

The background image shows a waterfall flowing down a rocky cliff face. The surrounding trees are in full autumn colors, with shades of red, orange, and yellow. The water is white and turbulent as it falls.

電子回路論第5回

Electric Circuits for Physicists

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物性研究所  
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# Outline

Introduction of a freeware “Scilab”

## **Ch.4 Amplification circuit**

4.1 Amplification and system stability

    4.1.1 What is amplifier?

    4.1.2 Feedback

    4.1.3 Stability of feedback

4.2 Operational amplifier (OP-amp)

    4.2.1 Linear model of OP-amp

    4.2.2 Package

    4.2.3 Circuit examples

    4.2.4 Datasheet

    4.2.5 Stability

# A convenient freeware: Scilab

The screenshot shows the official Scilab website. On the left, there's a large call-to-action button with a white arrow pointing right, containing the text "Download Scilab" and "Scilab 5.5.1 - 32-bit Windows • 127.92 MB Other Systems". Below this button is the tagline "Open source software for numerical computation". To the right of the button is a screenshot of the Scilab interface, which includes a "Scilab Console" window showing code execution and variable lists, and a "File Browser" window showing the local file system.

**News** : 10/16/2014 - Windows users, reinstall Scilab 5.5.1    10/6/2014 - Scilab at C. [f](#) [g+](#) [t](#) [in](#) [YouTube](#) [r](#)

**Professional Solutions**

 scilab enterprises

Scilab Enterprises, official publisher of Scilab software, also offers dedicated services for all its users: support, consulting, migration, training, development and implementation of specific applications...

**Open Source**

Scilab is open source software distributed under [CeCILL license](#). Many other [third-party projects](#) are also available.

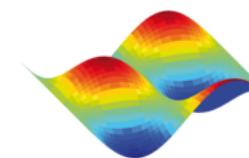
**Education**

Scilab is widely used in secondary and higher education institutions for teaching [mathematics](#), [engineering sciences](#) and [automatic control engineering](#).

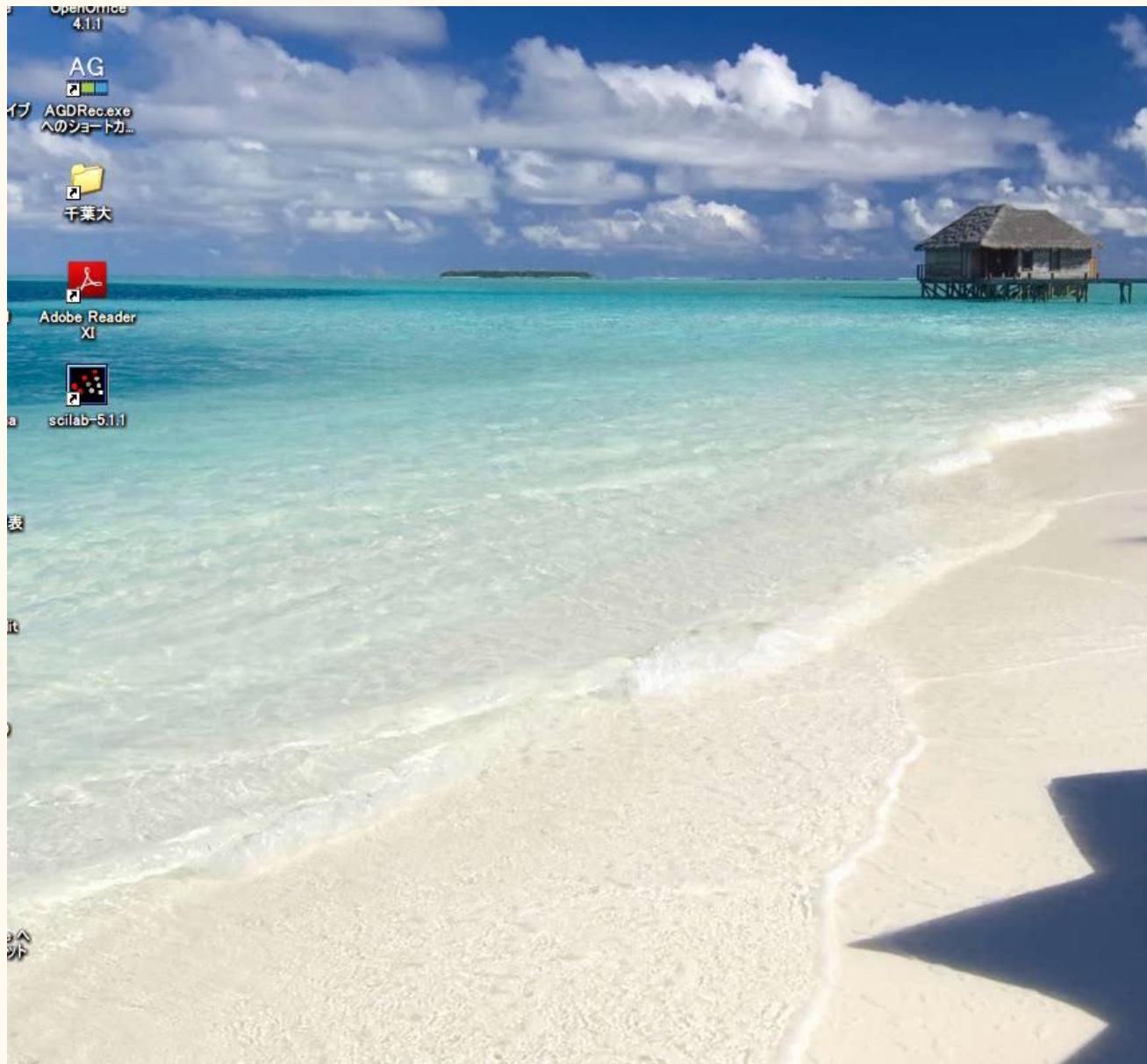
**To donate**

**Scilab**

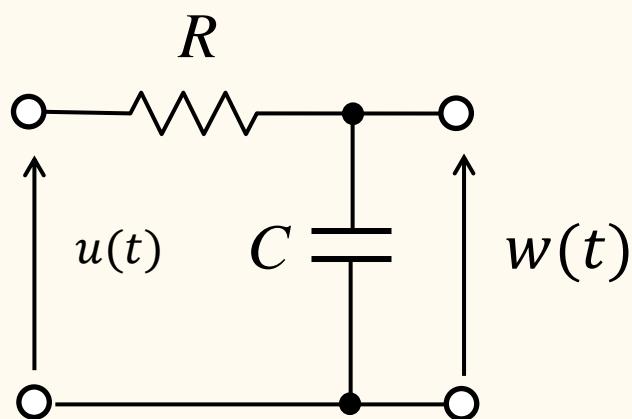
Overview  
New in Scilab 5.5.0  
New in Scilab 5.5.1  
Xcos  
Features  
Gallery  
System requirements  
Quality



# Transfer function analysis with Scilab



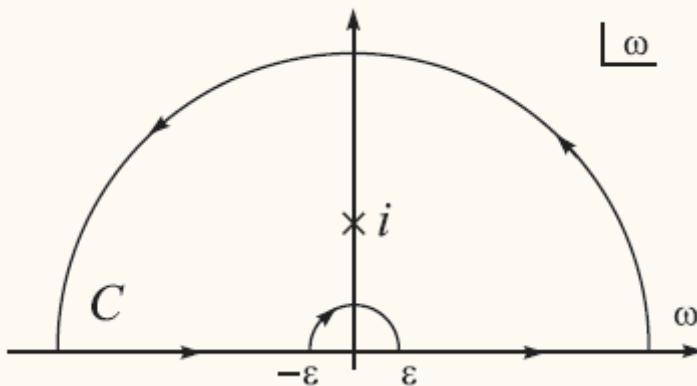
# Simple application



$$V = V_0 \left[ 1 - \exp \left( -\frac{t}{CR} \right) \right]$$

$$\begin{aligned} g(t) &= \int_{-\infty}^{\infty} \frac{1}{1+i\omega} \left[ \frac{1}{i\omega} + \pi\delta(\omega) \right] e^{i\omega t} \frac{d\omega}{2\pi} \\ &= \int_{-\infty}^{\infty} \frac{e^{i\omega t}}{(i-\omega)\omega} \frac{d\omega}{2\pi} + \frac{1}{2} \end{aligned}$$

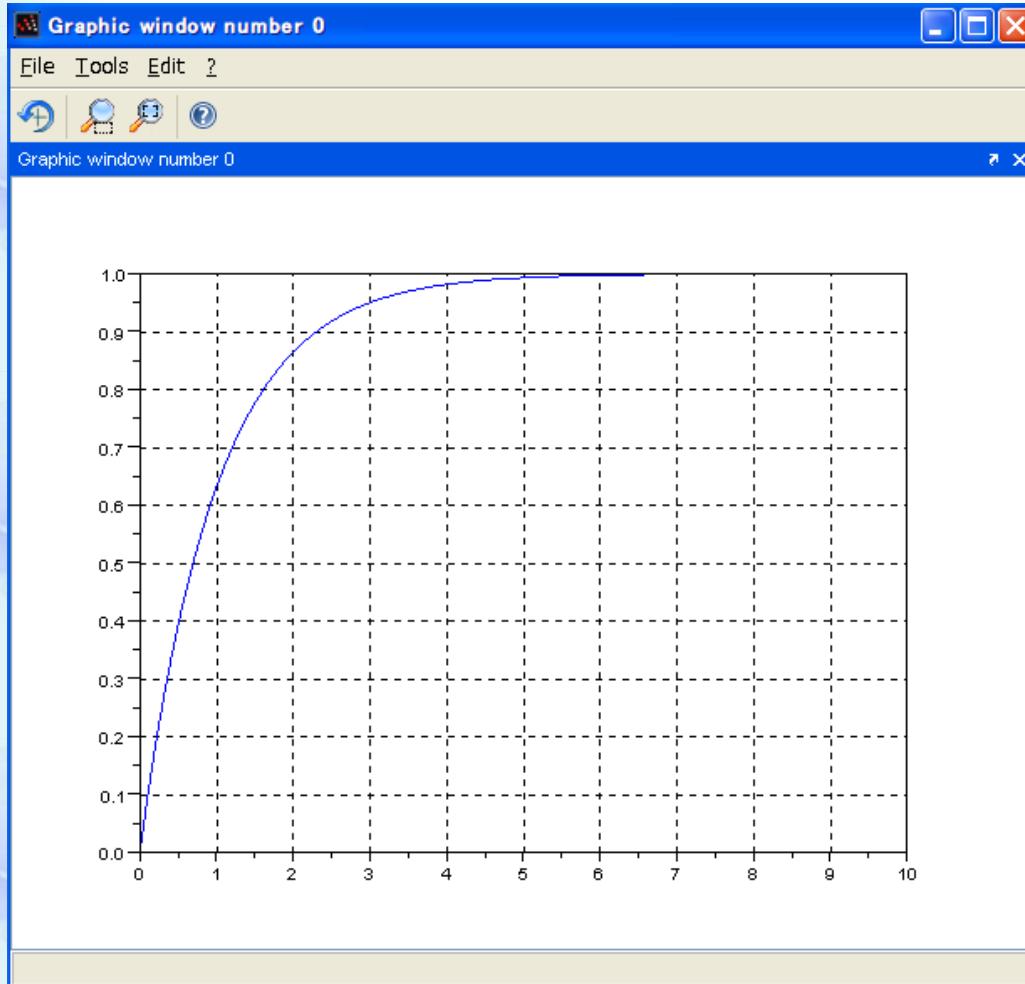
$$-2\pi i \frac{e^{-t}}{2\pi i} - \lim_{\epsilon \rightarrow 0} \left[ \int_{\pi}^0 \frac{e^{i\epsilon e^{i\theta} t}}{\epsilon e^{i\theta} (\epsilon e^{i\theta} - i)} \frac{i\epsilon e^{i\theta} d\theta}{2\pi} \right] = -e^{-t} - \frac{1}{2}$$



