電子回路論第7回

Electric Circuits for Physicists

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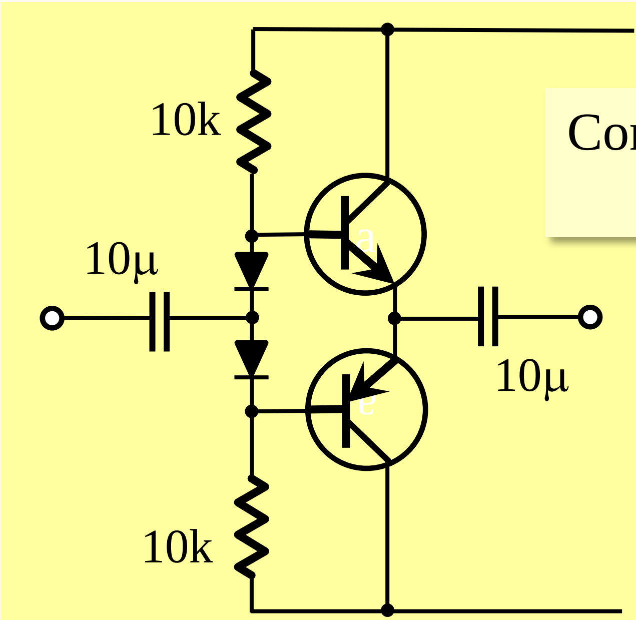
Outline

4.5 Field Effect Transistors (FETs)

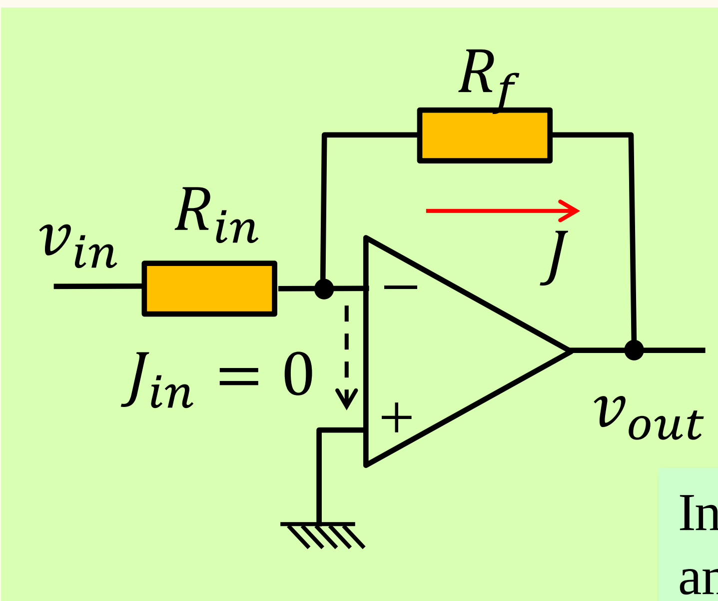
Ch.5 Distributed constant circuits

- 5.1 Transmission lines
  - 5.1.1 Coaxial cables
  - 5.1.2 Lecher lines
  - 5.1.3 Micro-strip lines
- 5.2 Wave propagation through transmission lines
  - 5.2.2 Connection and termination of transmission lines

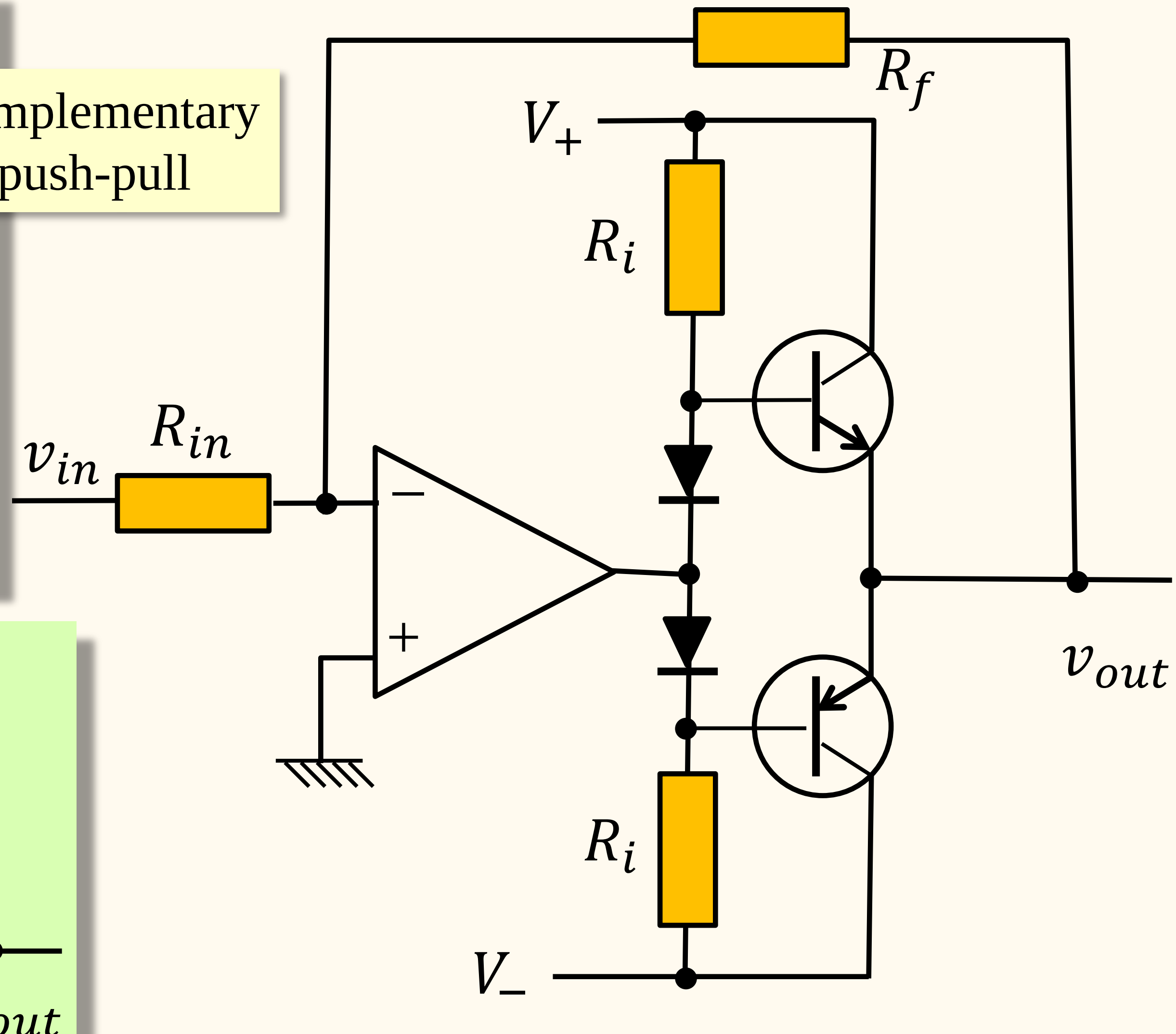
Combination of an OP-amp and discrete transistors



Complementary push-pull

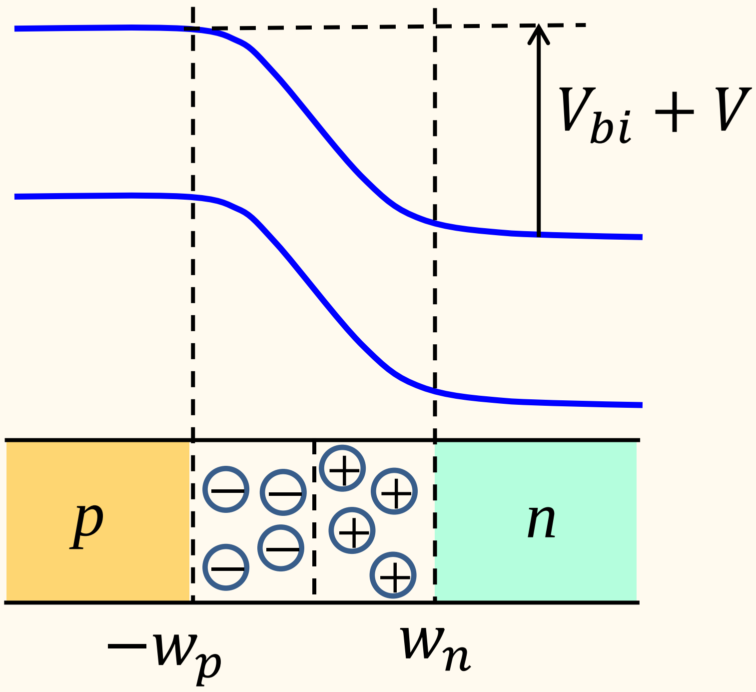


Inversion amplifier



Voltage, current booster

Depletion layer width with reverse bias voltage



Poisson equation

$$\frac{d^2\phi}{dx^2} = -aq(x) \quad (a \equiv (\epsilon\epsilon_0)^{-1})$$

$$\begin{cases} q = -eN_A & (-w_p \le x \le 0), \\ q = eN_D & (0 \le x \le w_n) \end{cases}$$

