電子回路論第5回

Electric Circuits for Physicists

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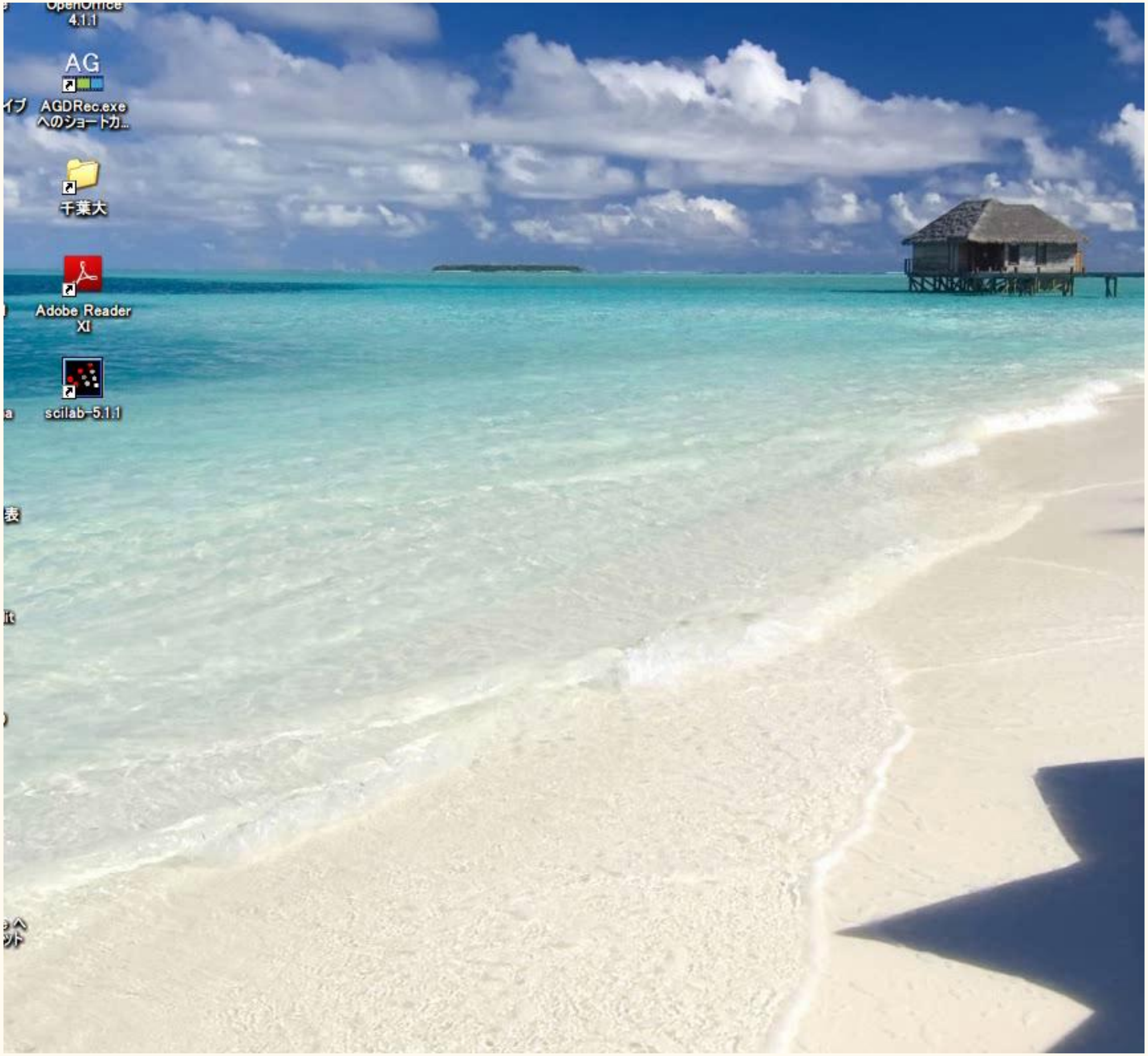
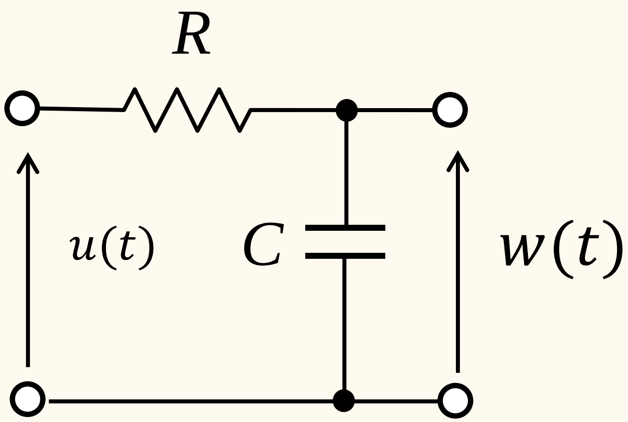
Outline

A convenient freeware: Scilab

Transfer function analysis with Scilab



Simple application

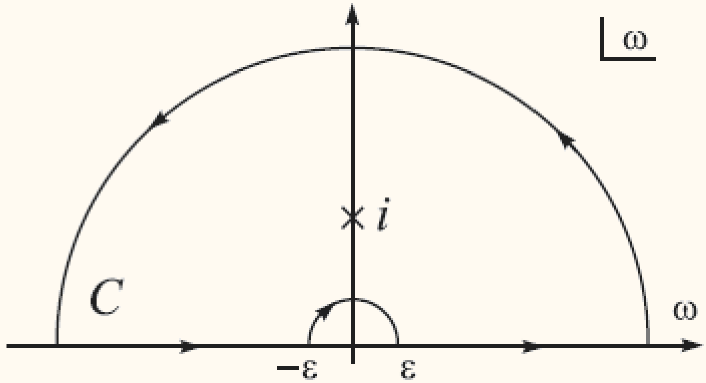


$$V = V_0\left[1 - \exp\left(-\frac{t}{CR}\right)\right]$$

$$g(t) = \int_{-\infty}^{\infty} \frac{1}{1+i\omega}\left[\frac{1}{i\omega} + \pi\delta(\omega)\right] e^{i\omega t}\frac{d\omega}{2\pi}$$

$$= \int_{-\infty}^{\infty} \frac{e^{i\omega t}}{(i-\omega)\omega}\frac{d\omega}{2\pi} + \frac{1}{2}$$

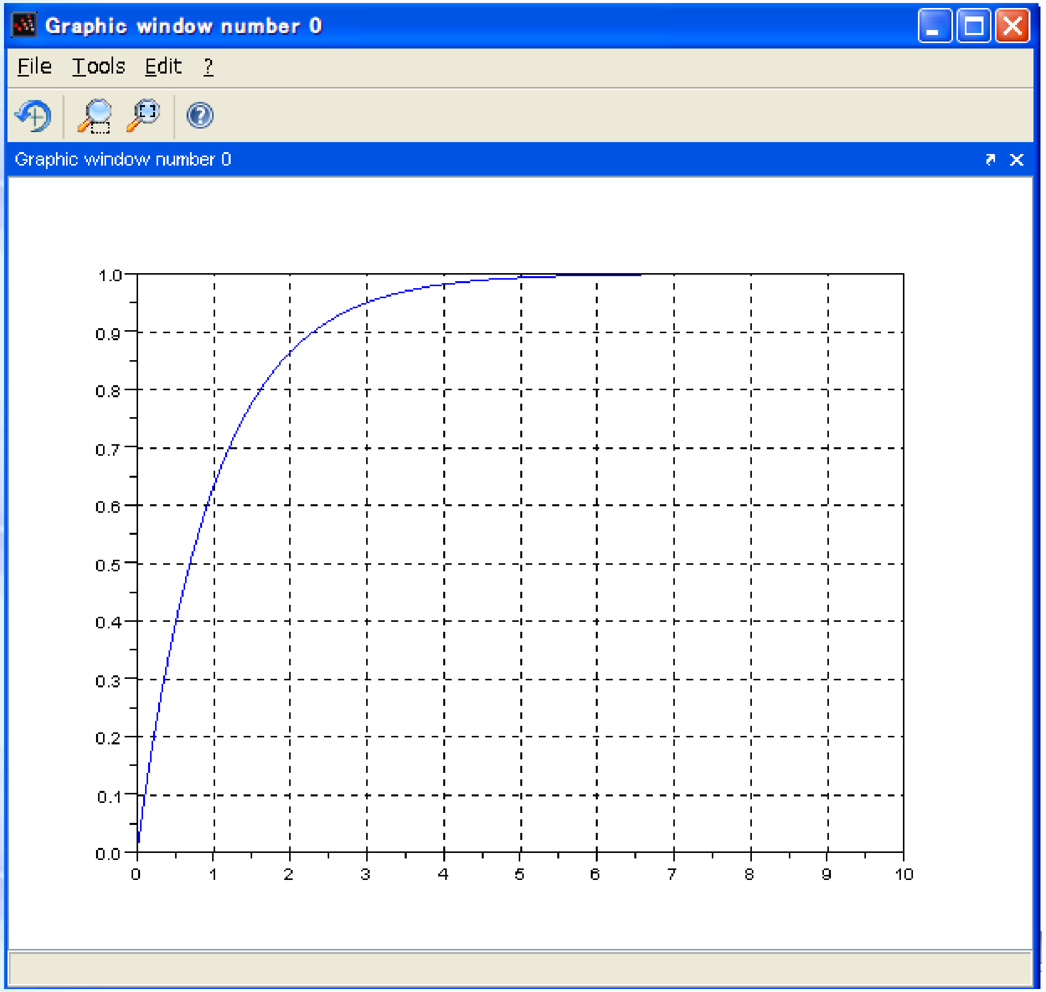
$$-2\pi i\frac{e^{-t}}{2\pi i} - \lim_{\epsilon\to 0}\left[\int_{\pi}^{0} \frac{e^{i\epsilon e^{i\theta} t}}{\epsilon e^{i\theta}(\epsilon e^{i\theta} - i)}\frac{i\epsilon e^{i\theta} d\theta}{2\pi}\right] = -e^{-t} - \frac{1}{2}$$