$$g(t) = -e^{-t}$$

# Transient response: Use of Scilab

$$\Xi(s) = \frac{1}{1+s}$$



```
-->s=poly(0,'s');

-->G=1/(1+s);

-->sys=syslin('c',G);

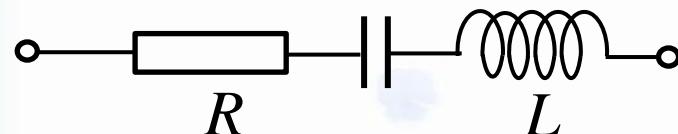
-->t=linspace(0,10,100);

-->y=csim('step',t,sys);

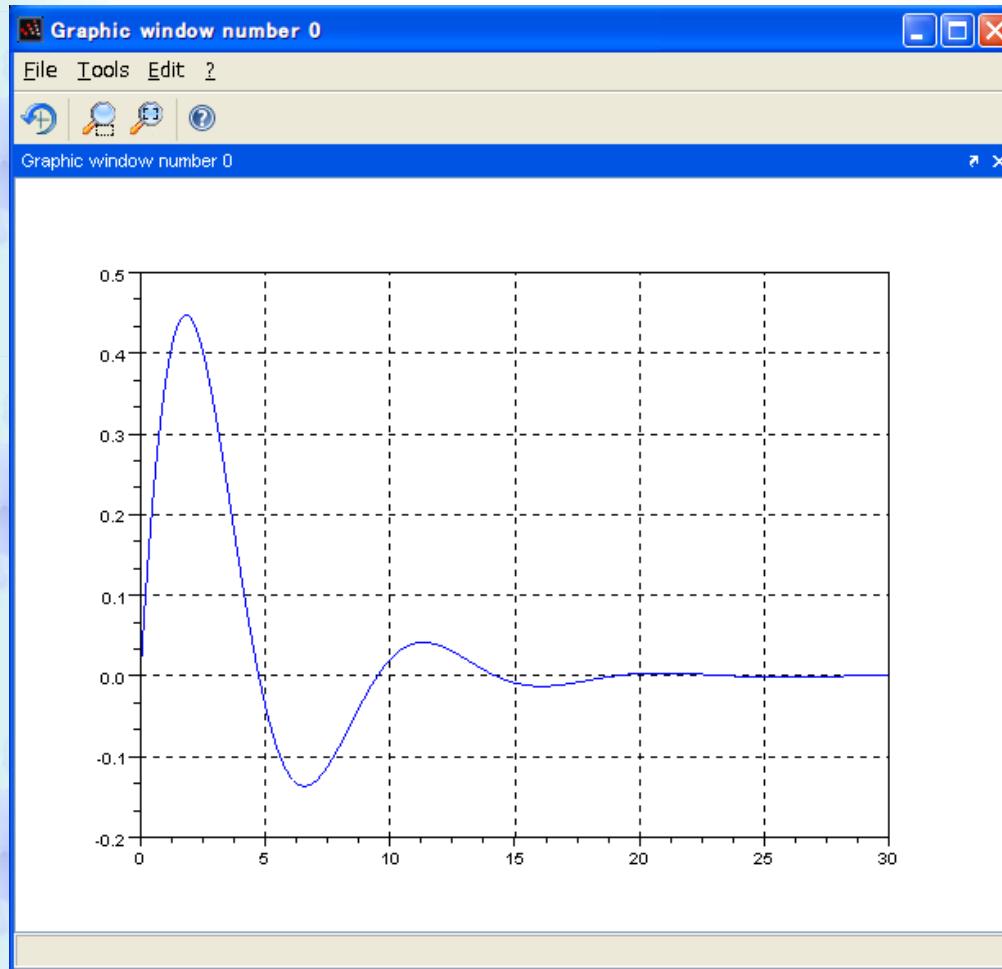
-->plot(t,y)

-->xgrid()
```

# Transient response: Use of Scilab



$$Y(s) = \frac{Cs}{LCs^2 + CRs + 1}$$



```
-->G=s/(1+s+2*s*s);  
  
-->sys=syslin('c',G);  
  
-->y=csim('step',t,sys);  
  
-->plot(t,y)
```

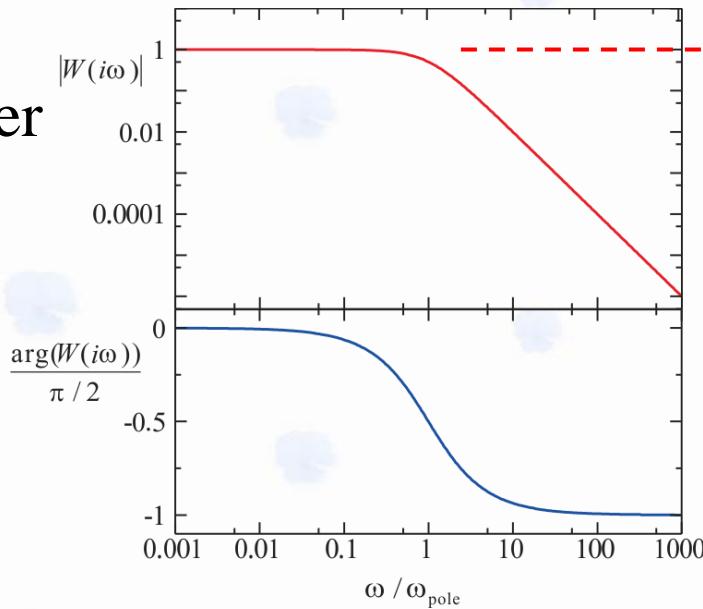


# Chapter 4

# Amplification circuits

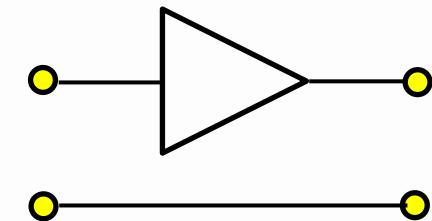
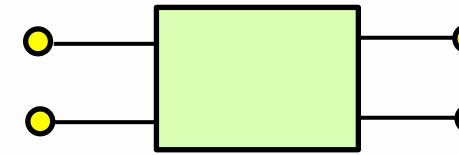
# Linear amplifier

passive filter



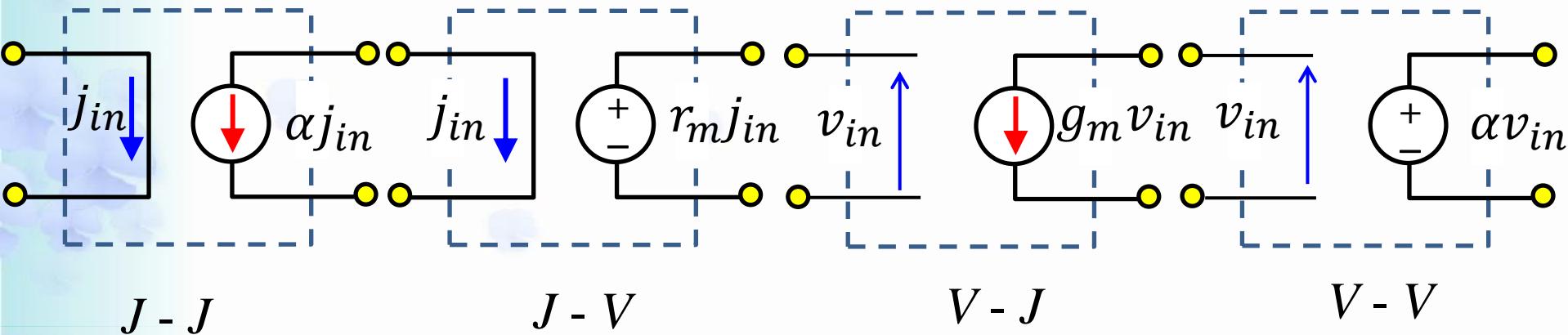
gain = 1  
gain > 1 → amplifier

four terminal circuit model



Circuit symbol

Controlled power source models



# Gain, and “Unit” for gain

$$\text{Voltage gain: } \left| \frac{v_{out}}{v_{in}} \right|$$

$$\text{Current gain: } \left| \frac{j_{out}}{j_{in}} \right|$$

$$\text{Power gain: } \left| \frac{v_{out} j_{out}}{v_{in} j_{in}} \right|$$

When we say “the gain of the amplifier ...”, the gain means power gain.

quantity  $Q$ , unit  $Q_0$  :  $Q$  in log scale:  $L = \log_{10} \frac{Q}{Q_0}$  (B, bel)

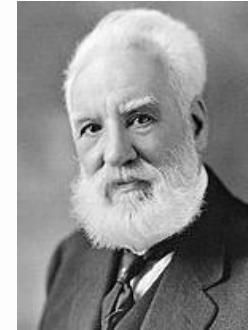
cf. deca- 10

1/10

dB : (decibel)

From: G. Bell

Alexander Graham Bell  
1847 - 1922



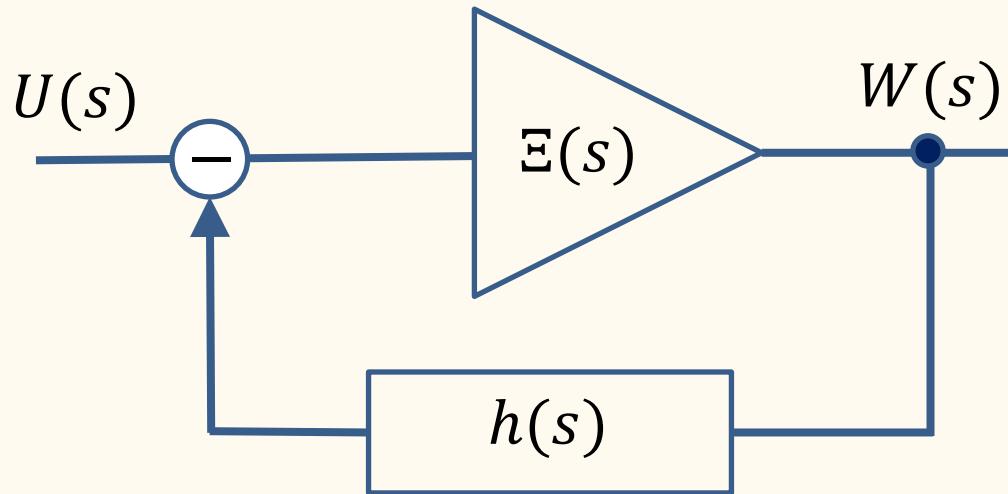
$$G = 10 \times \log_{10} \left( \frac{v_{out}}{v_{in}} \right)^2 = 20 \log_{10} \frac{v_{out}}{v_{in}}$$

---

dB units: dBm (1mW: 0dBm), dBv (1V: 0dBv), etc.