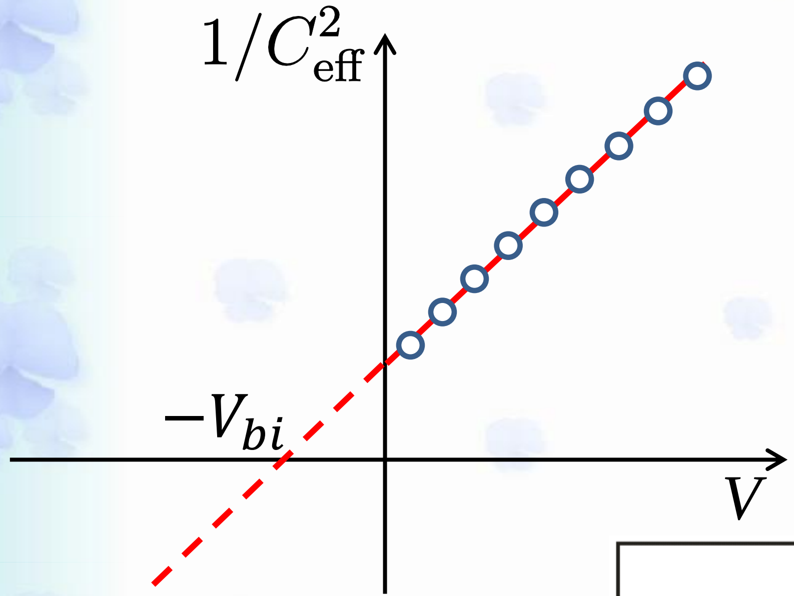
$$\phi(-\infty) = 0$$

$$\phi(-w_p) = 0, \qquad \left.\frac{d\phi}{dx}\right|_{-w_p} = 0,$$

$$\phi(w_n) = V + V_{\mathrm{bi}}, \qquad \left.\frac{d\phi}{dx}\right|_{w_n} = 0$$

$$\phi(x) = \begin{cases} (aeN_A/2)(x + w_p)^2 & (-w_p \le x \le 0), \\ V + V_{\mathrm{bi}} - (aeN_D/2)(x - w_n)^2 & (0 \le x \le w_n) \end{cases}$$

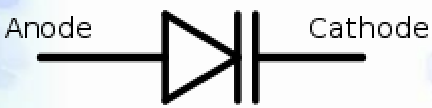
Effective capacitance and reverse bias voltage



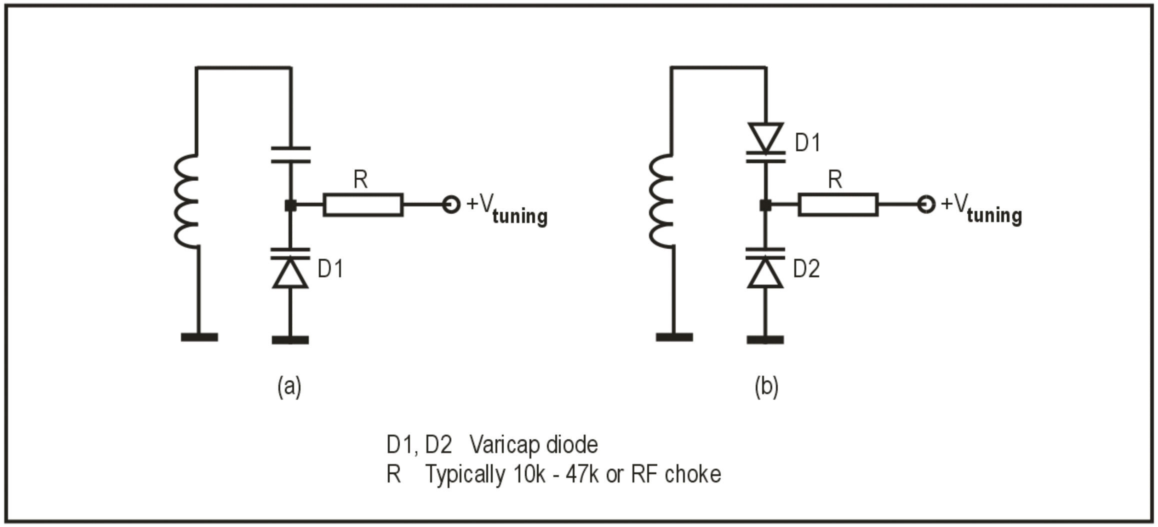
$$\frac{1}{C_{\rm eff}^2} = \frac{2}{\epsilon\epsilon_0 e N_{\rm D}}(V + V_{\rm bi})$$