$$g(t) = -e^{-t}$$

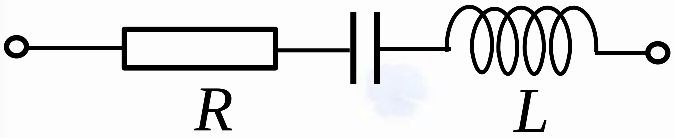
Transient response: Use of Scilab

$$\Xi(s) = \frac{1}{1+s}$$

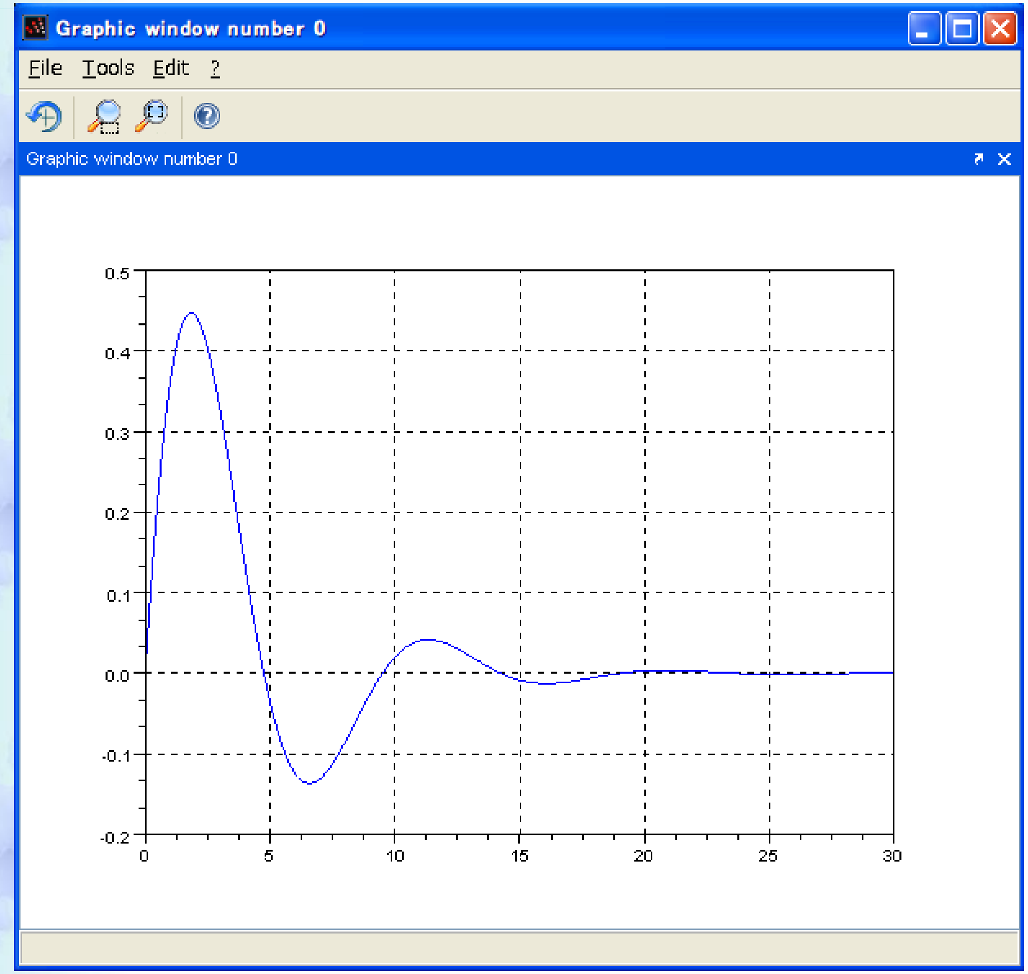



```
-->s=poly(0,'s');

-->G=1/(1+s);

-->sys=syslin('c',G);

-->t=linspace(0,10,100);

-->y=csim('step',t,sys);

-->plot(t,y)

-->xgrid()
```

Transient response: Use of Scilab

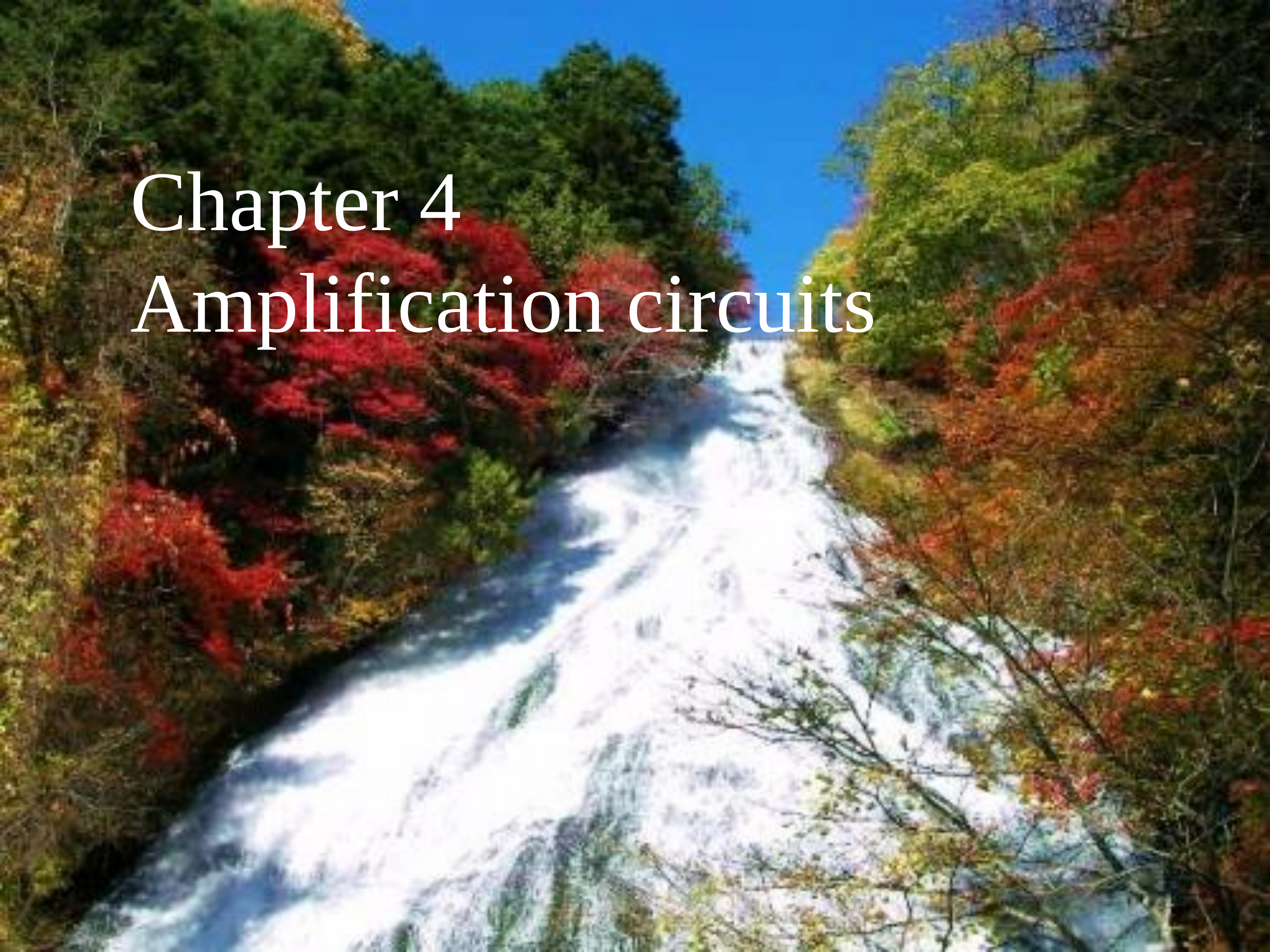


$$Y(s) = \frac{Cs}{LCs^2 + CRs + 1}$$



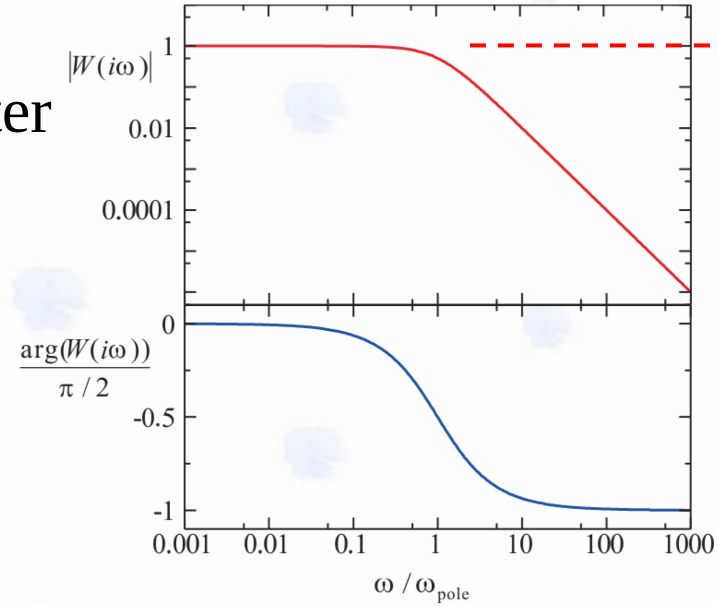

```
-->G=s/(1+s+2*s*s);

-->sys=syslin('c',G);

-->y=csim('step',t,sys);

-->plot(t,y)
```

Chapter 4
Amplification circuits



Linear amplifier

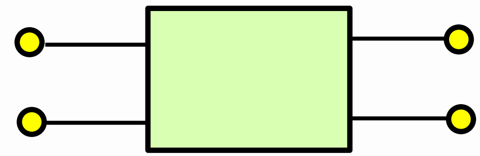
passive filter



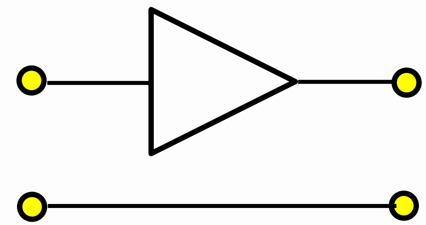
gain = 1

gain >1 ⟶ amplifier

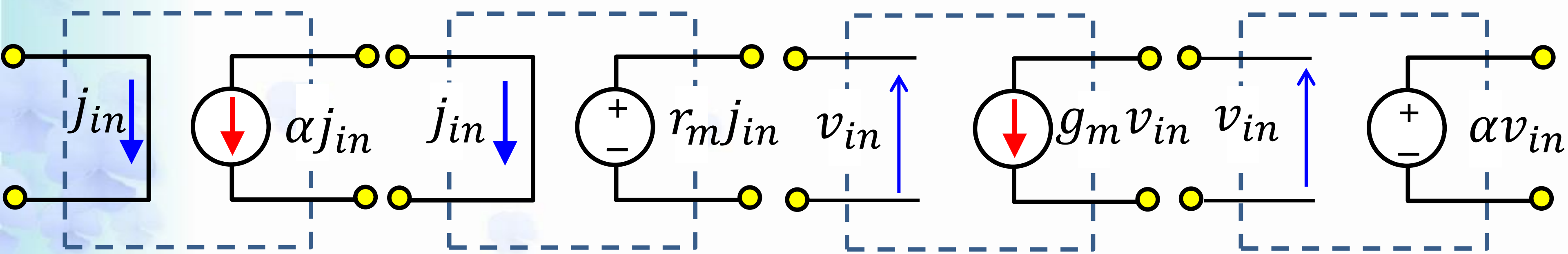
four terminal circuit model



Circuit symbol



Controlled power source models



J - *J*   *J* - *V*   *V* - *J*   *V* - *V*

Gain, and "Unit" for gain

Voltage gain: $\left|\frac{v_{out}}{v_{in}}\right|$ Current gain: $\left|\frac{j_{out}}{j_{in}}\right|$ Power gain: $\left|\frac{v_{out}j_{out}}{v_{in}j_{in}}\right|$

When we say "the gain of the amplifier …", the gain means power gain.

quantity Q, unit Q_0 : Q in log scale: $L = \log_{10}\frac{Q}{Q_0}$ (B, bel)

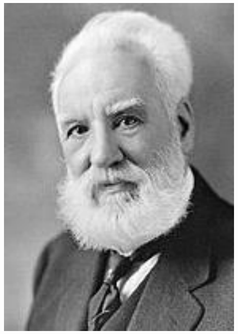
cf. deca- 10

1/10

dB : (decibel)

From: G. Bell

Alexander Graham Bell
1847 - 1922



$$G = 10 \times \log_{10}\left(\frac{v_{\mathrm{out}}}{v_{\mathrm{in}}}\right)^2 = 20\log_{10}\frac{v_{\mathrm{out}}}{v_{\mathrm{in}}}$$

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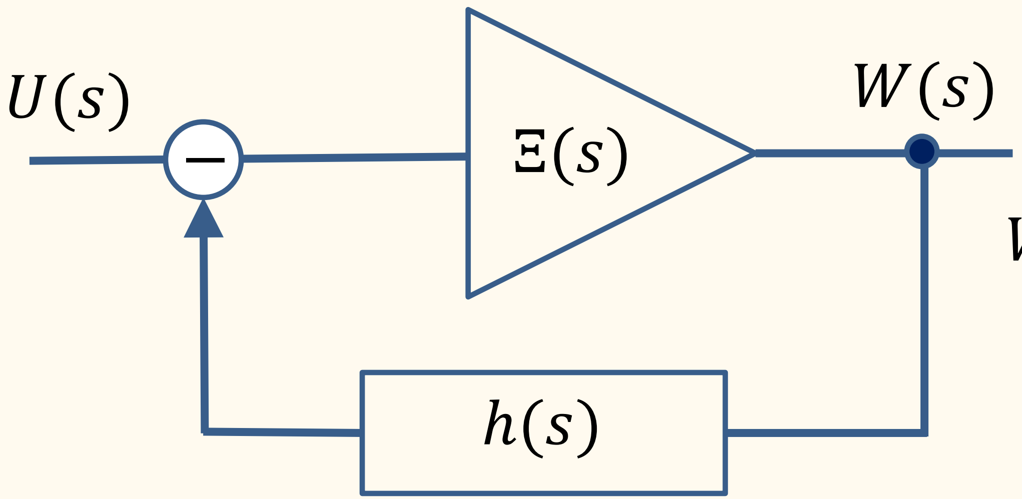
dB units: dBm (1mW: 0dBm), dBv (1V: 0dBv), etc.

