電子回路論　第2回

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2016.10.5

Electric Circuits No.2

Shingo Katumoto

ノート・資料等の置き場

http://kats.issp.u-tokyo.ac.jp/kats/

勝本信吾

Shingo Katsumoto

研究紹介
メンバー
実験装置
投稿
出版リスト

自己紹介

現在の研究テーマ

論文リスト

「ポケットに電磁気を」が単行本になりました

出版された書籍

物理屋のための「電子回路論」講義ノート（2015 Oct.-2016 Jun.）

2週に１回簡単な練習問題を出題　→　2週間以内に解答を提出

TA

試験は期末レポート．練習問題と合わせて採点します

Review

Ch.1 Electromagnetic field and electric circuits

Metals: super-screening material

(but not superconducting. The difference is important in designing superconducting circuits.)

Local electromagnetic field

→ Lumped constant circuits (集中定数回路)

local magnetic fields (parts) are connected by metallic wires → Circuit diagrams

Resistors, Capacitors and Inductors

Ch.2 Introduction to linear response systems

Outline Today

1. Transfer function (伝達関数) (continued)
2. Representative passive devices in the linear treatment
3. Impedance, admittance and other parameters in the linear treatment
4. Power sources
5. Circuit networks
6. Four terminal (two terminal-pair) circuits
7. Circuit theorems

Linear response: Transfer function

$$\overset{\text{response}}{w(t)} = \mathscr{R}\{\overset{\text{input}}{u(t)}\} = \mathscr{R}\left\{\int_{-\infty}^{\infty} u(t')\delta(t-t')dt'\right\} = \int_{-\infty}^{\infty} u(t')\underbrace{\mathscr{R}\{\delta(t-t')\}}_{\text{impulse response}}dt'$$

$$= \int_{-\infty}^{\infty} u(t')\xi(t-t')dt' = \int_{-\infty}^{\infty} u(t-t')\xi(t')dt'$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt \qquad W(\omega) = U(\omega)\boxed{\Xi(\omega)}$$

Transfer function

Laplace:

$$X(s) = \int_{0}^{\infty} e^{-st}x(t)dt \qquad W(s) = U(s)\boxed{\Xi(s)}$$

Expansion to the complex plane: $s \longrightarrow \sigma + i\omega$

On the imaginary axis (the frequency space)

$$W(i\omega) = U(i\omega)\Xi(i\omega)$$

Impedance

Current to voltage

$$V_{12} = \hat{A} I_{12} \quad \begin{cases} V_{12} = R I_{12} & \text{resistor} \\ V_{12} = \dfrac{q(t)}{C} = \dfrac{1}{C} \displaystyle\int^{t} I_{12}(t') dt' & \text{capacitor} \\ V_{12} = L \dfrac{d I_{12}}{dt} & \text{inductor} \end{cases}$$

$$\Xi(i\omega) = \begin{cases} \displaystyle\int_{-\infty}^{\infty} e^{-st} \left[R \delta(t) \right] dt = R & \text{resistor} \\ \displaystyle\int_{-\infty}^{\infty} e^{-i\omega t} \left[\frac{1}{C} \int^{t} \delta(t') dt' \right] dt = \frac{1}{i\omega C} & \text{capacitor} \\ \displaystyle\int_{-\infty}^{\infty} e^{-st} \left[L \frac{d}{dt} \delta(t) \right] dt = i\omega L & \text{inductor} \end{cases}$$

$$\downarrow$$

Impedance $Z(i\omega)$

Admittance

Voltage to current

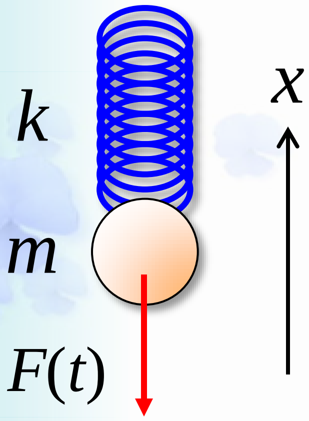
$$\mathcal{J}(i\omega) = Y(i\omega)\mathcal{V}(i\omega)$$

$\boxed{\text{Admittance } Y(i\omega)}$

$$Y(i\omega) = \frac{1}{Z(i\omega)}$$

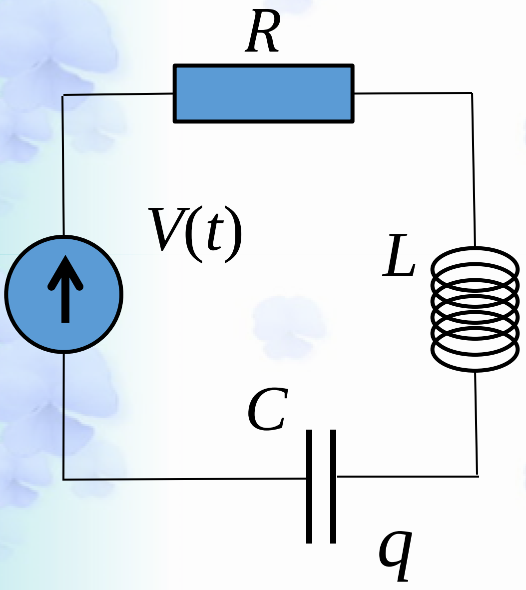
Example of equivalent circuit 「等価回路」の例

Spring pendulum (ばね振り子)



$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=F(t)$$

RLC circuit with electromotive force
(電源を接続したRLC回路)



$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{C}q=V(t)$$

Parallelism

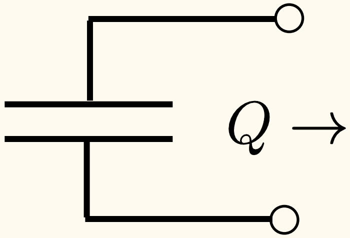
並行論

2.2 Power sources

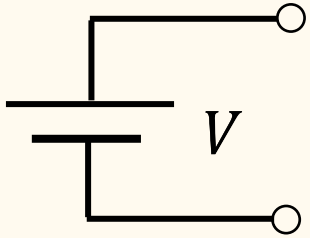
An active device: electric power source, electromotive force

Realistic power source: ideal power source + non-ideal factors

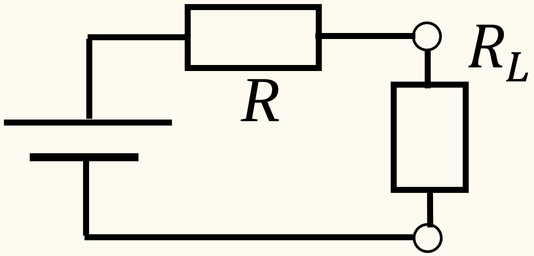
Ideal voltage source



$$Q \to \infty, \quad C \to \infty, \quad V = \frac{Q}{C} = \text{const.}$$



Voltage source
+resistor

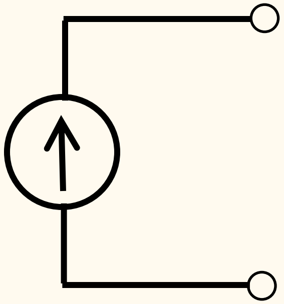


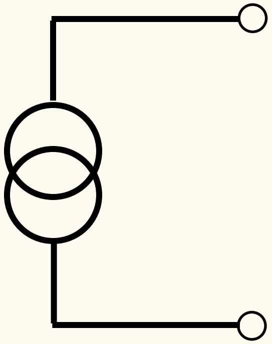
$$J = \frac{V}{R + R_L}$$

$$V_{out} = \frac{R_L}{R_L + R} V$$

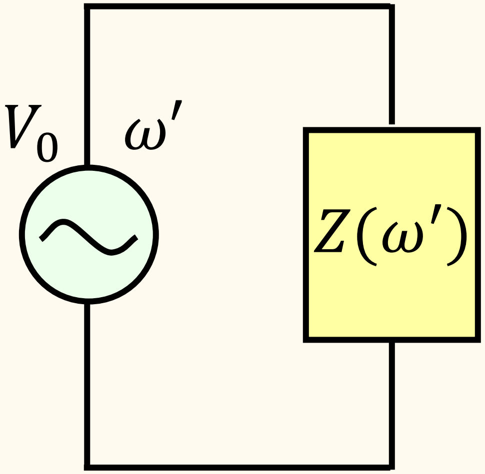
Ideal current source

$$R \to \infty, \; V \to \infty, \; J = \frac{V}{R} = \text{const.}$$





Power consumption



Energy dissipation per unit time: $P = VJ$

Electric power consumption

$$V(\omega',t) = V_0 e^{i\omega' t}$$

$$\mathcal{V}(i\omega) = \mathscr{F}\{V\} = 2\pi V_0 \delta(\omega - \omega')$$

$$J(\omega',t) = 2\pi \int_{-\infty}^{\infty} \frac{V_0}{Z(i\omega)} \delta(\omega - \omega') e^{i\omega t} \frac{d\omega}{2\pi} = \frac{V_0}{Z(i\omega')} e^{i\omega' t} = \frac{V(\omega',t)}{Z(i\omega')}$$

$$V = V_0 \cos \omega' t \qquad W(\omega',t) = V(\omega',t) J(\omega',t) = V_0^2 \cos^2 \omega t / Z(\omega')$$