Doping profiler

Varicap diode



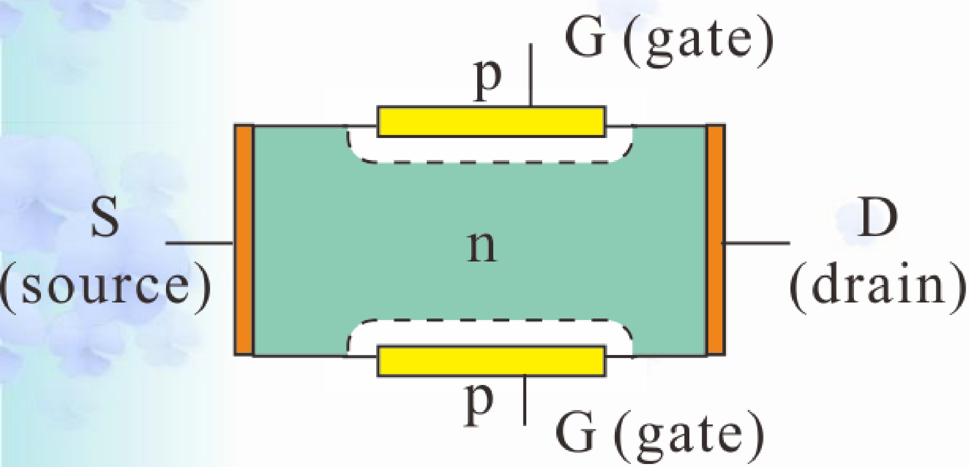
KB505

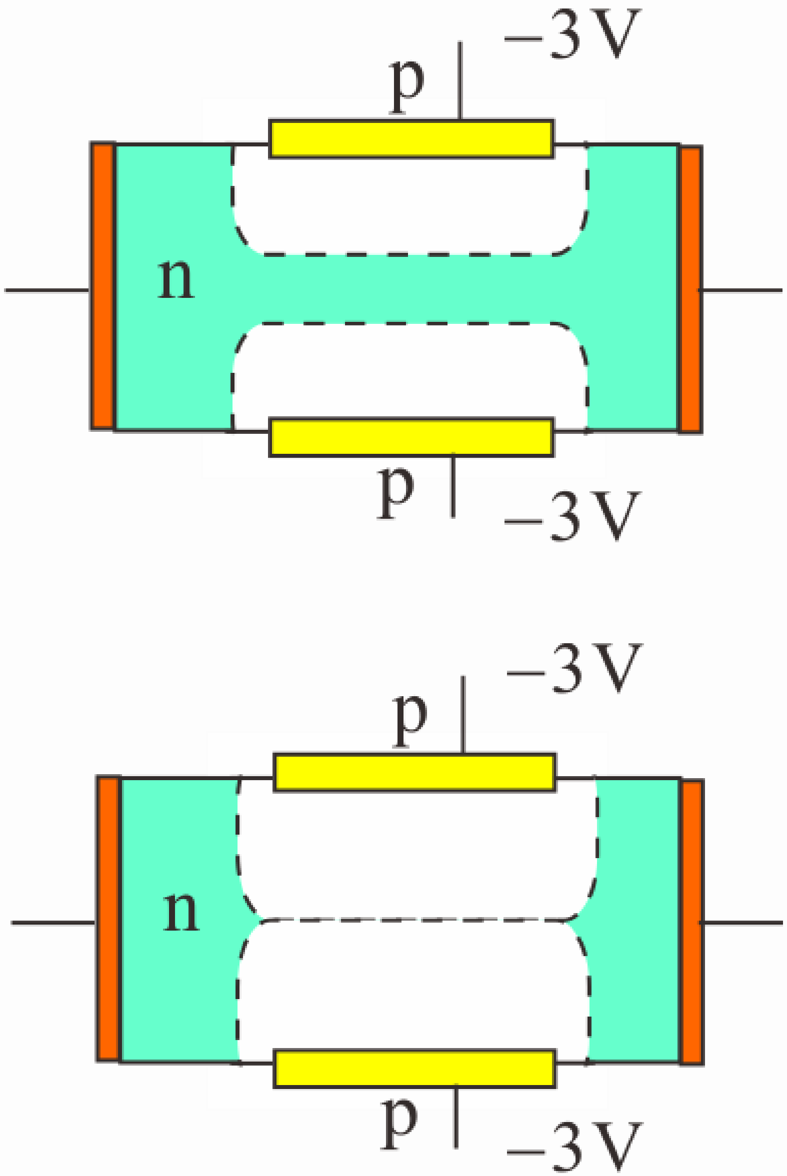


Frequency modulation
Phase lock loop

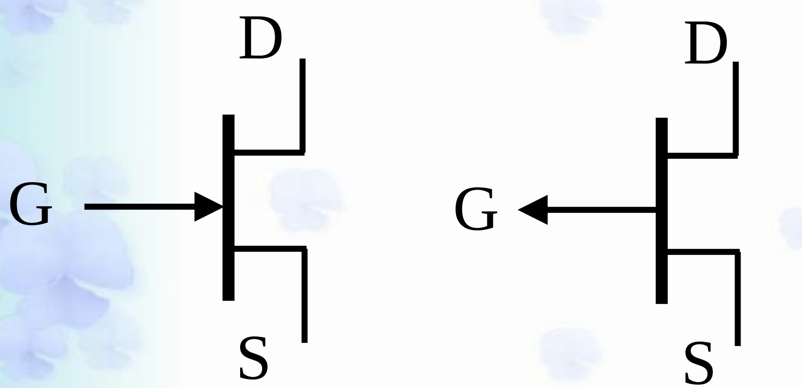
4.4 Field effect transistor (FET)

Junction FET (JFET)