# Feedback circuit

Feedforward  
  
Feedback



$$W(s) = \Xi(s)U(s)$$

$$W(s) = \Xi(s)[U(s) - h(s)W(s)]$$

$$W(s) = \frac{\Xi(s)}{1 + \Xi(s)h(s)} U(s)$$

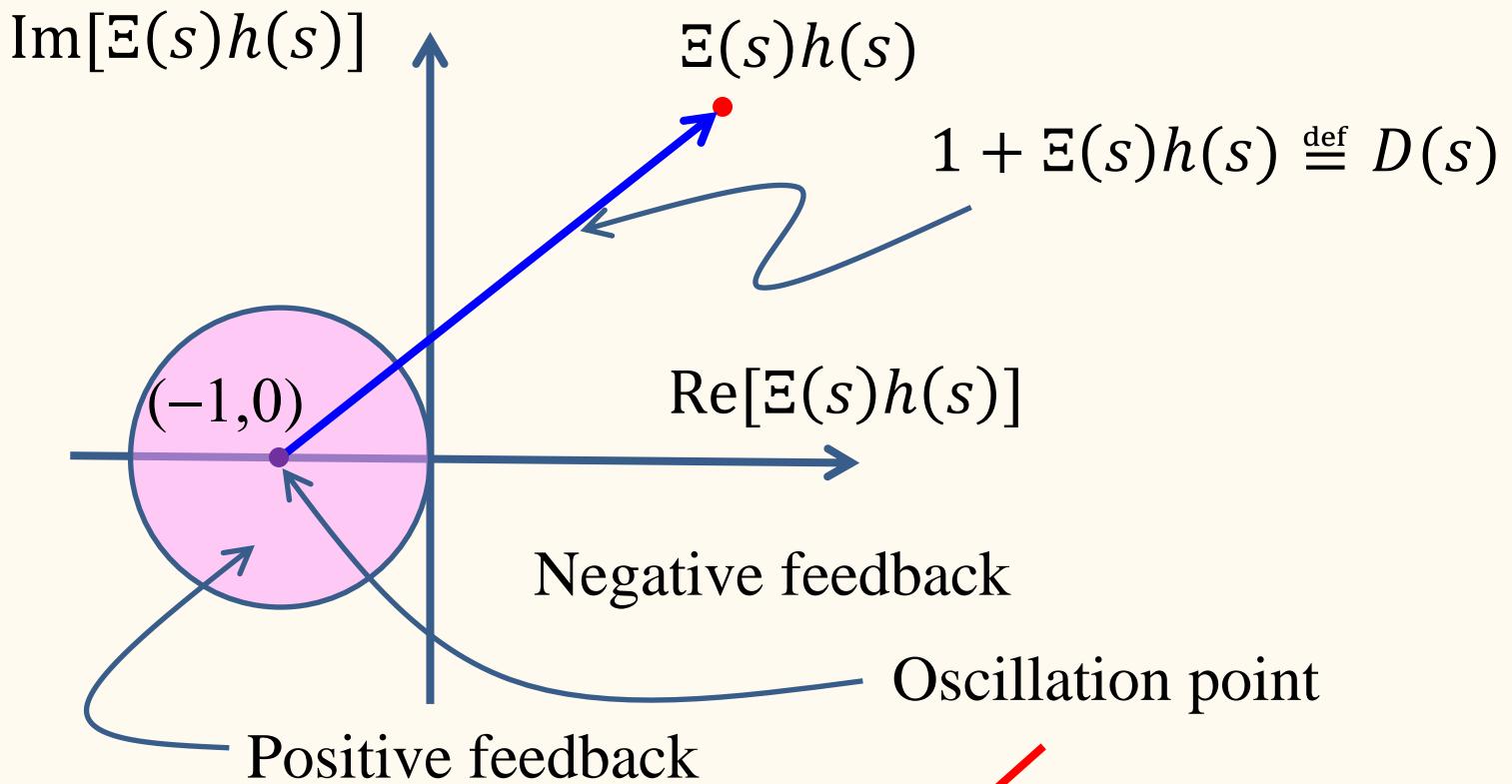
$$\stackrel{\text{def}}{=} G(s)U(s)$$

$|1 + \Xi(s)h(s)| > 1$ : Negative feedback,  $< 1$ : Positive feedback

$$|\Xi(s)| \gg 1 \rightarrow G(s) \approx \frac{1}{h(s)}$$

# Condition for negative feedback

$|1 + \Xi(s)h(s)| > 1$ : Negative feedback,  $< 1$ : Positive feedback



If  $\Xi(s)h(s) = -1$  has solutions, the circuit may be unstable.

How can we judge?



Criteria

(Routh-Hurwitz, Nyquist, Liapunov, ...)

# Zeros and poles of $D(s)$

Assumption 1:  $\Xi(s), \Xi(s)h(s)$  are stable  
→ Poles are on the left half plane of  $s$ .

Assumption 2:  $\Xi(i\omega), \Xi(i\omega)h(i\omega) \rightarrow 0$  for  $|\omega| \rightarrow \infty$

$$\Xi(s) = \frac{Q(s)}{P(s)}, \quad h(s) = \frac{q(s)}{p(s)} : P(s), Q(s), p(s), q(s) \text{ polynomials}$$

$$\deg(P) > \deg(Q), \deg(p) \geq \deg(q)$$

$$D(s) = 1 + \Xi(s)h(s) = \frac{P(s)p(s)}{P(s)p(s) + Q(s)q(s)}$$

$$D(s) = D_0 \frac{(s - \beta_1) \cdots (s - \beta_{\textcolor{red}{n}})}{(s - \alpha_1) \cdots (s - \alpha_{\textcolor{red}{n}})} \xrightarrow{\text{The same order}}$$

# Zeros and poles of $D(s)$

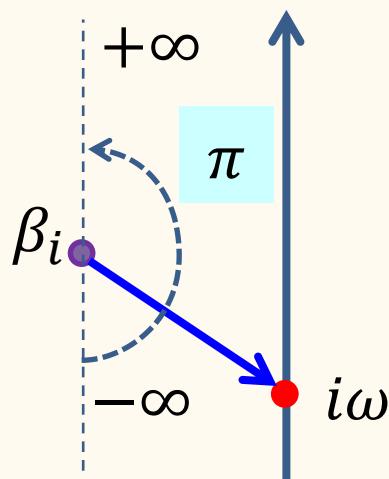
$$D(s) = D_0 \frac{(s - \beta_1) \cdots (s - \beta_n)}{(s - \alpha_1) \cdots (s - \alpha_n)}$$

$\{\beta_i\}$  : Zeros of  $D(s)$  → Poles of  $G(s)$

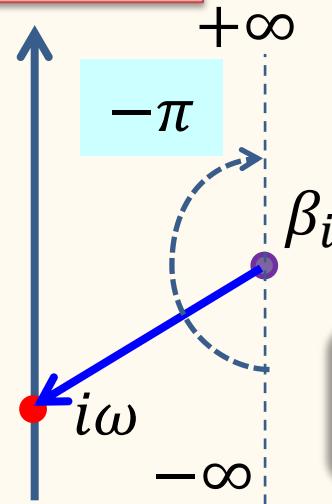
$\exists \beta_i \in$  right half plane of  $s \rightarrow$  The circuit is unstable.

$$\arg(D) = \sum_{i=1}^n \arg(s - \beta_i) - \sum_{i=1}^n \arg(s - \alpha_i)$$

Left half plane



Right half plane

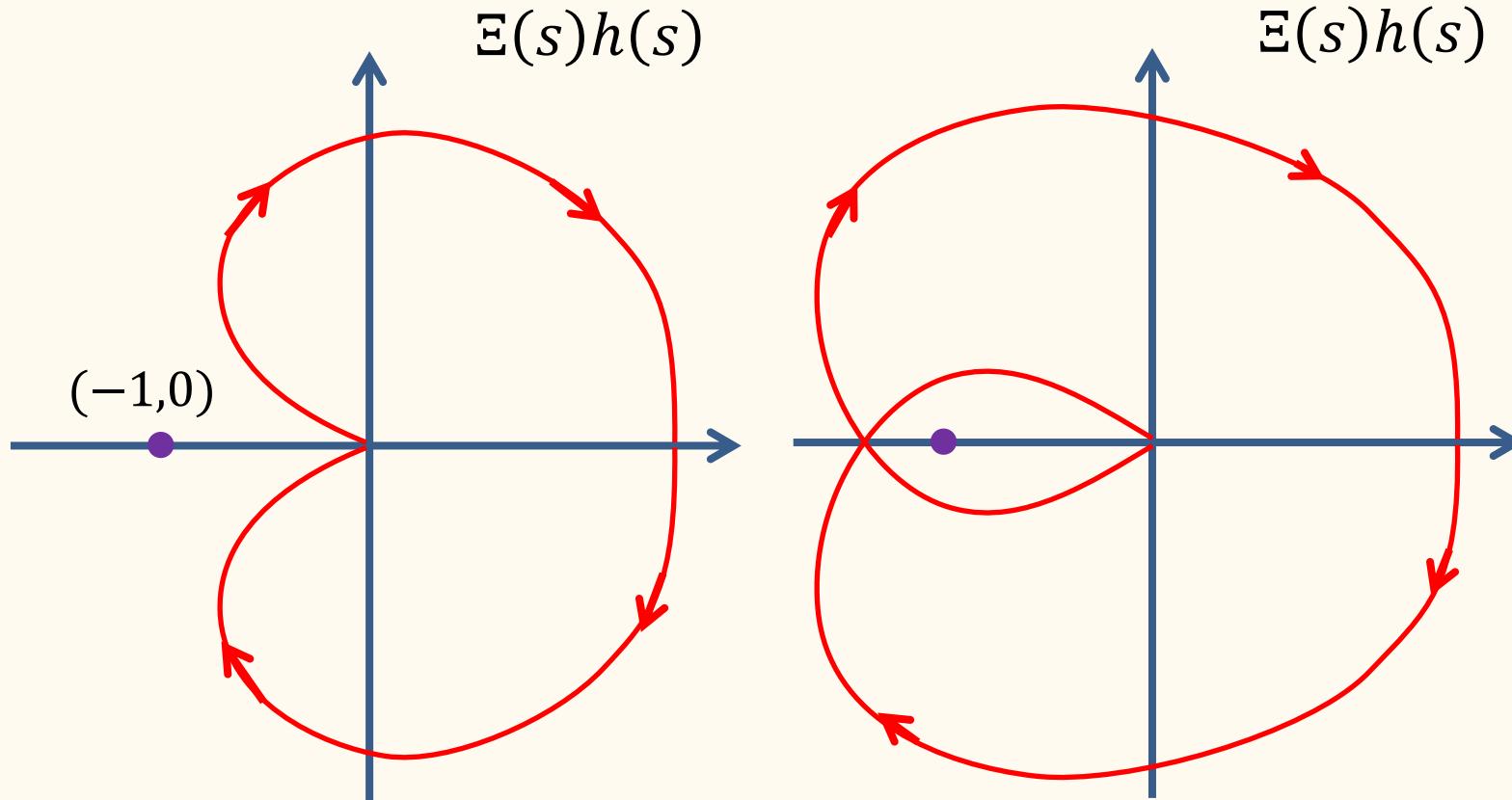


$s = i\omega$  (on imaginary axis)  
 $\omega: -\infty \rightarrow +\infty$

Number of zeros on the  
right half plane:  $m$

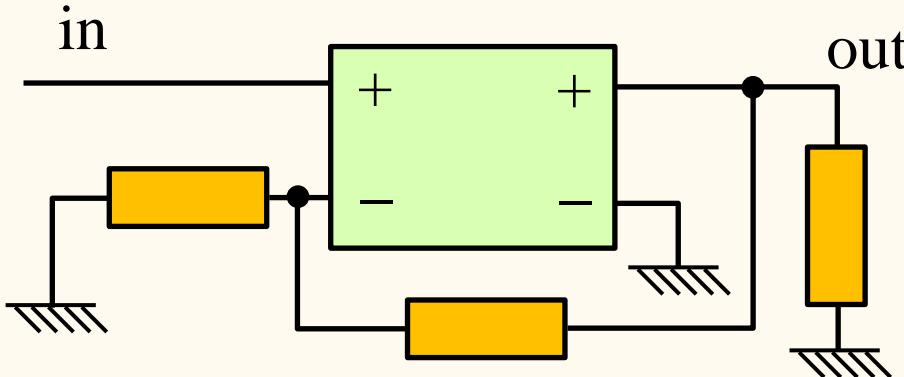
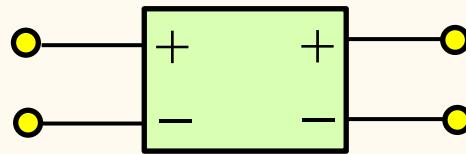
$$\begin{aligned}\Delta \arg(D) &= (n - m)\pi - m\pi \\ &\quad - n\pi = -2m\pi\end{aligned}$$