Complex instantaneous power

$$\overline{W}(\omega') = \frac{V_0^2}{2Z(\omega')}$$

$$P(\omega') \equiv \mathrm{Re}[\overline{W}(\omega')]$$

Effective power
(有効電力)

$$Q(\omega') \equiv \mathrm{Im}[\overline{W}(\omega')]$$

Reactive power
(無効電力)

Power consumption (2)

$|\overline{W}(\omega')|$ Apparent power (皮相電力）

$$I_{\mathrm{M}} \equiv \frac{\mathrm{Re}[\overline{W}(\omega')]}{|\overline{W}(\omega')|} = \cos\left[\arg(\overline{W}(\omega'))\right] \equiv \cos\phi$$

Moment (力率)

ϕ: Phase shift between voltage and current

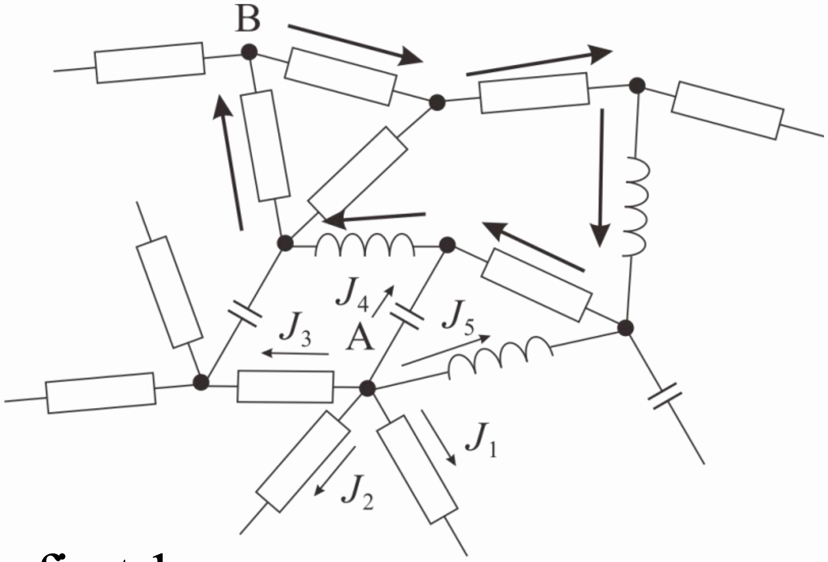
$$\overline{W}(\omega') = |\overline{W}(\omega')|e^{i\phi}$$: generally holds

$$\overline{W}(\omega') = V^*(\omega')J(\omega')$$

$$W = P = \frac{V_0^2}{2R}$$

$$\frac{V_0}{\sqrt{2}}$$: effective value

2.3 Circuit network



2.3.1 Kirchhoff's law

At all nodes $\sum_i J_i = 0$ Kirchhoff's first law

↑ Charge conservation $$\underbrace{\frac{\partial \rho}{\partial t}}_{=0} + \mathrm{div}\boldsymbol{J} = 0$$

For all looping paths $\sum_j V_j = 0$ Kichhoff's second law

↑ Single-valuedness of electric potential

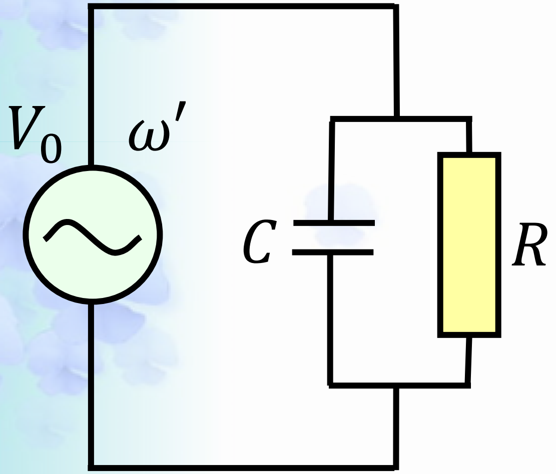
2.3 Circuit network (2)

From Kirchhoff's law, synthetic admittance and impedance are

for parallel connection: $$Y_{\text{tot}} = \sum_{i=1}^{n} Y_i, \qquad Z_{\text{tot}} = \left(\sum_{i=1}^{n} Z_i^{-1} \right)^{-1}$$

for series connection: $$Y_{\text{tot}} = \left(\sum_{i=1}^{n} Y_i^{-1} \right)^{-1}, \qquad Z_{\text{tot}} = \sum_{i=1}^{n} Z_i$$

Ex.)



$$Z(i\omega) = \left(\frac{1}{R} + i\omega C \right)^{-1}, \quad Y(i\omega) = \frac{1}{R} + i\omega C$$

$$P(\omega) = \frac{V_0^2}{R} \cos^2 \omega t, \quad Q(\omega) = \omega C V_0^2 \cos^2 \omega t$$

$$\frac{P(\omega)}{Q(\omega} = \frac{1}{\omega C R} = \tan \delta$$: Dissipation factor

2.3.3 Superposition theorem

Network: node, (directional) branch : directional graph (digraph)

All the branches: electromotive force E_i, resistance R_i

$$A\{(R)\}\begin{pmatrix} J_1 \\ \vdots \\ J_m \end{pmatrix} = \begin{pmatrix} E_1 \\ \vdots \\ E_m \end{pmatrix} \qquad \boldsymbol{R} = \begin{pmatrix} R_1 \\ \vdots \\ R_m \end{pmatrix}$$

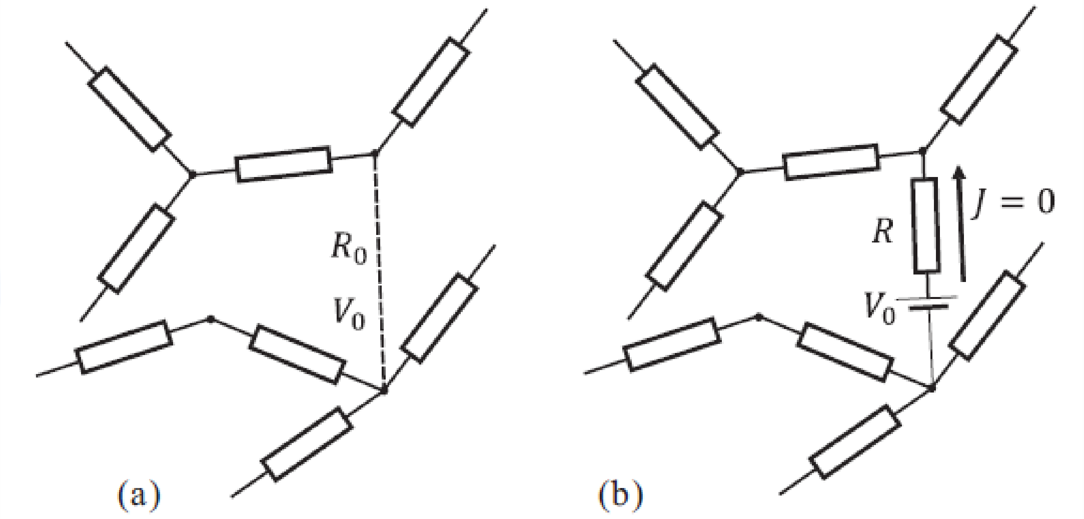
Superposition theorem:

The total current distribution is the superposition of those for single electromotive forces.

2.3.4 Ho (鳳) – Tevenin's theorem

Pick up two nodes in the network under consideration.
The voltage between these two nodes is V_0.
Set all the electromotive forces to zero and measure the resistance between the two nodes. The result is R_0.
Now connect the two nodes with resistance R and reset the electromotive forces to the original values. Then the current through resistance R is

$$J = V_0/(R + R_0)$$



2.3.5 Tellegen's theorem

$i = 1, \cdots, n$: index of nodes, $j = 1, \cdots, m$: index of branches

$$a_{ij} = \begin{cases} 1: & i \text{ is the start of } j, \\ -1: & i \text{ is the end of } j, \\ 0: & \text{others} \end{cases}$$

incidence matrix

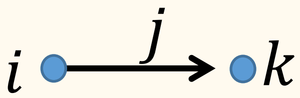
$$\forall j: \quad \sum_{i=1}^{n} a_{ij} = 0$$

: redundancy in $\{a_{ij}\}$

→ $(n-1) \times m$ matrix D : irreducible incidence matrix

Kirchhoff's first law: $D\boldsymbol{J} = 0$ J_j: current along branch j

$\boldsymbol{W}$: W_i electrostatic potential of node i, $\boldsymbol{V}$: V_j voltage across branch j



$$V_j = W_i - W_k = a_{ij} W_i - a_{kj} W_k$$

$$\boldsymbol{V} = {}^t\mathcal{D}\boldsymbol{W} \quad ({}^t\mathcal{D}: \text{ transpose})$$