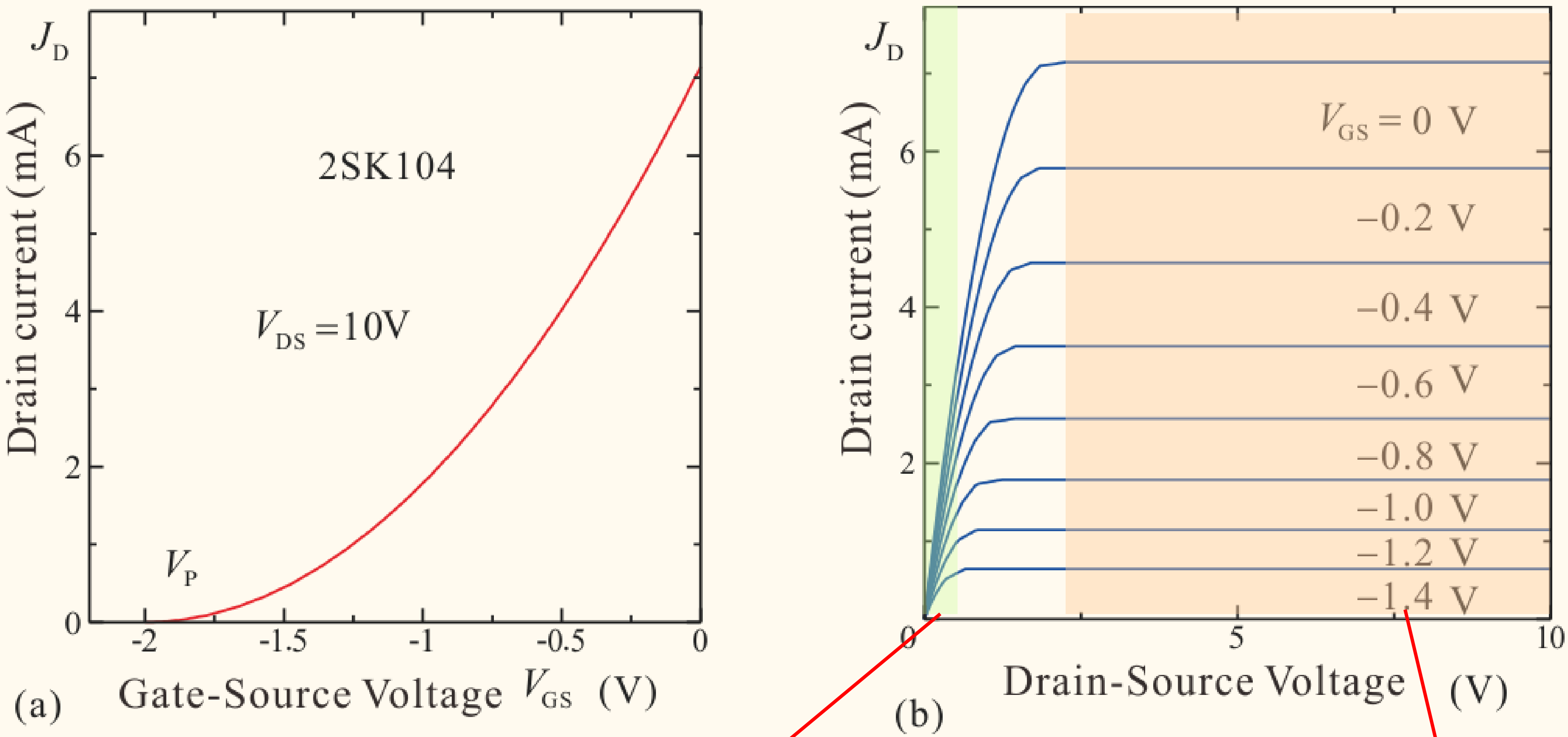
Circuit symbols



n-channel *p*-channel

Pinch-off

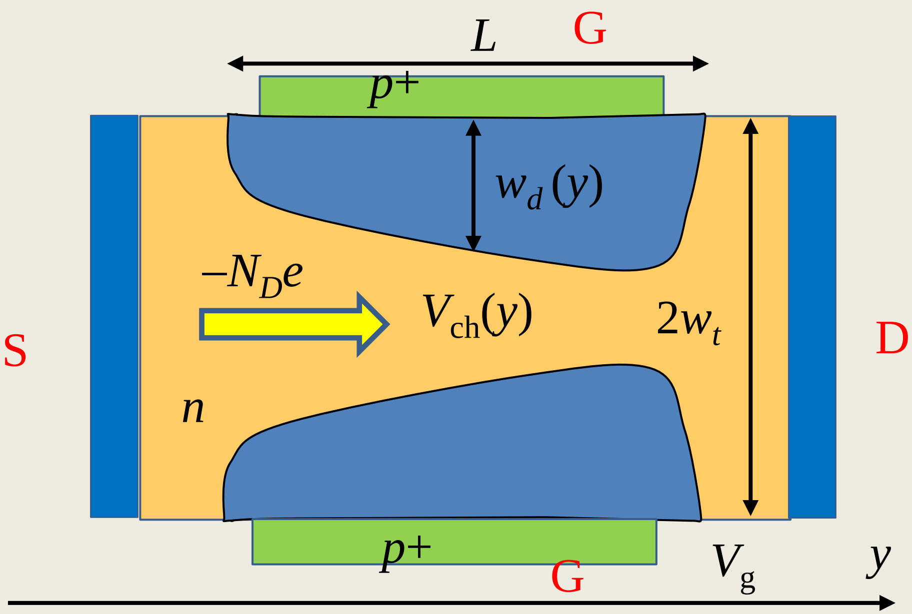
Static characteristics of FET



Ohmic area

Space charge limited area

Space-charge limitation of source-drain current



$$V(y) = V_g + V_{vi} - V_{ch}(y)$$

$$w_d(y) = \sqrt{\frac{2\epsilon\epsilon_0 V(y)}{eN_D}}$$

$$J_{ch} = eN_D\mu_n \frac{dV_{ch}}{dy} \cdot 2[w_t - w_d(y)]W$$

conductivity — electric field — channel width

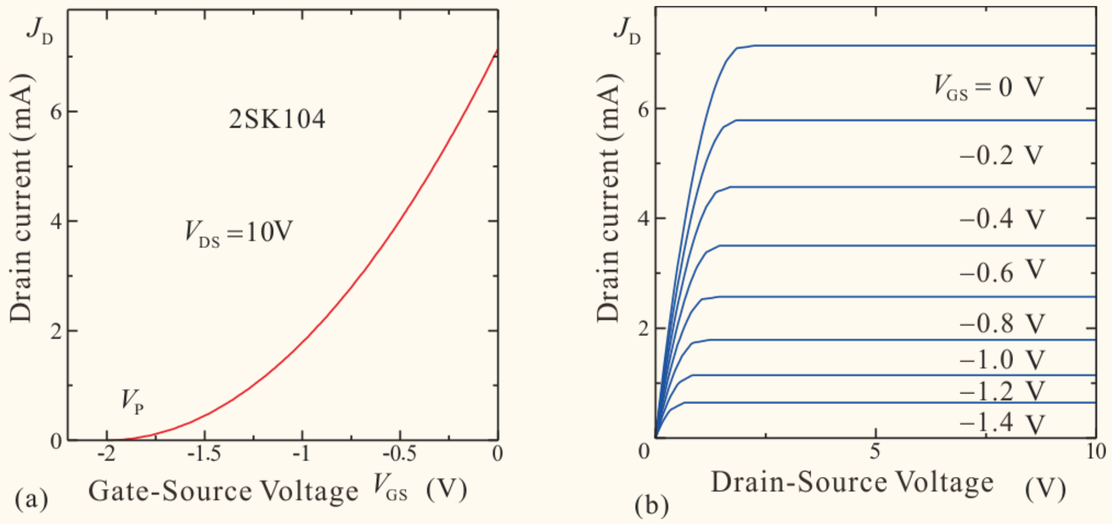
$$J_{ch}L = \int_0^L J_{ch}dy = 2eN_D\mu_n W \int_0^L (w_t - w_d)\frac{dV}{dy}dy = 2w_t eN_D\mu_n W \int_{V_0}^{V_L} \left(1 - \frac{w_d}{w_t}\right) dV$$

pinch off (internal) voltage: $w_d(V_c) = w_t \quad V_c = \frac{eN_D w_t^2}{2\epsilon\epsilon_0}$

$$J_{ch} = \frac{2N_D e\mu_n W w_t}{L}\left[V_L - V_0 + \frac{2}{3\sqrt{V_c}}\left(V(V_0)^{3/2} - V(V_L)^{3/2}\right)\right]$$

Only valid for $w_d < w_t/2$.

Static characteristics of FET



$$J_G \simeq 0,$$
$$J_D = f(V_G, V_D)$$

