_r \cos[\omega_c t + \phi(t)]}_{\text{Signal}} + \underbrace{n_l(t)\cos\omega_c t - n_r(t)\sin\omega_c t}_{\text{Noise}}$$

$$= A_r \cos[\omega_c t + \phi(t)] + A_n(t)\cos[\omega_c t + \phi_n(t)]$$

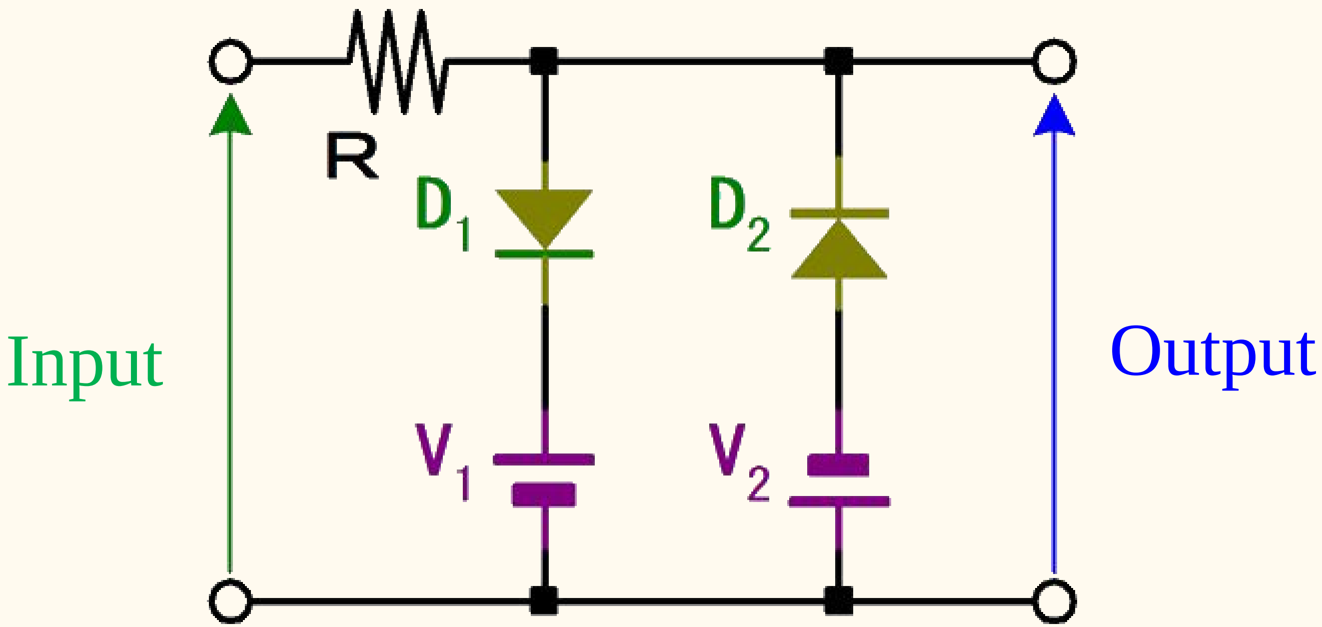
$$= V_r(t)\cos[\omega_c t + \theta(t)] \quad (\theta(t) = \phi(t) + \underbrace{\phi_{no}(t)}_{\text{Phase noise}})$$

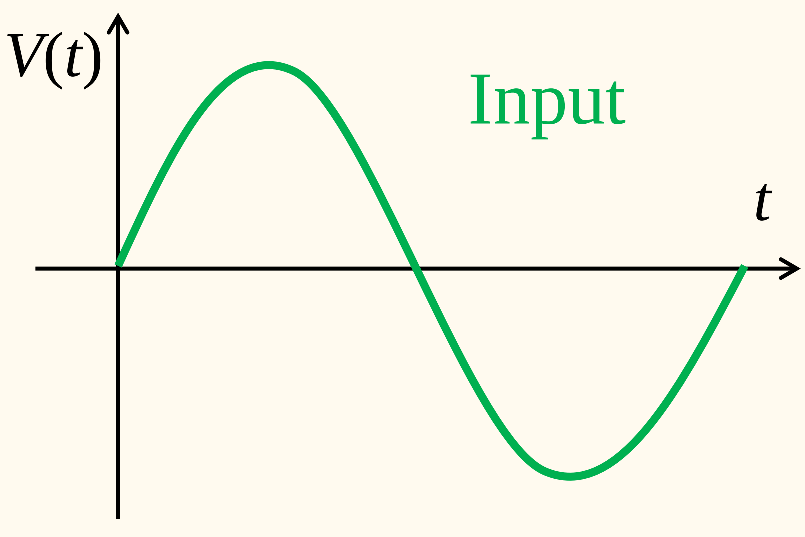
$$V_r(t) = \sqrt{A_r^2 + A_n^2(t) + 2A_r A_n(t)\cos[\phi_n(t) - \phi(t)]},$$