Feedback circuit

Feedforward

⇕

Feedback



$$W(s) = \Xi(s)U(s)$$

$$W(s) = \Xi(s)[U(s) - h(s)W(s)]$$

$$W(s) = \frac{\Xi(s)}{1 + \Xi(s)h(s)} U(s)$$

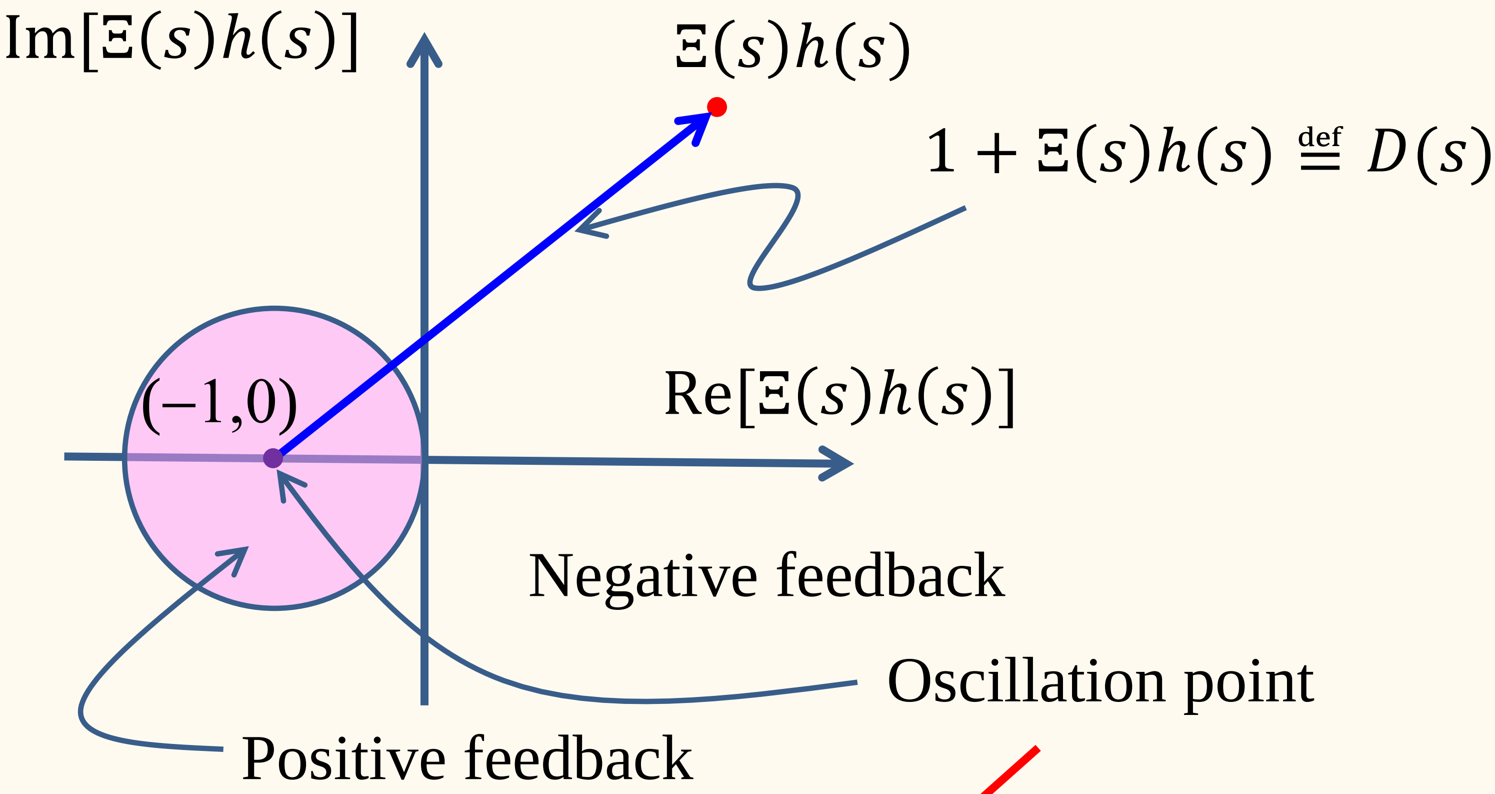
$$\stackrel{\text{def}}{=} G(s)U(s)$$

$|1 + \Xi(s)h(s)| > 1$: Negative feedback, < 1: Positive feedback

$$|\Xi(s)| \gg 1 \rightarrow G(s) \approx \frac{1}{h(s)}$$

Condition for negative feedback

$|1 + \Xi(s)h(s)| > 1$: Negative feedback, < 1: Positive feedback



If $\Xi(s)h(s) = -1$ has solutions, the circuit may be unstable.

How can we judge? → Criteria (Routh-Hurwitz, Nyqust, Liapunov, …)

Zeros and poles of $D(s)$

Assumption 1: $\Xi(s), \Xi(s)h(s)$ are stable

→ Poles are on the left half plane of s.

Assumption 2: $\Xi(i\omega), \Xi(i\omega)h(i\omega) \to 0$ for $|\omega| \to \infty$

$\Xi(s) = \frac{Q(s)}{P(s)},\ h(s) = \frac{q(s)}{p(s)}$: $P(s), Q(s), p(s), q(s)$ polynomials

$\deg(P) > \deg(Q), \deg(p) \geq \deg(q)$

$$D(s) = 1 + \Xi(s)h(s) = \frac{P(s)p(s)}{P(s)p(s) + Q(s)q(s)}$$

$$D(s) = D_0 \frac{(s-\beta_1)\cdots(s-\beta_n)}{(s-\alpha_1)\cdots(s-\alpha_n)}$$

The same order

Zeros and poles of $D(s)$

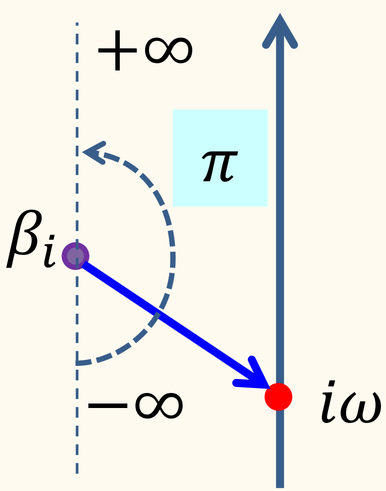
$$D(s) = D_0 \frac{(s-\beta_1)\cdots(s-\beta_n)}{(s-\alpha_1)\cdots(s-\alpha_n)}$$

$\{\beta_i\}$: Zeros of $D(s)$ → Poles of $G(s)$

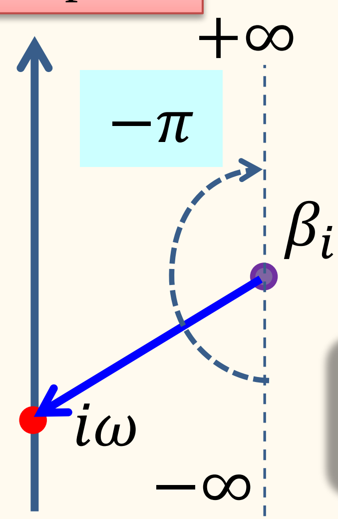
$\exists \beta_i \in$ right half plane of s → The circuit is unstable.

$$\arg(D) = \sum_{i=1}^{n} \arg(s-\beta_i) - \sum_{i=1}^{n} \arg(s-\alpha_i)$$

Left half plane



Right half plane



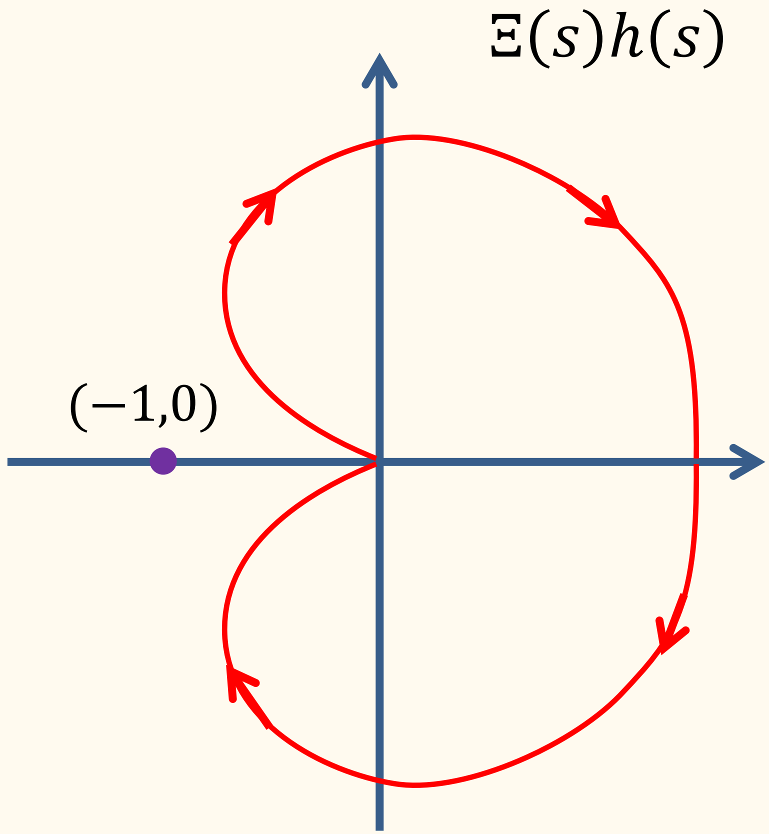
$s = i\omega$ (on imaginary axis)

$\omega: -\infty \rightarrow +\infty$

Number of zeros on the right half plane: m

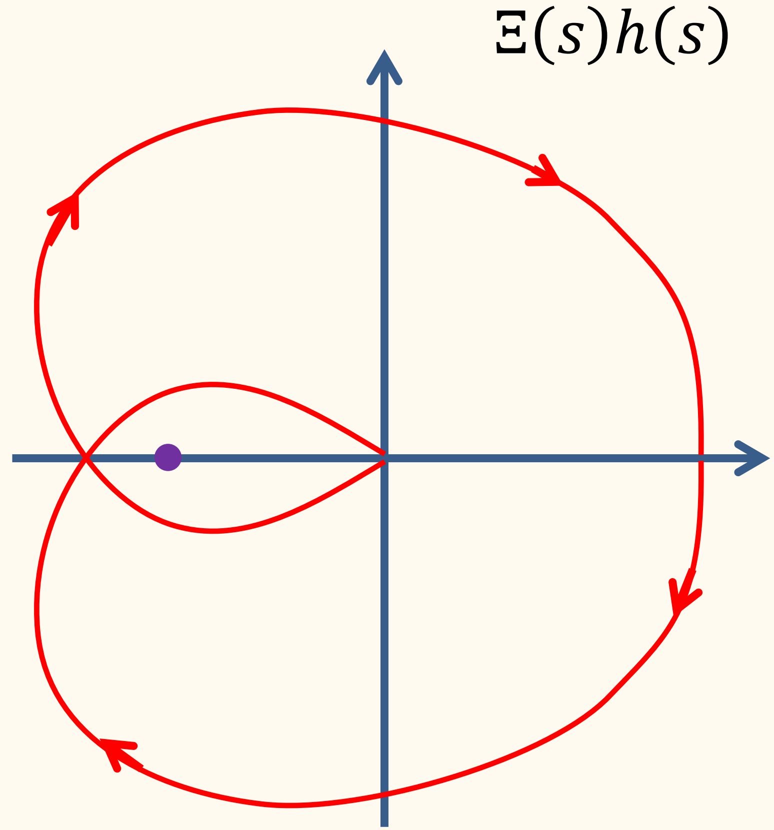
$$\Delta \arg(D) = (n-m)\pi - m\pi - n\pi = -2m\pi$$