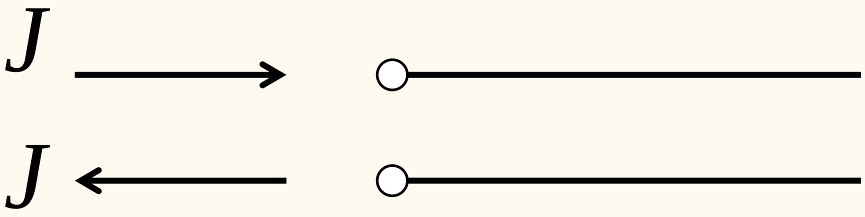
(Kirchhoff's second law)

$$\sum_{i=1}^{m} V_i J_i = ({}^t\mathcal{D}\boldsymbol{W}) \cdot \boldsymbol{J} = {}^t\boldsymbol{W}\mathcal{D}\boldsymbol{J} = 0 \qquad \boldsymbol{V} \perp \boldsymbol{J}$$

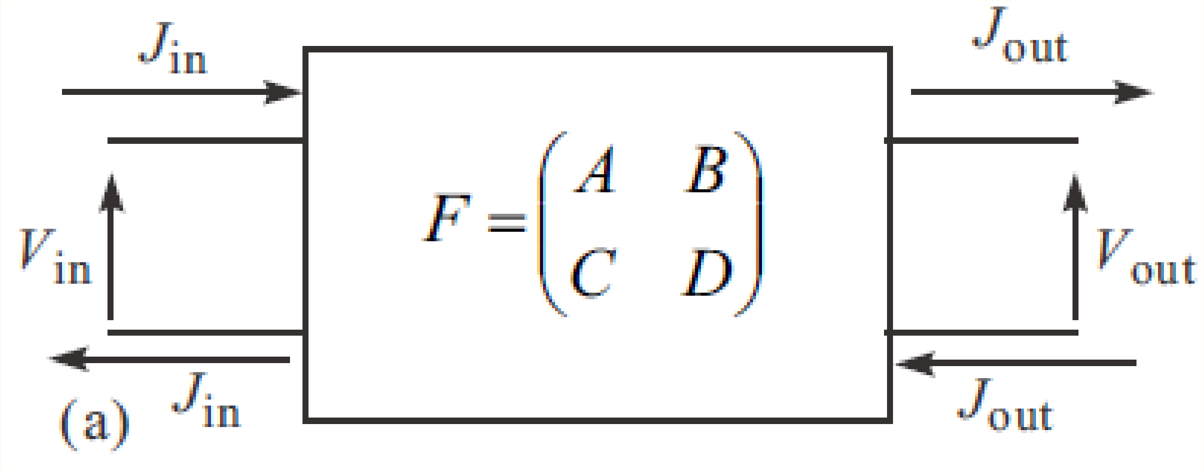
4-terminal (2-terminal pair) circuits 4端子回路

Terminal pair (端子対)

Current: circulation, no net current

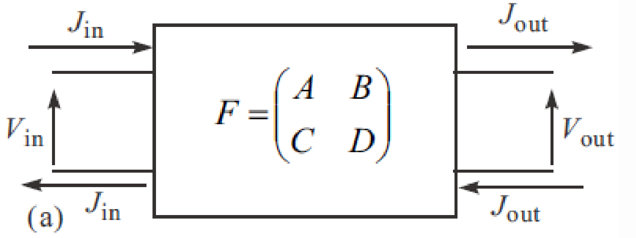


2-terminal pair (4-terminal) circuit



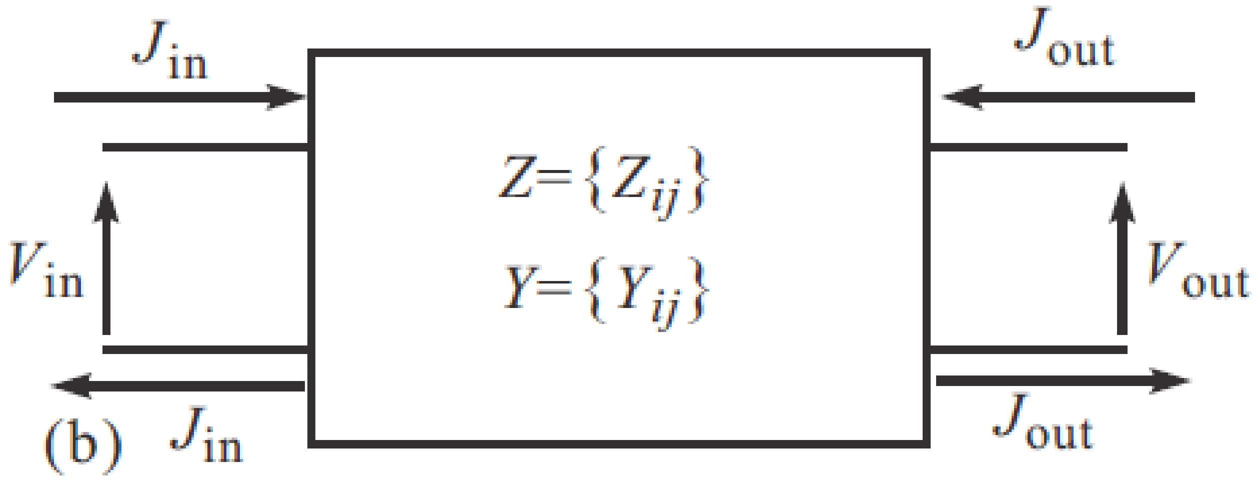
F-matrix of 4-terminal circuit

$$\begin{pmatrix} V_{\mathrm{in}} \\ J_{\mathrm{in}} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_{\mathrm{out}} \\ J_{\mathrm{out}} \end{pmatrix} \equiv F \begin{pmatrix} V_{\mathrm{out}} \\ J_{\mathrm{out}} \end{pmatrix}$$

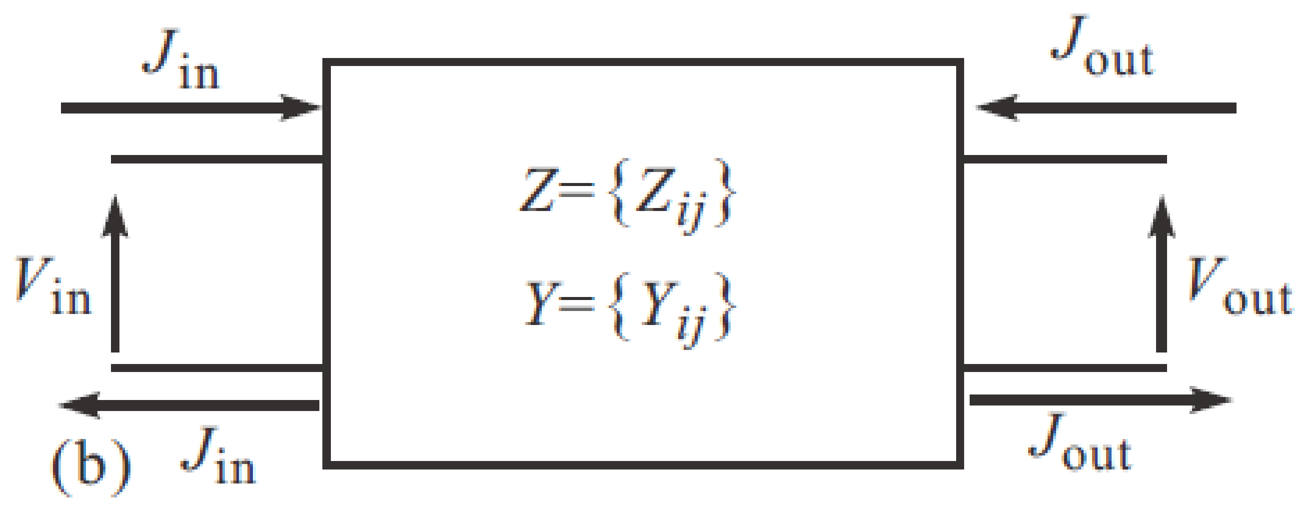


$$A = \left(\frac{V_{\mathrm{in}}}{V_{\mathrm{out}}} \right)_{J_{\mathrm{out}}=0}, \quad B = \left(\frac{V_{\mathrm{in}}}{J_{\mathrm{out}}} \right)_{V_{\mathrm{out}}=0}, \quad C = \left(\frac{J_{\mathrm{in}}}{V_{\mathrm{out}}} \right)_{J_{\mathrm{out}}=0}, \quad D = \left(\frac{J_{\mathrm{in}}}{J_{\mathrm{out}}} \right)_{V_{\mathrm{out}}=0}.$$

$$\begin{pmatrix} V_{\mathrm{out}} \\ J_{\mathrm{out}} \end{pmatrix} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} V_{\mathrm{in}} \\ J_{\mathrm{in}} \end{pmatrix} \equiv K \begin{pmatrix} V_{\mathrm{in}} \\ J_{\mathrm{in}} \end{pmatrix}$$