$$g_m \equiv \left(\frac{\partial J_D}{\partial V_{GS}}\right)_{V_D=\text{const.}}$$ transconductance

Low bias current:
small power consumption

$$r_d \equiv \left(\frac{\partial V_D}{\partial J_D}\right)_{V_{GS}=\text{const.}}$$ Drain resistance

Locally linear approximation

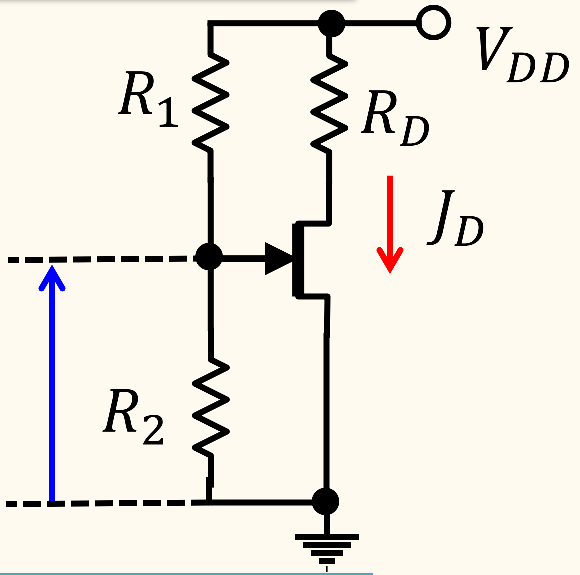
$$j_d = g_m v_{gs} + \frac{v_d}{r_d}$$

$$v_d = -r_d g_m v_{gs} + r_d j_d$$

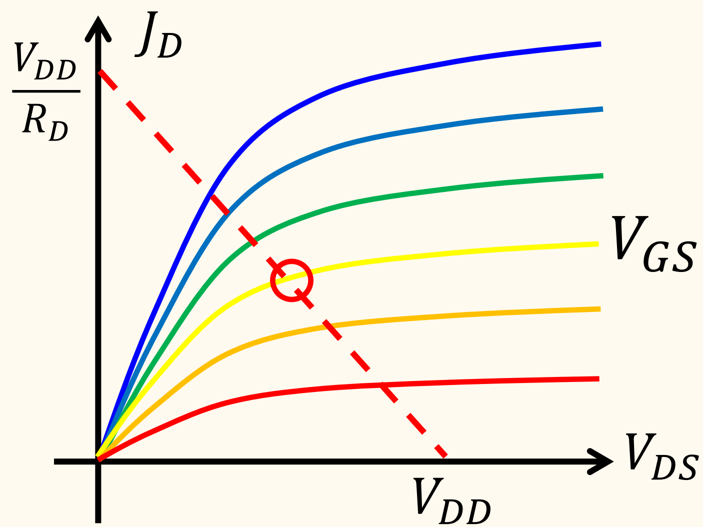
Amplification factor (voltage gain) $\equiv \mu$

Biasing circuits for FETs

Fixed bias circuit

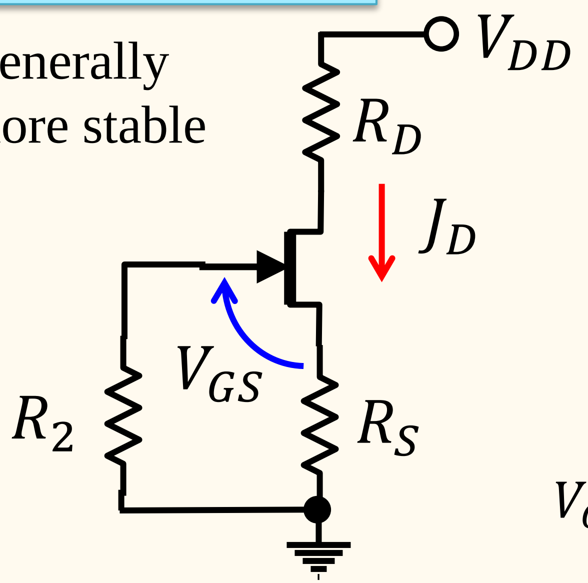


$$V_{\mathrm{GS}} = \frac{R_2}{R_1 + R_2} V_{\mathrm{DD}}, \quad V_{\mathrm{DS}} = V_{\mathrm{DD}} - R_{\mathrm{D}} J_{\mathrm{D}}$$

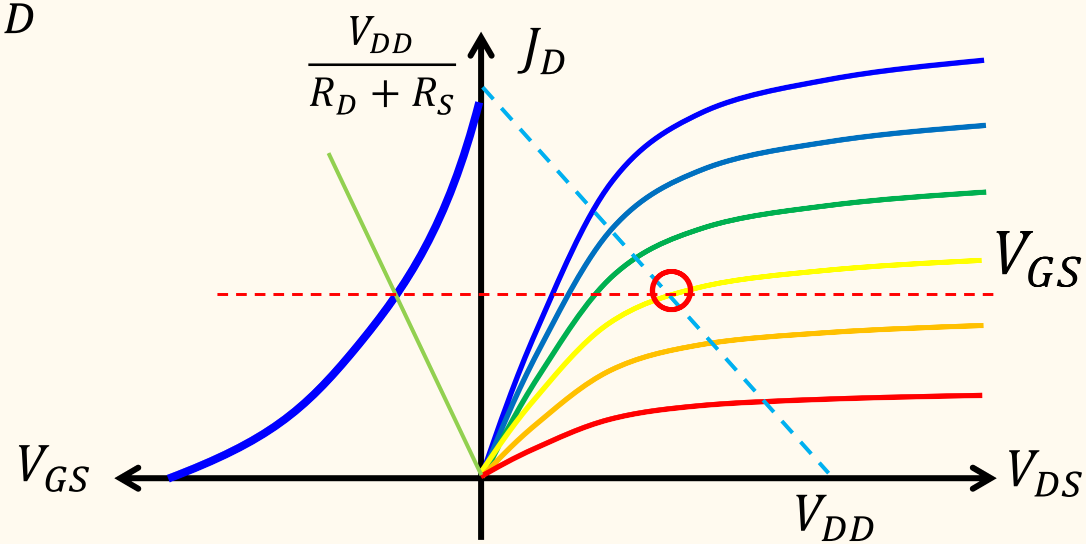


Self-biasing circuit

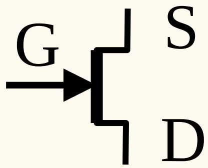
Generally more stable



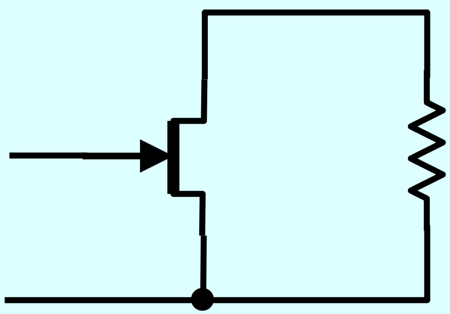
$$V_{\mathrm{GS}} = -R_{\mathrm{S}} J_{\mathrm{D}}, \quad V_{\mathrm{DS}} = V_{\mathrm{DD}} - (R_{\mathrm{D}} + R_{\mathrm{S}}) J_{\mathrm{D}}$$