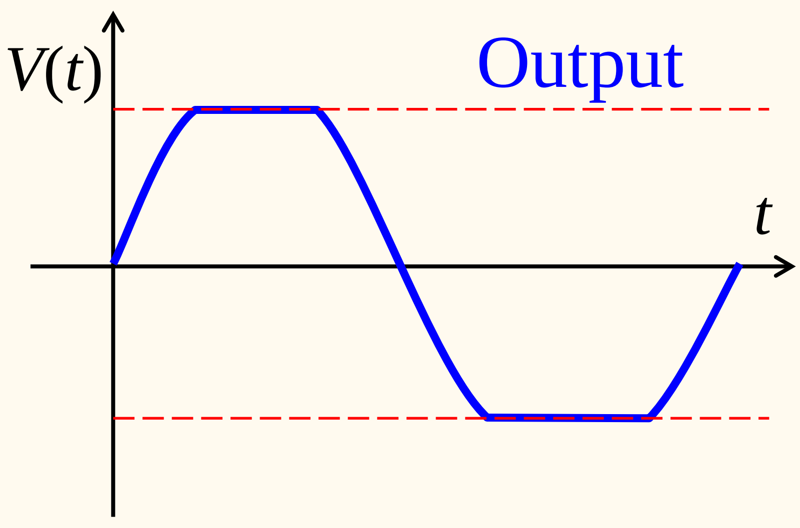
$$\phi_{no}(t) = \arctan\frac{A_n(t)\sin[\phi_n(t) - \phi(t)]}{A_r + A_n(t)\cos[\phi_n(t) - \phi(t)]}$$

Time-dependent part in $V_r(t)$ can be cut with a limiter circuit.

6.3.5 Modulation and noise (Diode limitter)







6.3.5 Modulation and noise

$$A_r \gg A_\mathrm{n}(t)$$

$$\phi_\mathrm{no} \cong \arctan\left[\frac{A_\mathrm{n}(t)}{A_r}\sin[\phi_\mathrm{n}(t)-\phi(t)]\right] \cong \frac{A_\mathrm{n}(t)}{A_r}\sin[\phi_\mathrm{n}(t)-\phi(t)]$$

Noise: White, Power spectrum density $n_a/2$, Band width ω_B

Noise power: $N_i = \dfrac{n_a \omega_B}{2\pi}$ Signal power: $\dfrac{A_r^2}{2}$ $\dfrac{S_\mathrm{i}}{N_\mathrm{i}} = \dfrac{\pi A_r^2}{n_a \omega_\mathrm{B}}$

Phase modulation $\phi[t, f(t)] = k_p f(t)$

Averaged signal power: $k_p^2 \langle f^2 \rangle$

Averaged noise power: $N_\mathrm{oPM} \cong \dfrac{1}{A_r^2}\langle A_\mathrm{n}(t)^2 \sin^2[\phi_\mathrm{n}(t)-\phi(t)]\rangle$

$\phi_n(t)$: Uniform in [0, 2π] → ignored

$$N_\mathrm{oPM} \cong \frac{1}{A_r^2}\langle A_\mathrm{n}(t)^2 \sin^2\phi(t)\rangle = \frac{n_a \omega_\mathrm{w}}{\pi A_r^2}$$

6.3.5 Modulation and noise

$$f(t) = A_p \cos \omega_p t, \quad \beta \equiv k_p A_p \to S_o = \frac{\beta^2}{2}, \quad \omega_\mathrm{B} = 2(\beta + \xi)\omega_\mathrm{w} \ (1 \le \xi \le 2)$$

$$\frac{S_\mathrm{o}}{N_\mathrm{o}} = \frac{\beta^2}{2}\frac{\pi A_r^2}{n_a \omega_\mathrm{w}} = \frac{\beta^2}{2}\frac{\omega_\mathrm{B}}{\omega_\mathrm{w}}\frac{\pi A_r^2}{n_a \omega_\mathrm{B}} = \beta^2(\beta + \xi)\frac{S_i}{N_i}$$