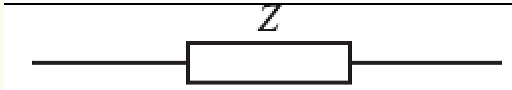
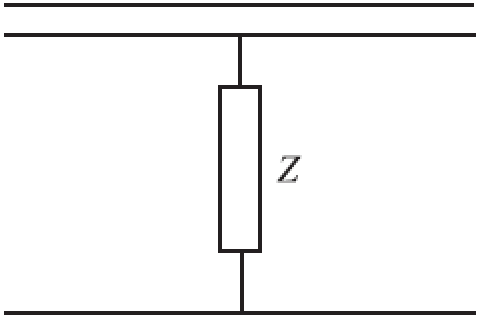
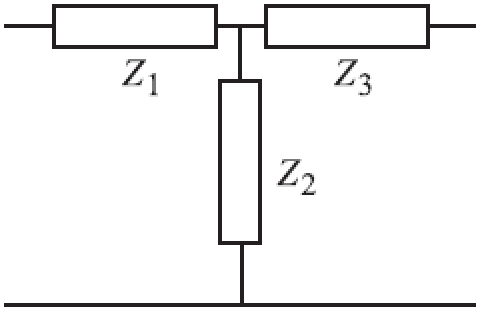
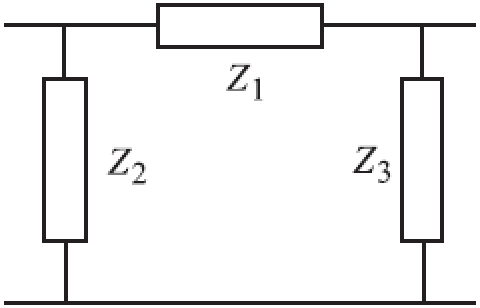
Impedance matrix, Admittance matrix



$$\begin{pmatrix} V_{\text{in}} \\ V_{\text{out}} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} J_{\text{in}} \\ J_{\text{out}} \end{pmatrix} \equiv Z \begin{pmatrix} J_{\text{in}} \\ J_{\text{out}} \end{pmatrix}$$

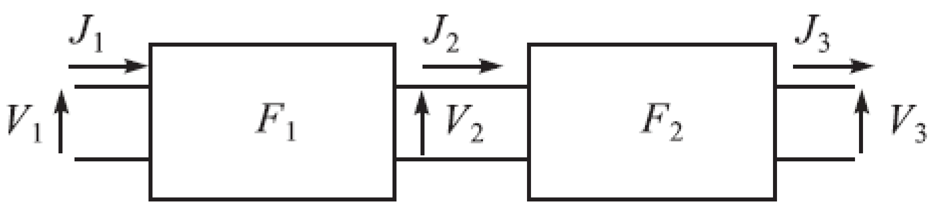
$$\begin{pmatrix} J_{\text{in}} \\ J_{\text{out}} \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_{\text{in}} \\ V_{\text{out}} \end{pmatrix} \equiv Y \begin{pmatrix} V_{\text{in}} \\ V_{\text{out}} \end{pmatrix}$$

Examples with impedances

	A	B	C	D
	1	Z	0	1
	1	0	$\frac{1}{Z}$	1
	$1+\frac{Z_1}{Z_2}$	$\frac{Z_1Z_2+Z_2Z_3+Z_3Z_1}{Z_2}$	$\frac{1}{Z_2}$	$1+\frac{Z_3}{Z_2}$
	$1+\frac{Z_1}{Z_3}$	Z_1	$\frac{Z_1+Z_2+Z_3}{Z_2Z_3}$	$1+\frac{Z_1}{Z_2}$

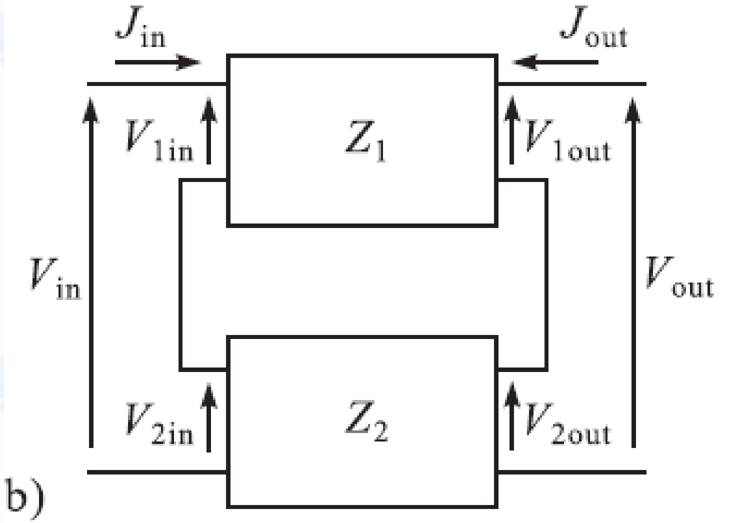
Connections of 4-terminal circuits

Cascade



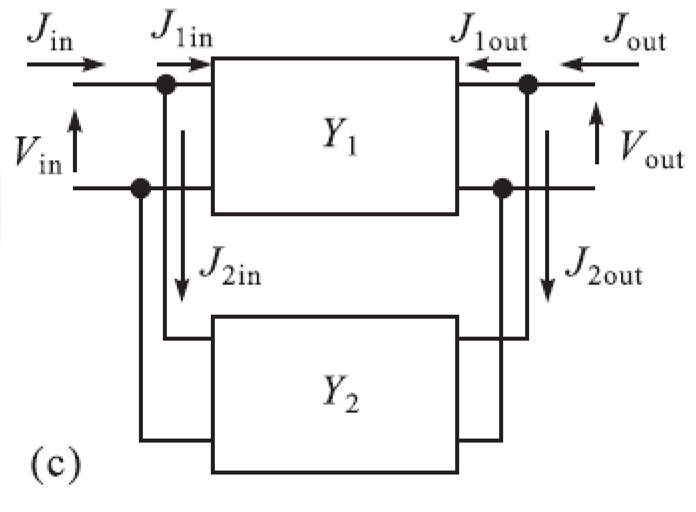
$$F_{\mathrm{tot}} = \prod_{i=1}^{N} F_i$$

Series



$$Z_{\mathrm{tot}} = \sum_{i=1}^{N} Z_i$$

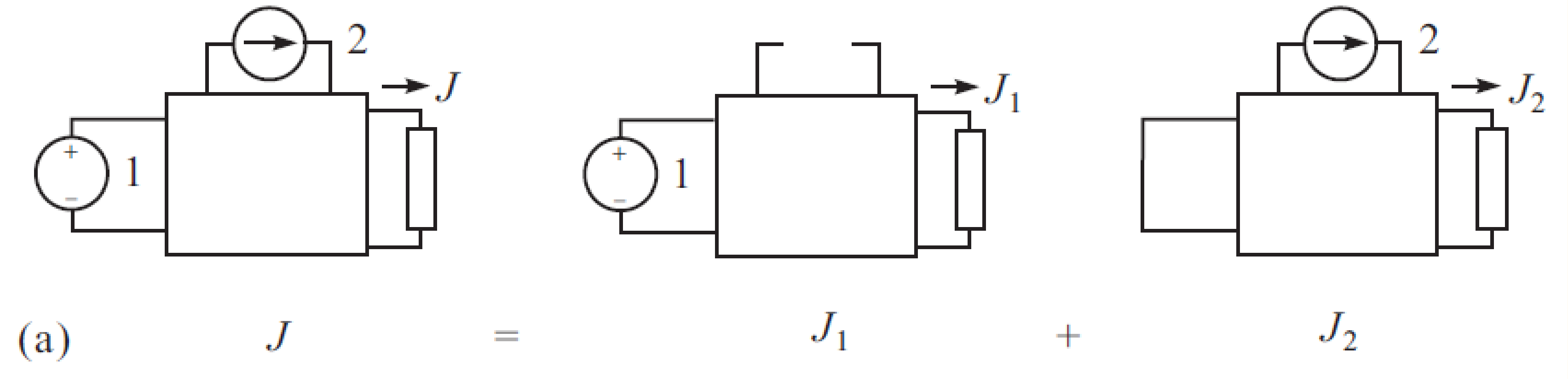
Parallel



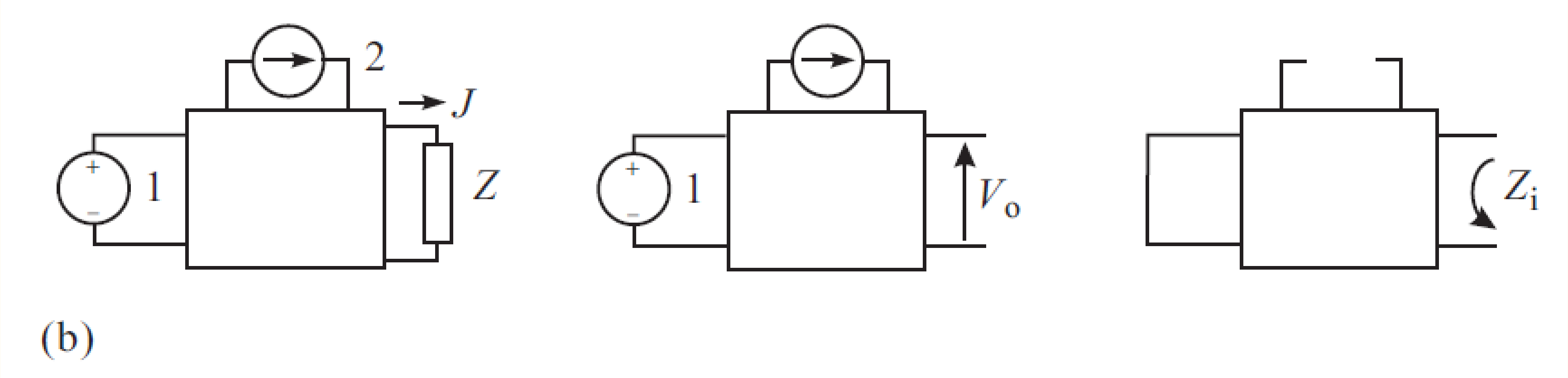
$$Y_{\mathrm{tot}} = \sum_{i=1}^{N} Y_i$$