Nyquist Plot and Criterion



$\Delta\,\arg(D) = 0$

Stable

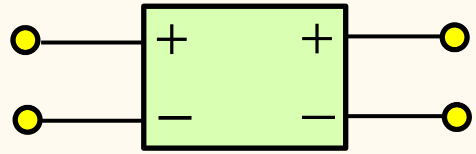


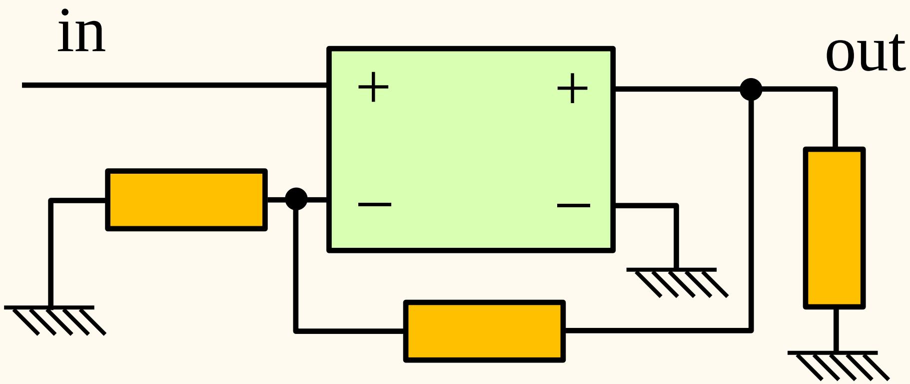
$\Delta\,\arg(D) = -4\pi$

Unstable

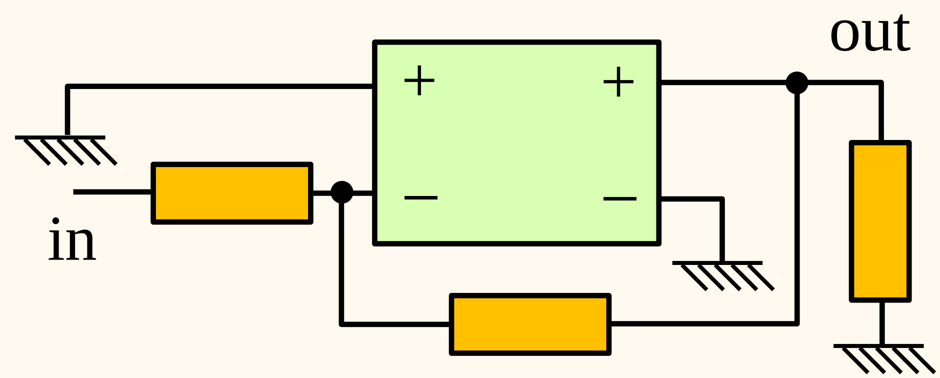
Harry Nyquist
(1889–1976)

Feedback in terminal-pair circuits with resistors

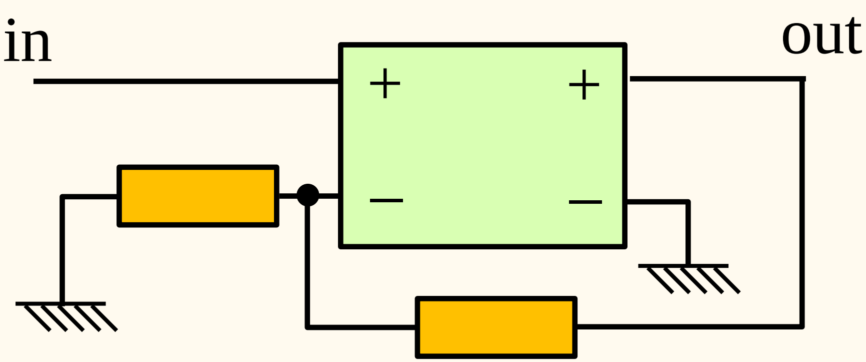




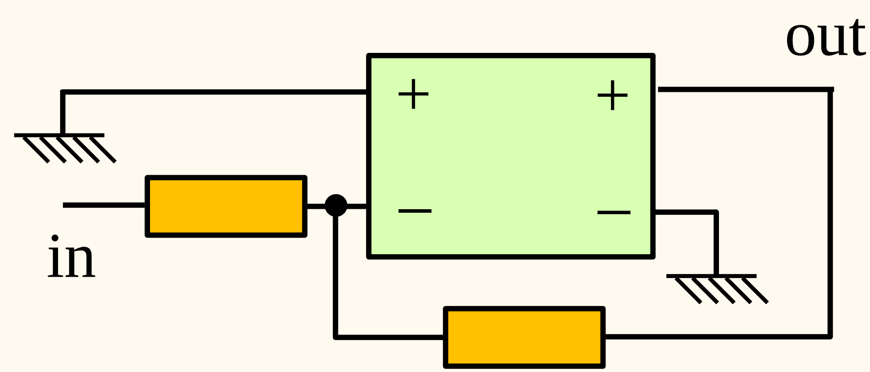
(i) input: parallel, output: parallel



(ii) input: series, output: parallel

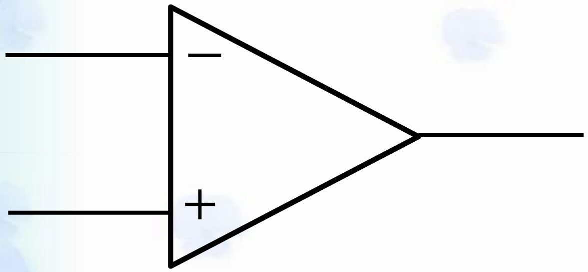


(iii) input: parallel, output: series



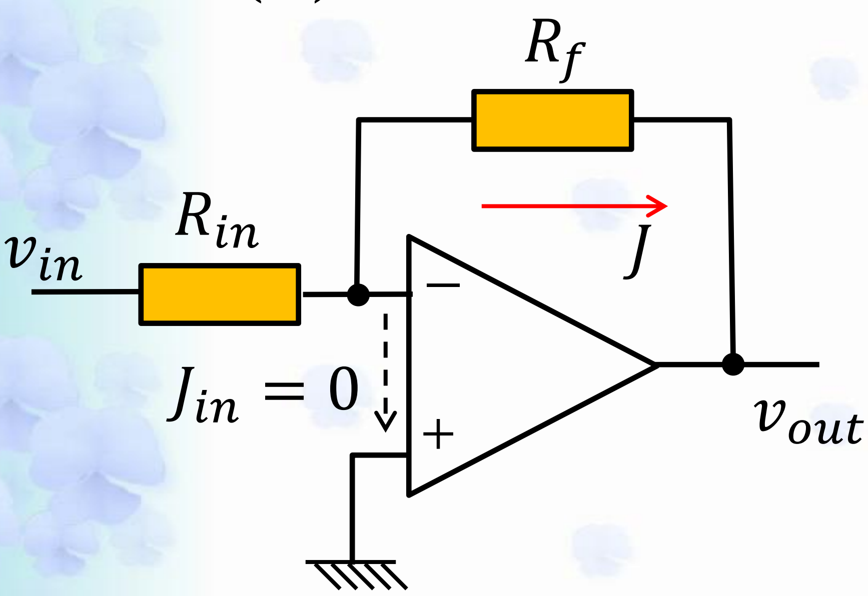
(iv) input: series, output: series

Operational amplifier (OP amp.)



- Differential amplifier
- Input impedance ~∞
- Open loop gain $A_o \gg 1$
- Output resistance ≈ 0

Case (iv)



$$A_\mathrm{o} \gg 1 \quad \therefore V_- \approx V_+ = 0$$

Virtual short circuit

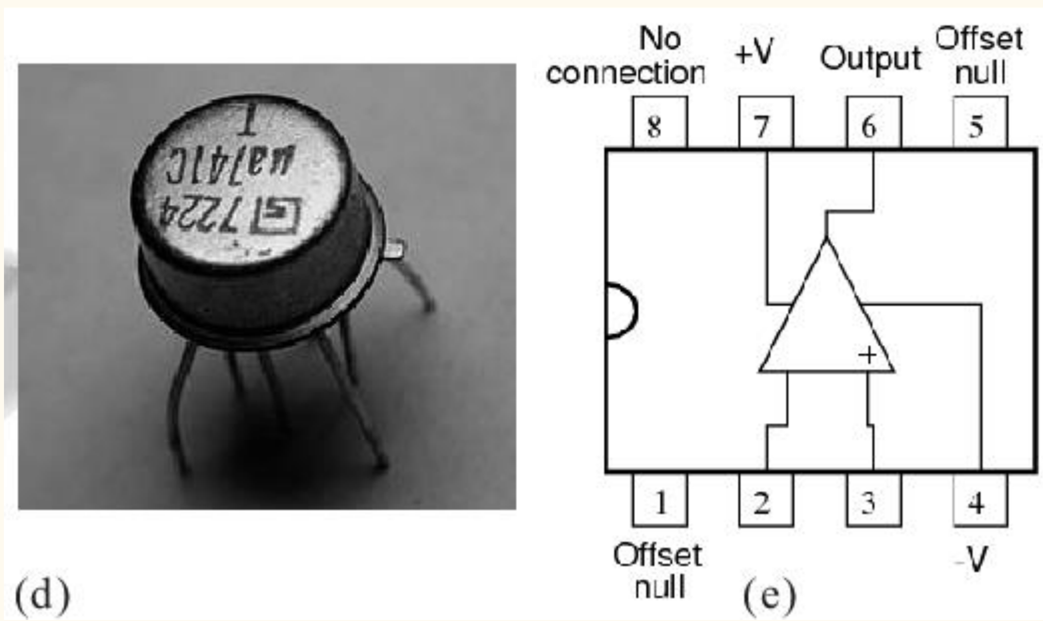
$$J = -\frac{v_\mathrm{out}}{R_f} = \frac{v_\mathrm{in}}{R_\mathrm{in}}$$

$$\therefore v_\mathrm{out} = -\frac{R_f}{R_\mathrm{in}} v_\mathrm{in}$$

Inverting amplifier

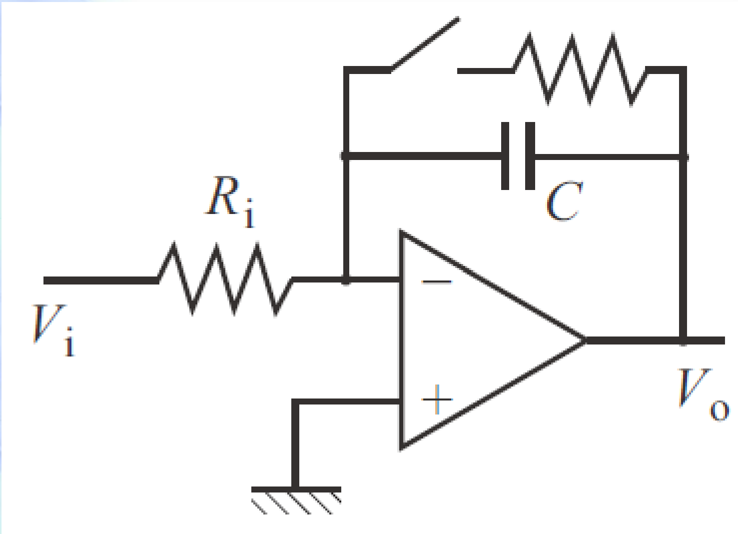

ANALOG
DEVICES
FastFET™
OPERATIONAL
AMPLIFIER
LMP8671
TEXAS
INSTRUMENTS
OPA627

(a) (b) (c)


ua741C
7224
No
connection
+V
Output
Offset
null
8
7
6
5
+
1
2
3
4
Offset
null
-V

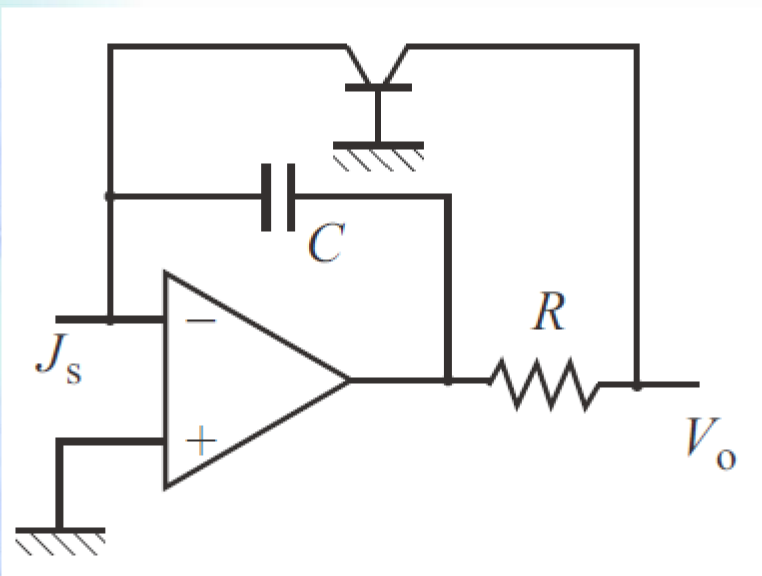
(d) (e)