Frequency modulation

Demodulated output $d\theta/dt$ Signal, power: $k_f f(t)$, $k_f^2 \langle f^2 \rangle$

$$N_\mathrm{oFM} = \left\langle \frac{dn_\mathrm{no}}{dt} \right\rangle = \frac{1}{A_r^2}\left\langle \frac{dn_l}{dt} \right\rangle = \frac{1}{A_r^2}\int_{-\omega_\mathrm{w}}^{\omega_\mathrm{w}} n_a \omega^2 \frac{d\omega}{2\pi} = \frac{n_a \omega_\mathrm{w}^3}{3\pi A_r^2}$$

$$\beta \equiv k_f A_p/\omega_\mathrm{w} \qquad \frac{S_\mathrm{o}}{N_\mathrm{o}} = 3\beta^2(\beta + \xi)\frac{S_\mathrm{i}}{N_\mathrm{i}}$$

$$\left.\frac{S_\mathrm{o}}{N_\mathrm{o}}\right|_\mathrm{FM} = 3\beta^2 \left.\frac{S_\mathrm{o}}{N_\mathrm{o}}\right|_\mathrm{AM}, \qquad \left.\frac{S_\mathrm{o}}{N_\mathrm{o}}\right|_\mathrm{PM} = \beta^2 \left.\frac{S_\mathrm{o}}{N_\mathrm{o}}\right|_\mathrm{AM}$$