Theorems for terminal-pair circuits

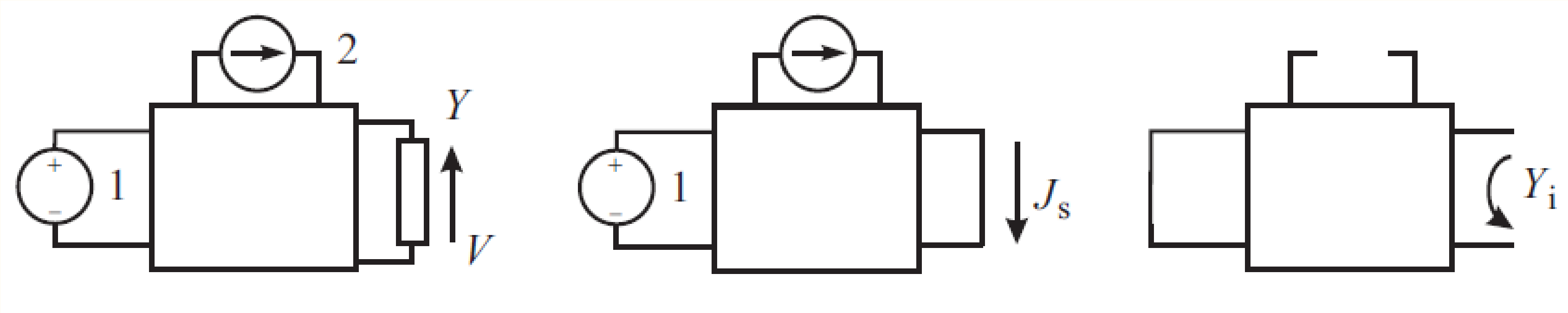
Superposition theorem



Ho-Thevenin's theorem

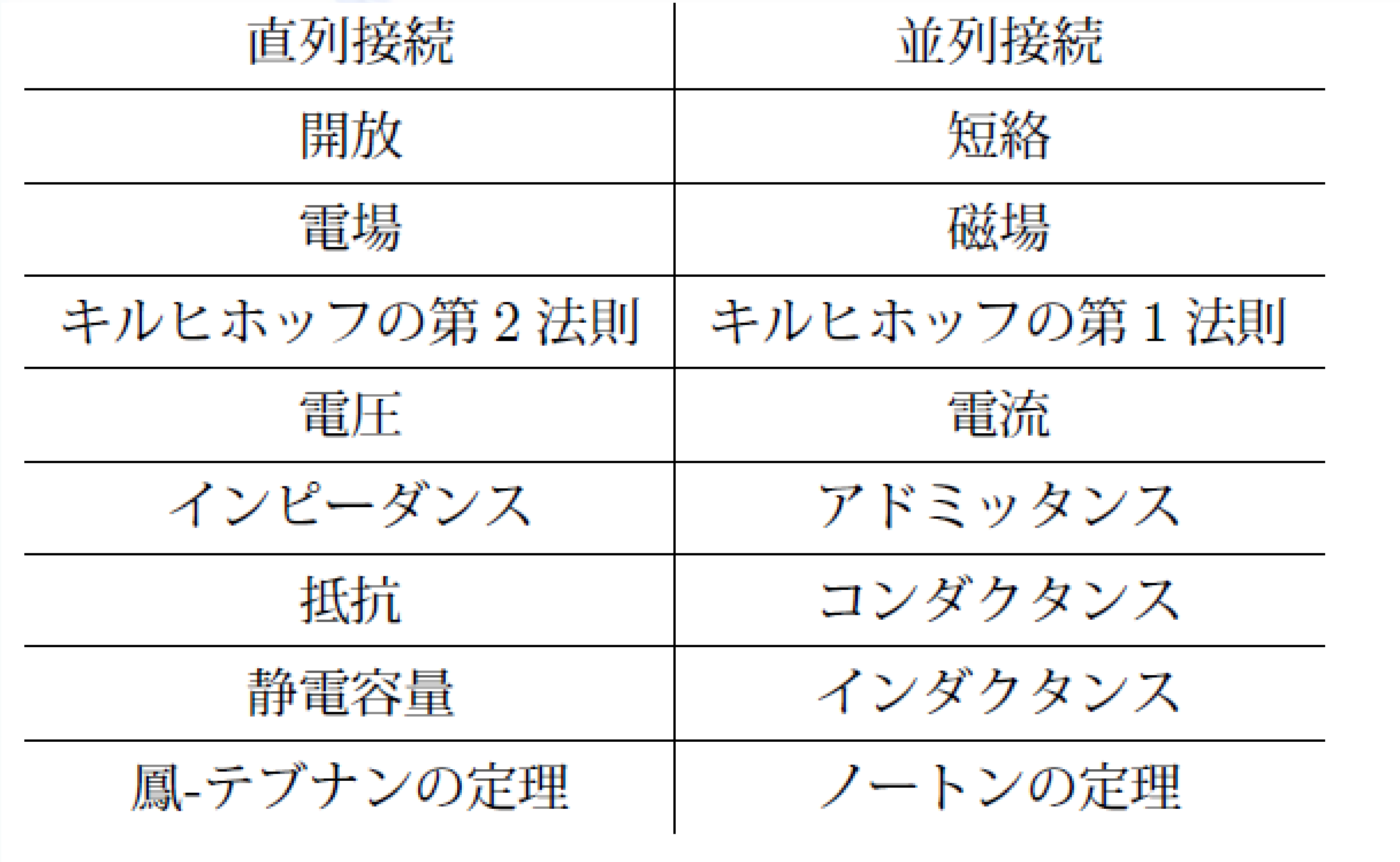


Norton's theorem



$$V = \frac{J_s}{Y + Y_i}$$

Duality 双対性



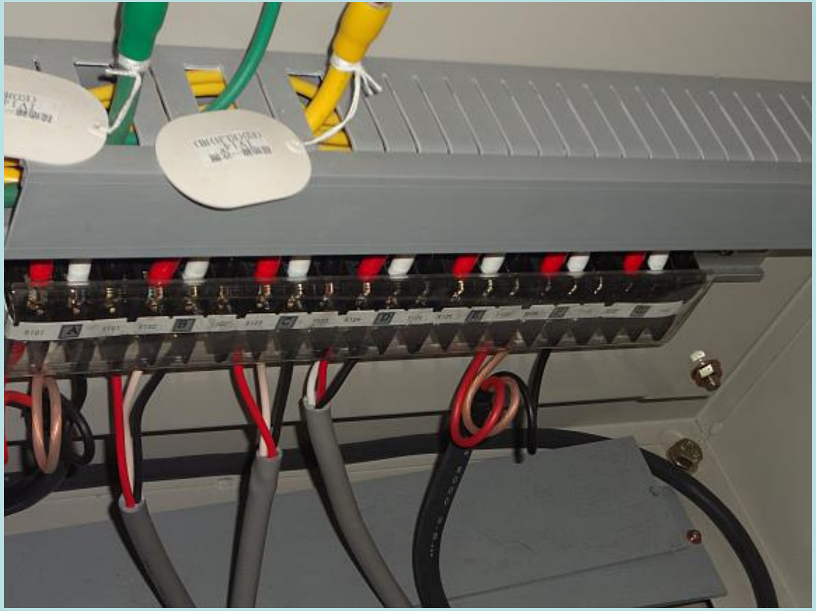
直列接続	並列接続
開放	短絡
電場	磁場
キルヒホッフの第2法則	キルヒホッフの第1法則
電圧	電流
インピーダンス	アドミッタンス
抵抗	コンダクタンス
静電容量	インダクタンス
鳳-テブナンの定理	ノートンの定理