# Nyquist Plot and Criterion

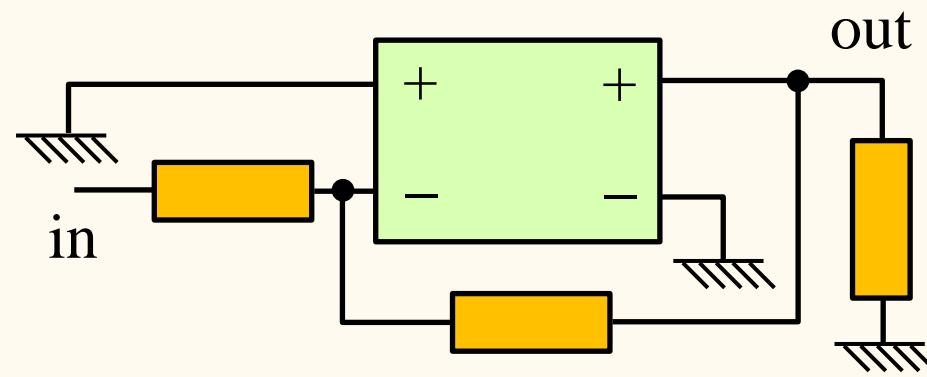


Harry Nyquist  
(1889–1976)

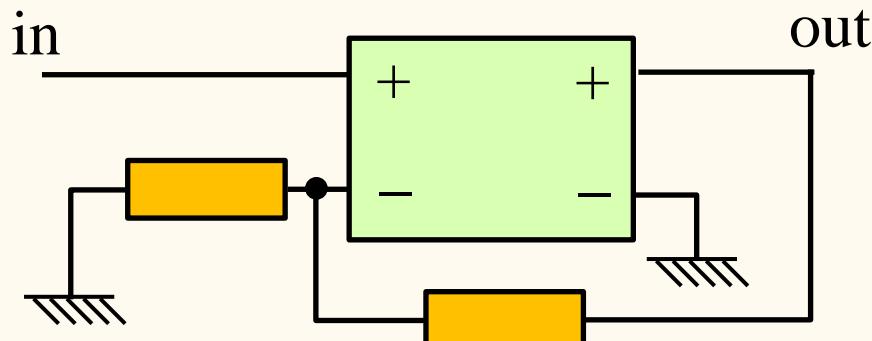
# Feedback in terminal-pair circuits with resistors



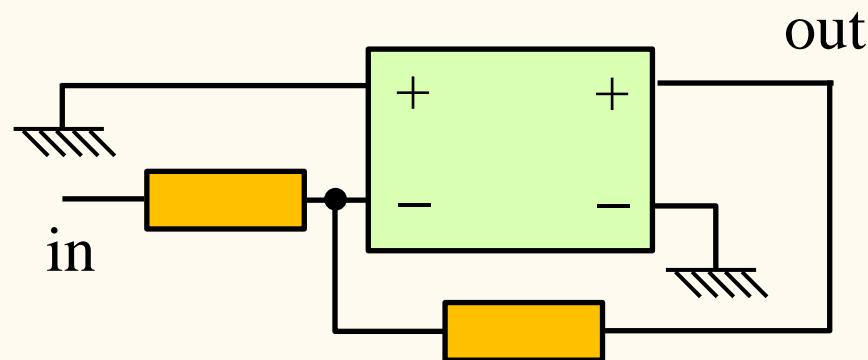
(i) input: parallel, output: parallel



(ii) input: series, output: parallel

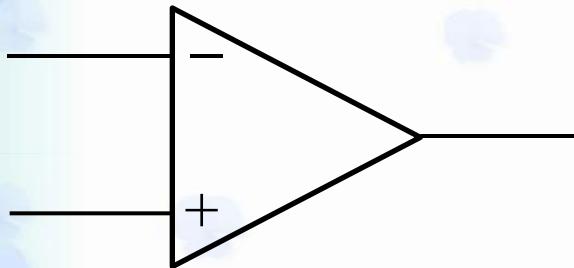


(iii) input: parallel, output: series



(iv) input: series, output: series

# Operational amplifier (OP amp.)



- Differential amplifier
- Input impedance  $\sim \infty$
- Open loop gain  $A_o \gg 1$
- Output resistance  $\approx 0$

Case (iv)

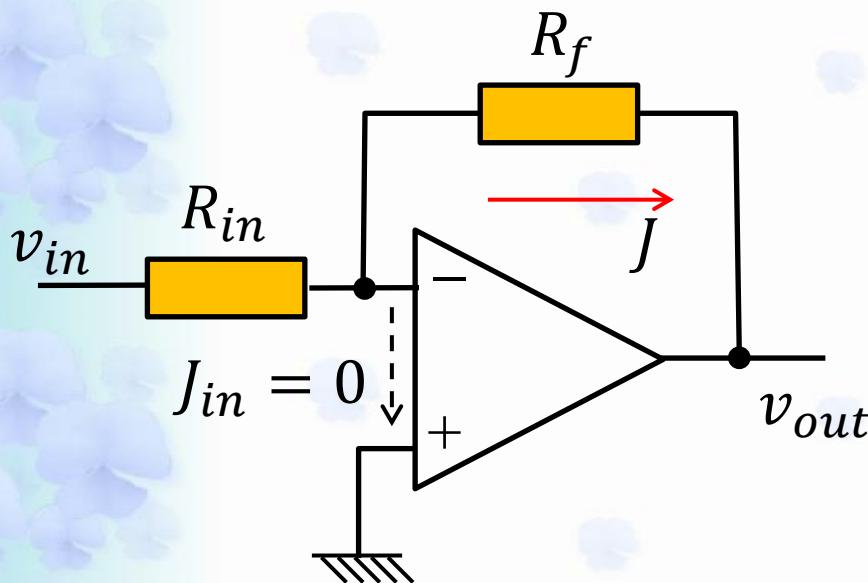
$$A_o \gg 1 \therefore \underline{V_- \approx V_+ = 0}$$

Virtual short circuit

$$J = -\frac{v_{out}}{R_f} = \frac{v_{in}}{R_{in}}$$

$$\therefore v_{out} = -\frac{R_f}{R_{in}} v_{in}$$

Inverting amplifier



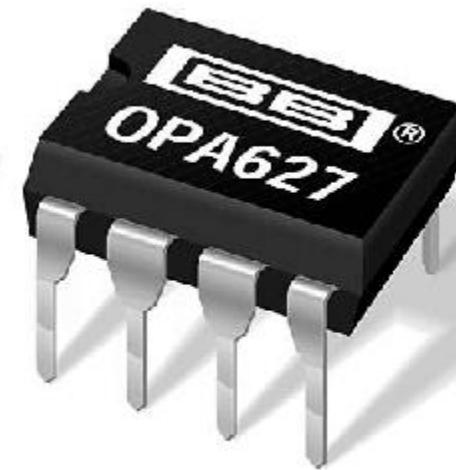
# Opamp packages



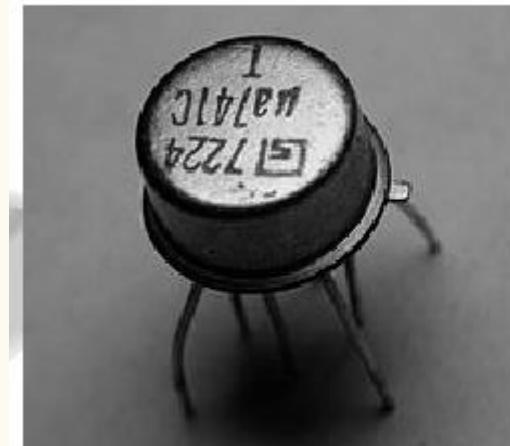
(a)



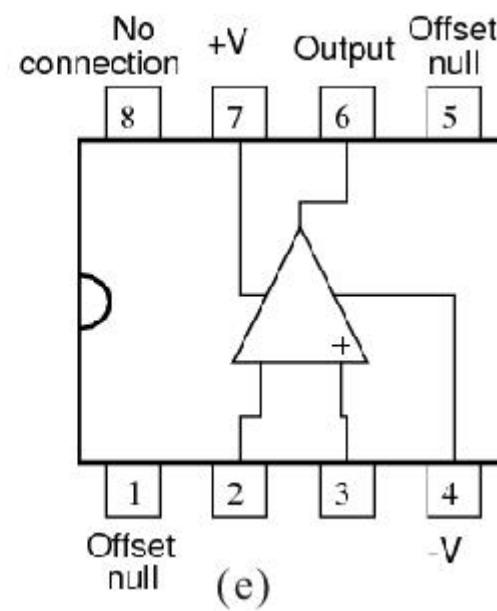
(b)



(c)

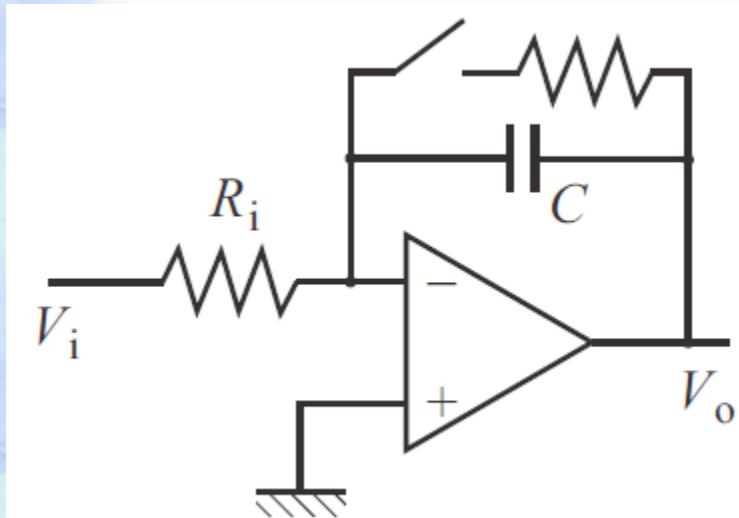


(d)



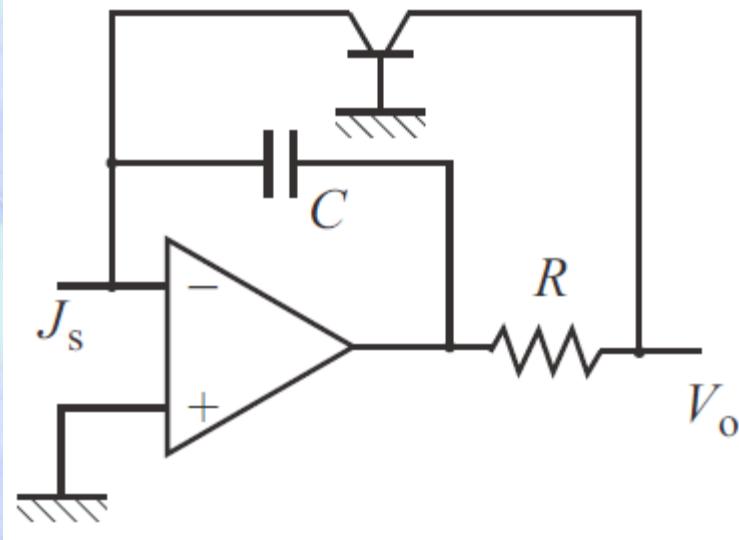
(e)

# Various applications of OP amps



$$\begin{aligned}V_{\text{out}}(t) &= -\frac{Q}{C} = -\frac{1}{C} \int_0^t \frac{V_i(\tau)}{R_i} d\tau \\&= -\frac{1}{CR_i} \int_0^t V_i(\tau) d\tau\end{aligned}$$

Integration circuit



$$V_{\text{out}} = -V_{\text{BE}} = -\frac{k_B T}{e} \ln \left( \frac{J_s}{J_0} + 1 \right)$$