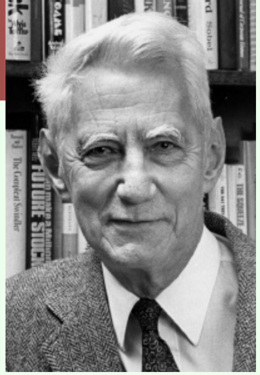
6.4 Discrete signal

6.4.1 Sampling theorem

Sampled signal $\tilde{x}(t) = x(t)\underline{\delta_\tau(t)}$

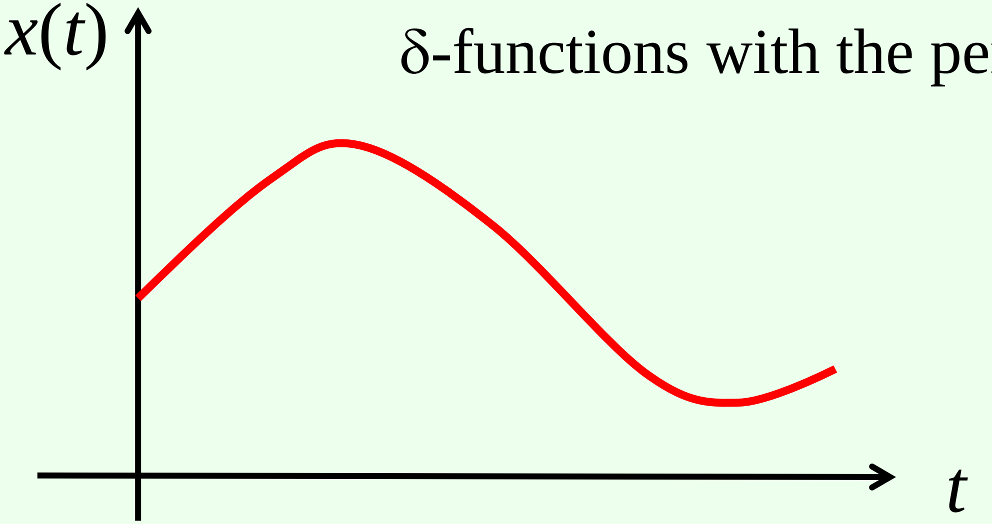
δ-functions with the period τ

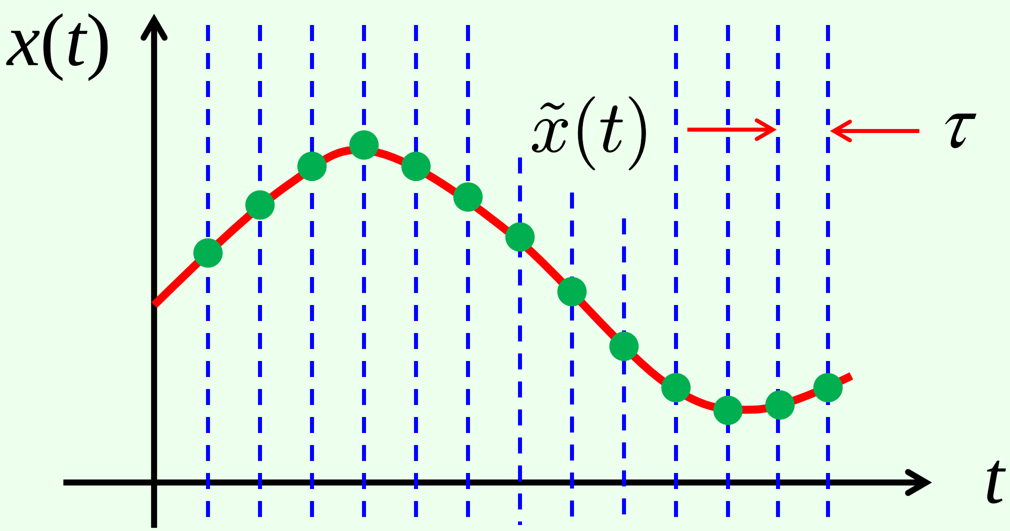
Isao Someya
1915-2007

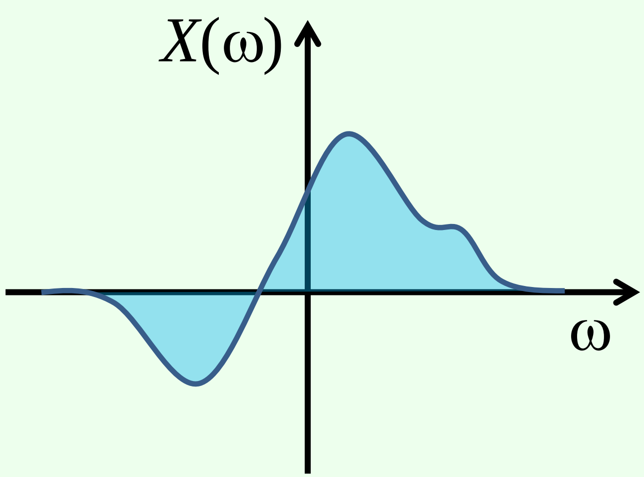


Claude Shannon
1916-2001

1928 H. Nyquist
1949 C. Shannon
染谷勲







6.4.1 Sampling theorem

$$\delta_\tau(t) = \sum_{j=-\infty}^{\infty} \delta(t - j\tau) = \sum_{n=-\infty}^{\infty} \left[\frac{1}{\tau}\int_{-\pi/\tau}^{\pi/\tau} \delta(s)ds\right] \exp\left(-in\frac{2\pi}{\tau}t\right)$$

$$= \frac{1}{\tau}\sum_{n=-\infty}^{\infty} \exp\left(-in\frac{2\pi}{\tau}t\right)$$

$$\mathscr{F}\{\delta_\tau(t)\} = \int_{-\infty}^{\infty}\left[\frac{1}{\tau}\sum_{-\infty}^{\infty} e^{-in(2\pi/\tau)t}\right] e^{i\omega t}dt = \frac{1}{\tau}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[i\left(\omega - n\frac{2\pi}{\tau}\right)t\right]dt$$

$$= \frac{2\pi}{\tau}\sum_{n=-\infty}^{\infty} \delta\left(\omega - n\frac{2\pi}{\tau}\right) = \frac{2\pi}{\tau}\delta_{2\pi/\tau}(\omega)$$