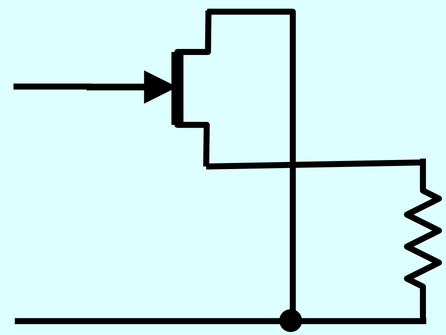
Equivalent signal circuits for FET



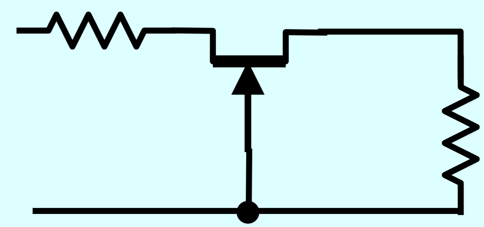
$$\mu = r_d g_m$$



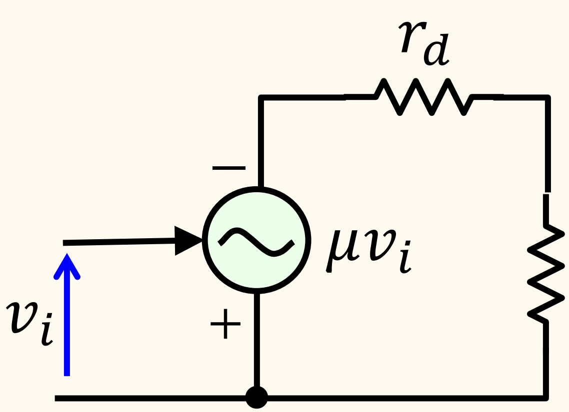
Source grounded

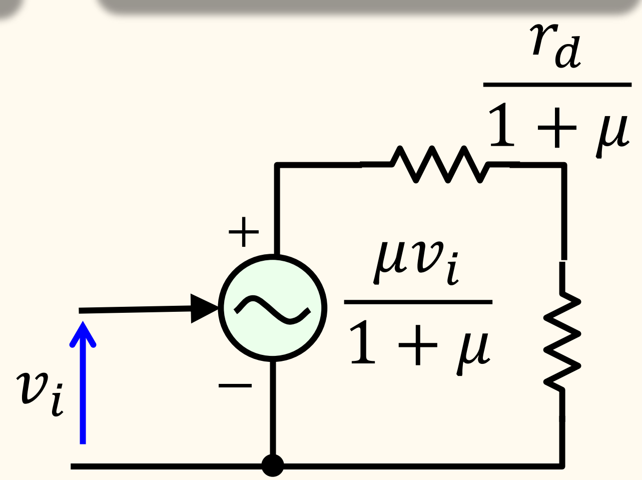


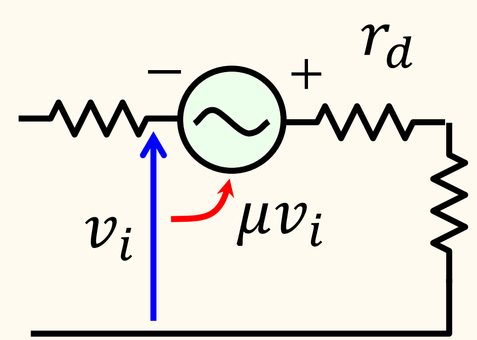
Drain grounded



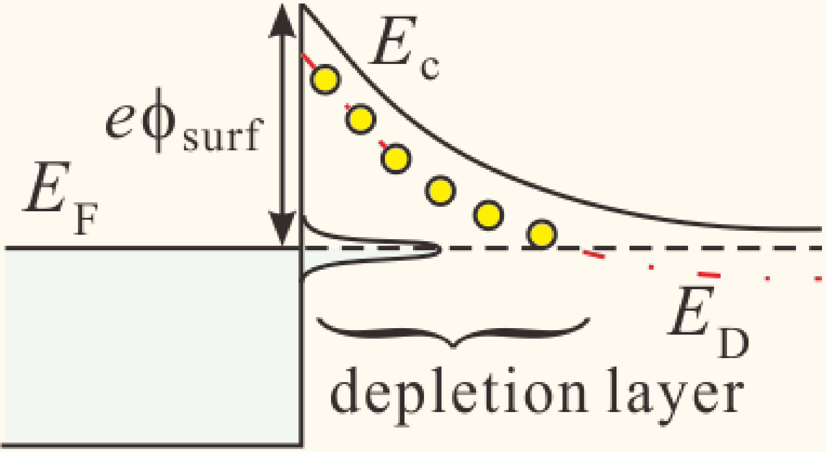
Gate grounded





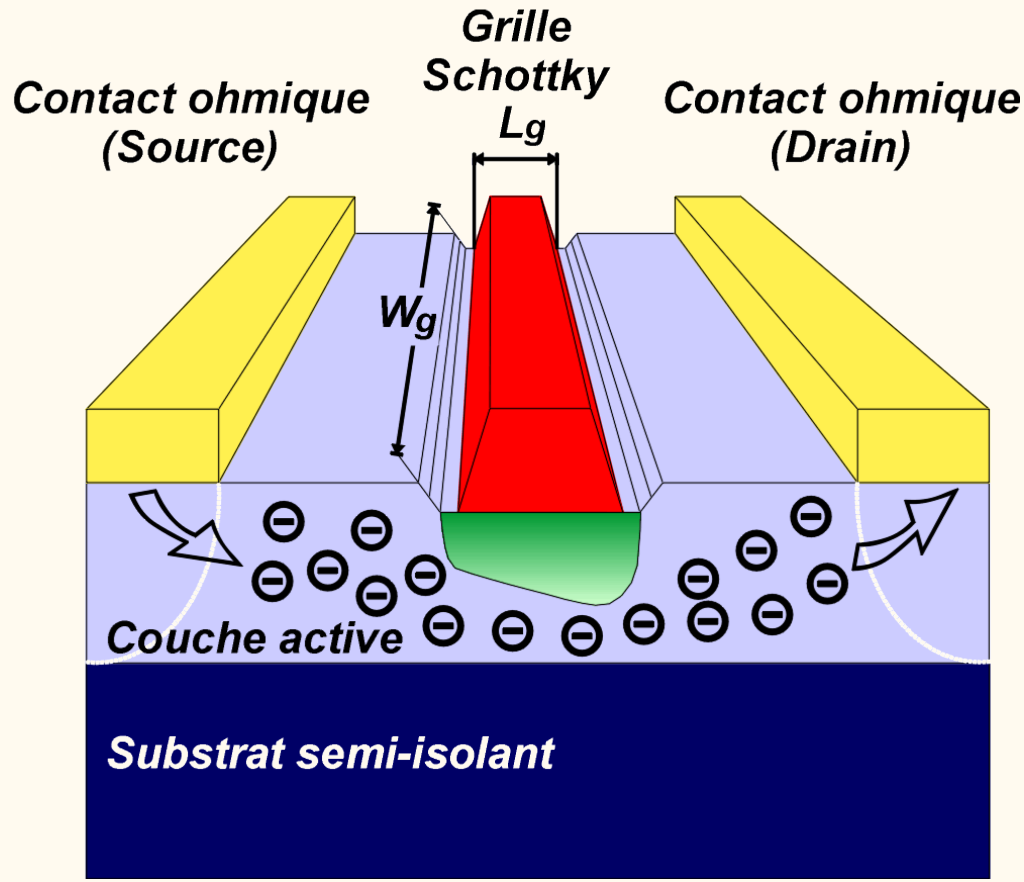


MES-FET



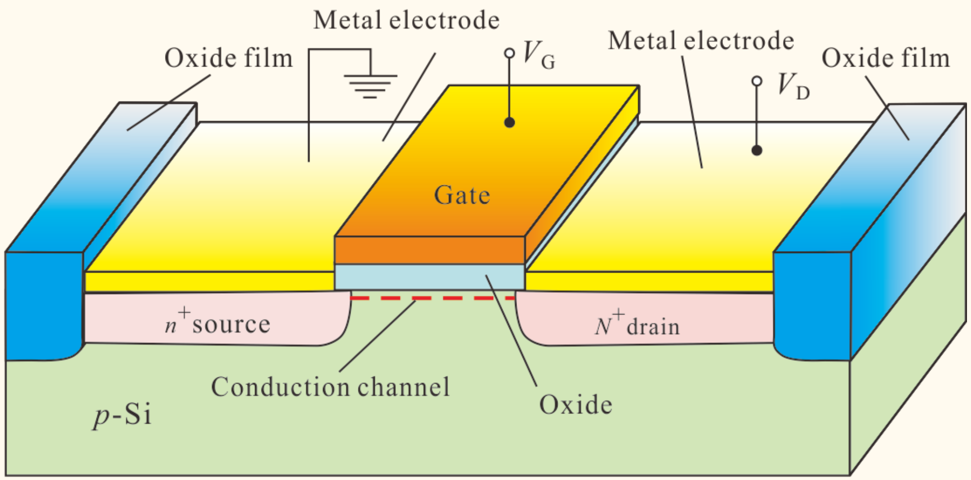
E_c
$e\phi_{surf}$
E_F
E_D
depletion layer

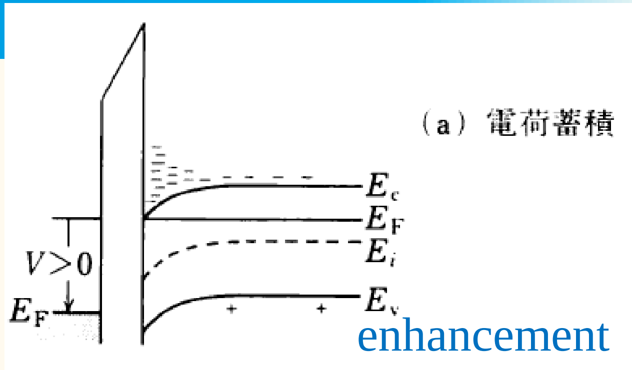
HWS2542
HL8052
HL27
H L36
H 1686

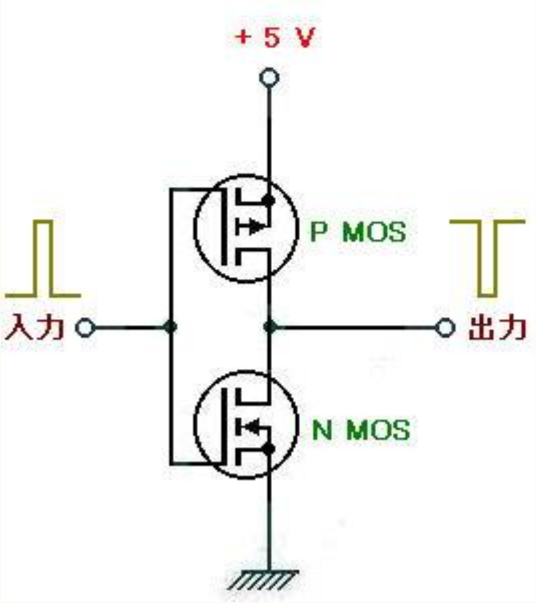


Grille Schottky
L_g
Contact ohmique (Source)
Contact ohmique (Drain)
W_g
Couche active
Substrat semi-isolant

