

電子回路論第14回 (最終回)

Electric Circuits for Physicists

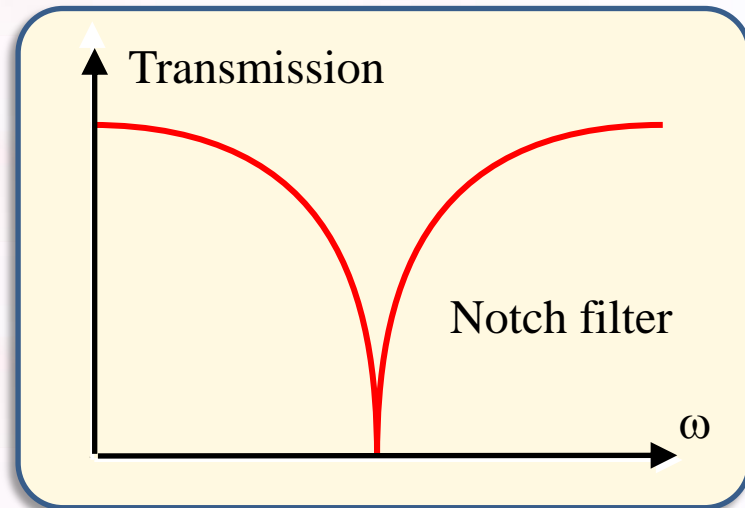
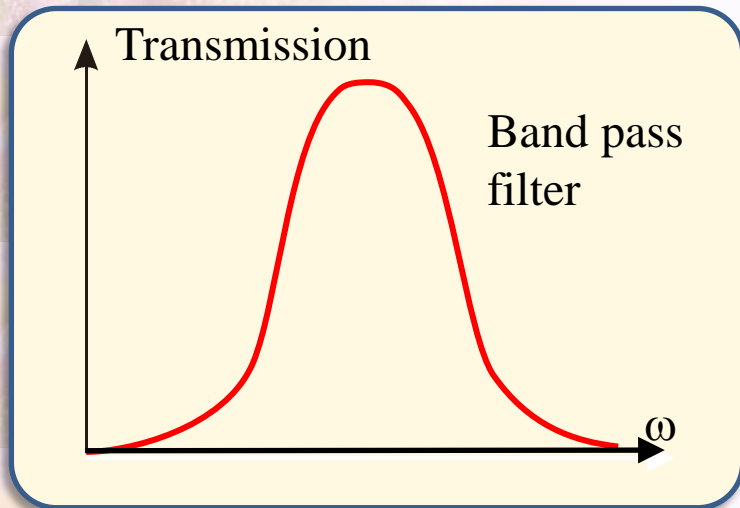
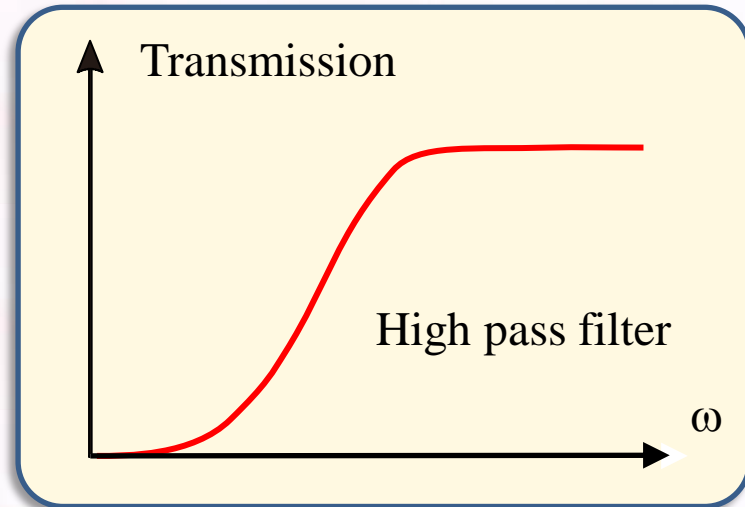
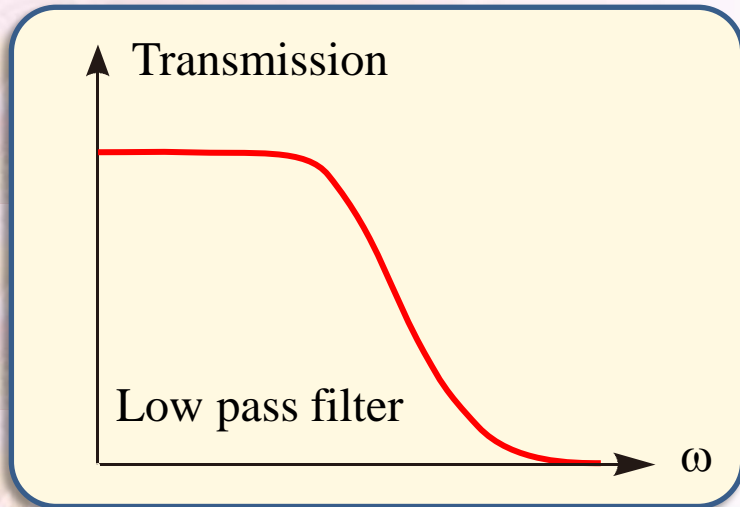
東京大学理学部・理学系研究科

物性研究所

勝本信吾

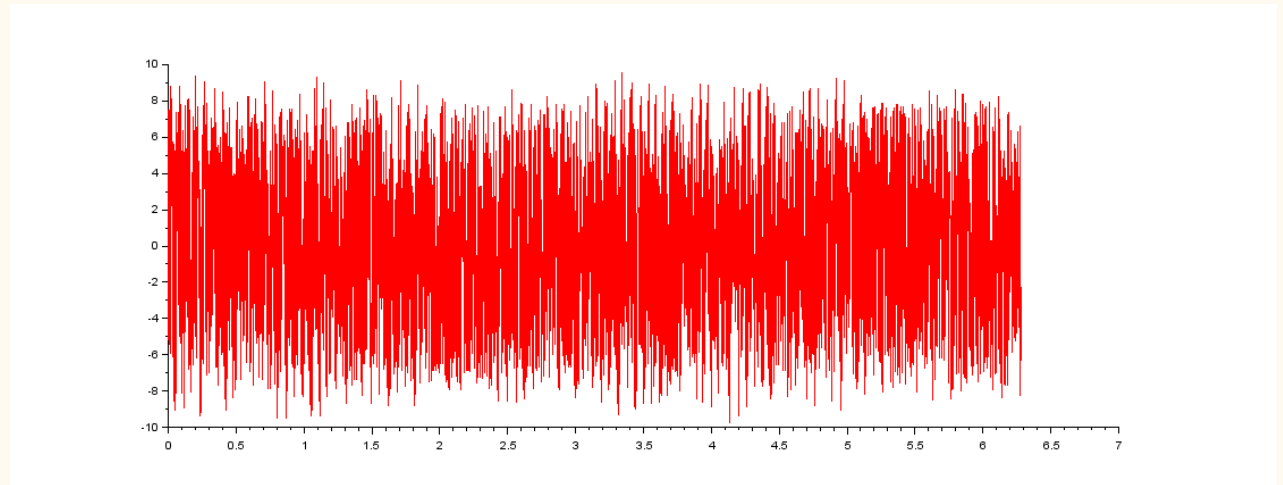
Shingo Katsumoto

Filter Circuit

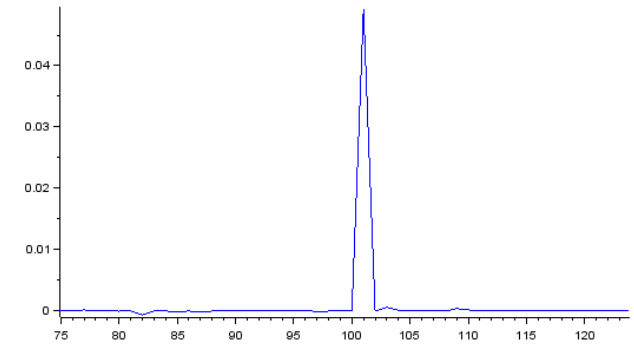
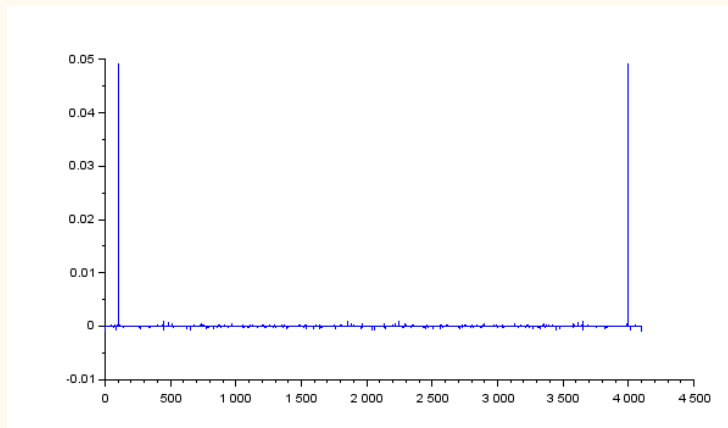


An example for retrieving data from noise

Signal with noise

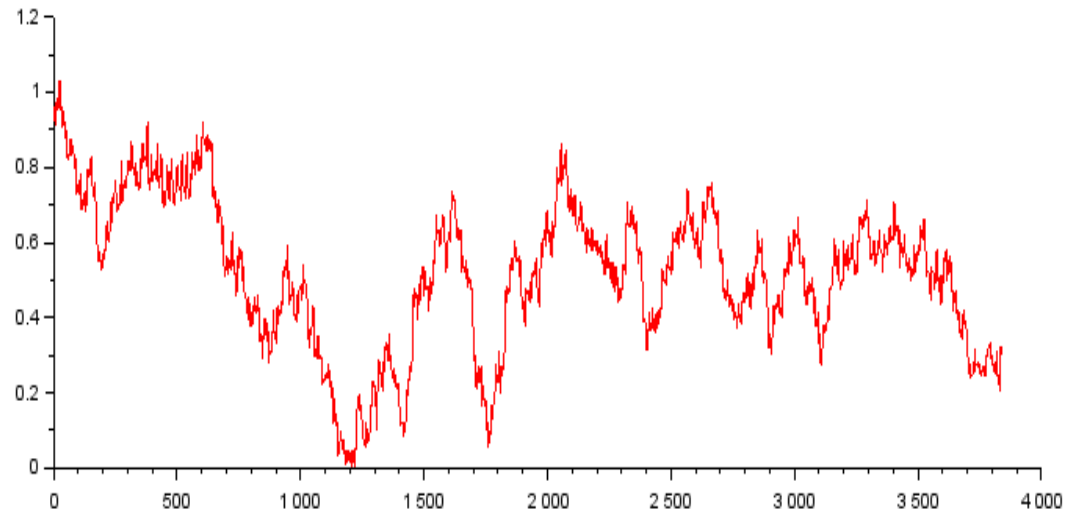


Detection of carrier

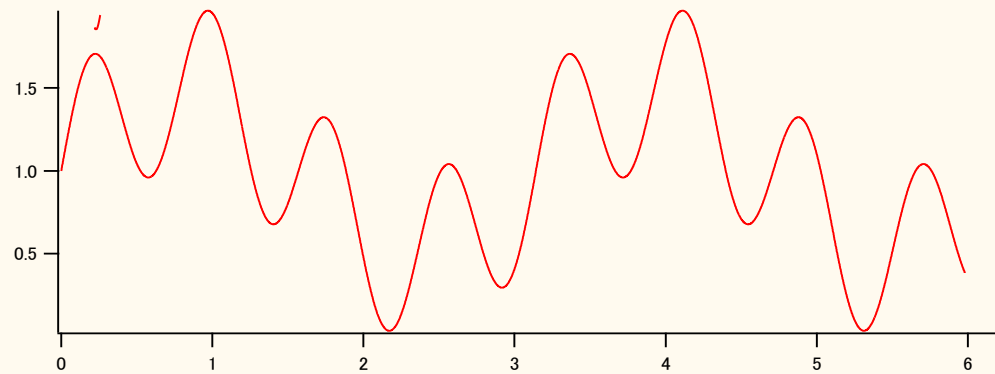


Results

Frequency filter



Original signal



7.6 Digital filter (as a digital signal processing)

Digital filtering:

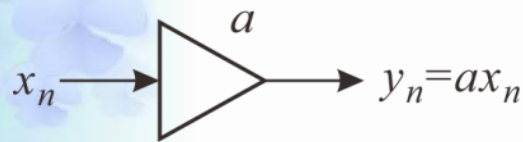
$$\{x_i\} = (x_0, x_1, \dots)$$



$$\{y_i\} = (y_0, y_1, \dots)$$

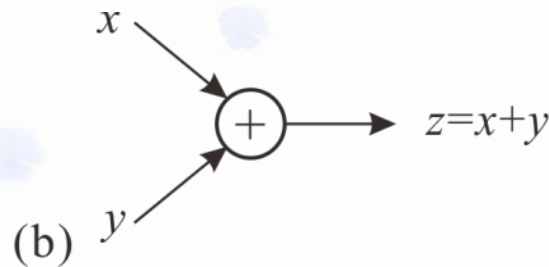
$$y_n = F(x_{n-k}, x_{n-k+1}, \dots, x_n)$$