Various applications of OP amps



$$V_{out}(t) = -\frac{Q}{C} = -\frac{1}{C}\int_0^t \frac{V_i(\tau)}{R_i}d\tau$$
$$= -\frac{1}{CR_i}\int_0^t V_i(\tau)d\tau$$

Integration circuit



$$Vout = -VBE = -\frac{k_B T}{e}\ln\left(\frac{J_s}{J_0}+1\right)$$

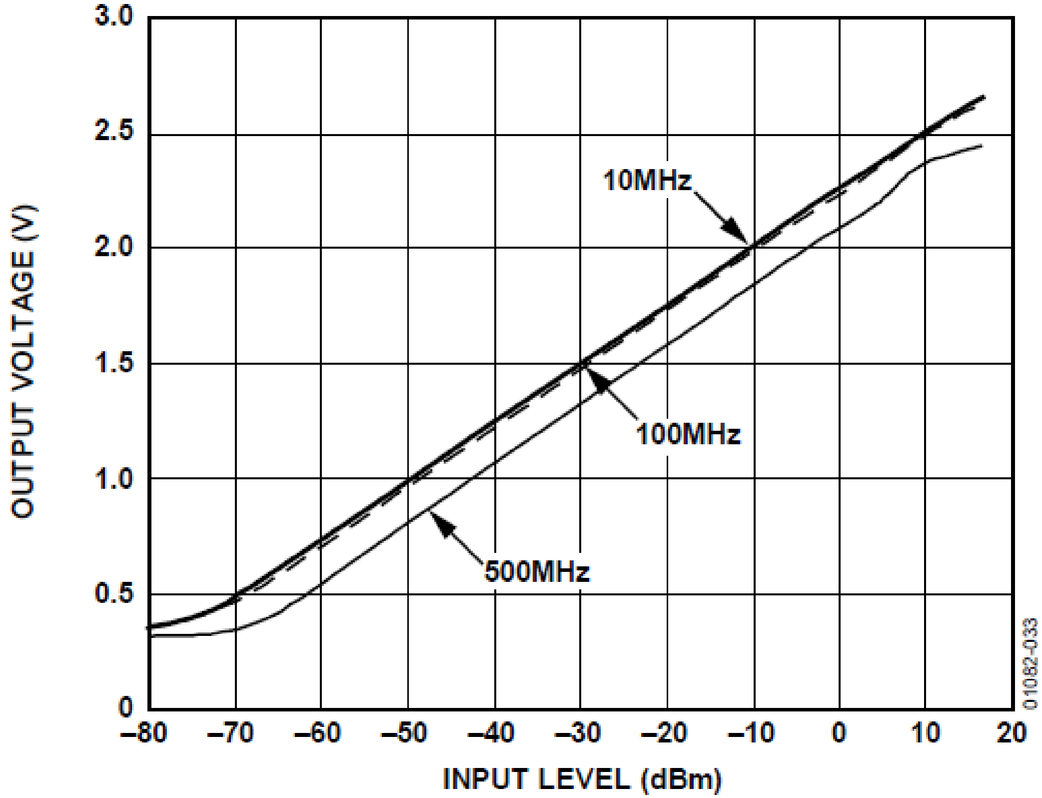
Logarithmic amplifier

Logarithmic Amplifier

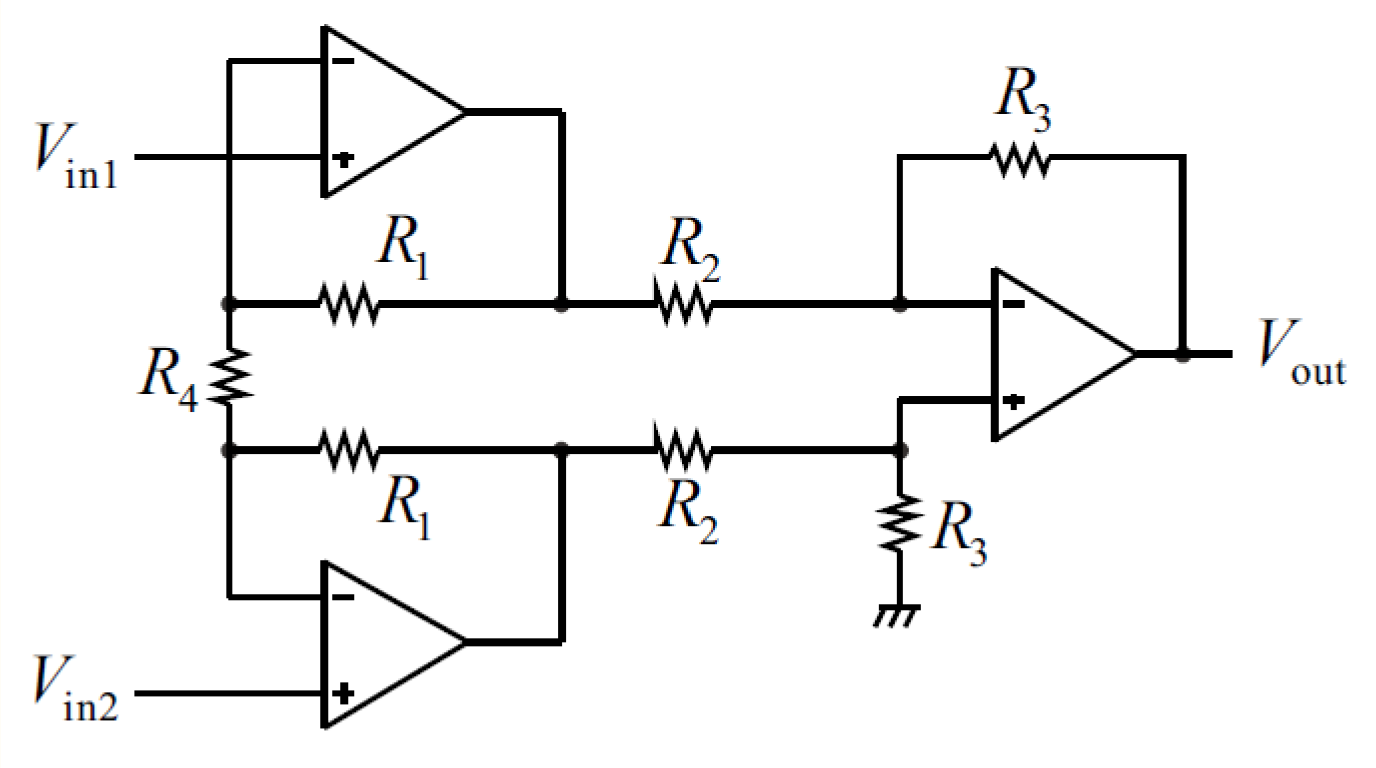
ANALOG DEVICES

Low Cost, DC to 500 MHz, 92 dB Logarithmic Amplifier

Data Sheet **AD8307**



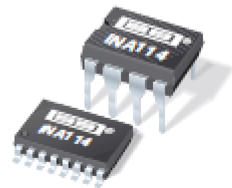
Instrumentation amplifier



$$V_{\text{out}} = -\frac{R_3}{R_2}\left(\frac{2R_1 + R_4}{R_4}\right)(V_{\text{in1}} - V_{\text{int2}})$$

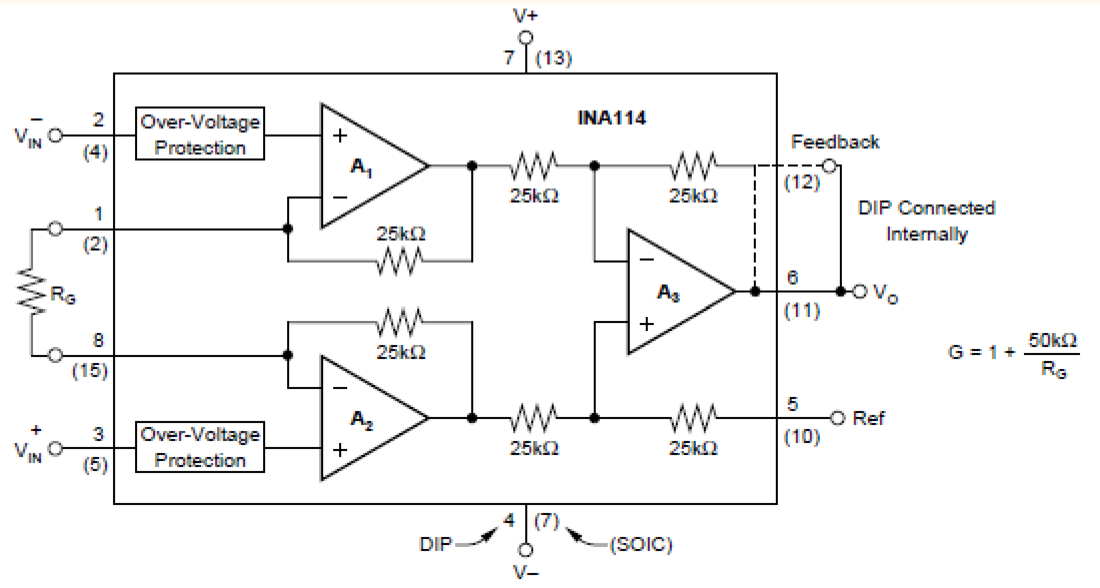
Instrumentation amplifier

BURR - BROWN®



INA114

Precision
INSTRUMENTATION AMPLIFIER



OP-amp data sheet

ANALOG DEVICES

Ultralow Offset Voltage Operational Amplifier

Data Sheet **OP07**

FEATURES

- Low V_{OS}: 75 µV maximum
- Low V_{OS} drift: 1.3 µV/°C maximum
- Ultrastable vs. time: 1.5 µV per month maximum
- Low noise: 0.6 µV p-p maximum
- Wide input voltage range: ±14 V typical
- Wide supply voltage range: ±3 V to ±18 V
- 125°C temperature-tested dice