Logarithmic amplifier

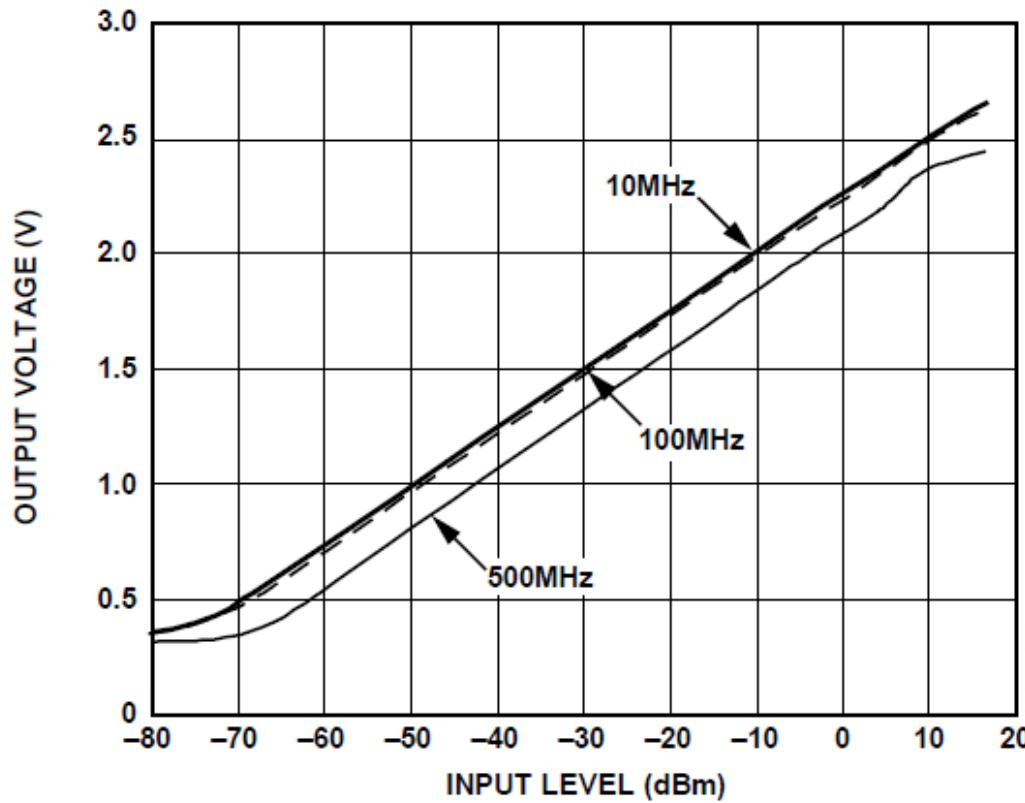
# Logarithmic Amplifier



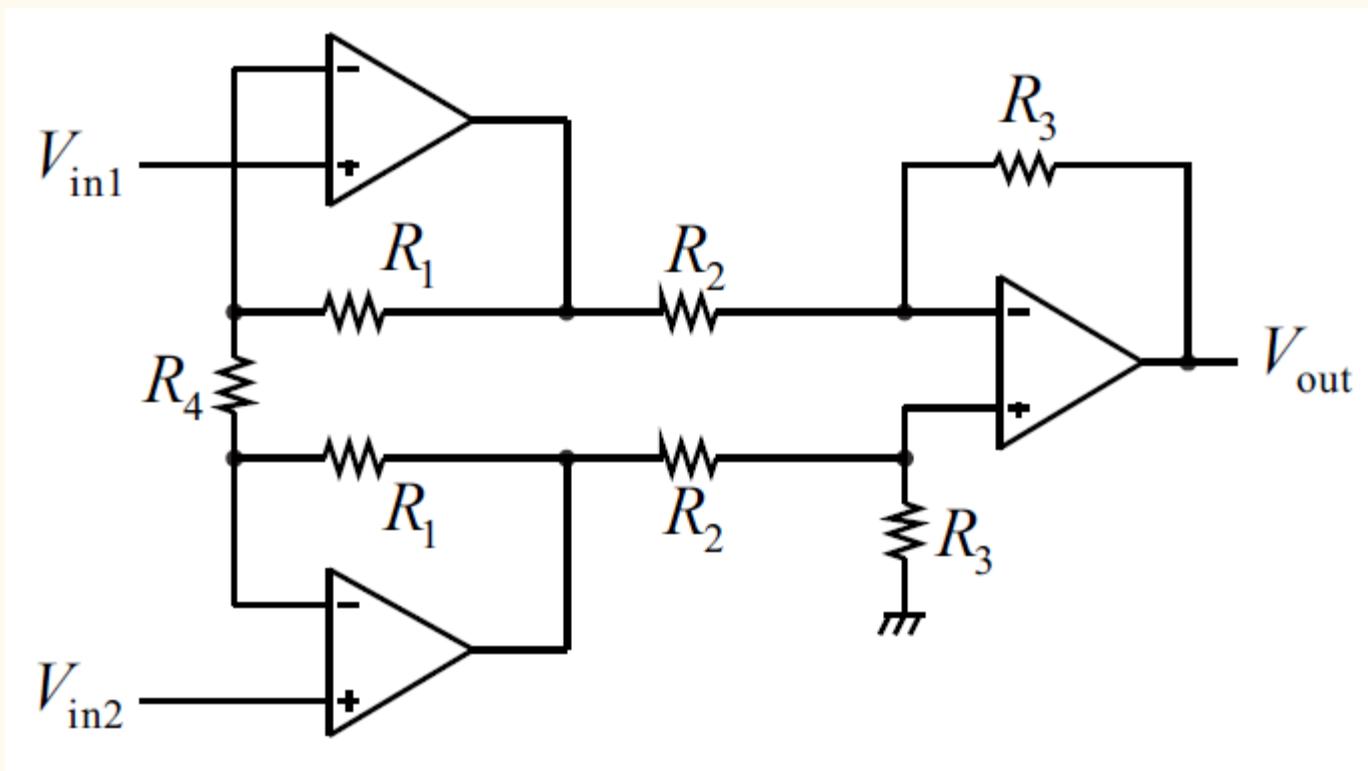
Low Cost, DC to 500 MHz, 92 dB  
Logarithmic Amplifier

Data Sheet

AD8307

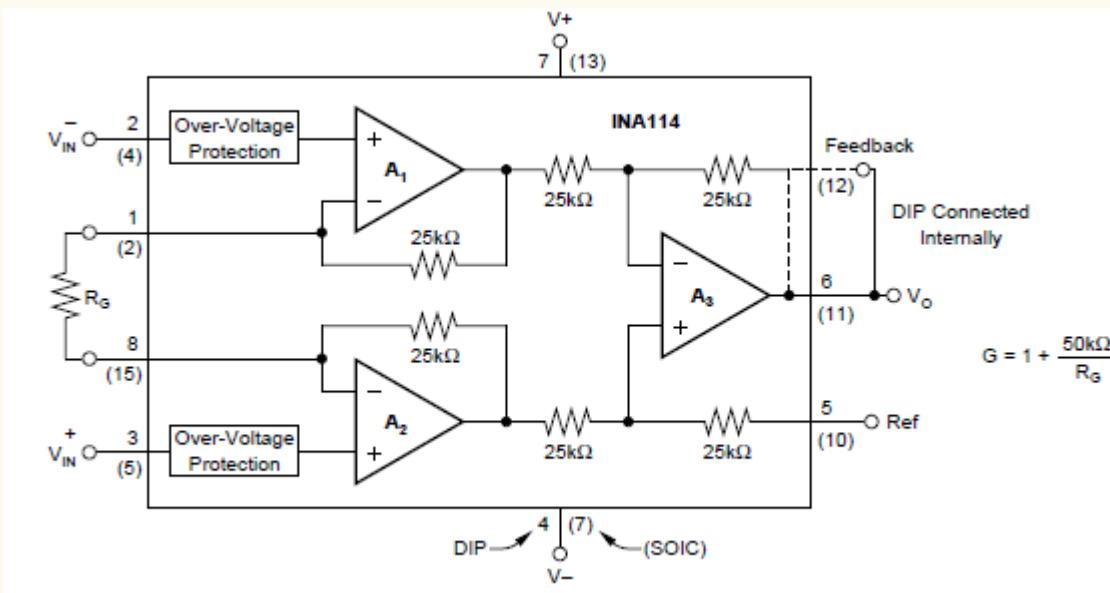
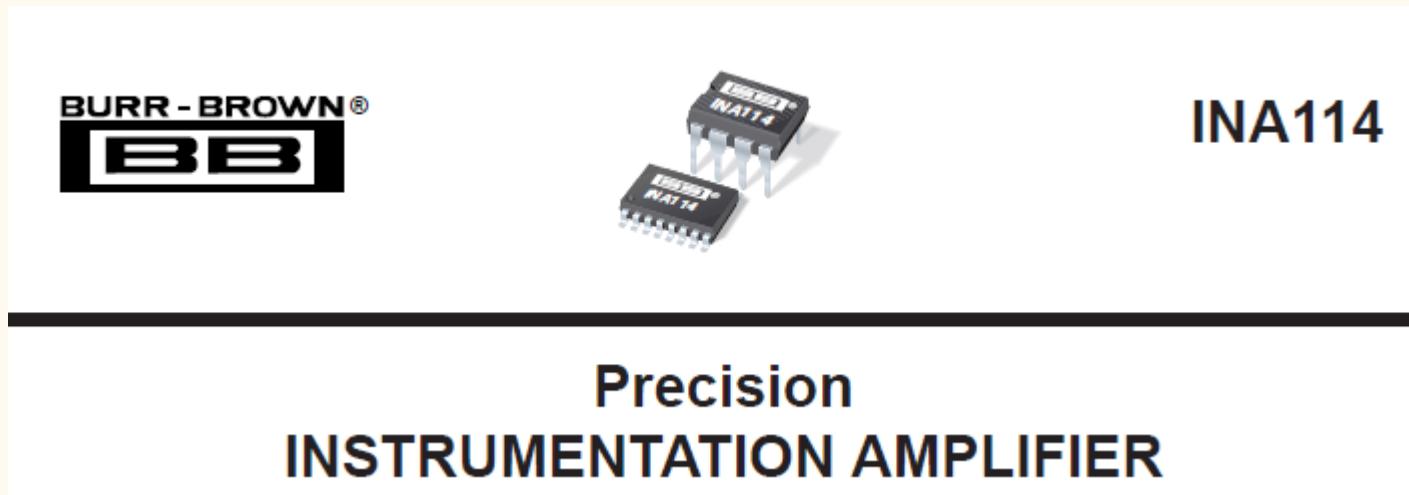


# Instrumentation amplifier



$$V_{out} = -\frac{R_3}{R_2} \left( \frac{2R_1 + R_4}{R_4} \right) (V_{in1} - V_{in2})$$

# Instrumentation amplifier



# OP-amp data sheet



## Data Sheet

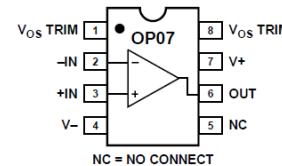
# Ultralow Offset Voltage Operational Amplifier