Block diagram representation of operations



(a)

constant multiplier



(b)

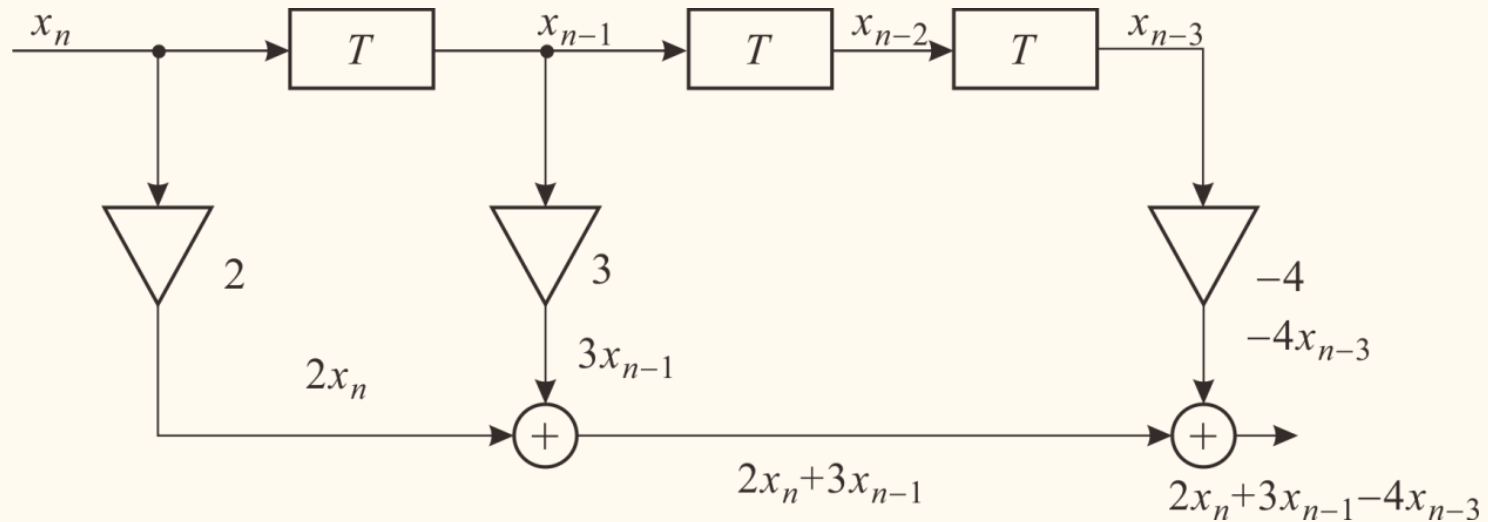
adder



(c)

delay (shift resistor)

Block diagram example



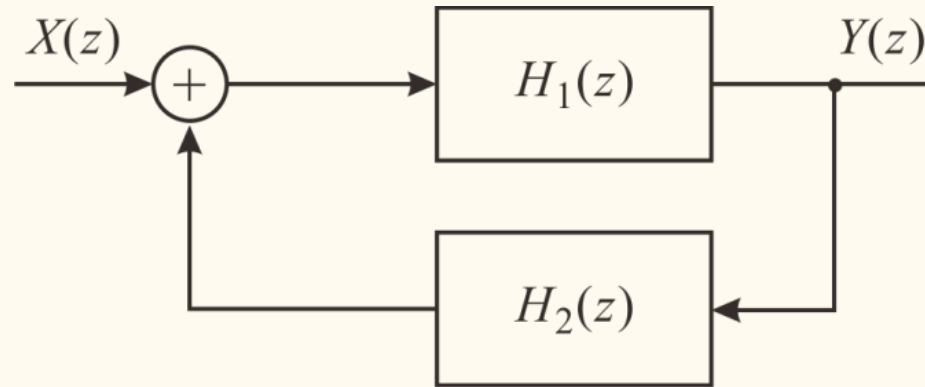
$$y_n = 2x_n + 3x_{n-1} - 4x_{n-3}$$

$$X(z) = \sum_{n=0}^{\infty} x_n z^{-n}, \quad Y(z) = \sum_{n=0}^{\infty} y_n z^{-n}$$

$$\begin{aligned} Y(z) &= 2X(z) + 3z^{-1}X(z) - 4z^{-3}X(z) \\ &= (2 + 3z^{-1} - 4z^{-3})X(z) \end{aligned}$$

$$\therefore H(z) \text{ (transfer function)} = 2 + 3z^{-1} - 4z^{-3}$$

Feedback and transfer function



$$Y(z) = H_1(z)W(z) = H_1(z)(X(z) + H_2(z)Y(z)),$$

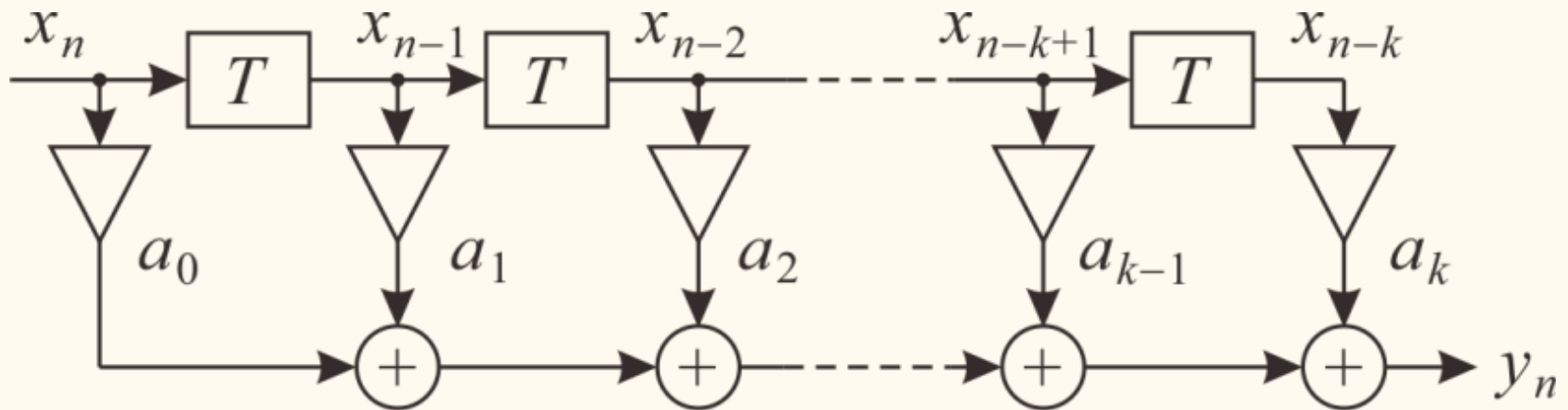
$$\therefore Y(z) = \frac{H_1(z)}{1 - H_1(z)H_2(z)}X(z)$$

$$H(z) = \frac{H_1(z)}{1 - H_1(z)H_2(z)}$$

$$\text{(transfer function)} = \frac{\text{(direct gain)}}{1 - \text{(feedback transfer gain)}}$$

FIR filter

Finite impulse response (FIR) filter

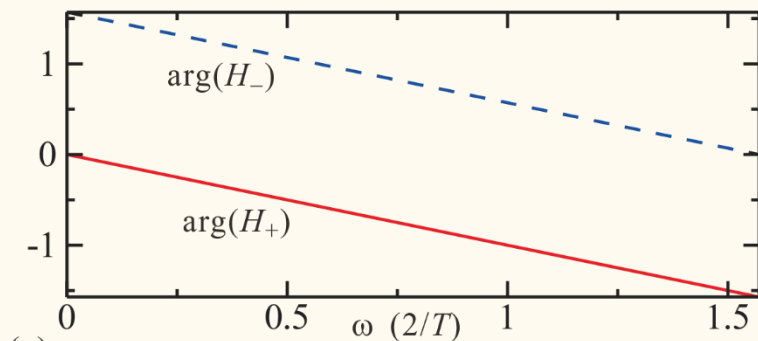
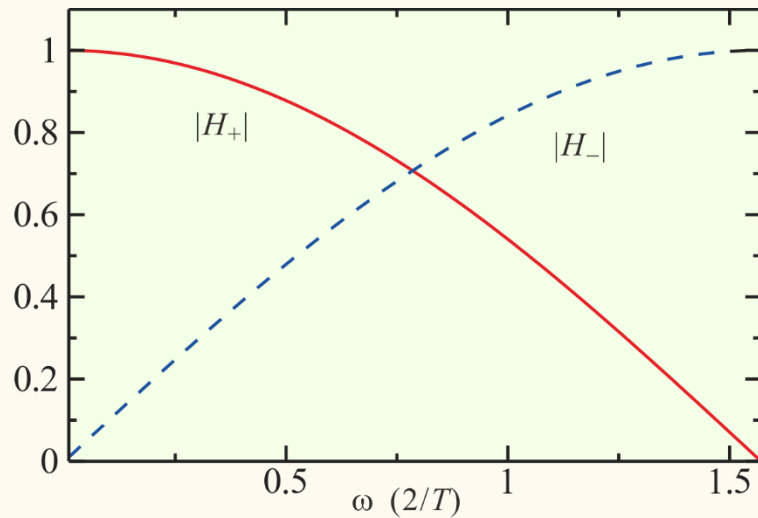


$$H(e^{i\omega\tau}) = \sum_{j=0}^k a_j e^{-ij\omega\tau}$$

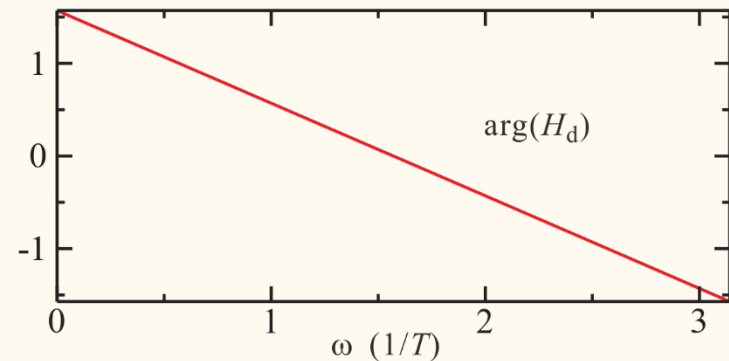
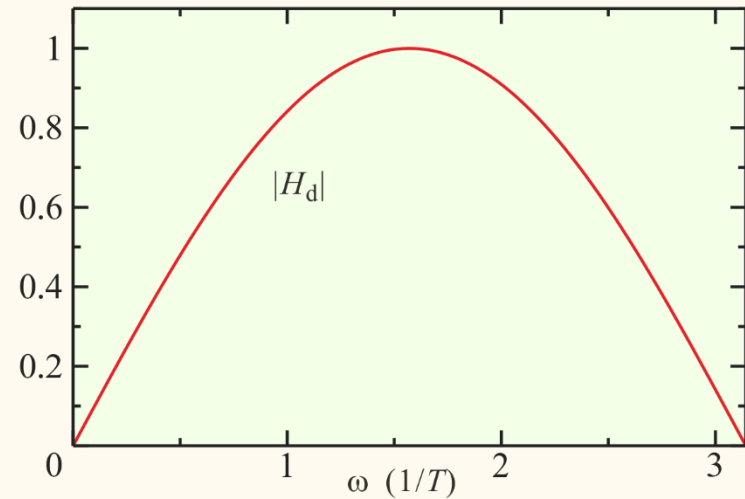
A Simple example of FIR filter

Moving average, differentiation: $F_{\pm}(x_n, x_{n-1}) = (x_n \pm x_{n-1})/2$

$$H_{\pm}(e^{i\omega\tau}) = e^{-i\omega\tau/2} \begin{pmatrix} \cos(\omega\tau/2) \\ i \sin(\omega\tau/2) \end{pmatrix}$$



(a)