PIN CONFIGURATION

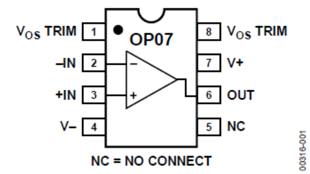
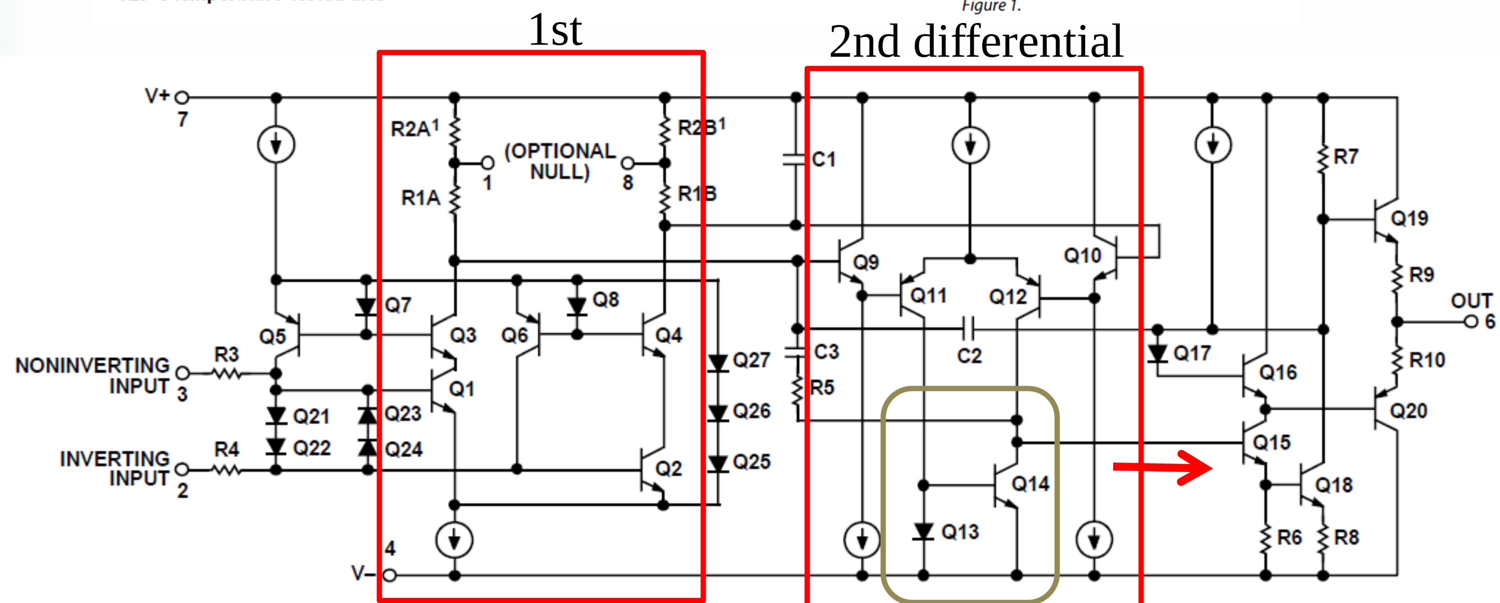


Figure 1.



[1] R2A AND R2B ARE ELECTRONICALLY ADJUSTED ON CHIP AT FACTORY FOR MINIMUM INPUT OFFSET VOLTAGE.

Figure 2. Simplified Schematic

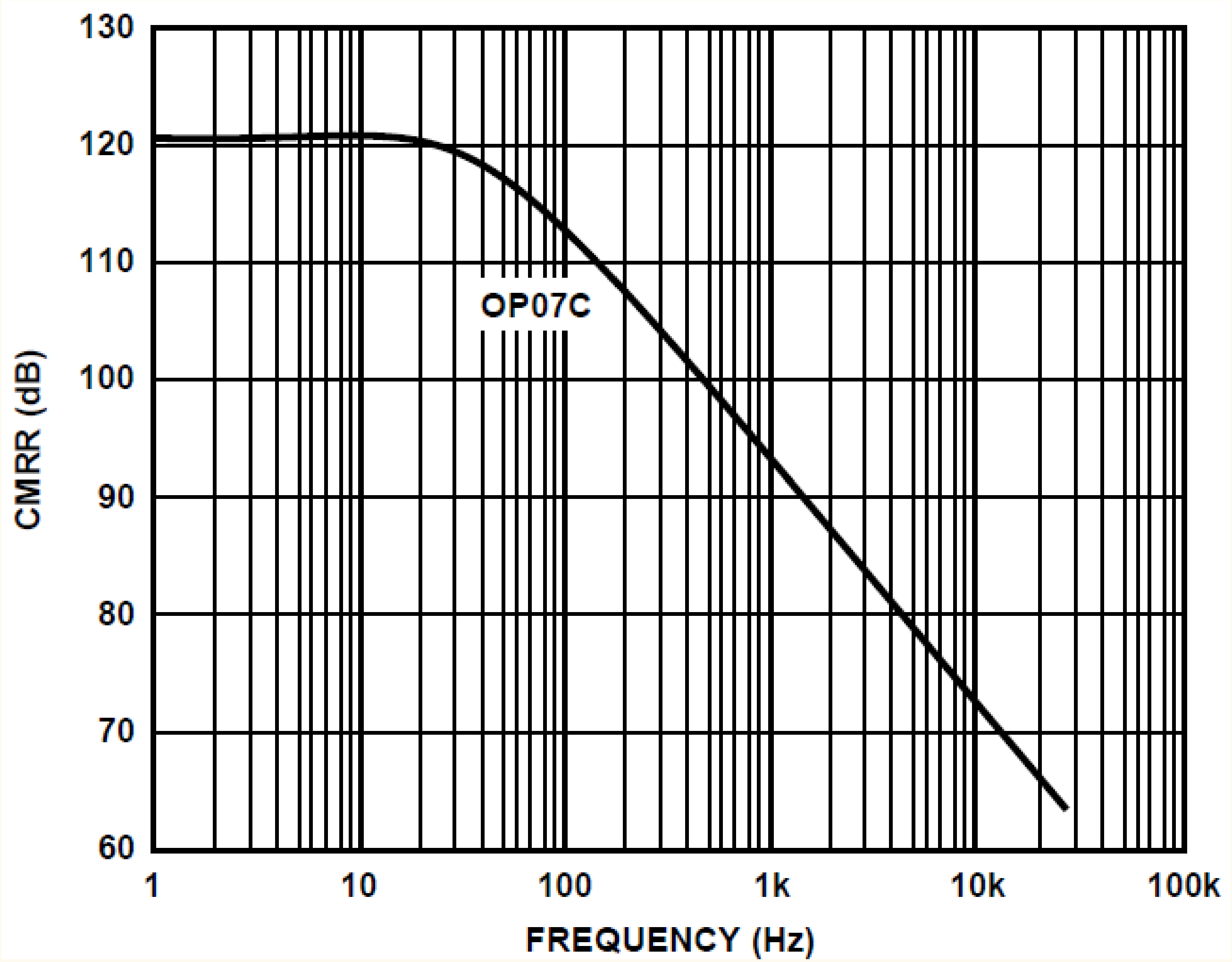
OP-amp data sheet

Parameters

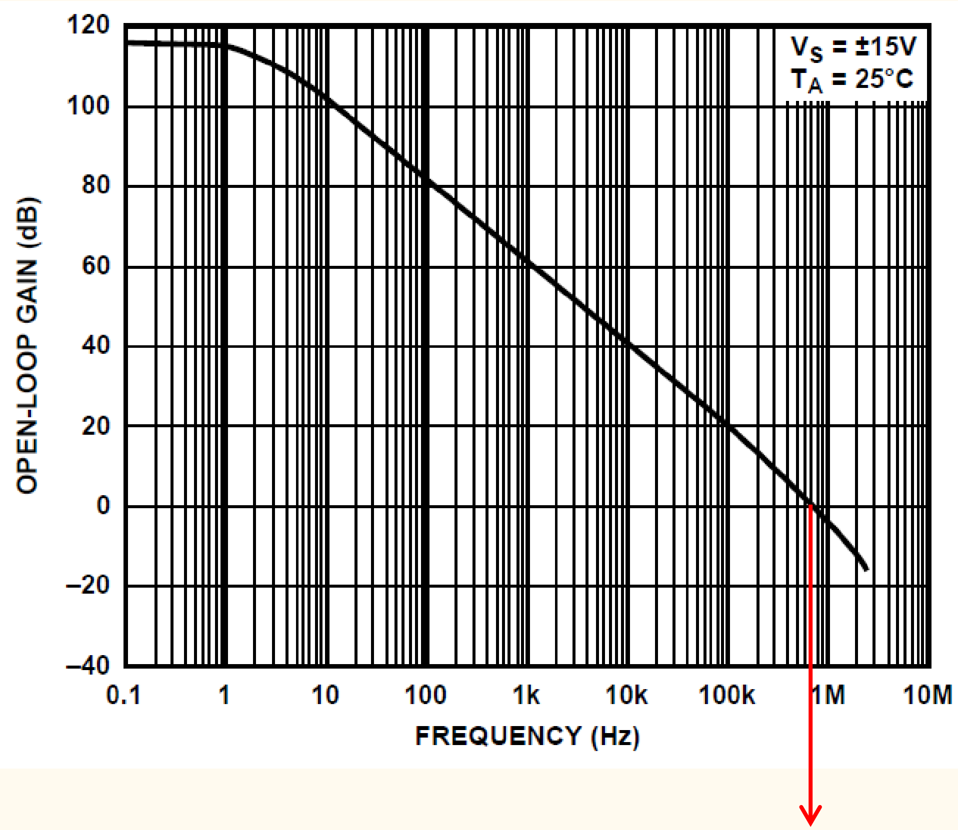
Parameter	Symbol	Conditions	Min	Typ	Max	Unit
INPUT CHARACTERISTICS						
T_A = 25°C						
Input Offset Voltage[1]	V_{OS}			60	150	μV
Long-Term V_{OS} Stability[2]	V_{OS}/Time			0.4	2.0	μV/Month
Input Offset Current	I_{OS}			0.8	6.0	nA
Input Bias Current	I_B			±1.8	±7.0	nA
Input Noise Voltage	e_n p-p	0.1 Hz to 10 Hz[3]		0.38	0.65	μV p-p
Input Noise Voltage Density	e_n	f_O = 10 Hz		10.5	20.0	nV/√Hz
		f_O = 100 Hz[3]		10.2	13.5	nV/√Hz
		f_O = 1 kHz		9.8	11.5	nV/√Hz
Input Noise Current	I_n p-p			15	35	pA p-p
Input Noise Current Density	I_n	f_O = 10 Hz		0.35	0.90	pA/√Hz
		f_O = 100 Hz[3]		0.15	0.27	pA/√Hz
		f_O = 1 kHz		0.13	0.18	pA/√Hz
Input Resistance, Differential Mode[4]	R_{IN}		8	33		MΩ
Input Resistance, Common Mode	R_{INCM}			120		GΩ

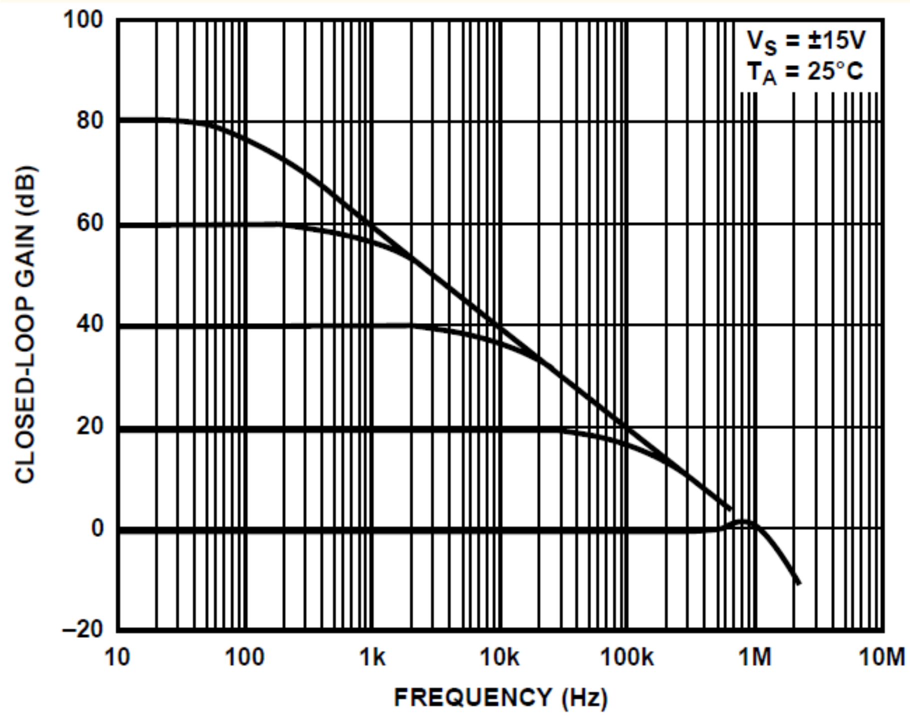
OP-amp data sheet

Common mode rejection ratio (CMRR)