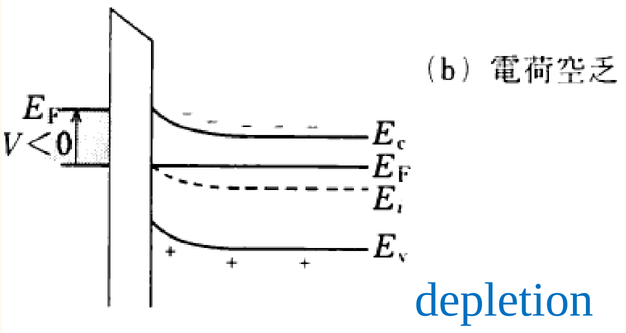
MOS-FET





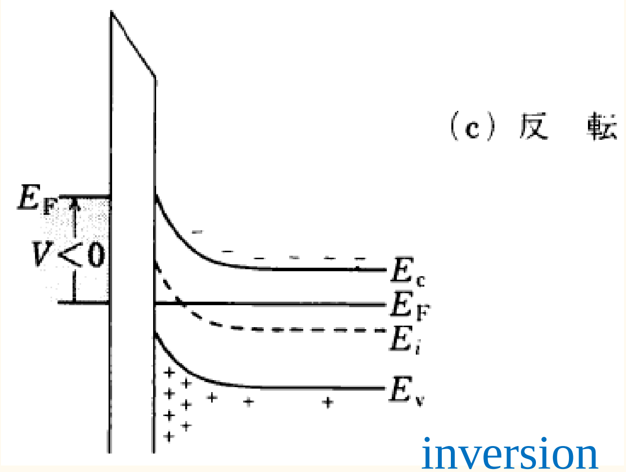


Simplified CMOS inverter circuit



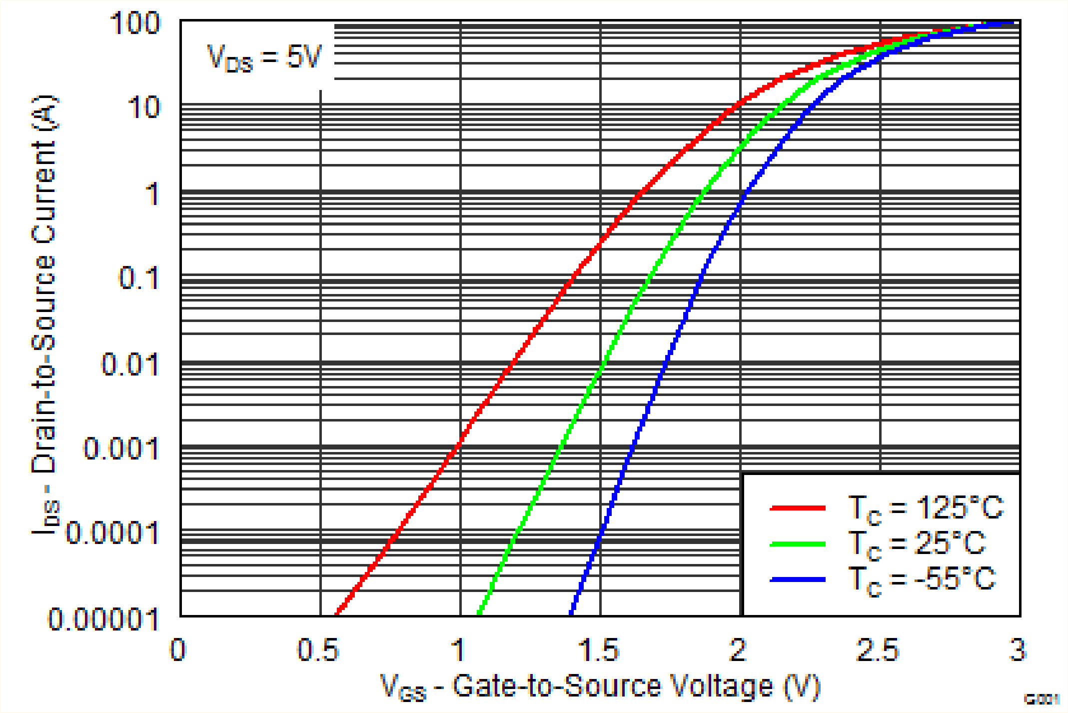
Low leakage current

Single gate input both on/off switch



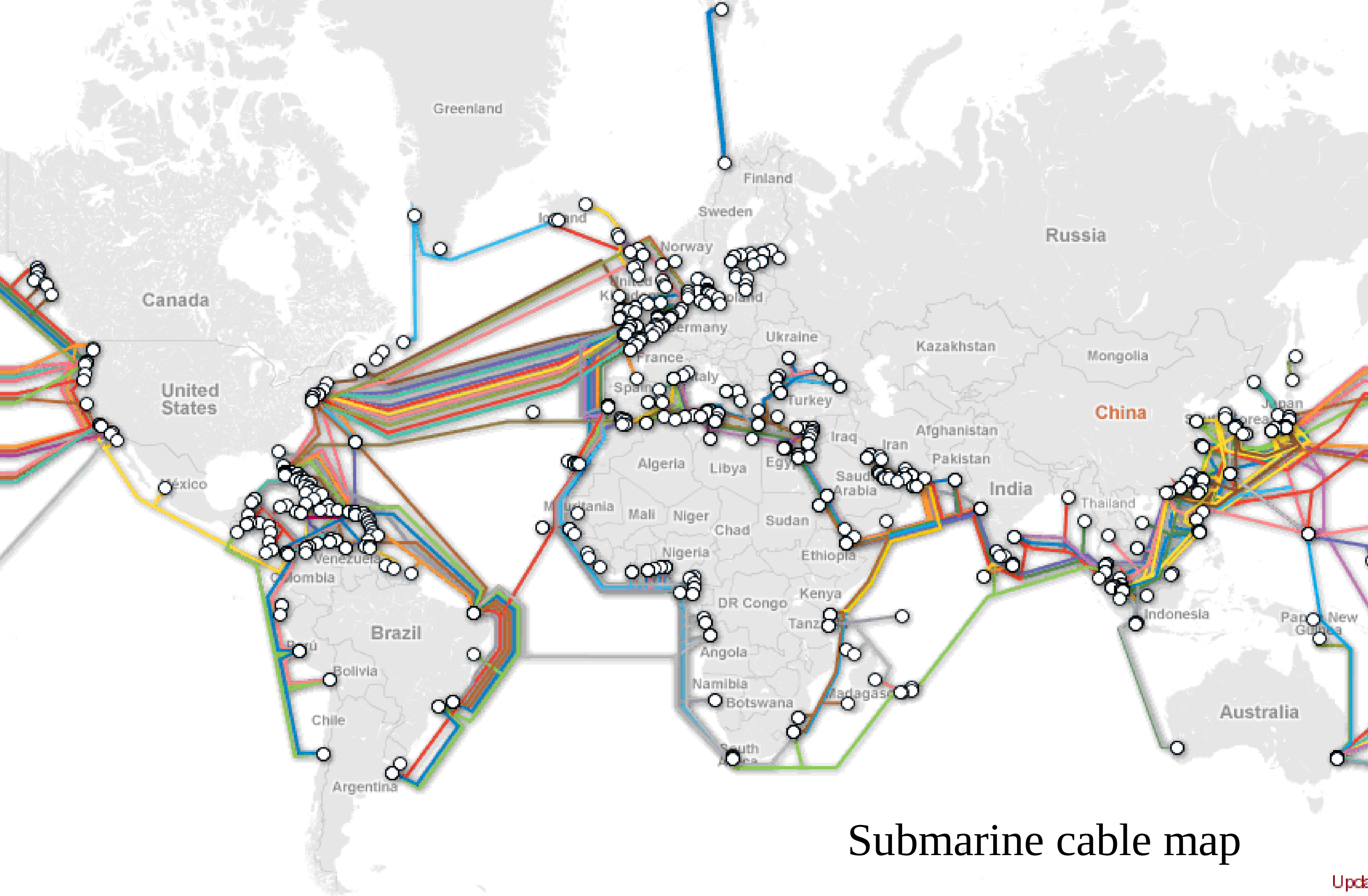
MOSFET switching characteristics

From datasheet CSD87381P power MOSFET (Texas instr.).



More than 7 orders change in J_D within 3 V change of V_{GS}.

Ch.5 Distributed constant circuits



Submarine cable map

Distributed constant circuit concept

1. In what case we need to consider distributed constant circuits?

Characteristic sizes of devices $\gtrsim$ wavelength of electromagnetic signal

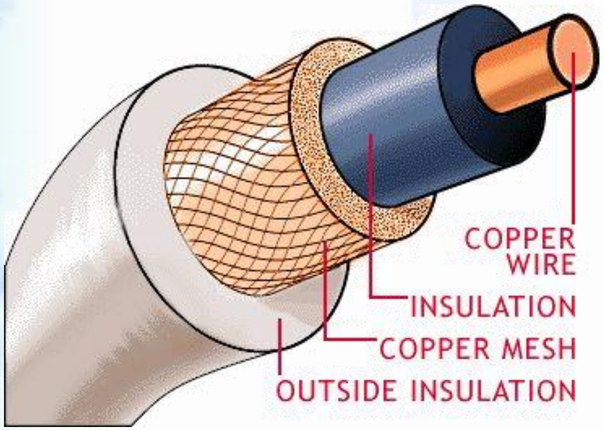
2. A typical scheme to make the shift for distributed circuit

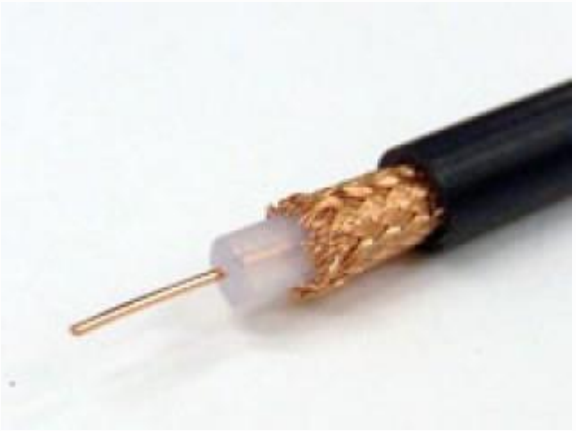
Lumped constant circuit → 1. Connection of unit circuits 2. Taking the infinitesimal limit → Distributed constant circuit

3. Distributed constant circuits : transmission lines

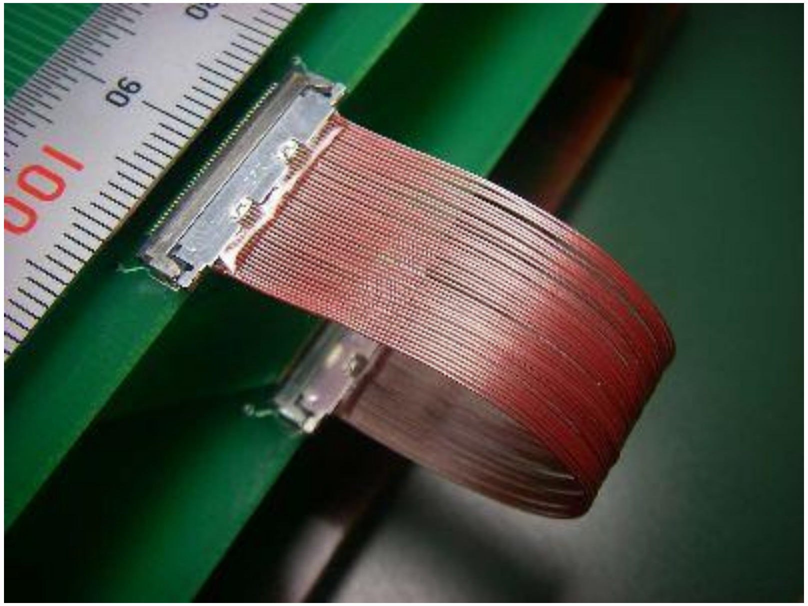
Coaxial cables, Lecher lines, micro-strip lines, waveguides, optical fibers

5.1.1 Coaxial cable



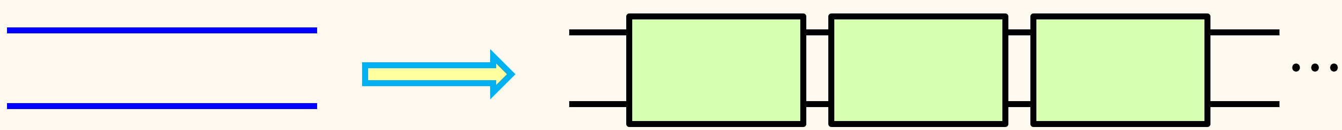


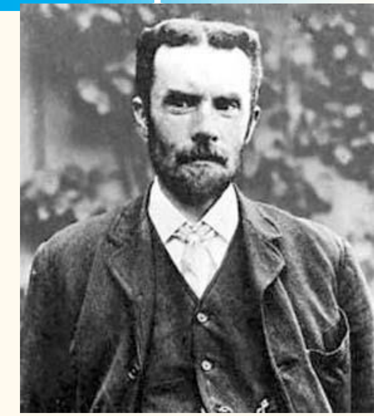
Thin coaxial cable AWG50 (ϕ25μm)