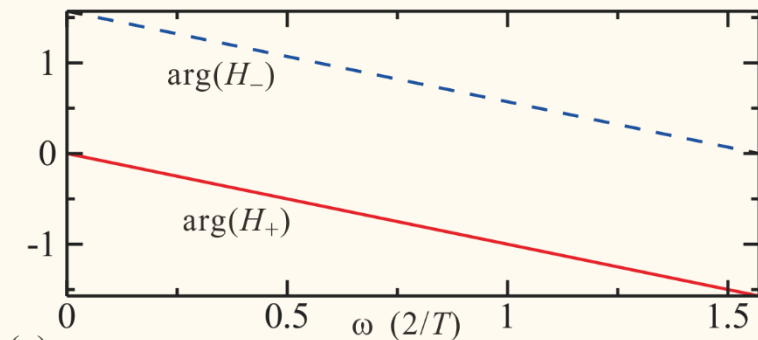
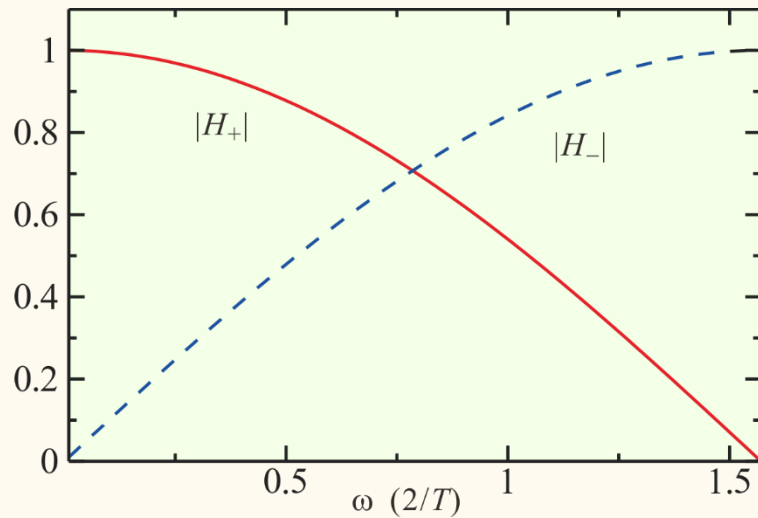
(b)

A Simple example of FIR filter

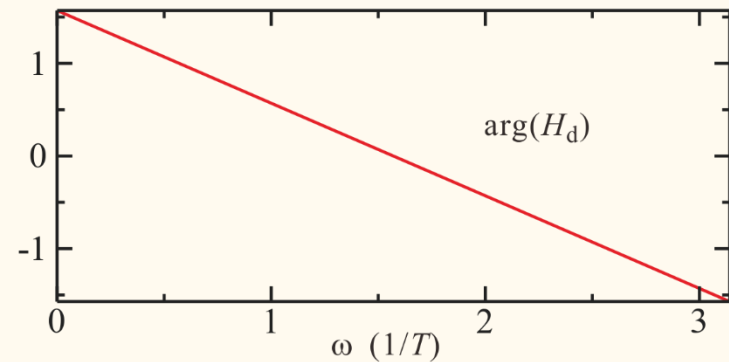
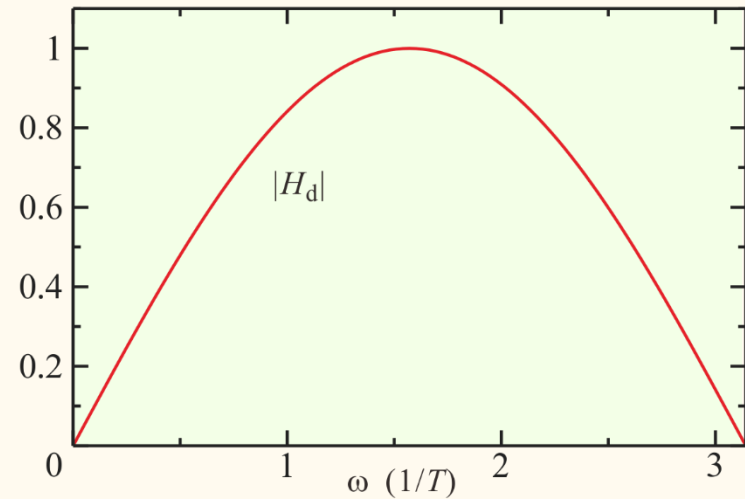
Differentiation of moving average:

$$F_d = [(x_n + x_{n-1}) - (x_{n-1} + x_{n-2})]/2 = [x_n - x_{n-2}]/2$$

$$H_d = (1 - e^{-2i\omega\tau})/2 = ie^{-i\omega\tau} \sin \omega\tau$$



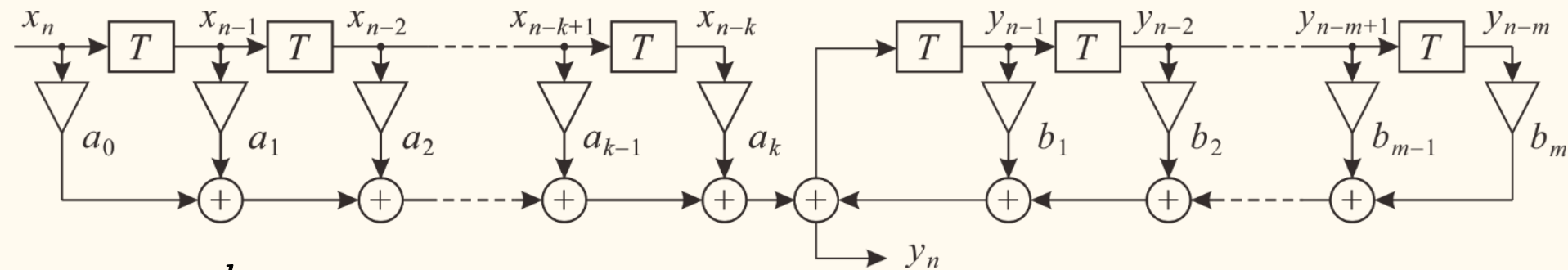
(a)



(b)

IIR Filter

Infinite impulse response (IIR) filter:



$$y_n = \sum_{l=0}^k a_l x_{n-l} + \sum_{j=1}^m b_j y_{n-j} \quad \text{Stability condition: } \lim_{n \rightarrow \infty} y_n = 0$$

$$Y(z) = X(z) \sum_{l=0}^k a_l z^{-l} + Y(z) \sum_{j=1}^m b_j z^{-j}$$

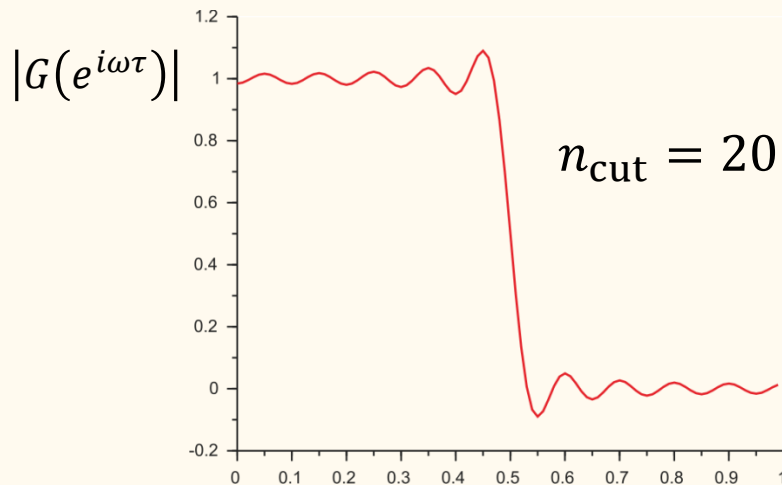
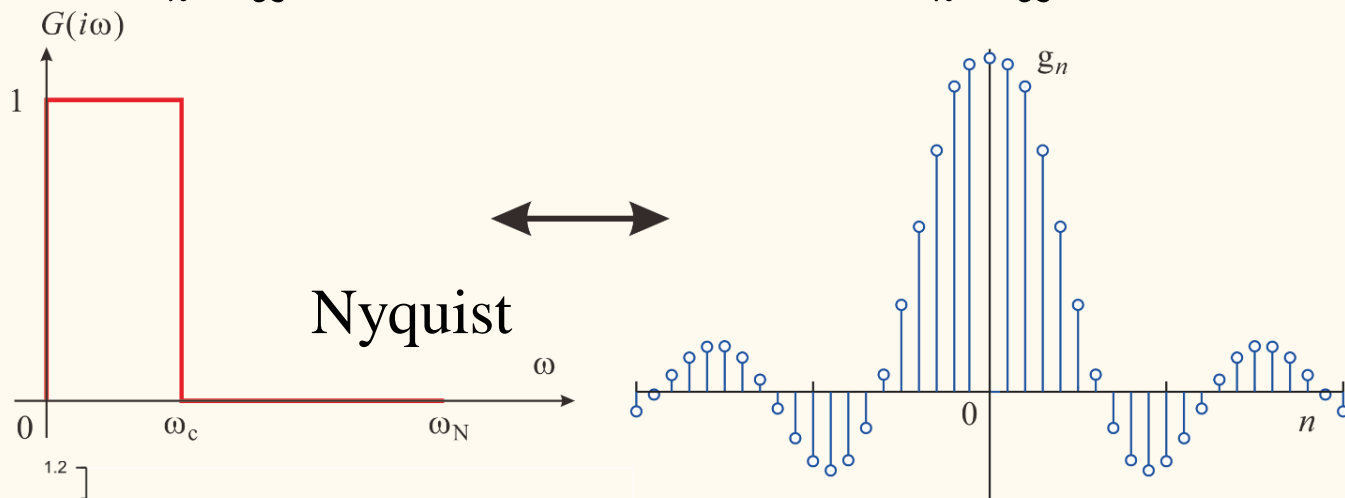
$$H(z) = \frac{Y(z)}{X(z)} = \sum_{l=0}^k a_l z^{-l} \bigg/ \left(1 - \sum_{j=1}^m b_j z^{-j} \right)$$

Conversion of z-transform: $|z| > 1$ the poles should be in $|z| < 1$

Design of FIR filter: Window function

Ideal low pass filter $G(e^{i\omega\tau}) = \begin{cases} 1, & |\omega| \leq \omega_c, \\ 0, & \omega_c < |\omega| \leq \omega_N \text{ Nyquist frequency} \end{cases}$

$$G(e^{i\omega\tau}) = \frac{\omega_c}{\omega_N} \sum_{n=-\infty}^{\infty} \frac{1}{n\pi} \text{sinc}\left(n \frac{\omega_c}{\omega_N}\right) e^{-ni\omega\tau} = \gamma_c \sum_{n=-\infty}^{\infty} \frac{1}{n\pi} \text{sinc}(n\gamma_c) z^{-n}$$



Cut the series at a finite number

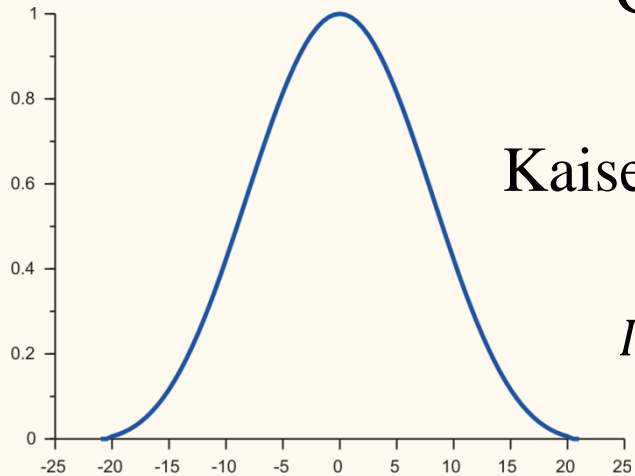
Ripples in frequency characteristics

ω/ω_N

Design of FIR filter: Window function

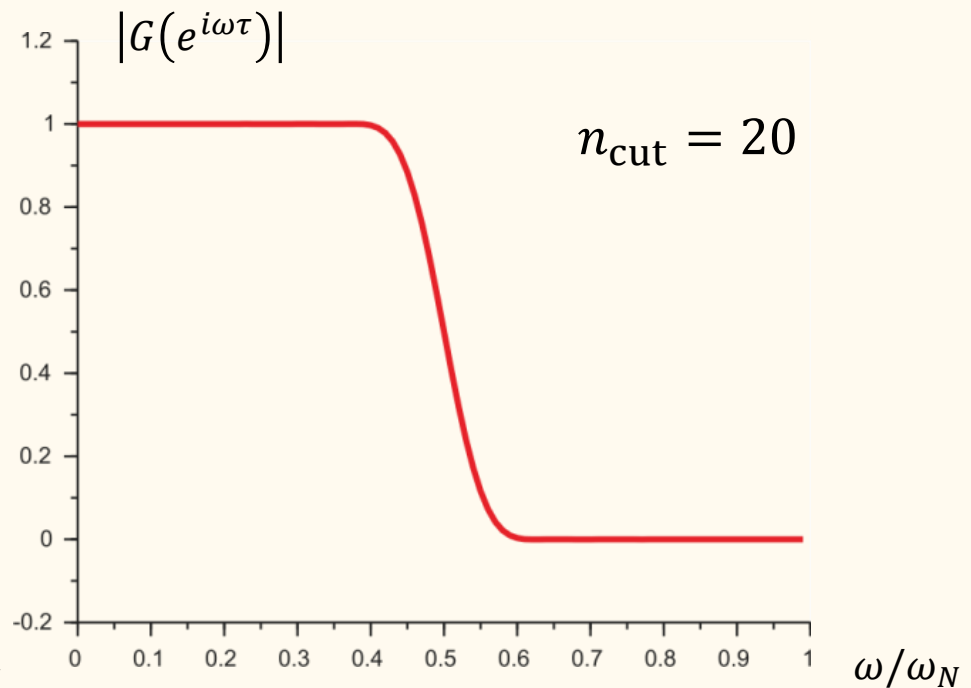
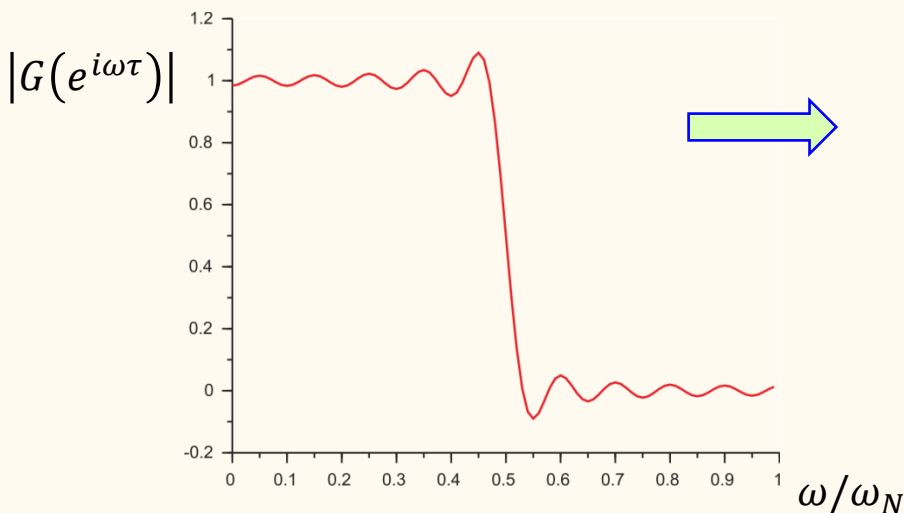
Sudden cutting of z-transform series \rightarrow Ripples