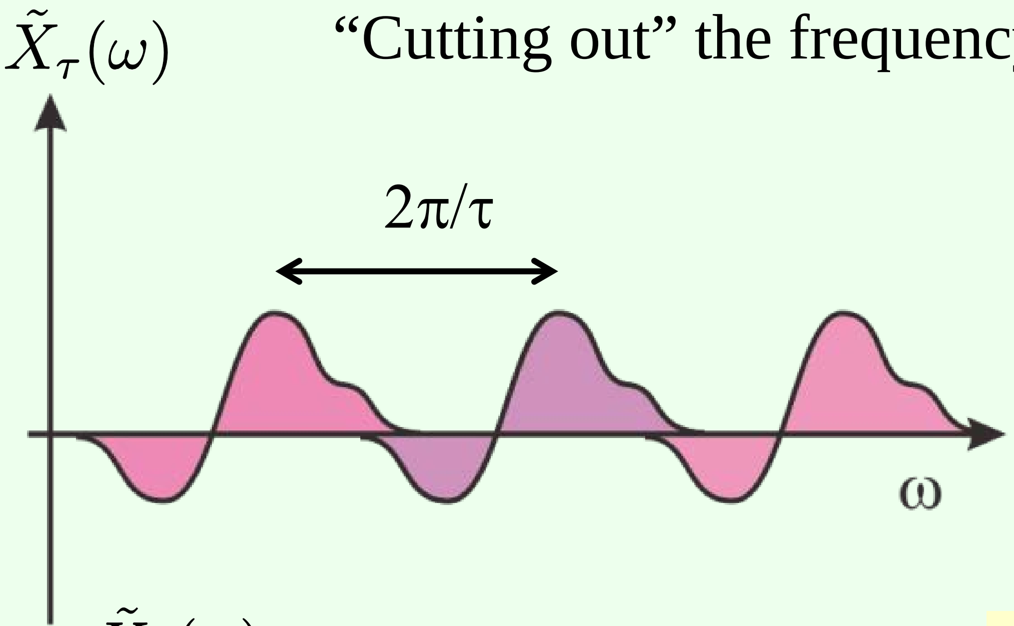
$$\mathscr{F}\{x(t)\} = X(\omega),\ \mathscr{F}\{\tilde{x}_\tau(t)\} = \tilde{X}_\tau(\omega)$$

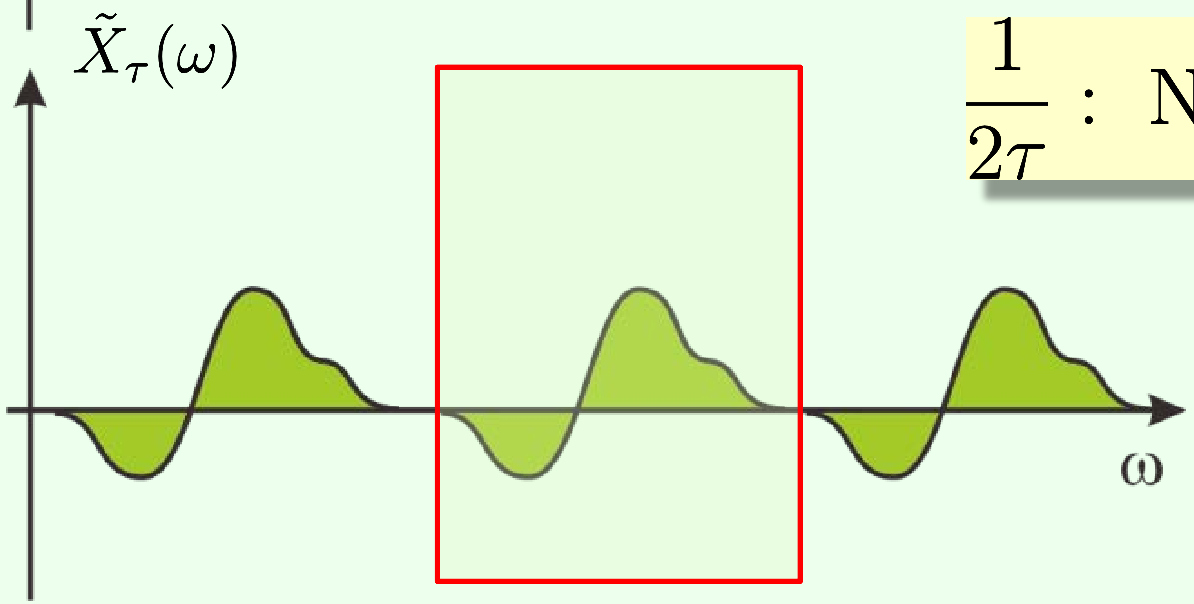
$$\tilde{X}_\tau(\omega) = \frac{1}{2\pi}X(\omega) * \frac{2\pi}{\tau}\delta_{2\pi/\tau}(\omega) = \frac{1}{\tau}X(\omega) * \sum_{n=-\infty}^{\infty}\delta\left(\omega - n\frac{2\pi}{\tau}\right)$$

$$= \frac{1}{\tau}\int_{-\infty}^{\infty} X(\omega')\left\{\sum_{n=-\infty}^{\infty}\delta\left(\omega - n\frac{2\pi}{\tau} - \omega'\right)\right\}d\omega' = \frac{1}{\tau}\sum_{n=-\infty}^{\infty} X\left(\omega - n\frac{2\pi}{\tau}\right)$$