OPO7

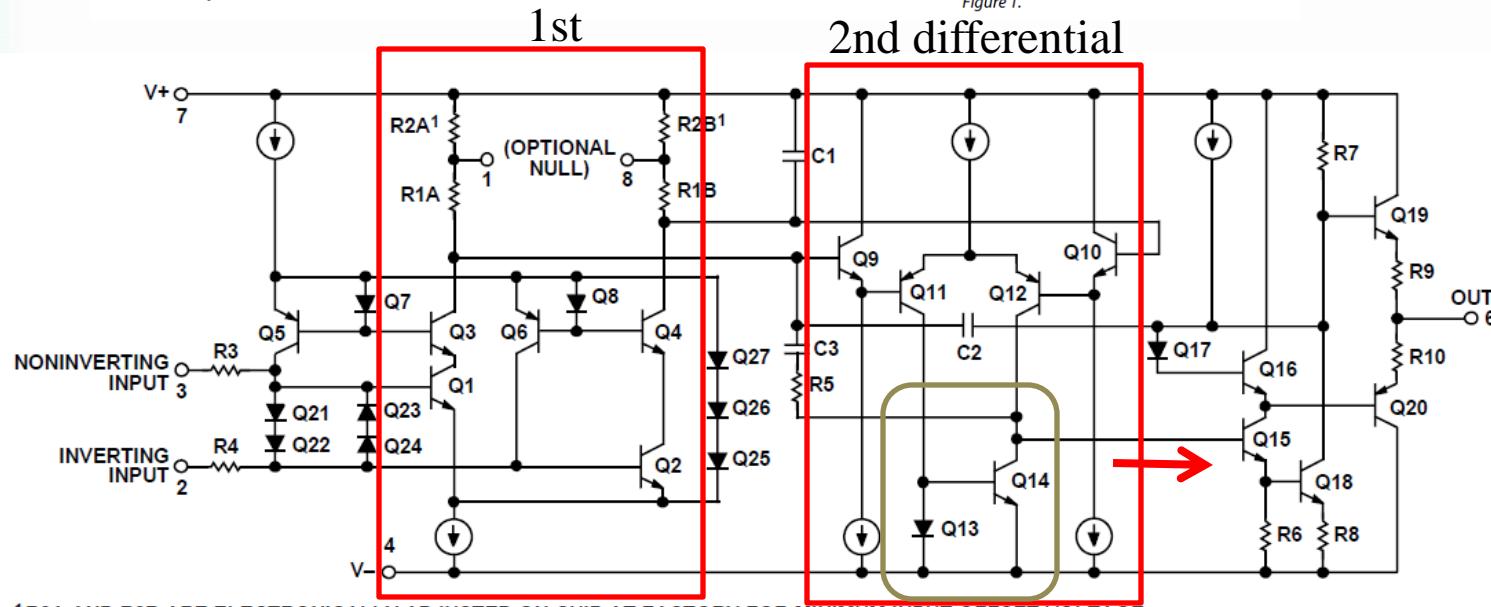
## FEATURES

- Low  $V_{OS}$ : 75  $\mu\text{V}$  maximum**  
**Low  $V_{OS}$  drift: 1.3  $\mu\text{V}/^\circ\text{C}$  maximum**  
**Ultrastable vs. time: 1.5  $\mu\text{V}$  per month maximum**  
**Low noise: 0.6  $\mu\text{V}$  p-p maximum**  
**Wide input voltage range:  $\pm 14 \text{ V}$  typical**  
**Wide supply voltage range:  $\pm 3 \text{ V}$  to  $\pm 18 \text{ V}$**   
**125°C temperature-tested dice**

## PIN CONFIGURATION



## Figure 1



**<sup>1</sup>R2A AND R2B ARE ELECTRONICALLY ADJUSTED ON CHIP AT FACTORY FOR MINIMUM INPUT OFFSET VOLTAGE**

*Figure 2. Simplified Schematic*

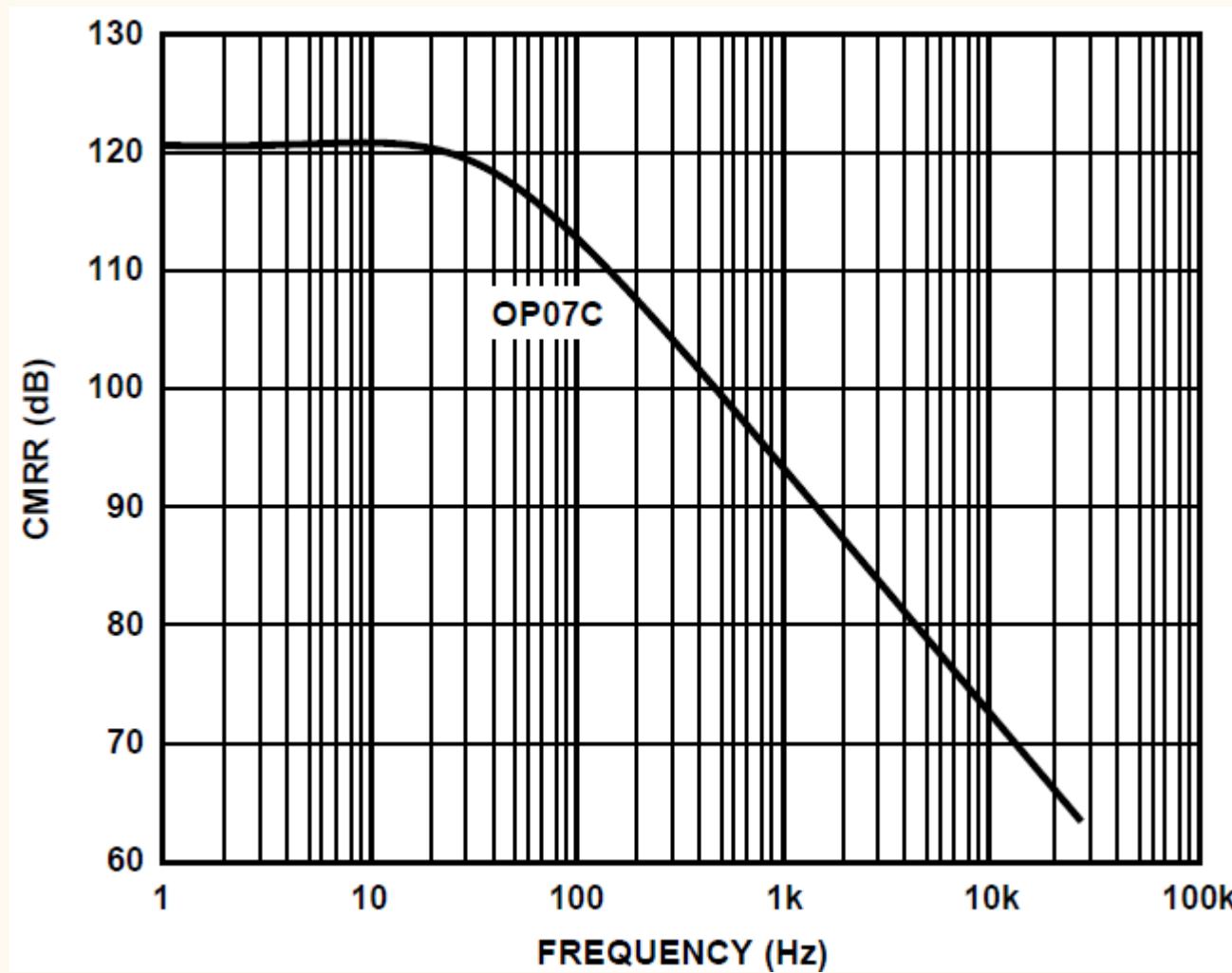
# OP-amp data sheet

## Parameters

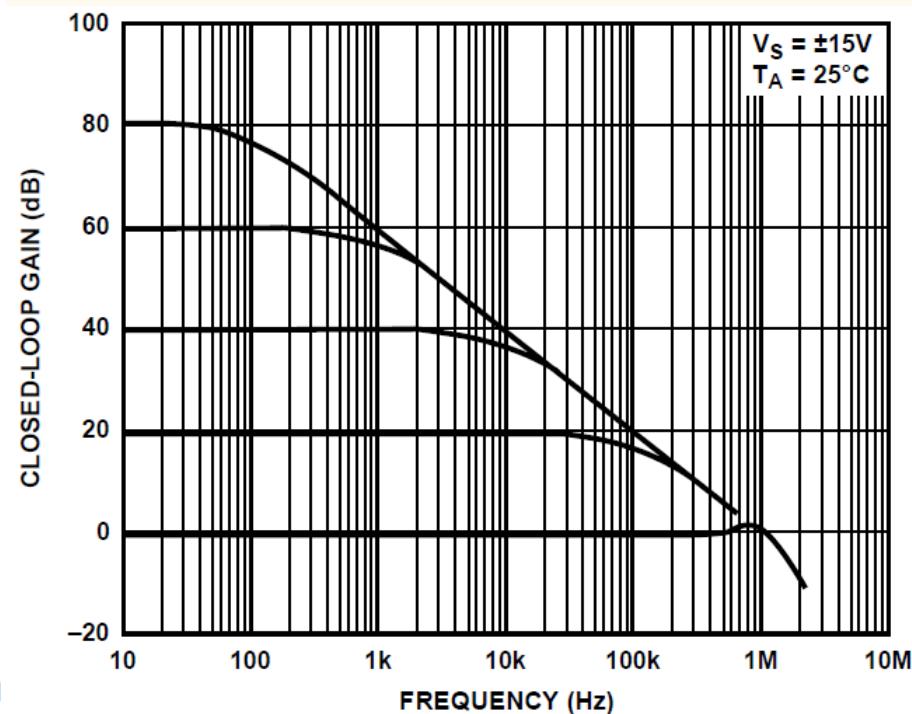
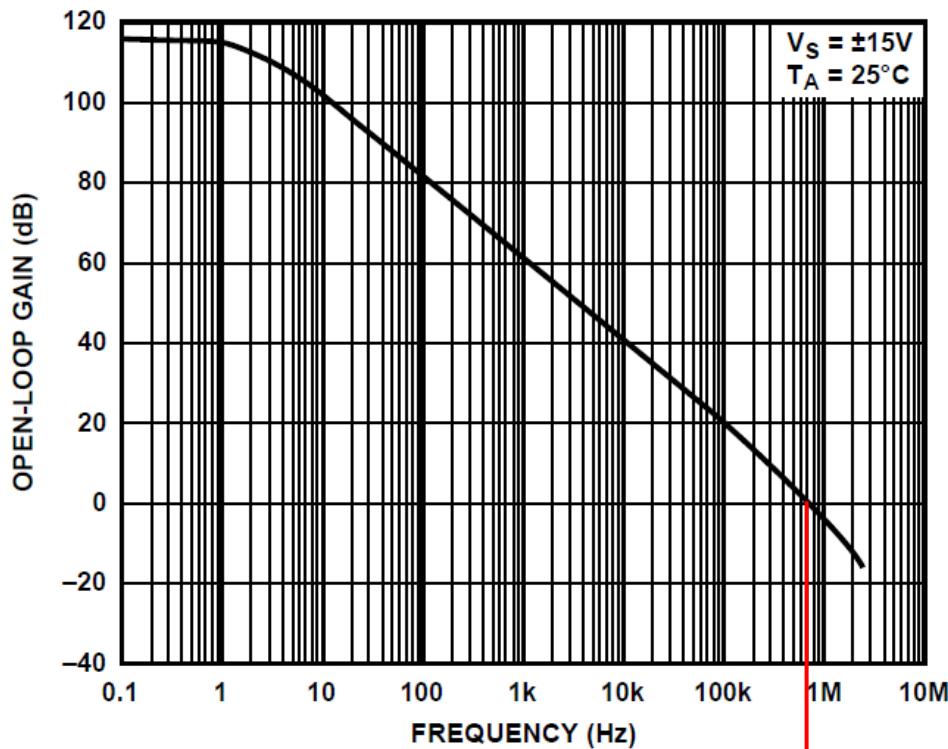
Parameter	Symbol	Conditions	Min	Typ	Max	Unit
INPUT CHARACTERISTICS						
$T_A = 25^\circ\text{C}$						
Input Offset Voltage <sup>1</sup>	$V_{OS}$		60	150		$\mu\text{V}$
Long-Term $V_{OS}$ Stability <sup>2</sup>	$V_{OS}/\text{Time}$		0.4	2.0		$\mu\text{V}/\text{Month}$
Input Offset Current	$I_{OS}$		0.8	6.0		$\text{nA}$
Input Bias Current	$I_B$		$\pm 1.8$	$\pm 7.0$		$\text{nA}$
Input Noise Voltage	$e_n \text{ p-p}$	$0.1 \text{ Hz to } 10 \text{ Hz}^3$	0.38	0.65		$\mu\text{V p-p}$
Input Noise Voltage Density	$e_n$	$f_0 = 10 \text{ Hz}$	10.5	20.0		$\text{nV}/\sqrt{\text{Hz}}$
		$f_0 = 100 \text{ Hz}^3$	10.2	13.5		$\text{nV}/\sqrt{\text{Hz}}$
		$f_0 = 1 \text{ kHz}$	9.8	11.5		$\text{nV}/\sqrt{\text{Hz}}$
Input Noise Current	$I_n \text{ p-p}$		15	35		$\text{pA p-p}$
Input Noise Current Density	$I_n$	$f_0 = 10 \text{ Hz}$	0.35	0.90		$\text{pA}/\sqrt{\text{Hz}}$
		$f_0 = 100 \text{ Hz}^3$	0.15	0.27		$\text{pA}/\sqrt{\text{Hz}}$
		$f_0 = 1 \text{ kHz}$	0.13	0.18		$\text{pA}/\sqrt{\text{Hz}}$
Input Resistance, Differential Mode <sup>4</sup>	$R_{IN}$		8	33		$\text{M}\Omega$
Input Resistance, Common Mode	$R_{INCM}$			120		$\text{G}\Omega$