Cut with a smooth function



Kaiser window $w_n = \begin{cases} \frac{I_0\left(\alpha\sqrt{1-(n/L)^2}\right)}{I_0(\alpha)} & |n| \leq L, \\ 0 & |n| > L \end{cases}$

I_0 : 0th order 1st type modified Bessel function



Design of IIR filter

Transfer function: a rational function (有理式)

A way to design IIR filter: modification of [analog filter](#) transfer function

Remember: Butterworth filter

$$\Xi(s) = \sum_{k=0}^{N-1} \frac{\omega_k}{s - s_k}, \quad s_k = r_c \exp \left[i \left\{ \frac{\pi}{2} + \frac{(2k+1)\pi}{2n} \right\} \right]$$

$$\xi(t) = \underbrace{u_H(t)}_{\swarrow} \sum_{k=0}^{n-1} w_k \exp(s_k t)$$

Heaviside function

$$h_n = h_{Hn} \sum_{k=0}^{n-1} w_k e^{n s_k},$$

Time discretization
with $\tau = 1$:

$$\therefore H(z) = \sum_{k=0}^{n-1} \frac{w_k}{1 - \exp(s_k) z^{-1}}$$

Design of IIR filter

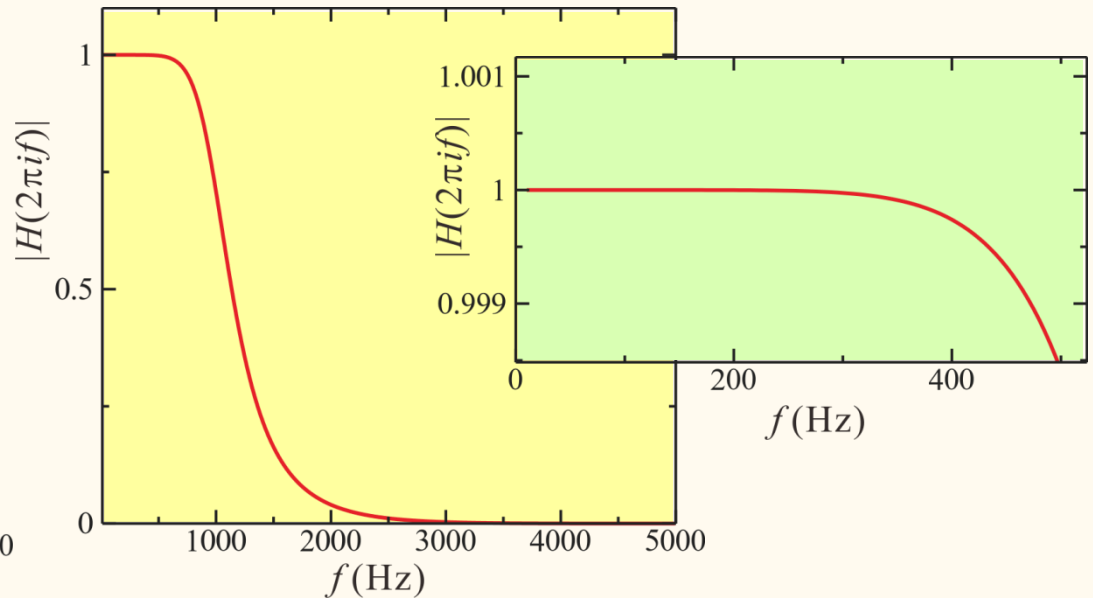
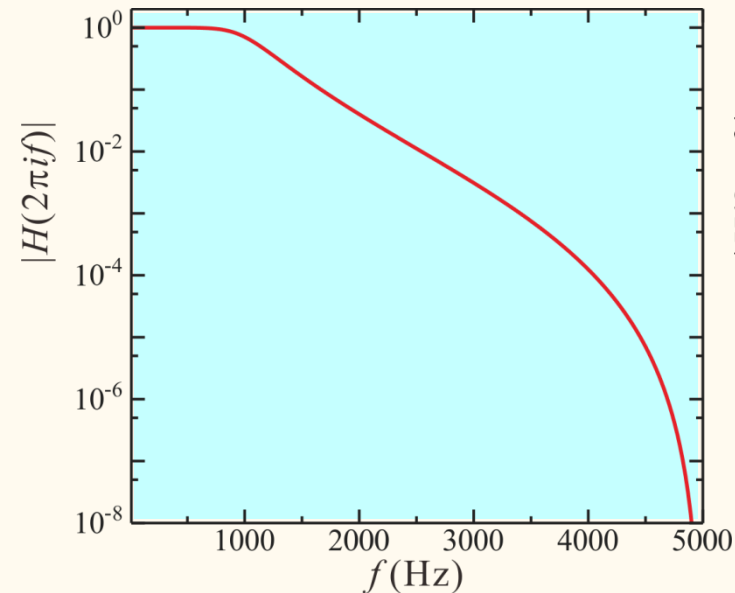
Impulse invariant method:

$$\frac{1}{s - s_k} \rightarrow \frac{1}{1 - \exp(s_k)z^{-1}}$$

Bilinear z-transform (双一次z变换法): $s \rightarrow \frac{1 - z^{-1}}{1 + z^{-1}}$

4th Butterworth:

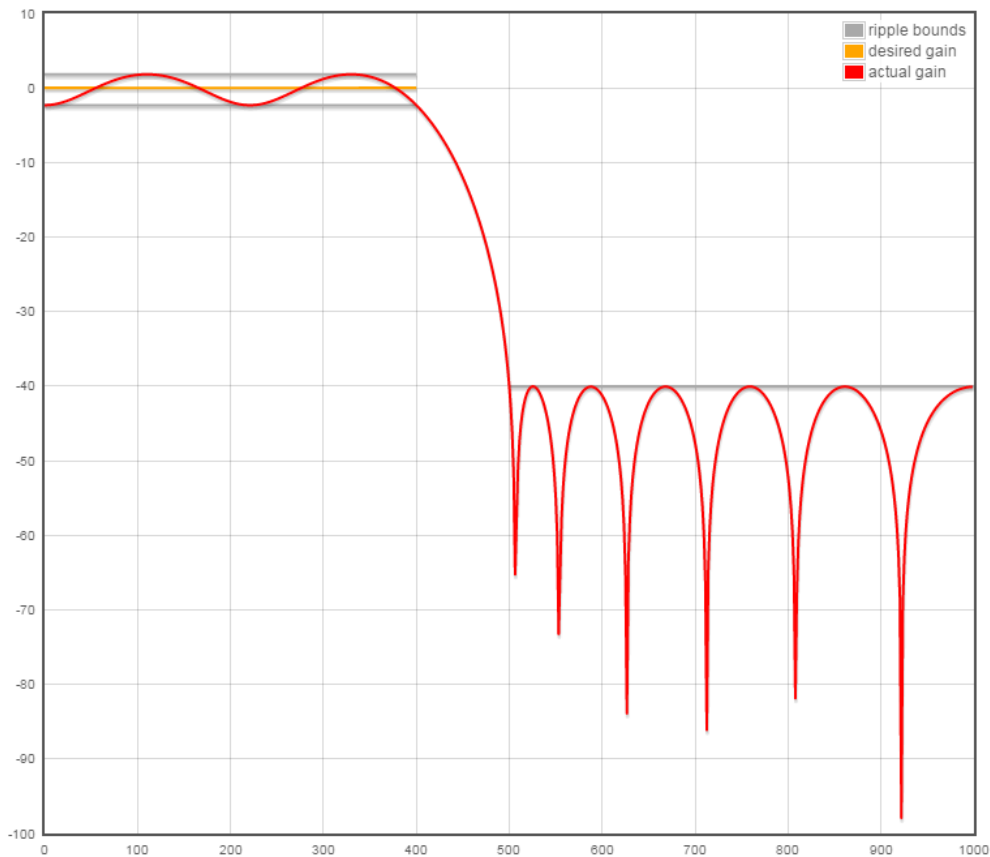
$$H(z) = \frac{b_0 + b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + b_4z^{-4}}{1 - a_1z^{-1} - a_2z^{-2} - a_3z^{-3} - a_4z^{-4}}$$



Digital filter design web application

<http://t-filter.engineerjs.com/>

Gain vs. Frequency Impulse Response Source Code Feature Request Enterprise IIR Design



Legend:
 - ripple bounds (grey)
 - desired gain (yellow)
 - actual gain (red)

```
-0.02010411882885732  
-0.05842798004352509  
-0.061178403647821976  
-0.010939393385338943  
0.05125096443534972  
0.033220867678947885  
-0.05855276971833928  
-0.08565500737264514  
0.063379599605449  
0.310854403656636  
0.4344309124179415  
0.310854403656636  
0.063379599605449  
-0.08565500737264514  
-0.05855276971833928  
0.033220867678947885  
0.05125096443534972  
-0.010939393385338943  
-0.061178403647821976  
-0.05842798004352509  
-0.02010411882885734
```

Buy me a beer Tweet

Copyright © 2011 Peter Isza

add passband add stopband predefined

from	to	gain	ripple/att.	act. rpl
0 Hz	400 Hz	1	5 dB	4.14 dB
500 Hz	1000 Hz	0	-40 dB	-40.07 dB

sampling freq. 2000 Hz
desired #taps minimum
actual #taps 21

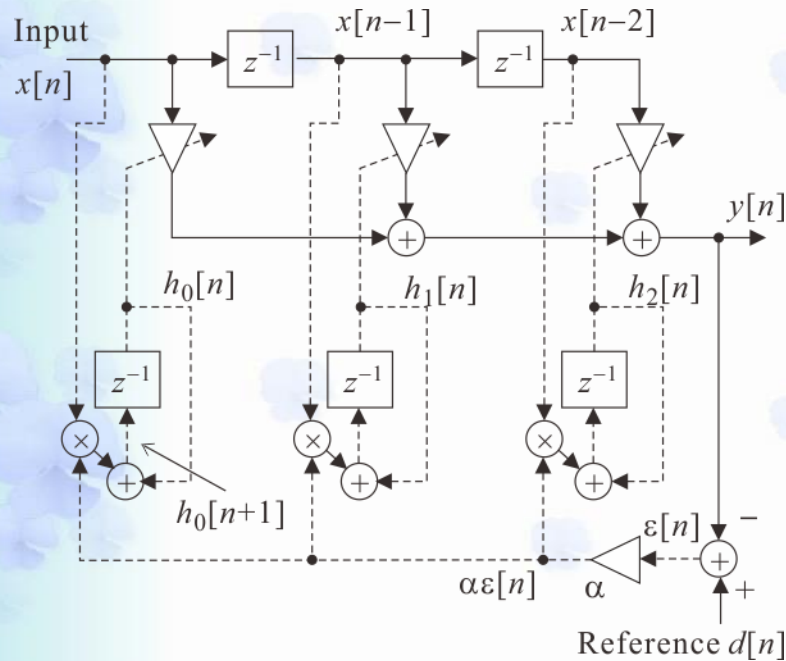
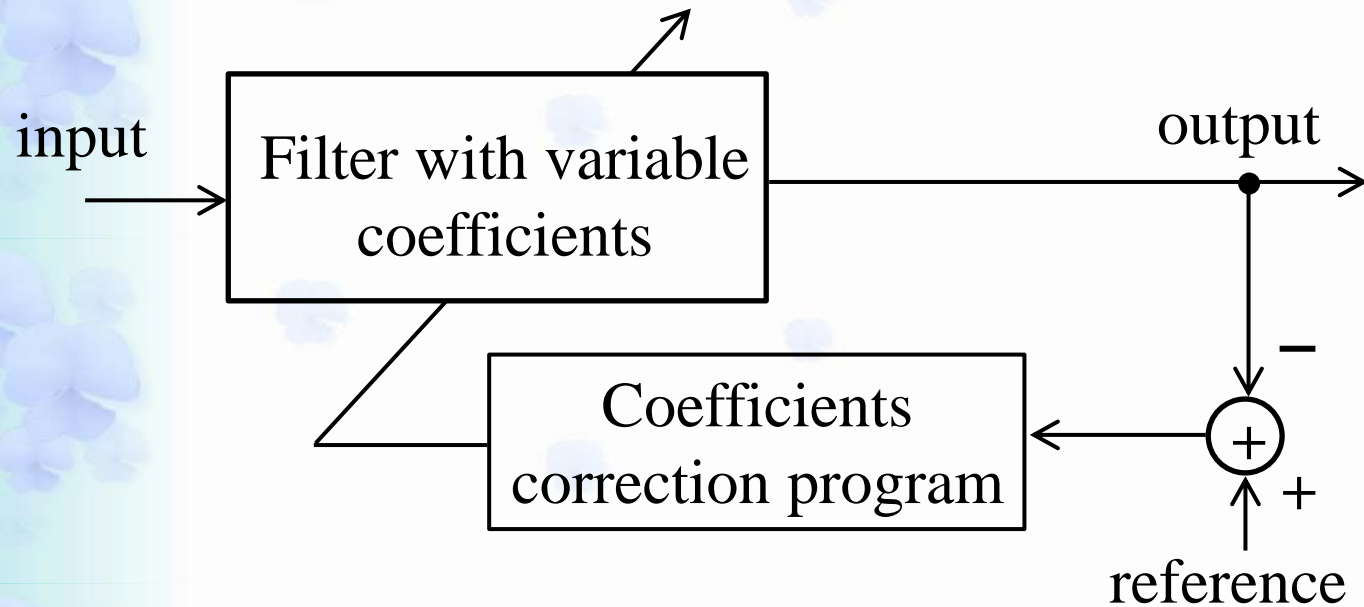
DESIGN FILTER