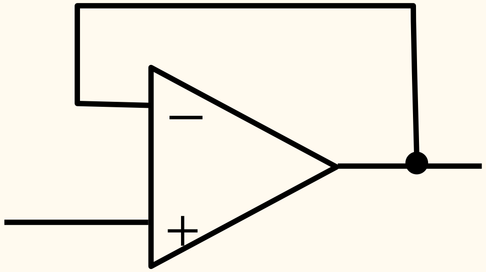


OP-amp data sheet



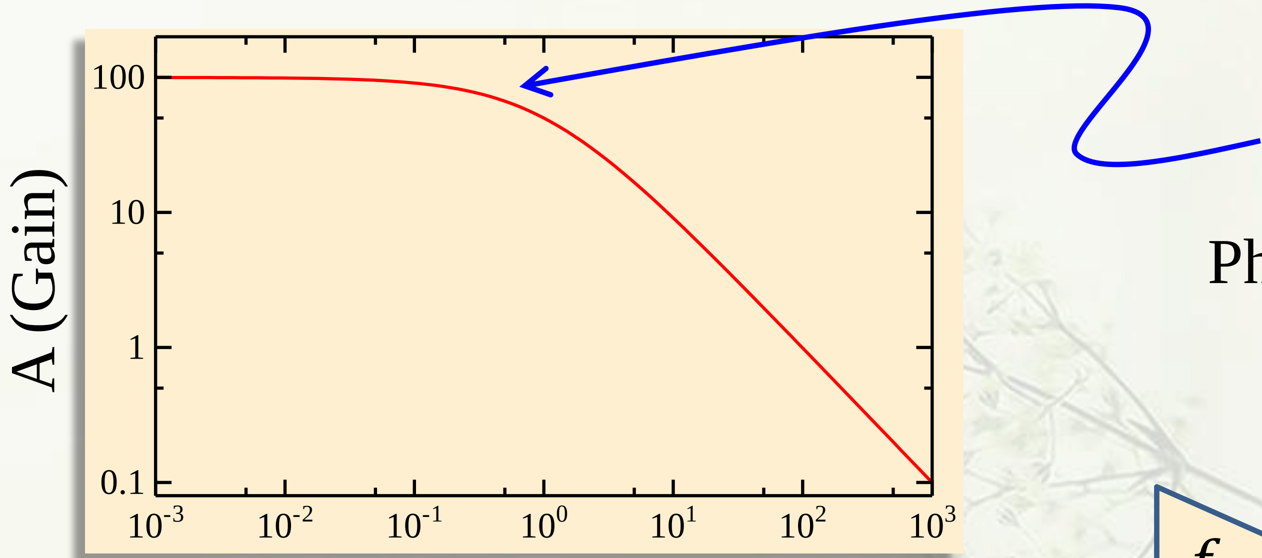


Unity gain frequency

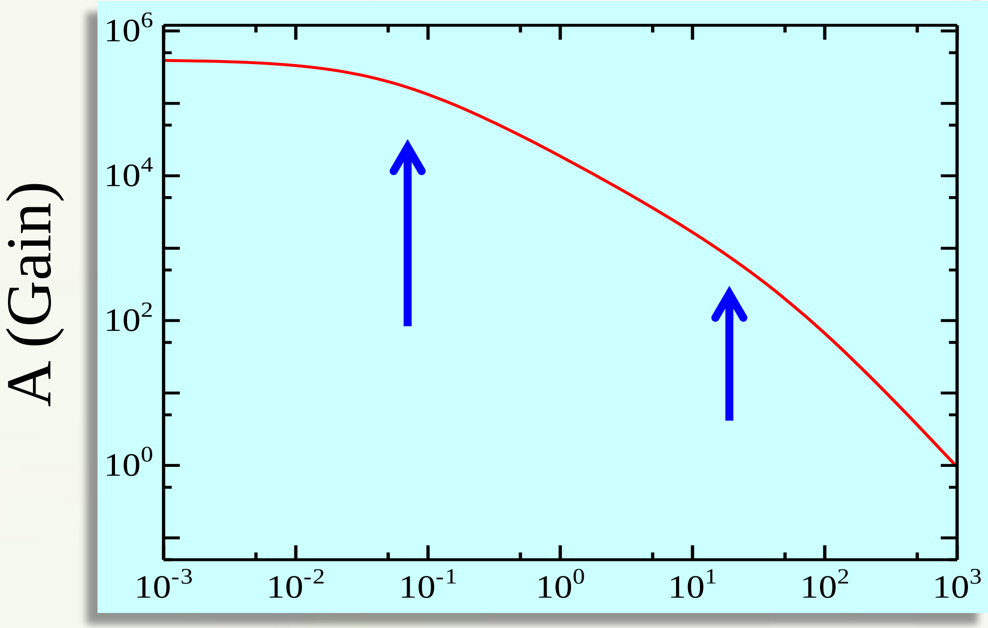


Voltage follower

Frequency Dependent Characteristics of OP-Amps



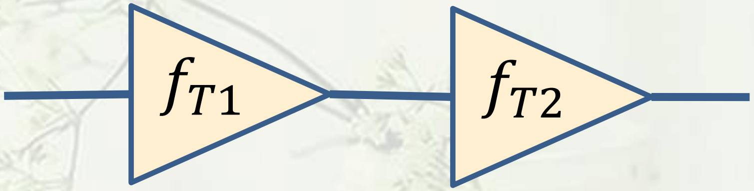
$\omega = 2\pi f$



Cut-off frequency

$\omega_T = 2\pi f_T$

Phase rotates by $\pi/2$



Multiple cut-off frequency:
Phase rotates more than π

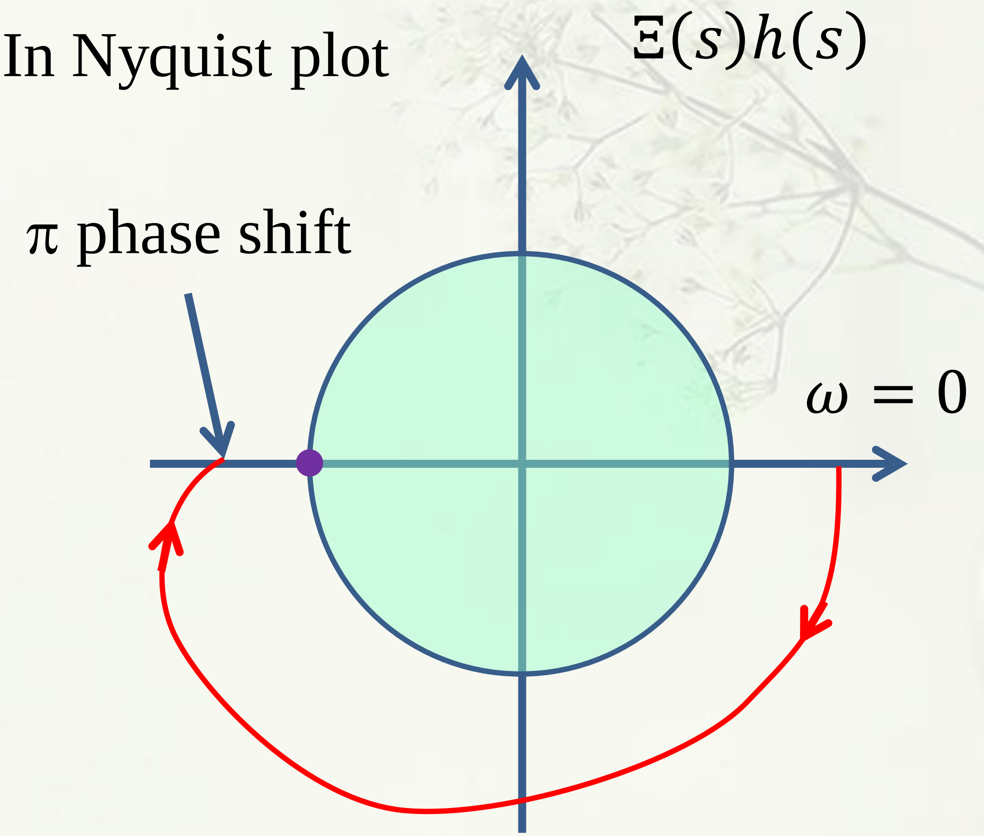
If gain is larger than 1 at
phase shift π :
Dangerous!

Phase compensation

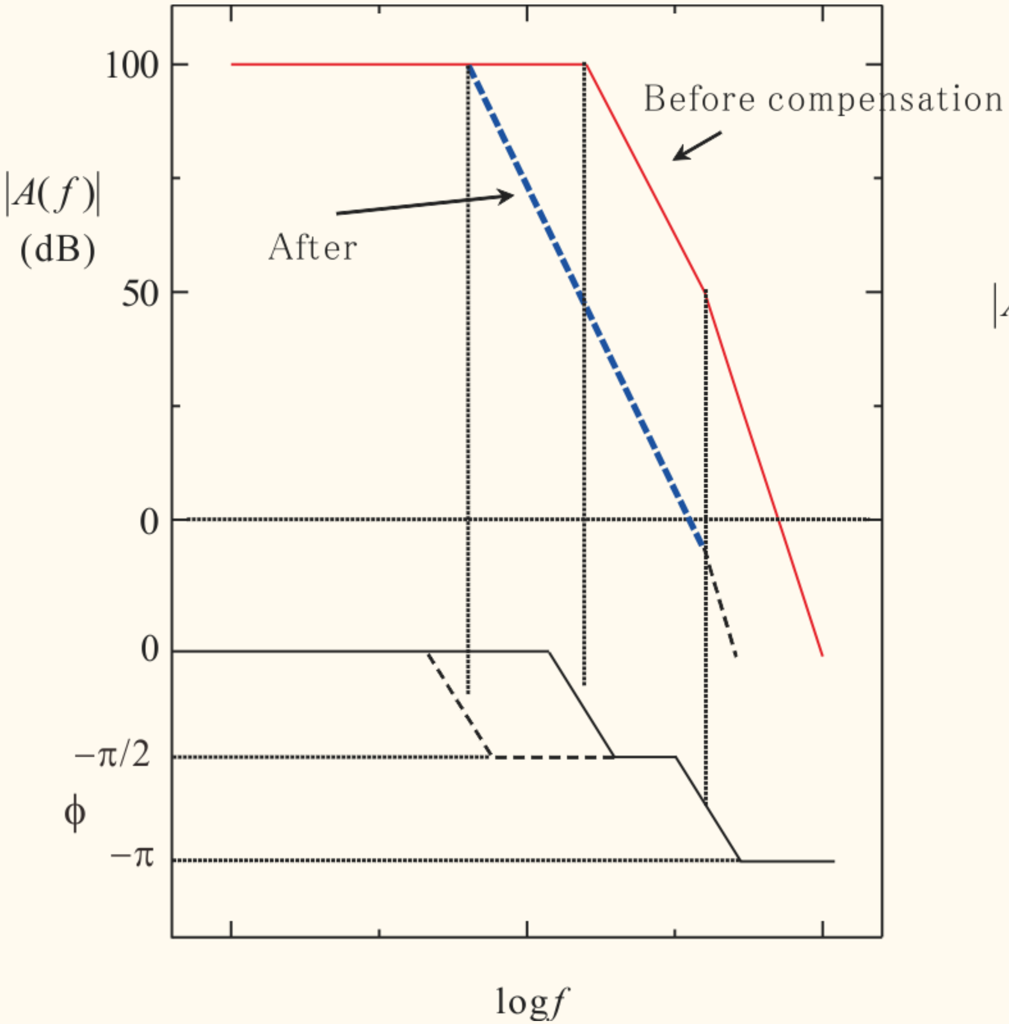
Why dangerous?