# OP-amp data sheet

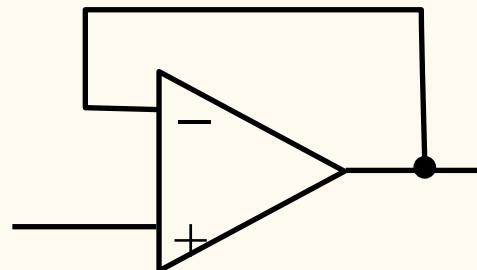
## Common mode rejection ratio (CMRR)



# OP-amp data sheet

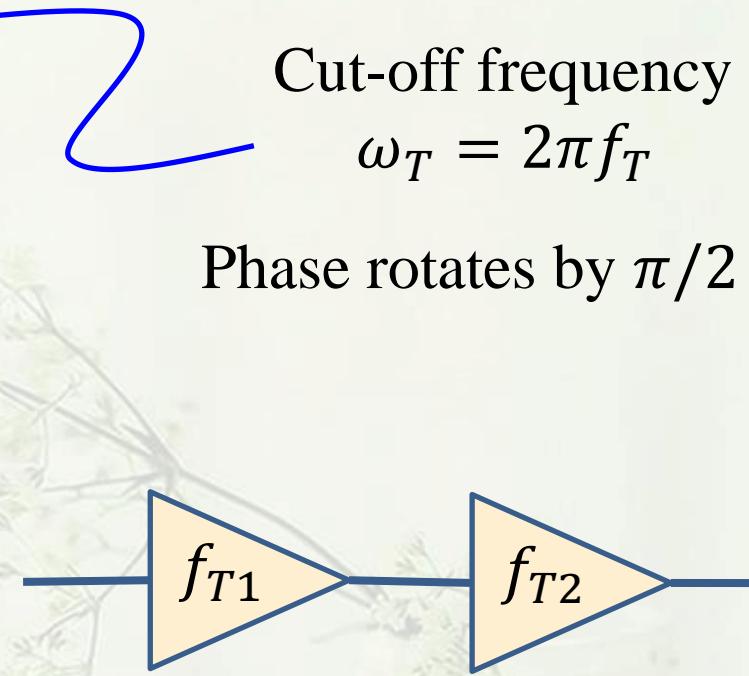
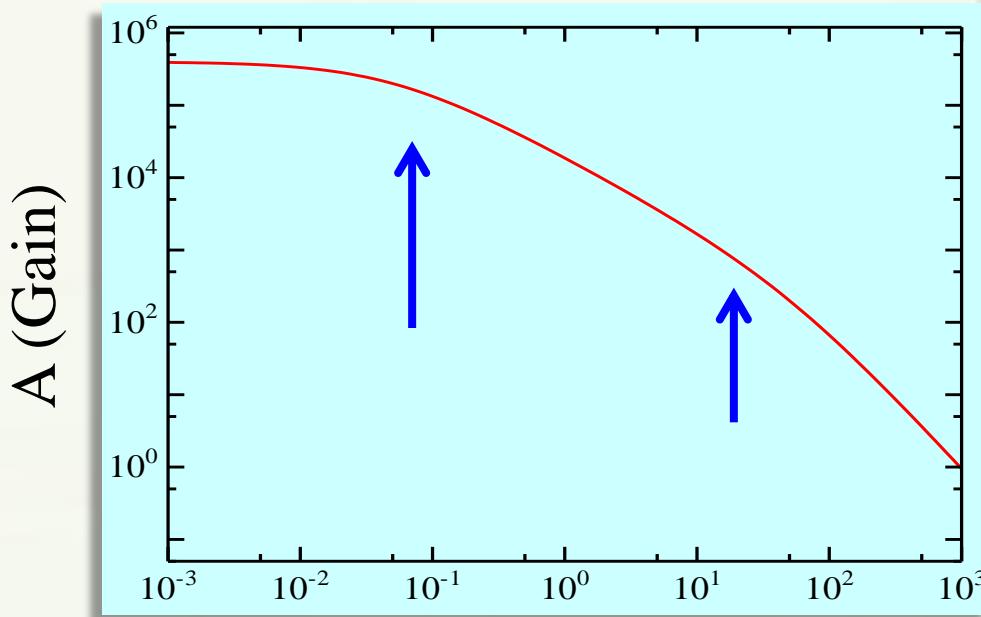
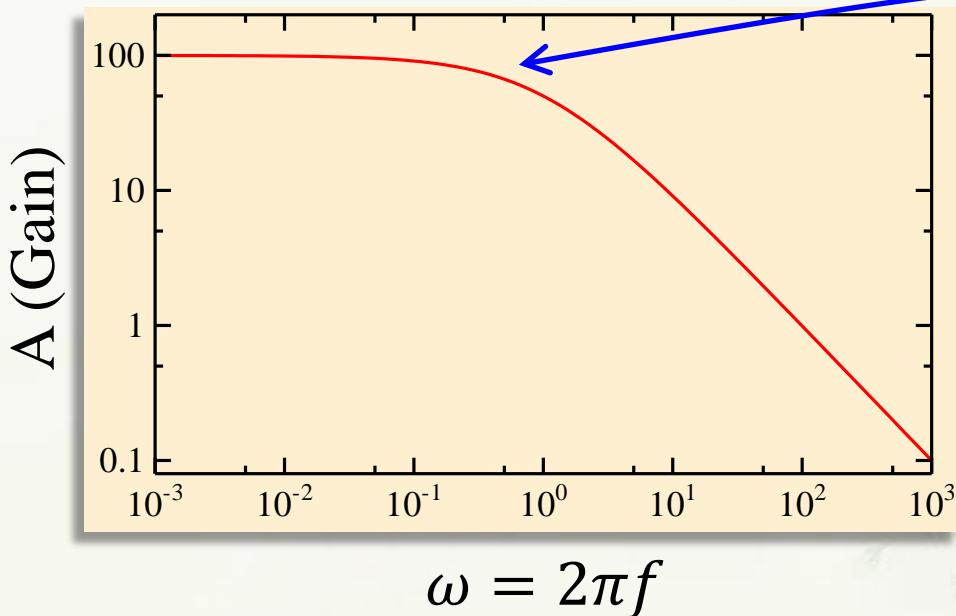


Unity gain frequency



Voltage follower

# Frequency Dependent Characteristics of OP-Amps



Multiple cut-off frequency:  
Phase rotates more than  $\pi$

If gain is larger than 1 at  
phase shift  $\pi$  :  
Dangerous!

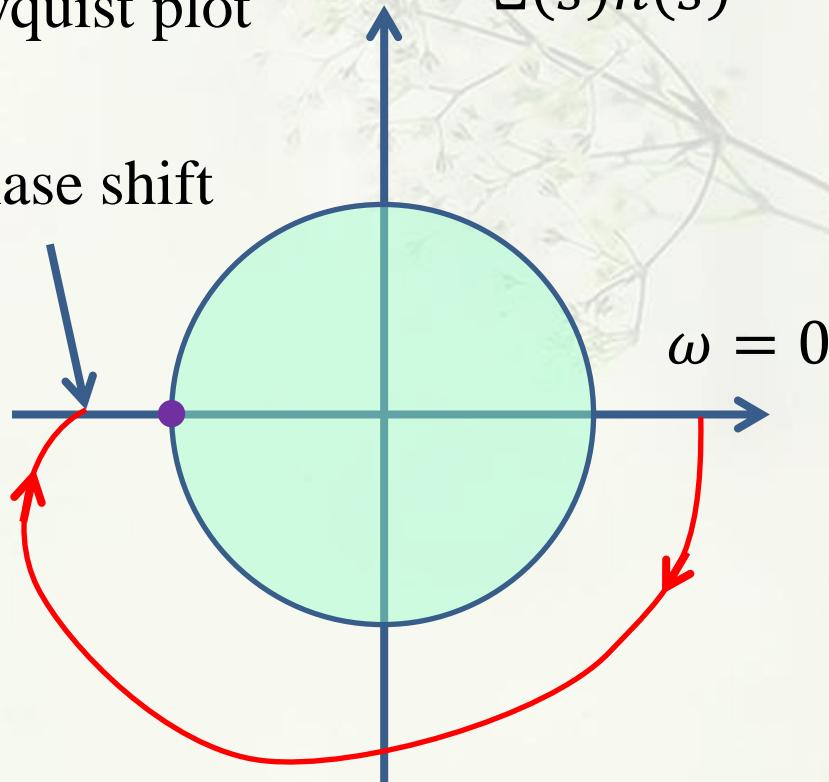
# Phase compensation

Why dangerous?

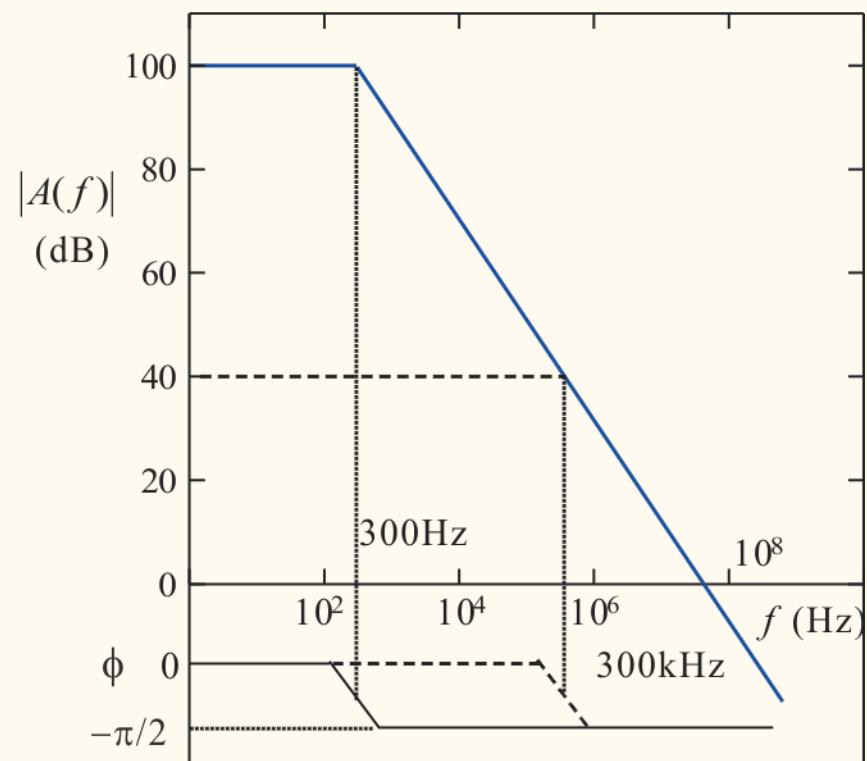
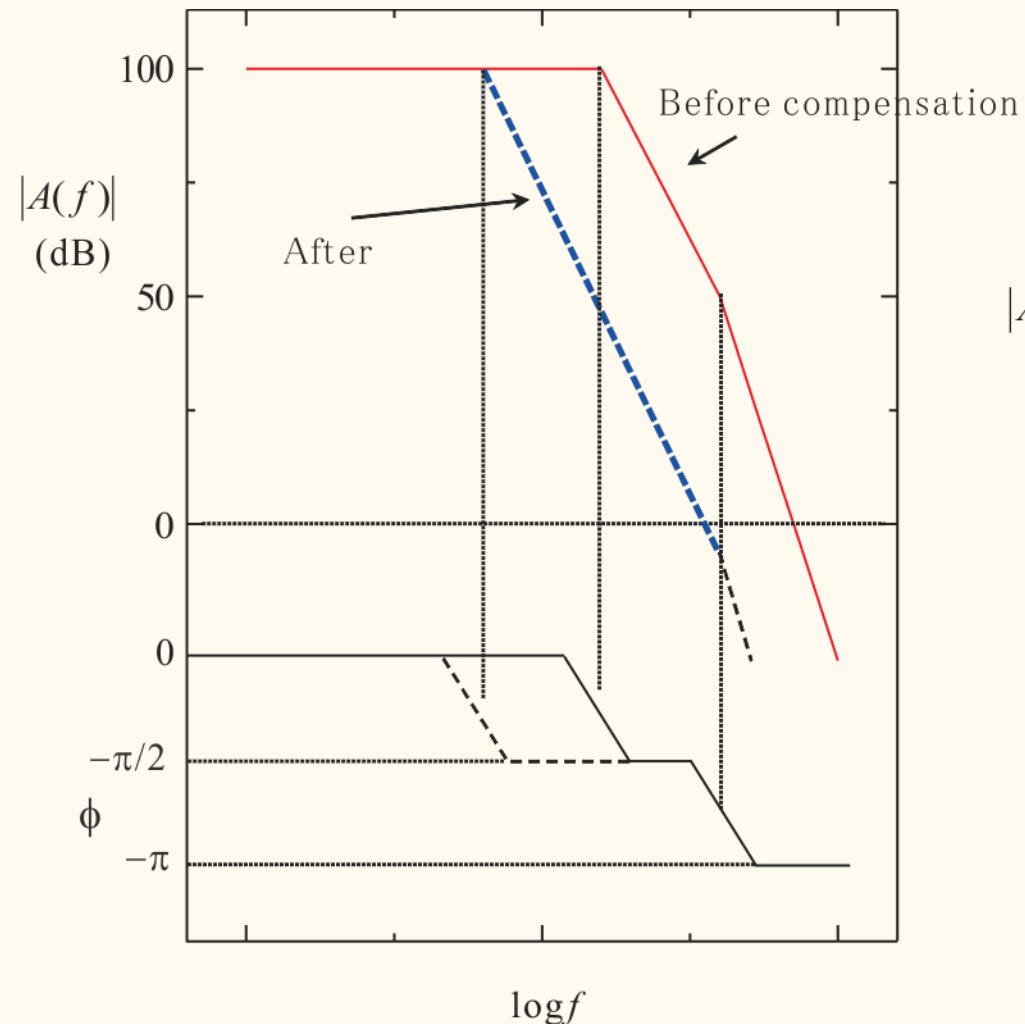
$\pi$  phase shift: negative feedback  $\rightarrow$  positive feedback