6.4.1 Sampling theorem

"Cutting out" the frequency spectrum





ω_h: Highest frequency in $\tilde{X}_\tau(\omega)$

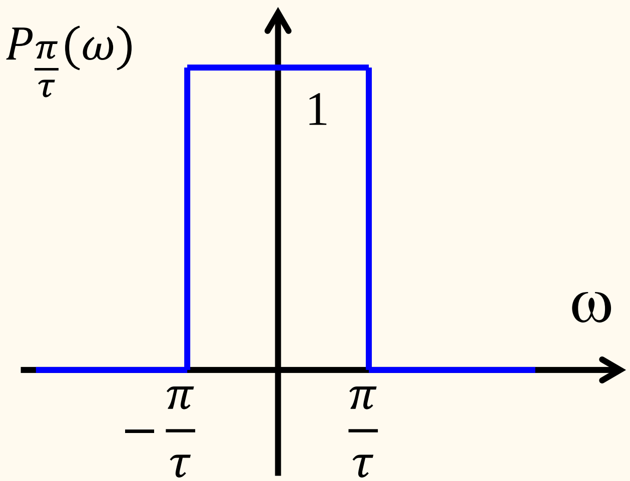
$$\frac{2\pi}{\tau} > 2\omega_h, \quad \tau < \frac{\pi}{\omega_h}$$

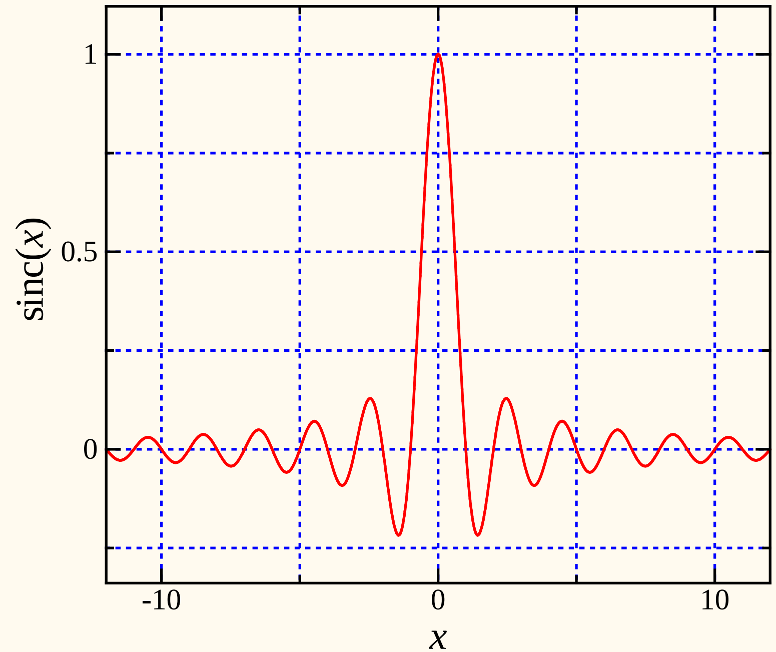
$\frac{1}{2\tau}$: Nyquist frequency

6.4.1 Sampling theorem: Reconstructing signal

$$P_{\pi/\tau}(\omega) = \begin{cases} 1 & |\omega| \le \dfrac{\pi}{\tau}, \\ 0 & |\omega| > \dfrac{\pi}{\tau} \end{cases}$$

$$x(t) = \mathscr{F}^{-1}\{\tau P_{\pi/\tau}(\omega)\tilde{X}_\tau(\omega)\}$$





$$x(t) = \tau\frac{1}{\tau}\mathrm{sinc}\left(\frac{t}{\tau}\right) * \tilde{x}_\tau(t) = \mathrm{sinc}\left(\frac{t}{\tau}\right) * \sum_{n=-\infty}^{\infty} x(t)\delta(t - n\tau)$$

$$= \int_{-\infty}^{\infty} \mathrm{sinc}\left(\frac{s}{\tau}\right) \sum_{n=-\infty}^{\infty} x(t-s)\delta(t - n\tau - s)ds = \sum_{n=-\infty}^{\infty} \mathrm{sinc}\left(\frac{t - n\tau}{\tau}\right) x(n\tau)$$