Duality

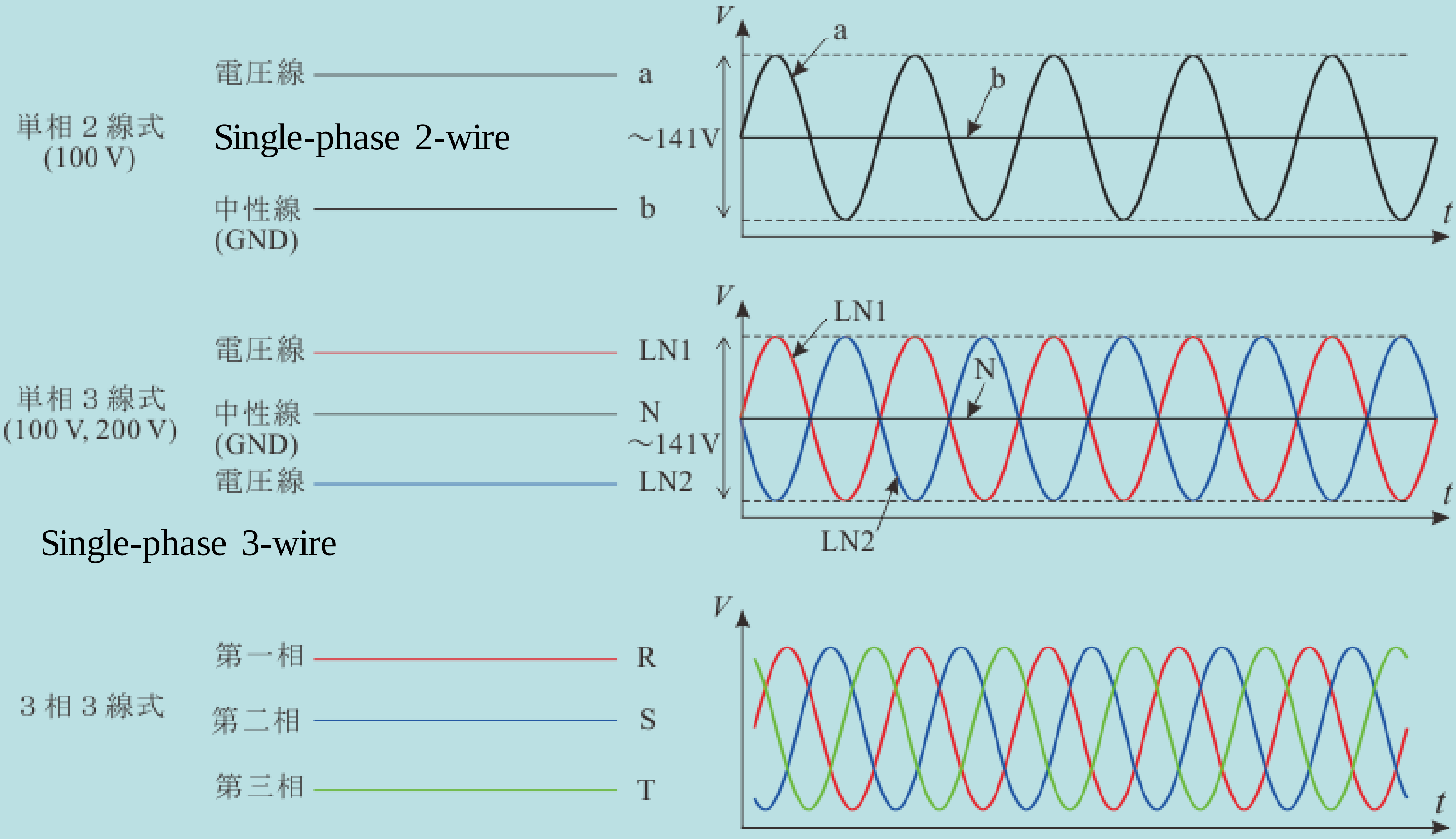
Series	Parallel
Open	Short
Voltage	Current
Impedance	Admittance
Capacitance	Inductance
Electric field	Magnetic field
Resistance	Conductance
Ho-Thevenin	Norton
Kirchhoff’s 2nd law	Kirchhoff’s 1st law