I am working on **TFilter2**. Screenshot here.

- CIC (Sinc) filters - faster than FIR for decimation
- the effect of quantization (fixed point) shown
- save/load/share configuration
- resampling, aliasing visualized
- signal chain

Buy me a beer to support the developer

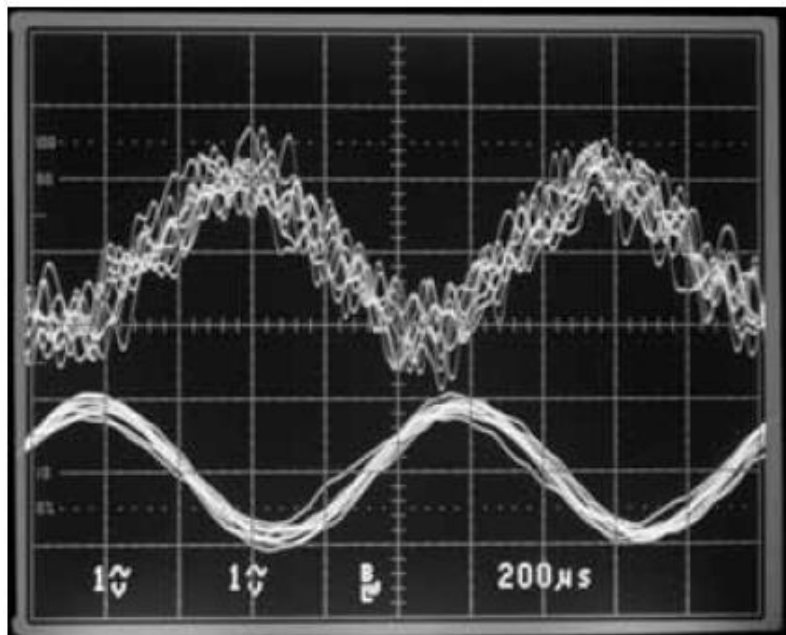
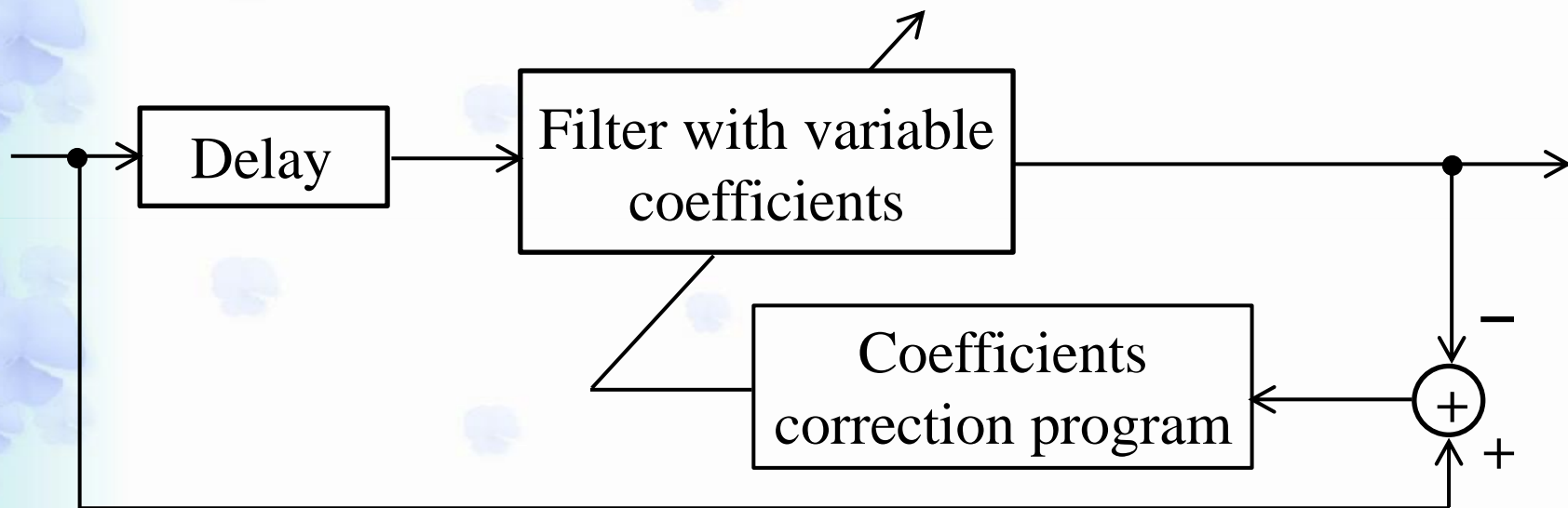
Adaptive filter



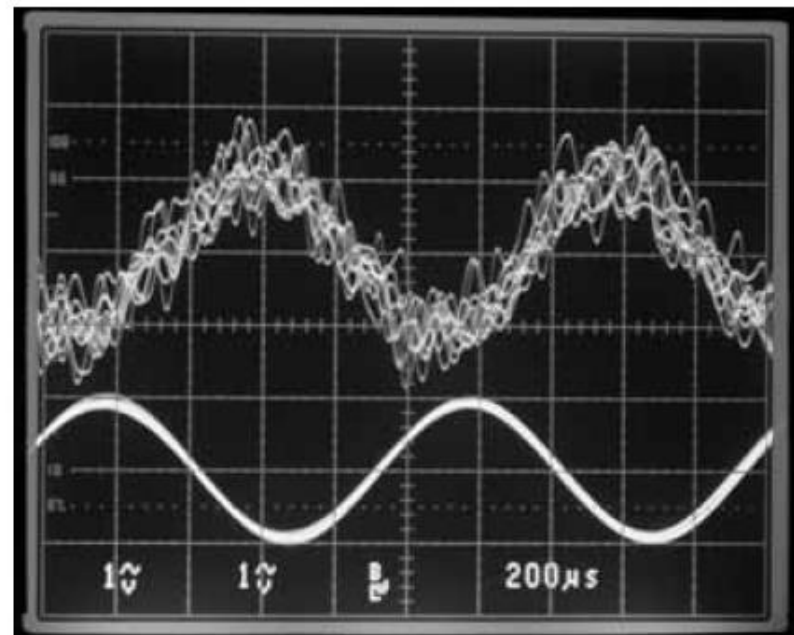
Least mean square method:

$$h_k[l + 1] = h_k[l] + 2\alpha\epsilon[l]x[l - k]$$

Adaptive filter (adaptive line enhancer)