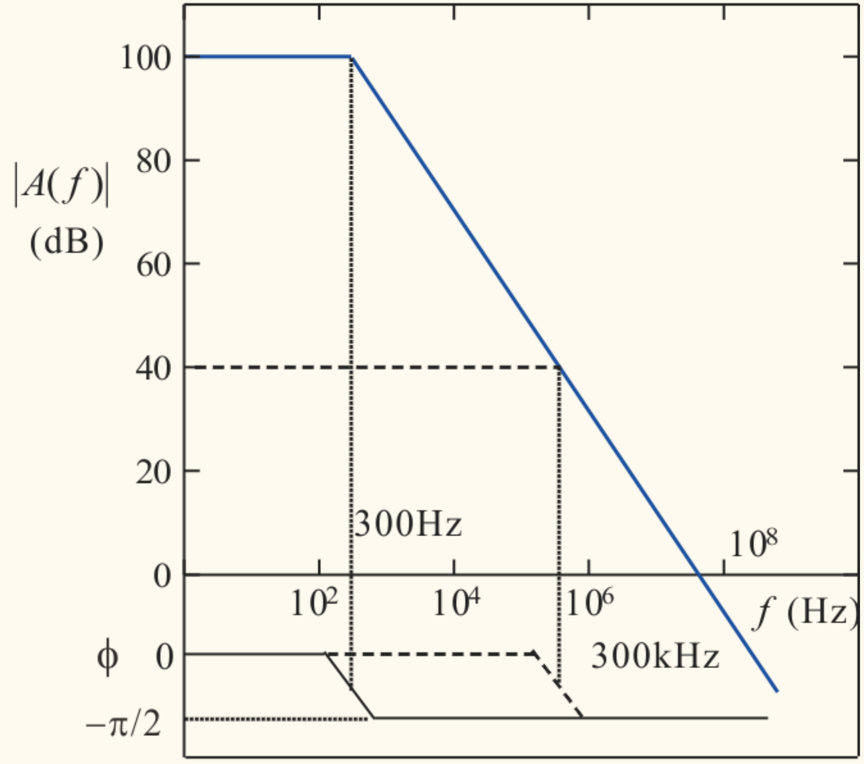
π phase shift: negative feedback → positive feedback

In Nyquist plot

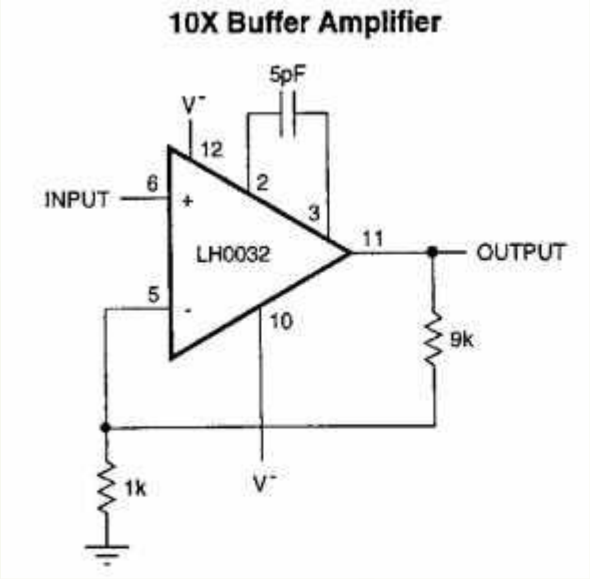


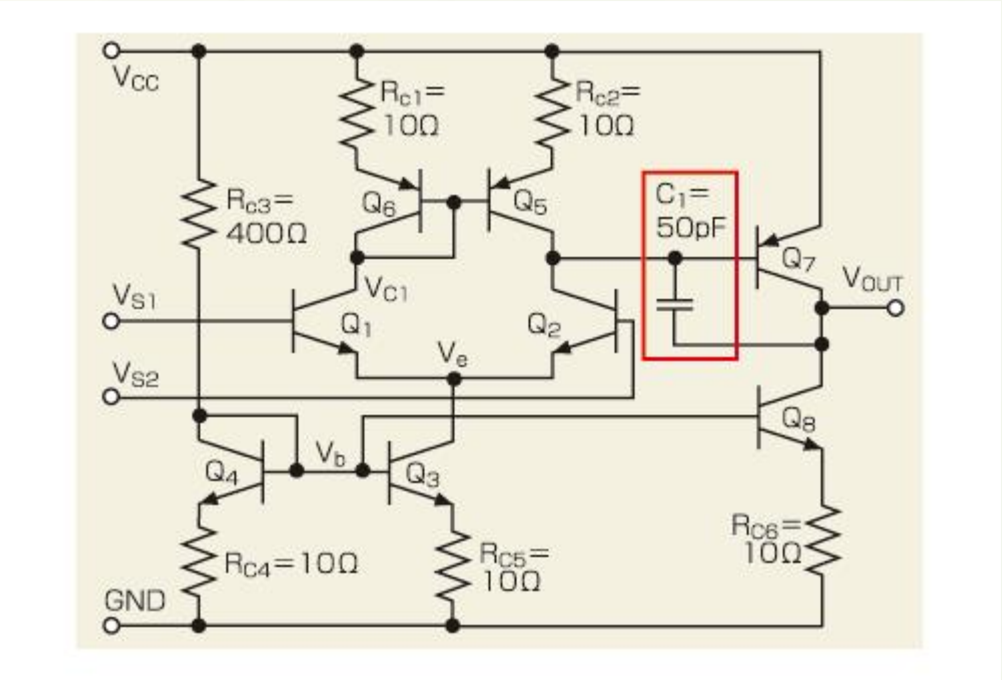
Phase compensation



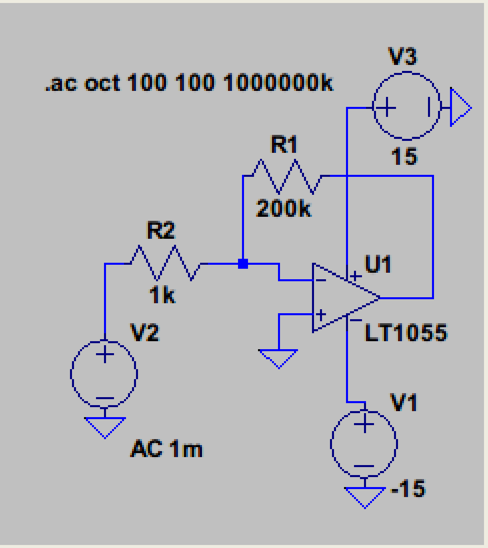


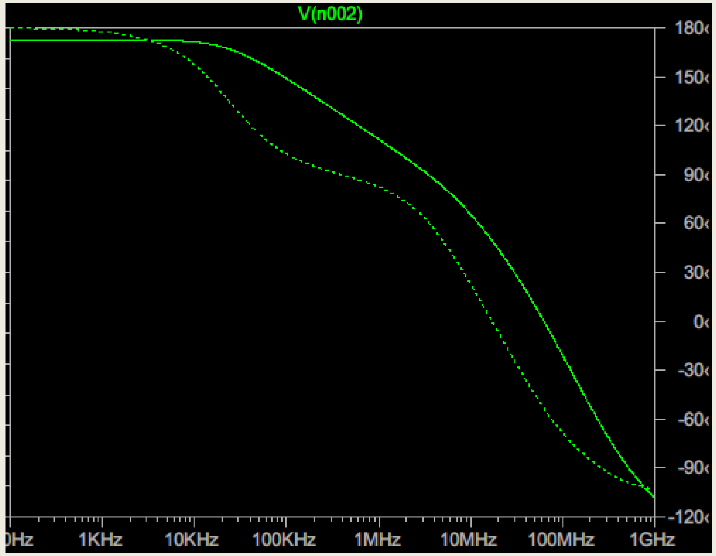
Phase compensation



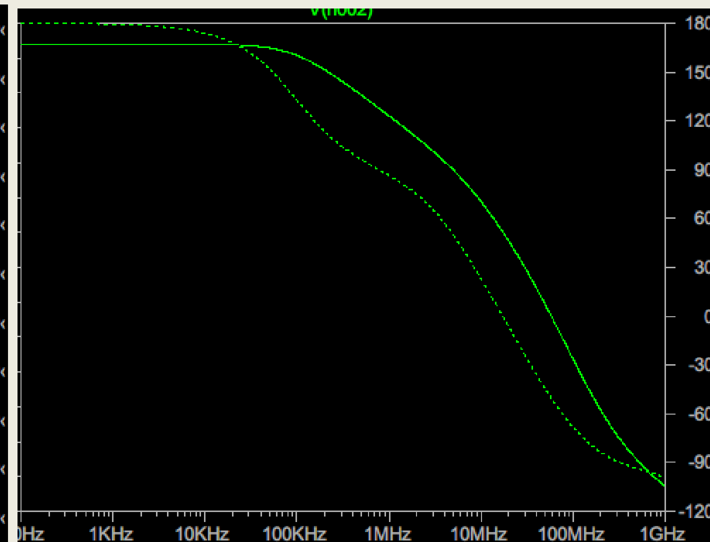


Inverting amplifier and cut-off frequency

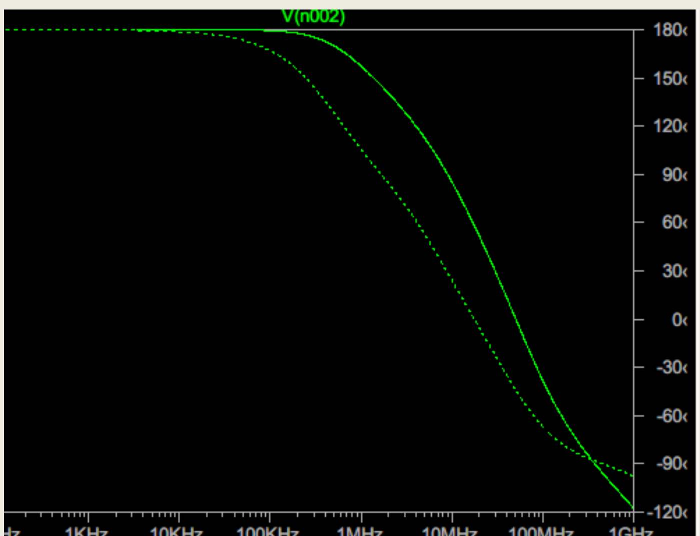




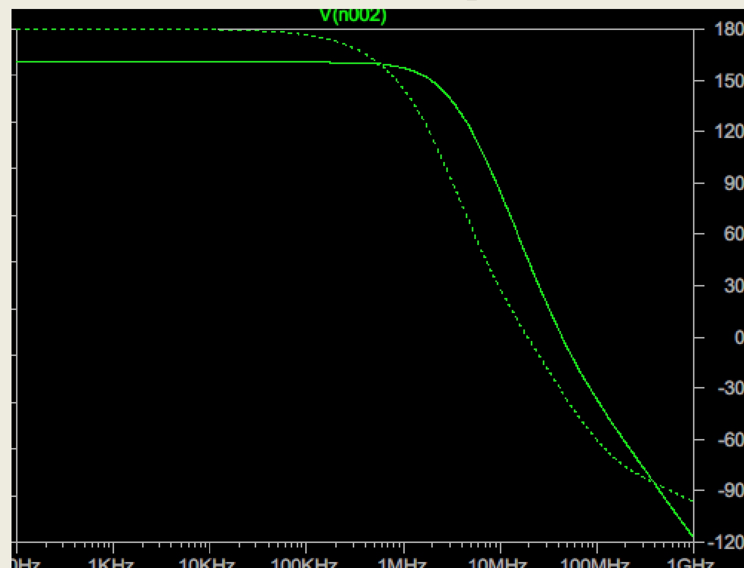
$A = 200\ f_T = 30\text{kHz}$



$A = 50\ f_T = 90\text{kHz}$



$A = 10\ f_T = 300\text{kHz}$



$A = 2\ f_T = 2\text{MHz}$

Oscillation of OPamp