Power Sources in Lab. 電源の雑知識

AC Power from distribution board 配電盤からの電力供給



AC Power from distribution board 配電盤からの電力供給

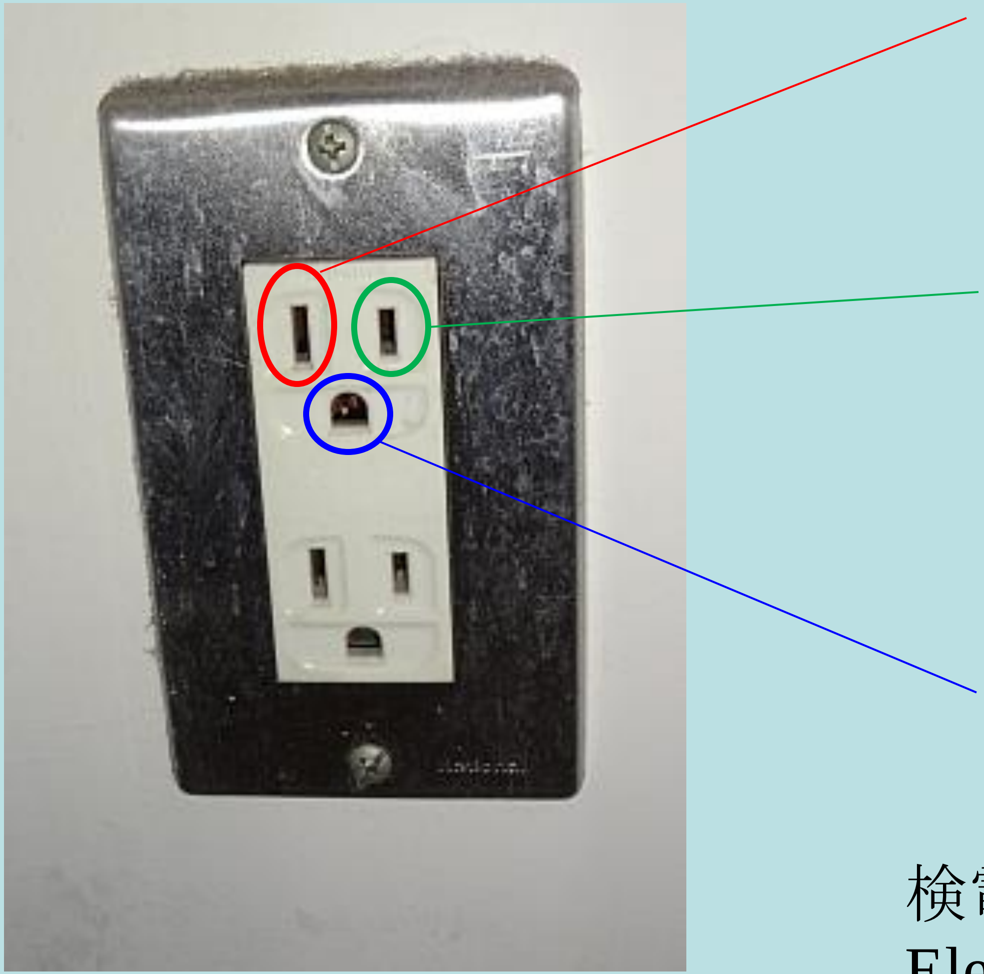


Single-phase 2-wire

Single-phase 3-wire

Three-phase 3-wire

Japanese outlet tap definition 日本式コンセント



Cold line 中性線

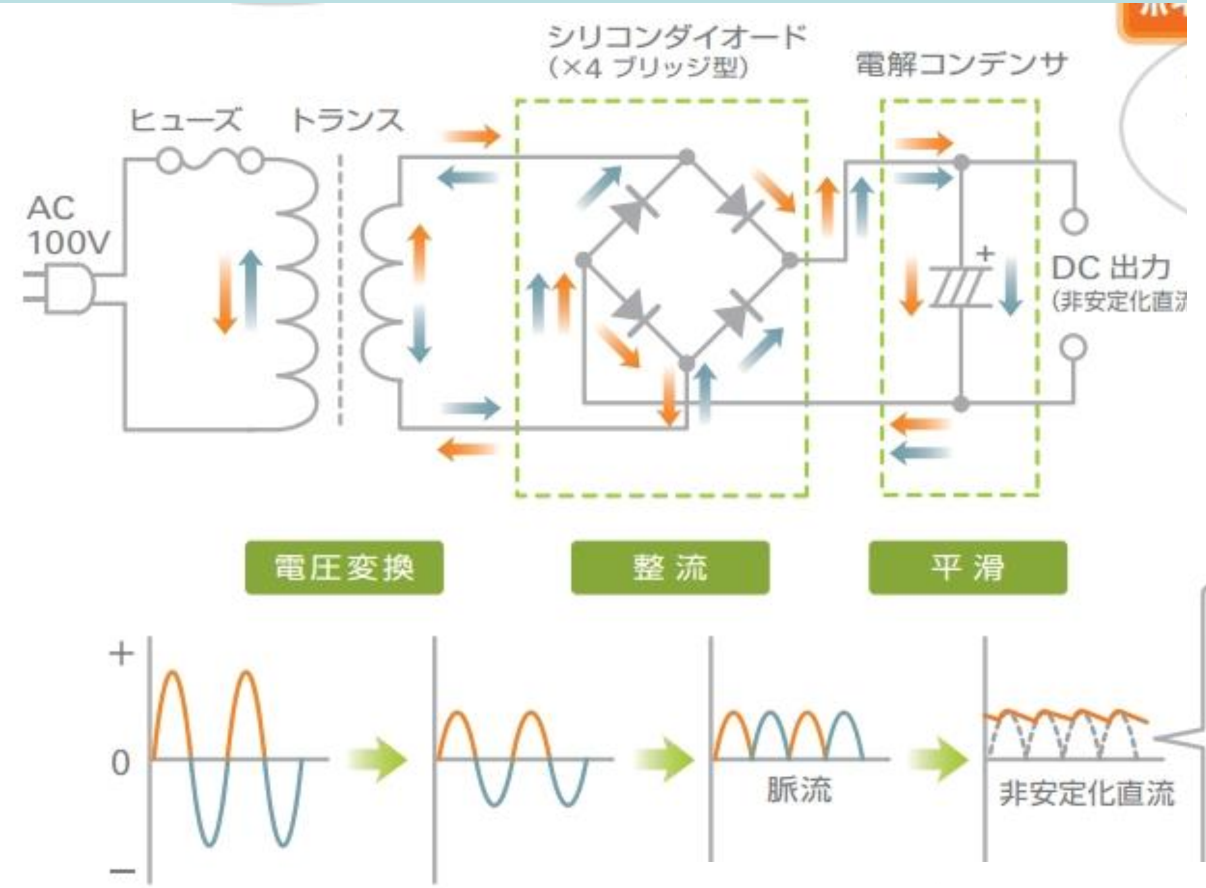
Hot line 電圧線

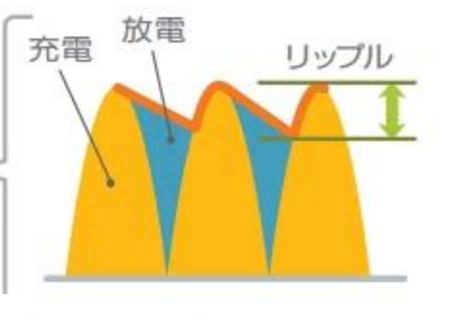
Ground 接地線

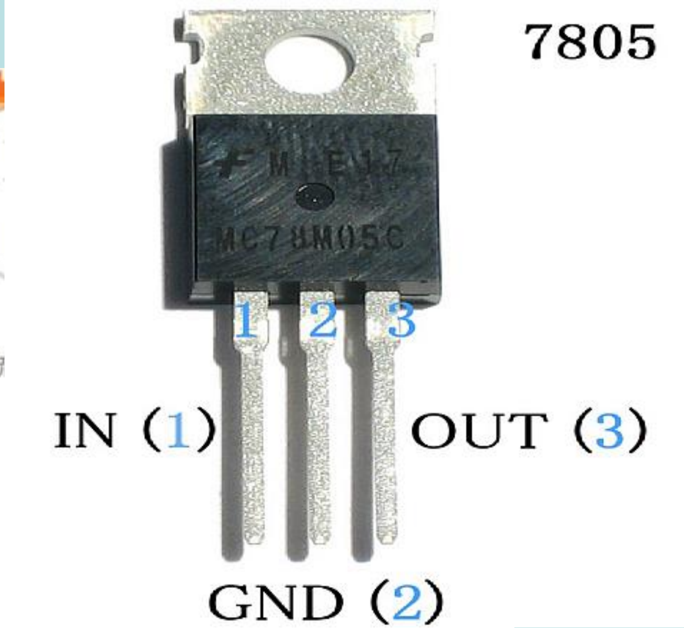
検電ドライバー
Electroscopic
Screwdriver

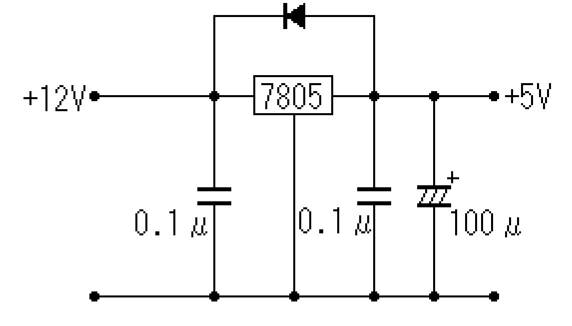
DC Stabilized Power Supply 直流安定化電源

Series (Dropper) regulation









From TDK web page

Series regulator power supply

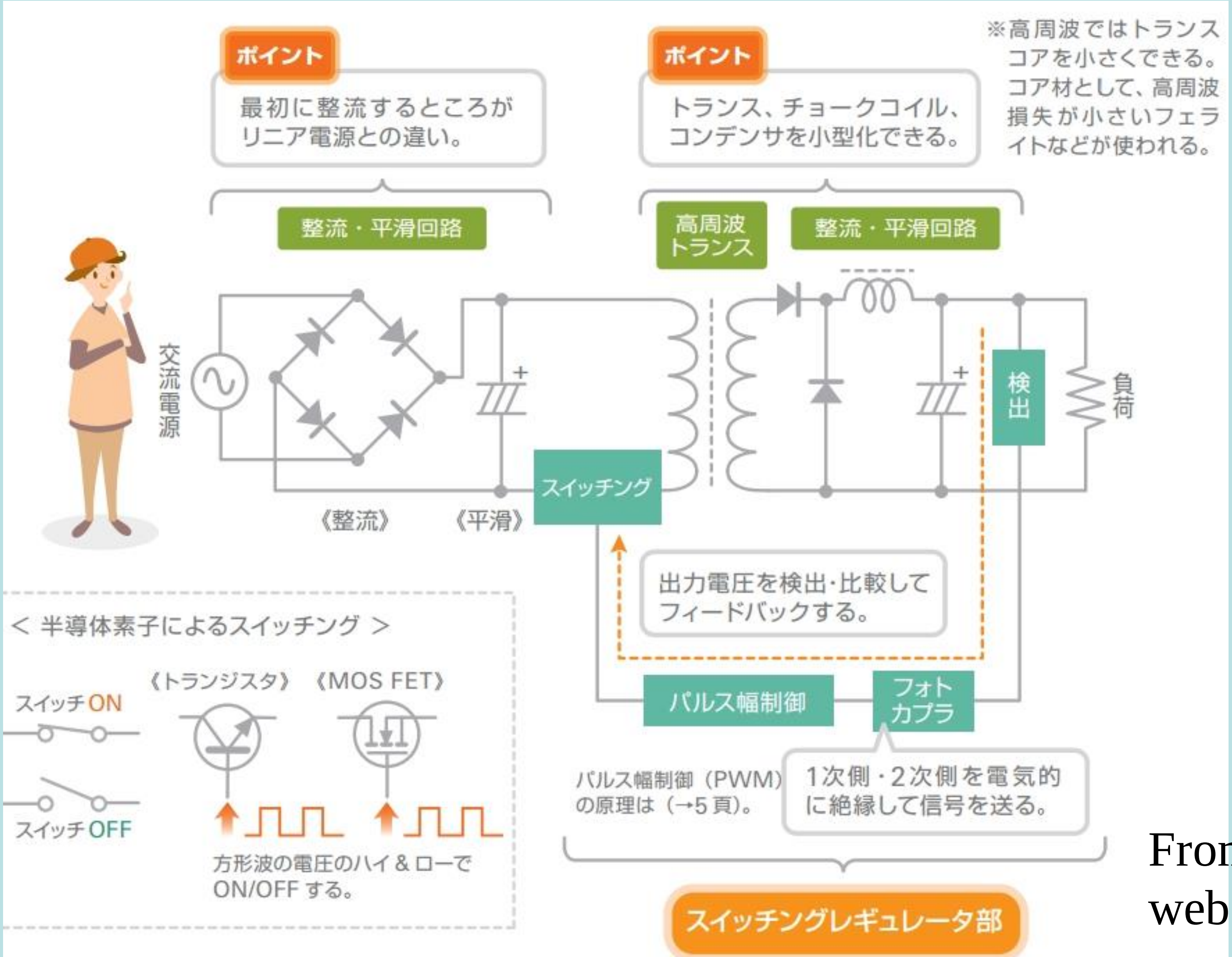
Uni-polar

Dual tracking

High precision