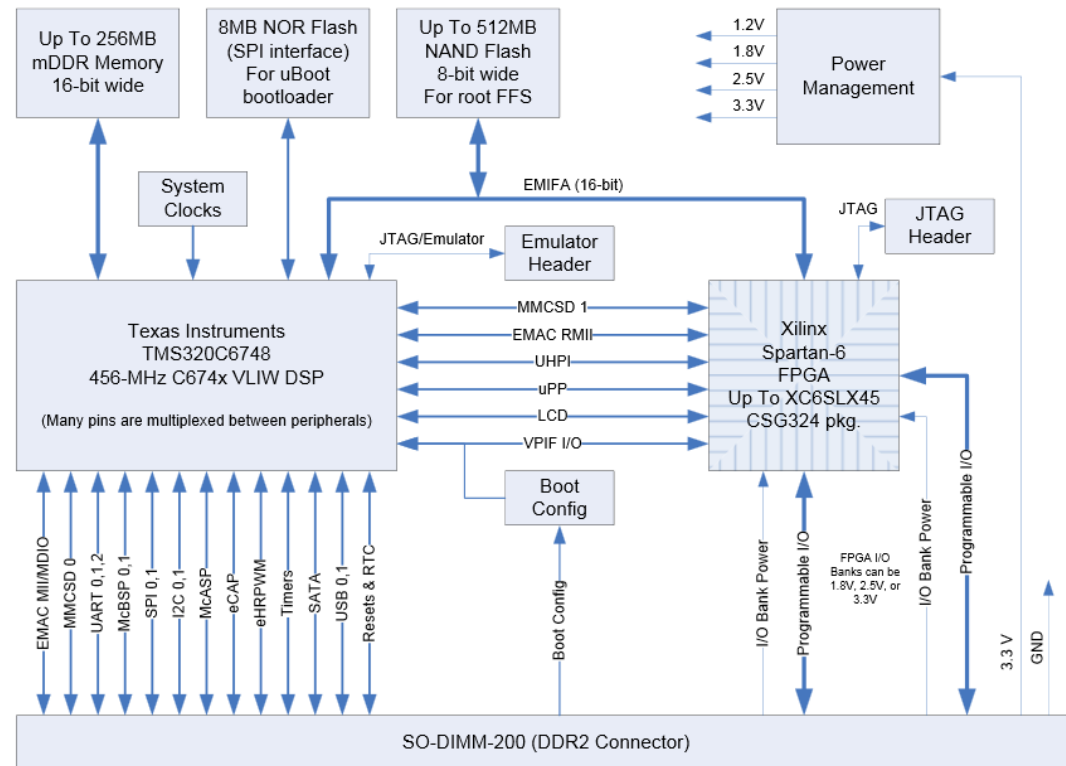
(a) $\mu = 1 \times 10^{-3}$ の場合



(b) $\mu = 1 \times 10^{-5}$ の場合

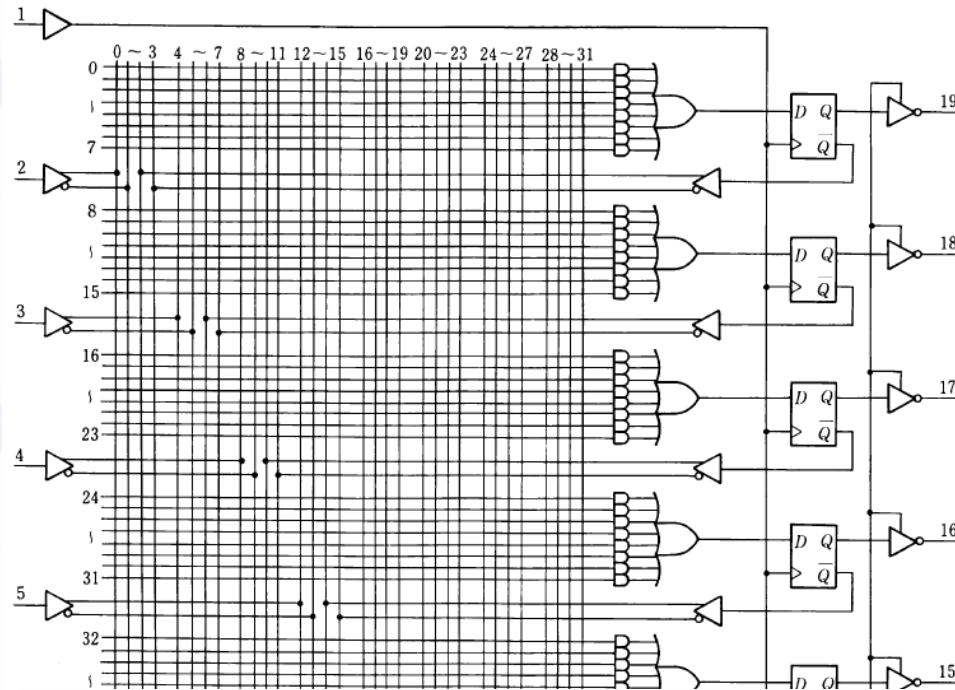
Digital filter implementation

Digital signal processing (DSP) board

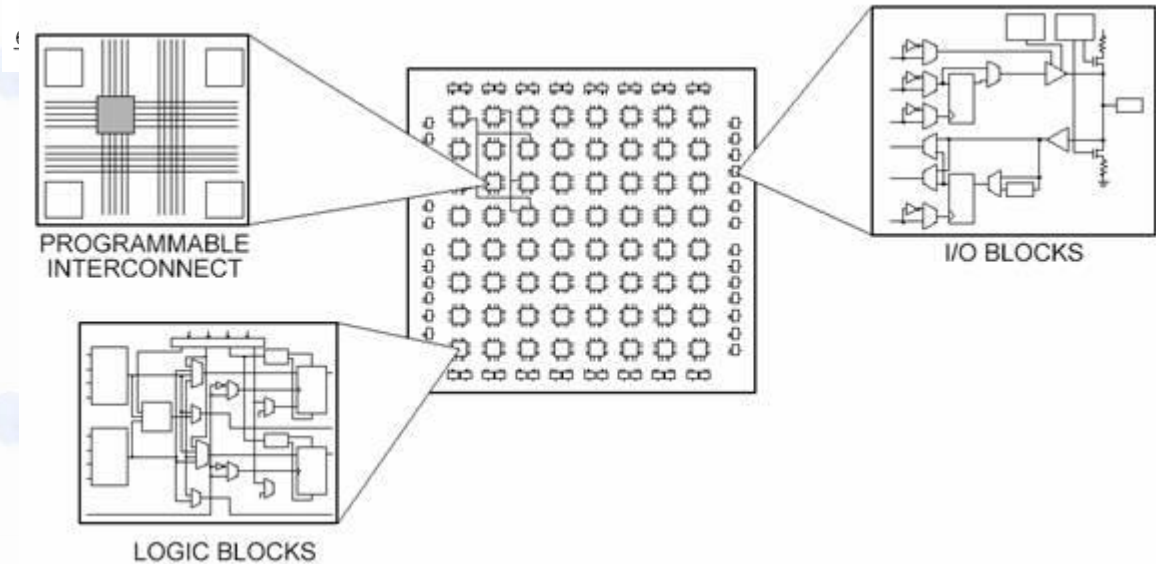


PLD/FPGA with HDL

Example of programmable logic device (PLD) circuit



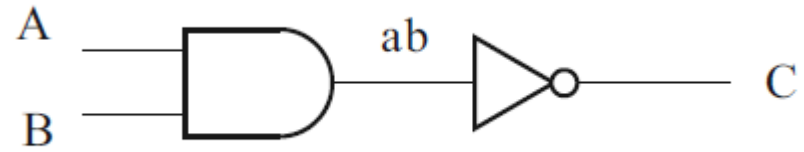
Example of field-programmable gate array (FPGA) circuit



FPGA \in PLD

Hardware description language, HDL

```
-- Library declaration -----  
library IEEE;  
use IEEE, STD_LOGIC_1164.ALL;  
-- Entity declaration -----  
entity NAND_CIRCUIT is  
port(  
  A : in std_logic;  
  B : in std_logic;  
  C : out std_logic  
);  
end NAND_CIRCUIT;  
-- Architecture declaration -----  
architecture RTL of NAND_CIRCUIT is  
  signal ab : std_logic;  
begin  
  ab <= A and B;  
  C <= not ab;  
end RTL;
```



RTL: register transfer level

Cf. SPICE

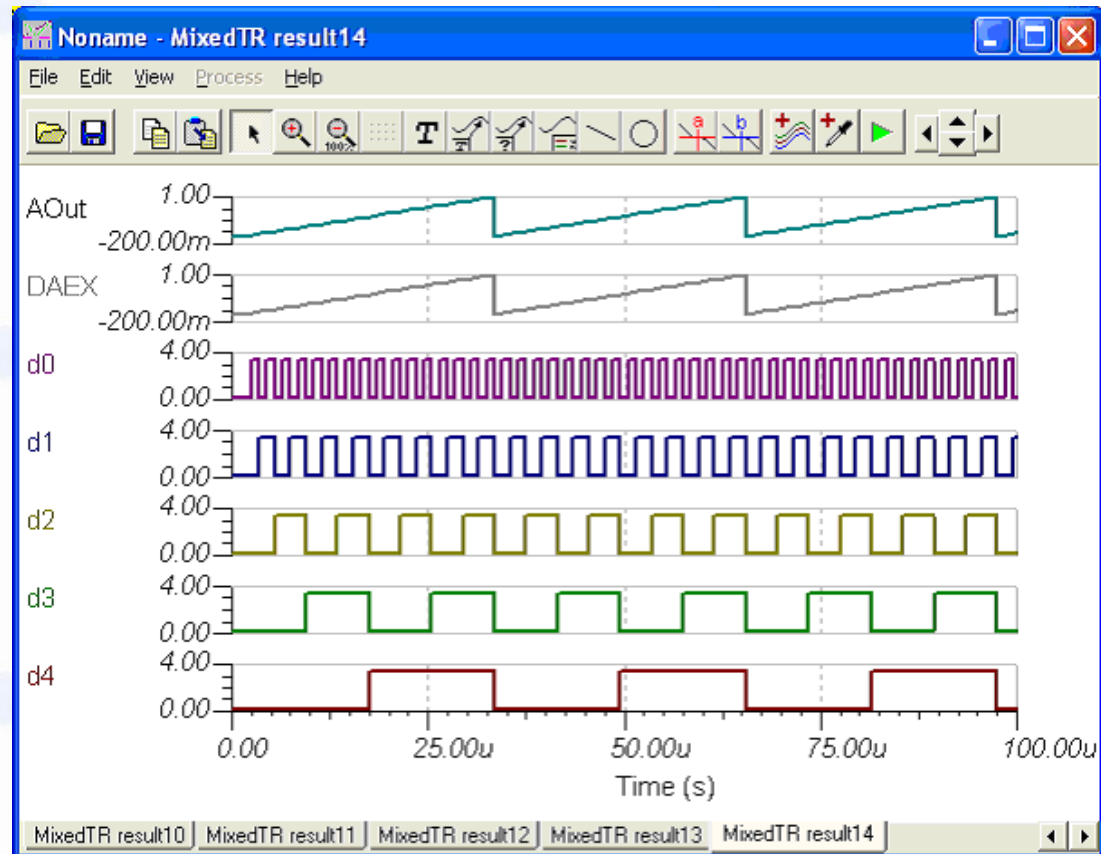
Spice+HDL mixed circuit simulation

Easier logic circuit design

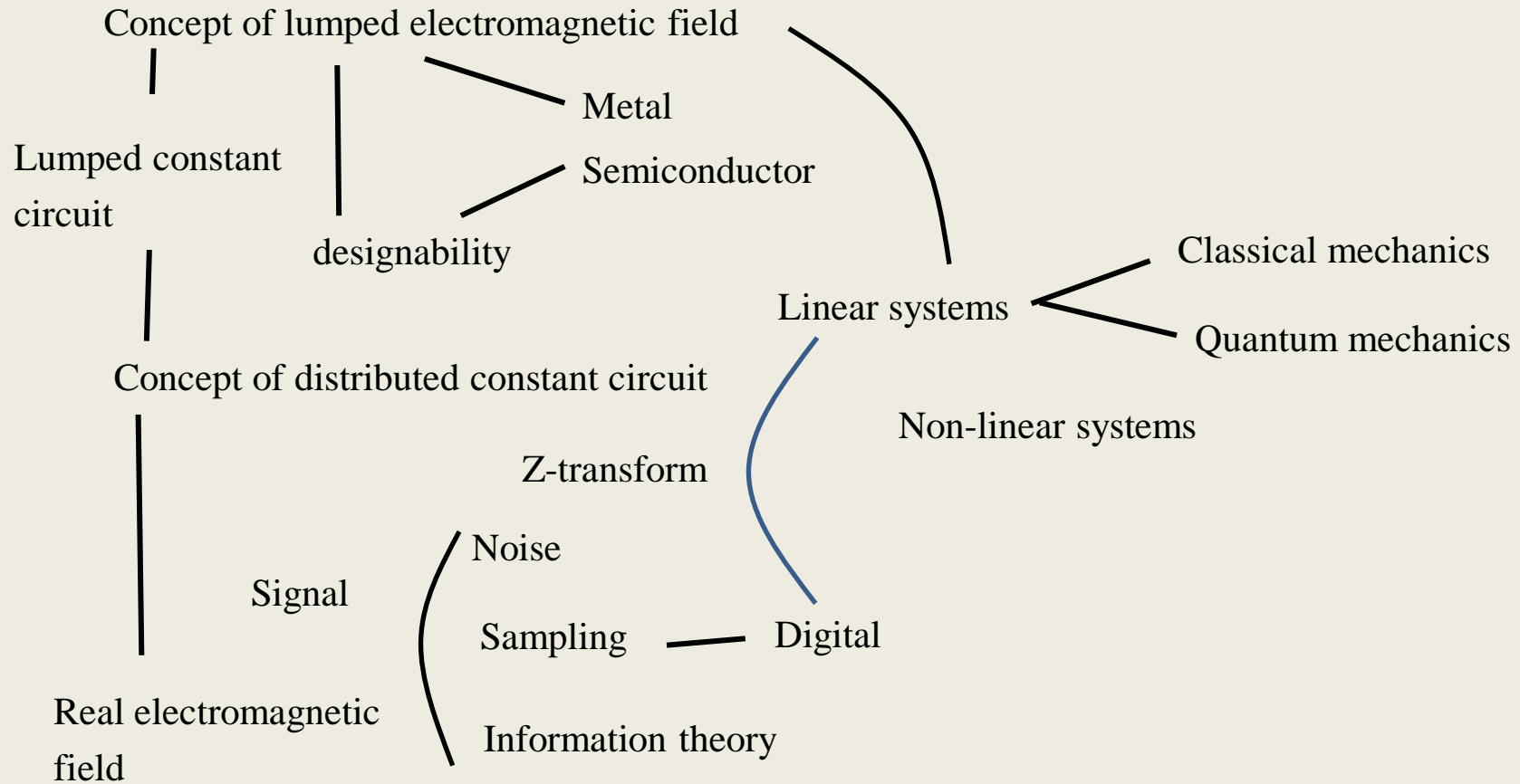
Polyphony: Python \rightarrow Verilog HDL <http://www.sinby.com/PolyPhony/index.html>

C-to-Hardware compiler (CHC) http://anvil.co.jp/?page_id=296

Mixed circuit simulation

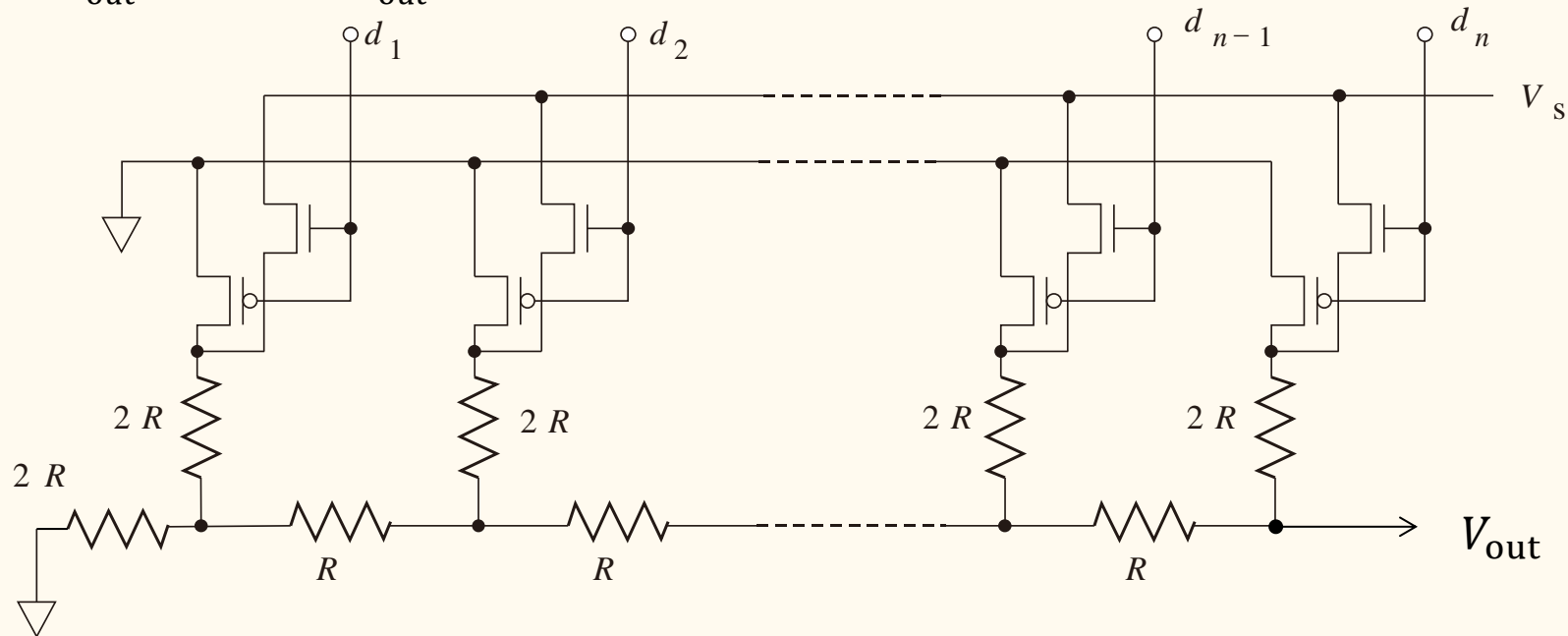


Electric Circuits: Treasury of Languages and Concepts

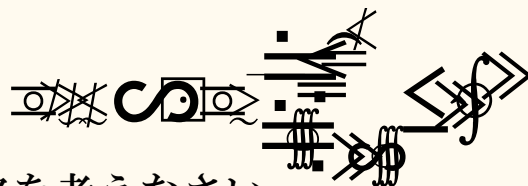


電子回路論レポート問題: 1. DA変換回路

(1) 次の図のような抵抗ラダー型DA変換回路を考える。講義で扱ったものと端の処理だけが違っている。この回路に2進数列 $\{d_k\}$ ($k = 1, \dots, n$)が入力されたとき、出力電圧 V_{out} を求めよ。 V_{out} は、高入力インピダンスアンプで受けるものとする。