In Nyquist plot  $\Xi(s)h(s)$

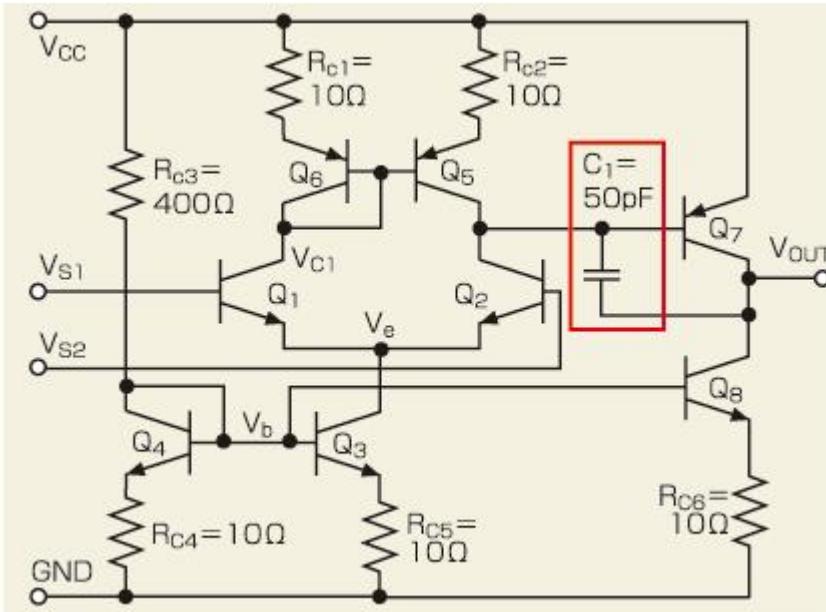
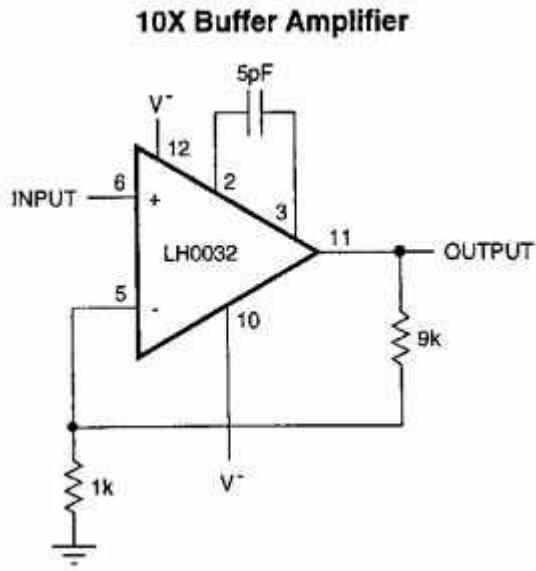
$\pi$  phase shift



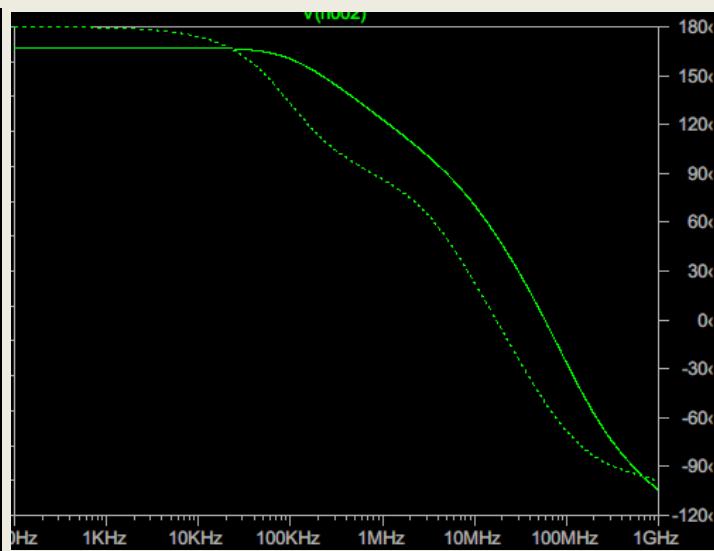
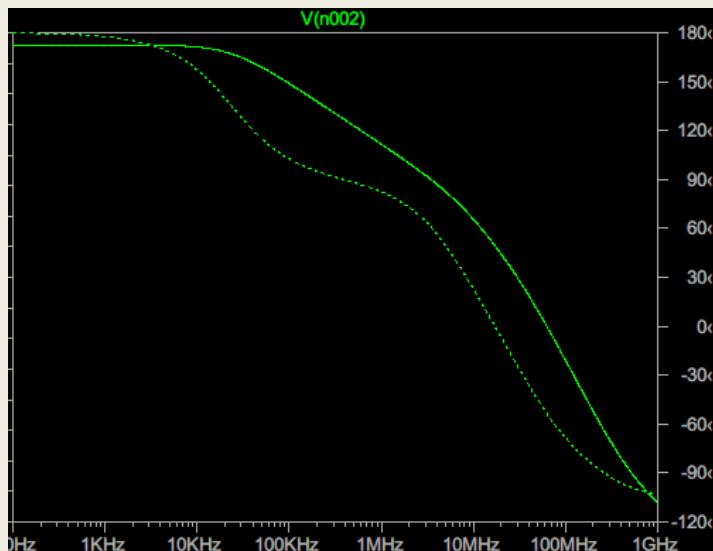
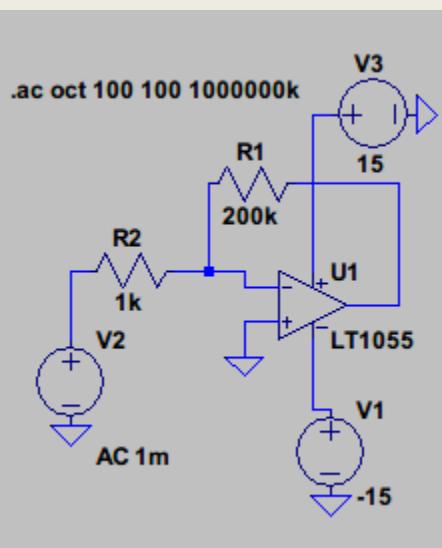
# Phase compensation



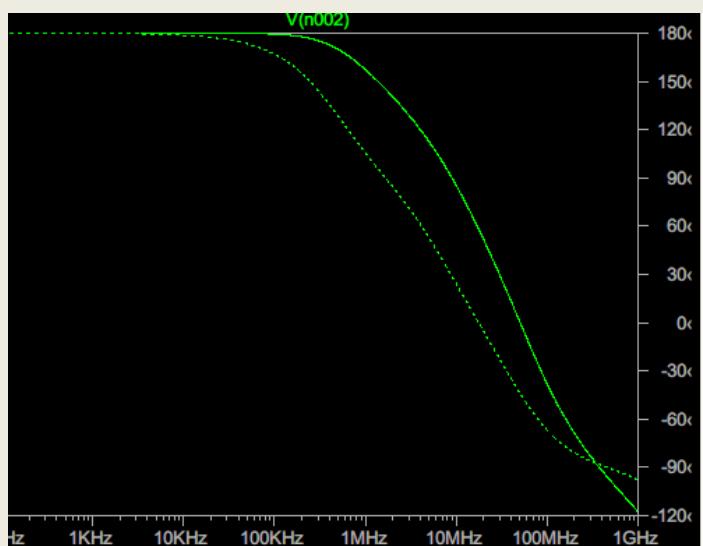
# Phase compensation



# Inverting amplifier and cut-off frequency

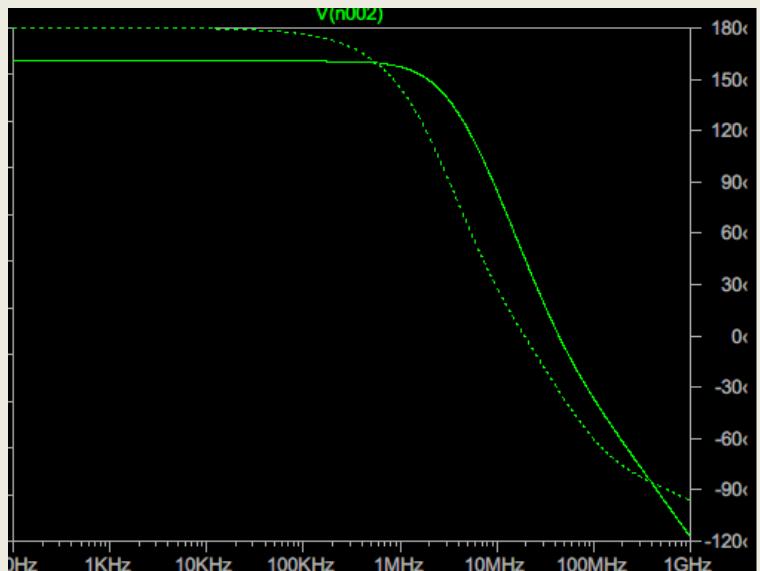


$$A = 200 \quad f_T = 30\text{kHz}$$



$$A = 10 \quad f_T = 300\text{kHz}$$

$$A = 50 \quad f_T = 90\text{kHz}$$



$$A = 2 \quad f_T = 2\text{MHz}$$

# Oscillation of OPamp