Bi-polar current source

Switching regulation



最初に整流するところがリニア電源との違い。

トランス、チョークコイル、コンデンサを小型化できる。

※高周波ではトランスコアを小さくできる。コア材として、高周波損失が小さいフェライトなどが使われる。

出力電圧を検出·比較してフィードバックする。

パルス幅制御（PWM）の原理は（→5頁）。

1次側・2次側を電気的に絶縁して信号を送る。

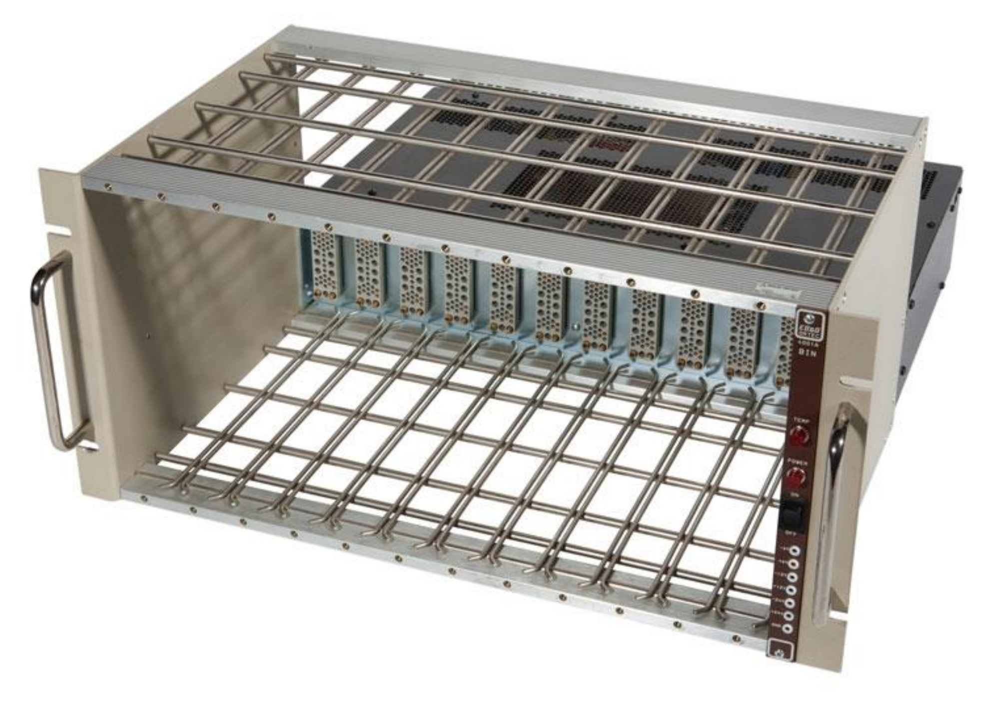
方形波の電圧のハイ＆ローでON/OFFする。

From TDK web page

Switching regulator power supply

Molecular beam epitaxy
Control panel

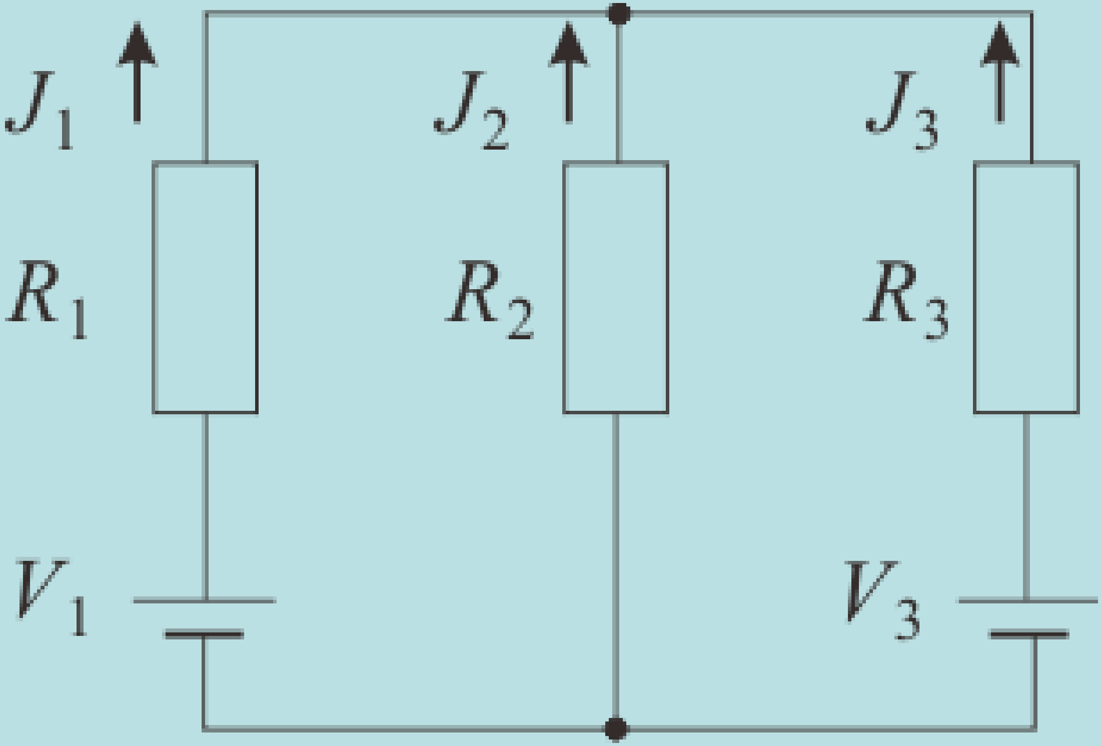
Bin 電源ビン



Complicated power lines ⟶ Bin

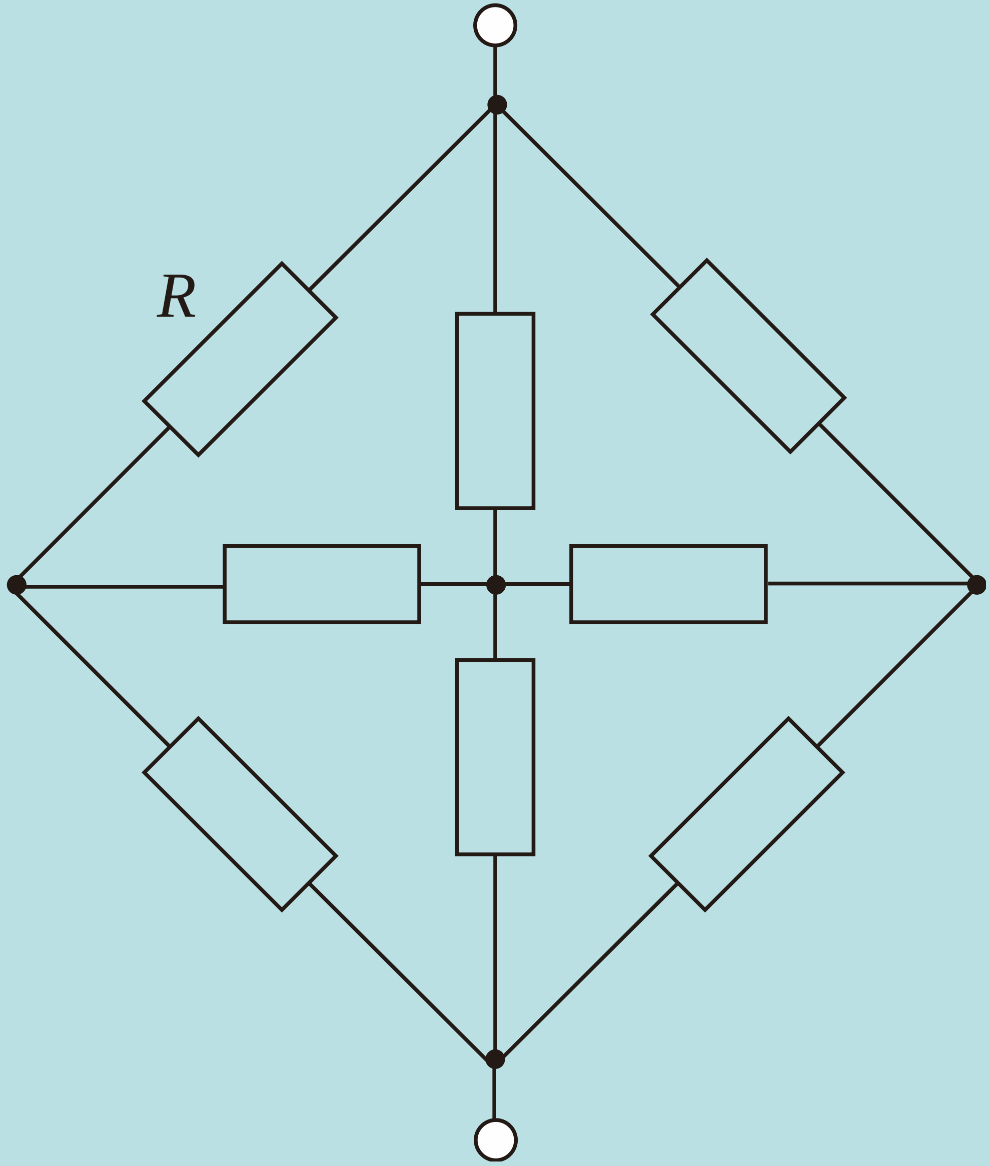
Exercise A-1

Express J_1, J_2, J_3 with other parameters.



Exercise A-2

All the resistors have the same resistance *R*. Obtain the combined resistance.



Exercise A-3

Obtain the effective value of voltage for the saw tooth wave.