(2) 手元に、 $\{R_0/2^k\}$ ($k=0, \dots, n$)の抵抗値列を持つ抵抗、抵抗値 R_f の抵抗、OPアンプ、電圧 V_s の標準電源、 $n+1$ 個の n チャンネルMOSスイッチ、同じく $n+1$ 個の p チャンネルMOSスイッチがある。これらを使って、2進数列 $\{d_k\}$ が入力された時に



を出力するDA変換器回路を考えなさい。

電子回路レポート問題 : 1. DA変換回路

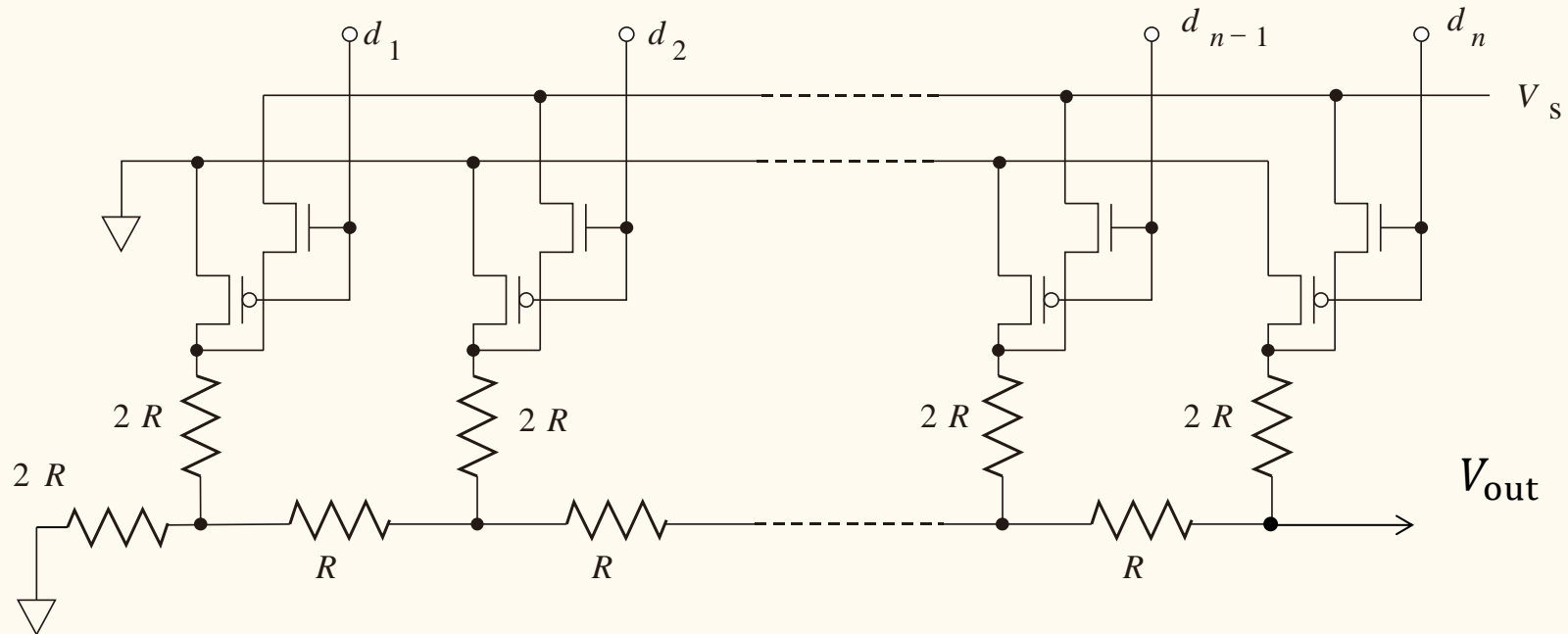
(3) (2) と同様、に手元に、 $\{2^k C_0\}$ ($k=0, \dots, n$)の抵抗値列を持つキャパシタ、電圧 V_S の標準電源、 $n+1$ 個の n チャンネルMOSスイッチ、同じく $n+1$ 個の p チャンネルMOSスイッチがある。これらを使って、2進数列 $\{d_k\}$ が入力された時に

$$V_{\text{out}} = \frac{V_S}{2^{n+1} - 1} \sum_{k=0}^n d_k 2^k$$

という出力が得られるようなDA変換回路を考えなさい。ただし、出力は入力インピダンスが高くバイアス電流が無視できるような増幅器で受けることとする。

Problems for the final report: 1. DA conversion circuits

(1) Let us consider the following resistance ladder DA conversion circuit. The right end is a bit different from the one we treated in the lecture. Calculate the output voltage V_{out} for the input $\{d_k\}$ ($k = 1, \dots, n$).



(2) We have resistors with values $\{R_0/2^k\}$ ($k = 0, \dots, n$), and R_f , an OP amp., a standard voltage source of the voltage V_S , $n + 1$ n-channel MOS switches, $n + 1$ p-channel MOS switches. With these components, design a DA conversion circuit which has the output

$$V_{\text{out}} = -V_S \frac{R_f}{R_0} \sum_{k=0}^n d_k 2^k \quad \text{for the binary input } \{d_k\}.$$

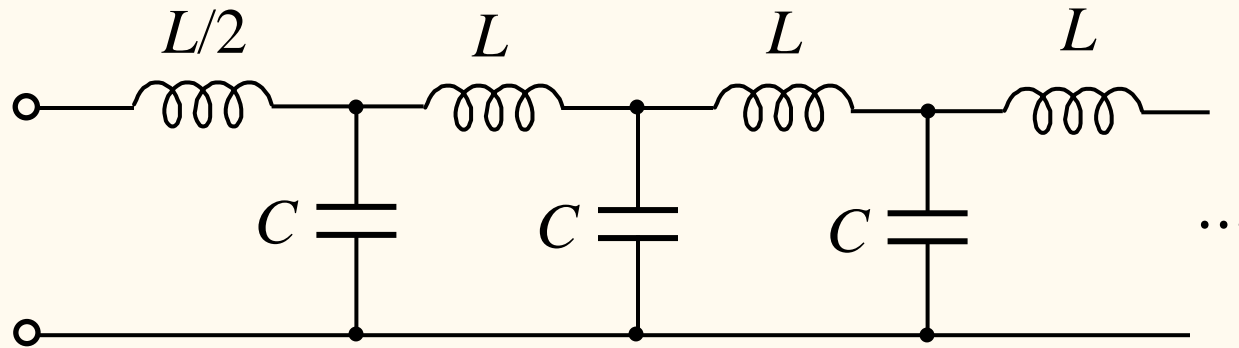
Problems for the final report: 1. DA conversion circuits

(3) We have capacitors with values $\{2^k C_0\}$ ($k = 0, \dots, n$), a standard voltage source of the voltage V_S , $n + 1$ n-channel MOS switches, $n + 1$ p-channel MOS switches. With these components, design a DA convertor circuit which has the output

$$V_{\text{out}} = \frac{V_S}{2^{n+1} - 1} \sum_{k=0}^n d_k 2^k$$

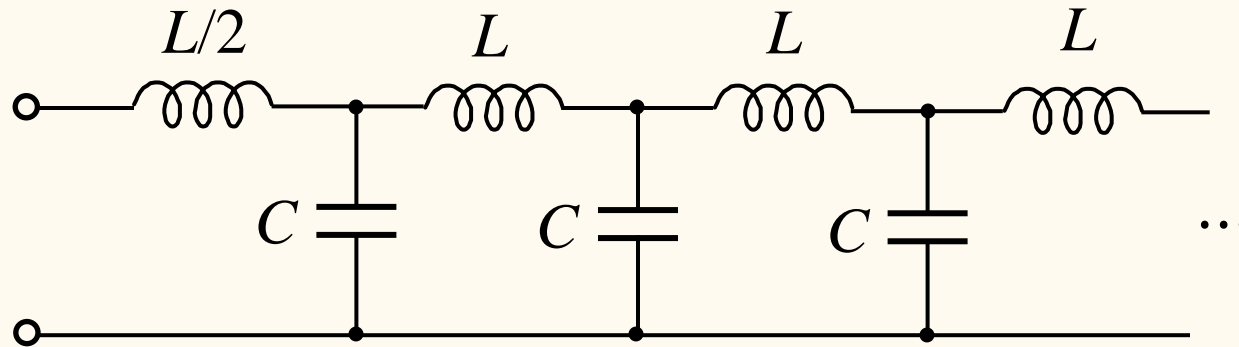
for the binary input $\{d_k\}$.

電子回路論レポート問題：2. 分布定数回路



上図のように，端のインダクタンス $L/2$ を除いて L と C が無限に繰り返す回路がある．この回路の，周波数軸上での透過域と減衰域を求めよ．また，左の端子から見たインピーダンスの周波数特性(周波数 ω に対するインピーダンス)を求めよ．

Problems for the final report: 2. Distributed constant circuit



Consider the above circuit with L, C infinite repetition to the right and the inductor $L/2$ at the left end. Obtain the transmission range and the attenuation range in the frequency domain. And what is the total impedance from the left end for the frequency ω .

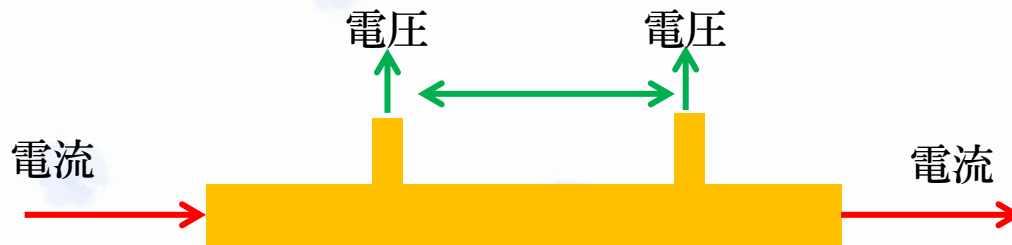
電子回路論レポート問題：3. OPアンプ回路

次のような回路部品がある。

高精度OPアンプ・・・4個

定電圧ダイオード(逆方向に電圧を印加するとツェナートンネルにより一定電圧を発生する) 2.5V・・・1個

これらを使って、低温で試料の電気抵抗を測定するための回路を構成しなさい。試料は下の図のように、電流端子、電圧端子が別に出た構造をしている。



ただし、

- (a) 抵抗, キャパシタ, インダクタの類の受動素子は適当に追加してよい。
- (b) OPアンプの電源を供給するためのトラッキング電源は準備されているものとする。
- (c) 測定抵抗の範囲は $100\Omega \sim 10\text{k}\Omega$ で、接触抵抗を含めて $50\text{k}\Omega$ 以内である。
- (d) OPアンプのオフセット電圧, バイアス電流は無視できるとする。従ってオフセット調整回路を入れる必要はない。

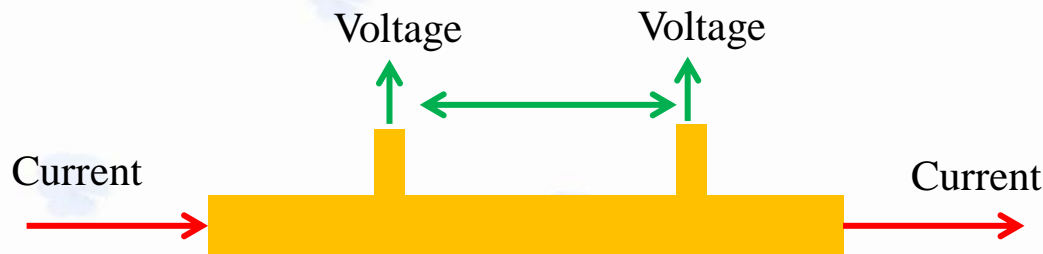
Problems for the final report: 3. OP amp. circuit

We have the following components:

4 high precision operational amplifiers,

1 high precision Zener tunnel diode with the constant voltage 2.5V (this diode provide precise 2.5V for the reverse bias).

With these components, design a circuit to measure the electric resistance of a sample at low temperatures. The shape of the sample is shown below:



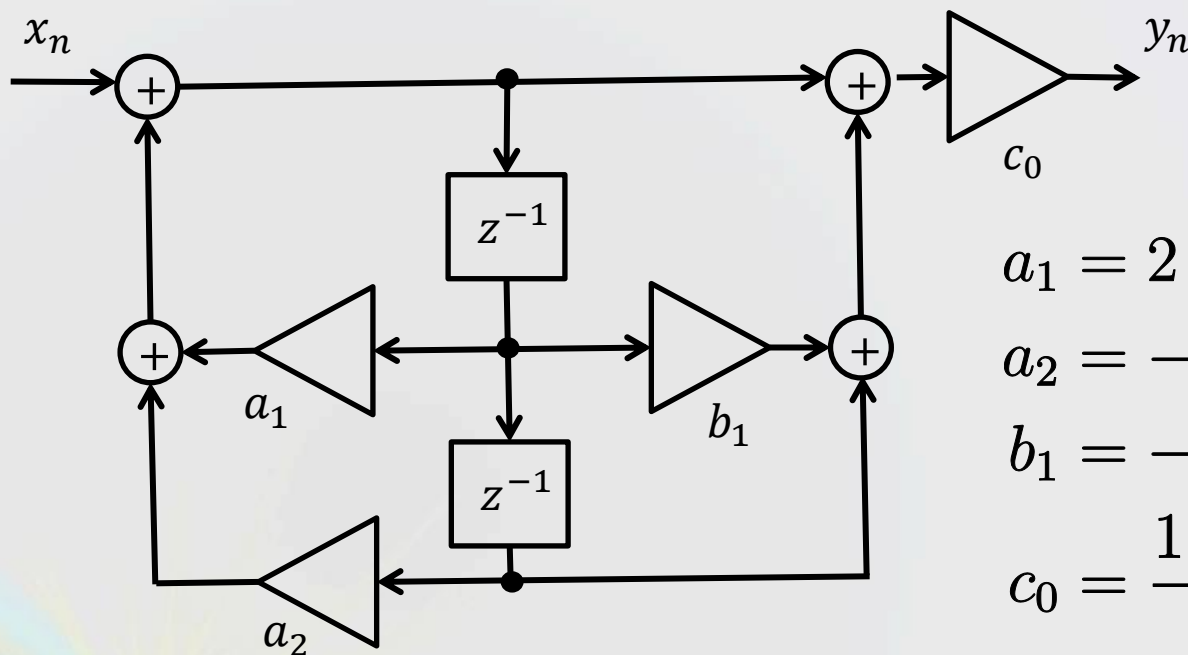
(a) You can add any passive elements (resistors, capacitors, inductors).

(b) The power supply for the OP amps. is ready.

(c) The sample resistance range is from 100Ω to $10\text{k}\Omega$, lower than $50\text{k}\Omega$ including the contact resistance.

(d) The offset voltages, the bias currents of the OP amps. can be ignored. No need for the offset cancellation circuit.

電子回路論レポート問題:4. デジタル・フィルタ



$$a_1 = 2 \exp(-\pi g_0 \tau) \cos(2\pi f_0 \tau)$$

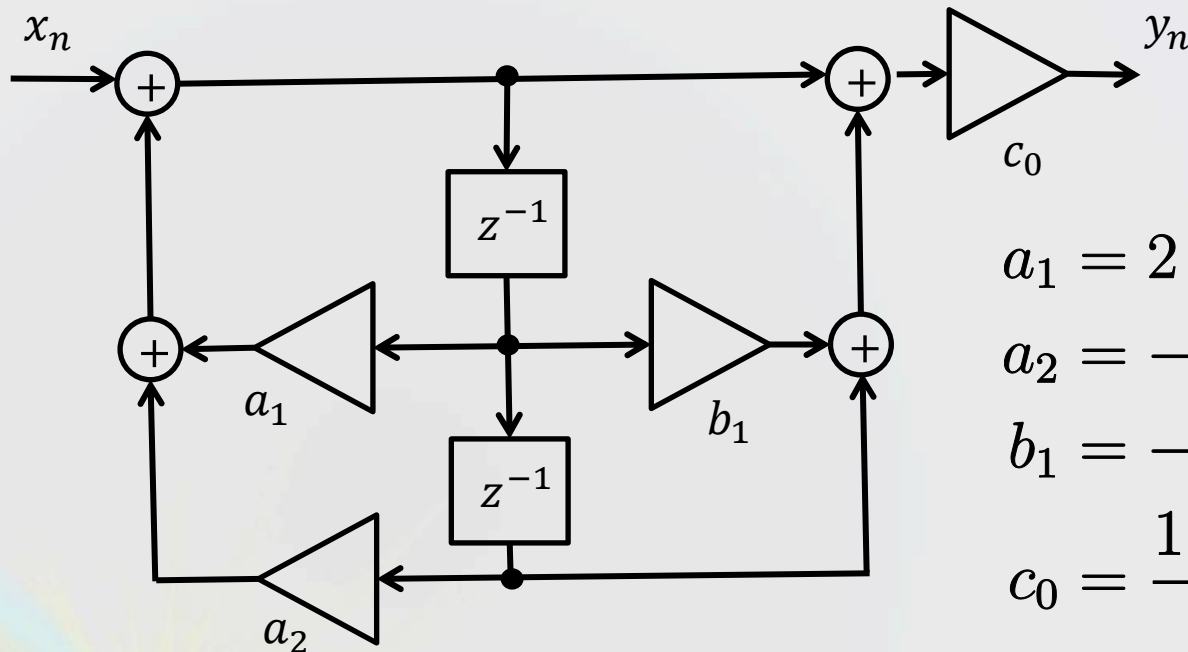
$$a_2 = -\exp(-2\pi g_0 \tau)$$

$$b_1 = -2 \cos(2\pi f_0 \tau)$$

$$c_0 = \frac{1 - a_1 - a_2}{2 + b_1}$$

- (1) 上のブロックダイアグラムから，出力 y_n, y_{n-1}, y_{n-2} と入力 x_n, x_{n-1}, x_{n-2} との関係式を示せ。
- (2) 係数 a_1, a_2, b_1, c_0 が右上のような関係を満たすとき，このフィルタはどのような周波数特性を示すか．ただし， $g_0 < f_0 < f_s/2$ (サンプリング周波数)を満たすとする．($20g_0 = 10f_0 = f_s$ としてグラフを描いてみよ．)

Problems for the final report: 4. Digital filter



$$a_1 = 2 \exp(-\pi g_0 \tau) \cos(2\pi f_0 \tau)$$

$$a_2 = -\exp(-2\pi g_0 \tau)$$

$$b_1 = -2 \cos(2\pi f_0 \tau)$$

$$c_0 = \frac{1 - a_1 - a_2}{2 + b_1}$$

- (1) From the above diagram, write down the relation between the output y_n, y_{n-1}, y_{n-2} and the input x_n, x_{n-1}, x_{n-2} .
- (2) When the coefficients a_1, a_2, b_1, c_0 satisfy the above relations, obtain the frequency characteristics of this filter. Draw a rough sketch of the graph for $20g_0 = 10f_0 = f_s$.

電子回路論レポート問題: 5 離散フーリエ変換

<http://kats.issp.u-tokyo.ac.jp/kats/electroniccircuit/report/reportdata.txt>

に、4096点のデータが入っている。各行に x $f(x)$ という形で納められており、セパレーターはTAB記号 (ASCII番号9番)である。

このデータは、ある特定の周波数で励起した出力を取ったもので、信号は特定の周波数成分で応答する。ただし、その振幅は完全に一定とは限らない。

(1) 離散フーリエ変換 (現実的には高速フーリエ変換, FFTをかけることになると思われる)を施し、パワースペクトルの主ピークから信号の周波数を特定せよ。

(2) 特定の窓 (幅 256点程度が適当)を使って部分的にフーリエ変換を施し、上で求めた周波数に最も近い周波数成分振幅を求める。窓の位置を x が小さい位置から100点刻み程度に動かし、各点での振幅を窓位置に対してプロットせよ。

解答のみ記せばよい。プログラム等を示す必要はない。

Problems for the final report: 5 Discrete Fourier Transform

<http://kats.issp.u-tokyo.ac.jp/kats/electroniccircuit/report/reportdata.txt>

contains data of 4096 points. Each line has a point in the form $x \ f(x)$.
The separator between x and $f(x)$ is TAB (ASCII No.9).

The data are signal responding to an excitation with a particular frequency. Hence the signal has the same central frequency but the amplitude is not necessarily constant.

(1) Apply discrete Fourier transformation (DFT, practically fast Fourier transformation) to the signal and extract the central frequency from the main peak.

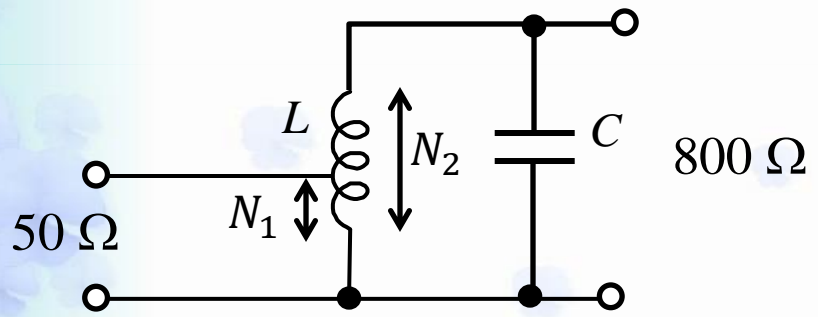
(2) Carry out DFT to a window with a shorter period (256 points is appropriate) and obtain the amplitude of the central frequency component. Shift the position of the window with a step size about 100 points. Plot the amplitude as a function of the window position.

You do not need to show your program codes for the analysis.

電子回路レポート問題：6 インピーダンス整合

FETを用いたアンプで信号ラインの特性インピーダンスとFETの入力インピーダンスとの間で整合を取りゲインを最大化すると、雑音も増幅され、雑音指数 (Noise figure, NF) が悪くなる。

そこで、 $50\ \Omega$ ラインの信号をFETから見たときのインピーダンスが $800\ \Omega$ 程度になるように変換する。入力は $80\ \text{MHz} \sim 90\ \text{MHz}$ の共鳴フィルターを構成する。

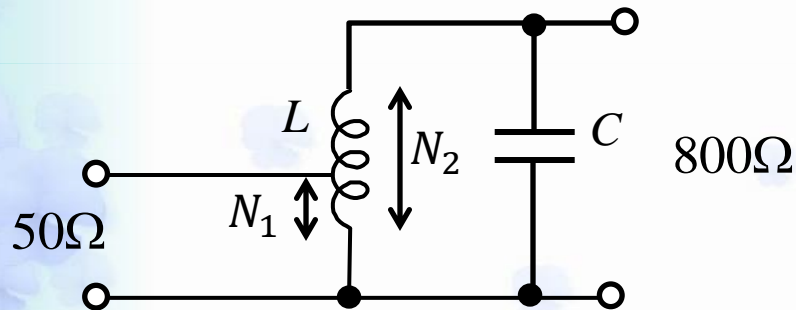


これを左図のようにコイルに中間タップを出すことで行う。全体のコイル巻き数 N_2 に対してタップの巻き数位置は N_1 とする。入力中心周波数は $85\ \text{MHz}$ 、共鳴幅(半値幅)は $10\ \text{MHz}$ とする。 C と L 及び N_1 の N_2 に対する比を求めよ。(有効数字3桁)

ここでは、コイルのインダクタンスは巻き数の2乗に比例するとする。

Problems for the final report: 6 Impedance matching

In amplifiers with FETs, the noise matching condition, that optimizes the noise figure, usually deviates from the power matching one. The impedance conversion circuit shown below thus converts the transmission line characteristic impedance $50\ \Omega$ into $800\ \Omega$. The input to FET also constitutes a resonance filter for 80 MHz to 90 MHz.



The coil in the left has a tap at the winding number N_1 in the total winding number N_2 . The central frequency of input is 85 MHz, the resonance width (peak width at half height) is 10 MHz. Calculate C , L , and the ratio of N_1 to N_2 . (significant digits =3)
Assume the inductances are proportional to squares of the winding numbers.