Transmission line as a series of infinitesimal terminal-pairs

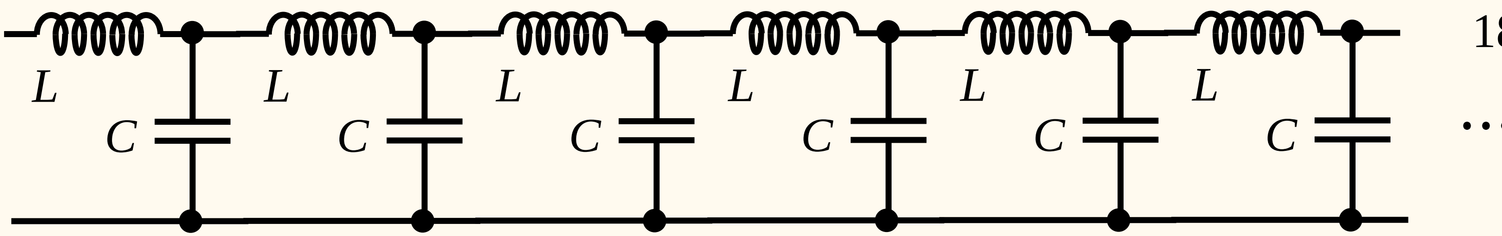
Transmission line → divide into four terminal circuits



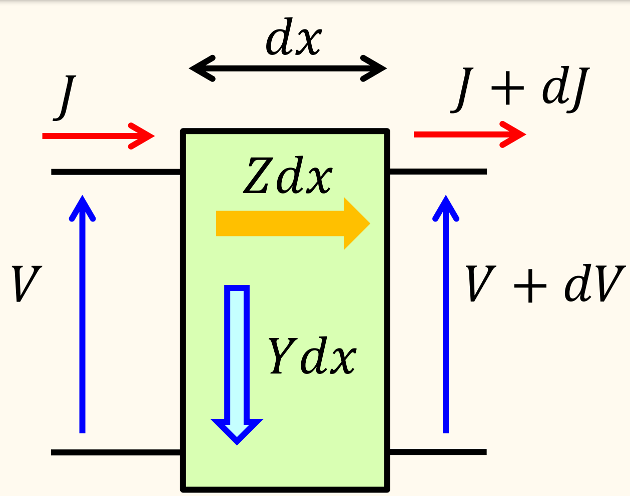


Oliver Heaviside
1850- 1925

Each unit should have delay. Ignore energy dissipation.



Then take the infinitesimal limit



Width → 0, Number → ∞

$$dV = -JZdx, \quad dJ = -VYdx$$

$$\begin{cases} \dfrac{d^2 J}{dx^2} = YZJ, \\ \dfrac{d^2 V}{dx^2} = YZV \end{cases}$$

Telegraphic equation

Characteristic impedance

$$\kappa \equiv \sqrt{YZ} \qquad \text{(dimension: } L^{-1}\text{)}$$

$$J(x,t) = J(0,t)\exp(\pm\kappa x), \quad V(x,t) = V(0,t)\exp(\pm\kappa x)$$

$-$: Progressive, $+$: Retrograde

$$\frac{V}{J} = \mp\frac{Z}{\kappa} = \mp\sqrt{\frac{Z}{Y}}$$

Characteristic impedance

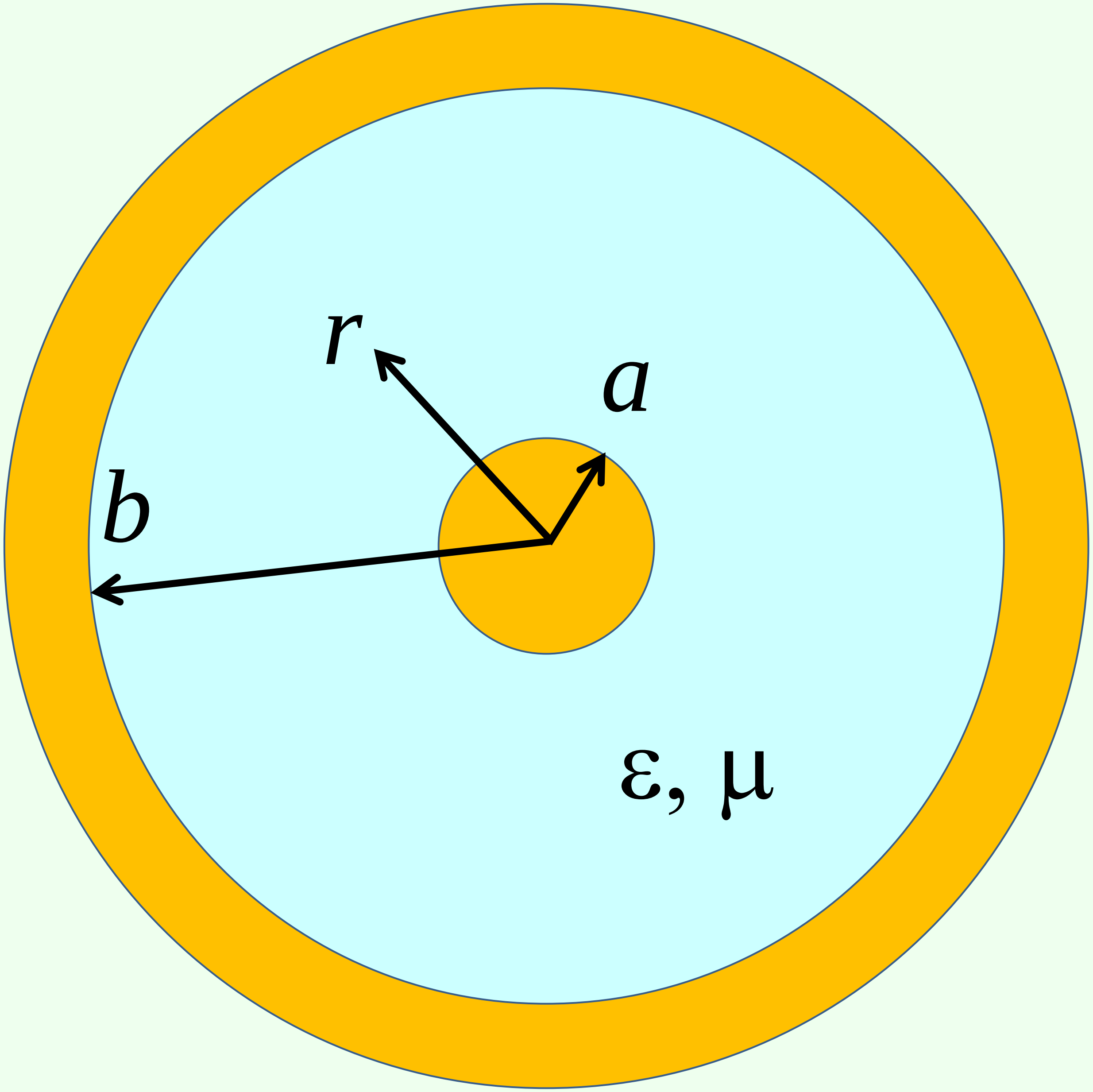
Pure reactance $Y = i\omega C,\ Z = i\omega L$ For L and C model

$$\kappa = \sqrt{-\omega^2 LC} = i\frac{\omega}{\omega_0}, \quad \omega_0 \equiv \frac{1}{\sqrt{LC}}$$

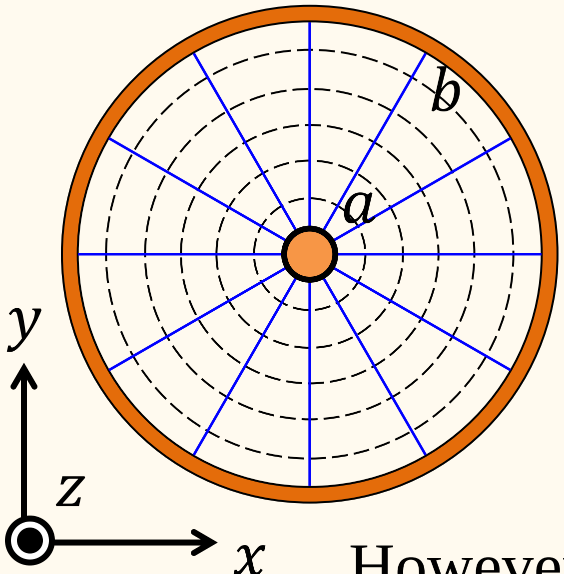
(dimension: velocity)

$$Z_0 = \sqrt{\frac{L}{C}}$$

Coaxial cable setup



Maxwell theory



$$E = E_0(x,y)e^{i\omega t-\gamma z}, \quad H = H_0(x,y)e^{i\omega t-\gamma z}$$

From Maxwell equations

$$(\omega^2\epsilon\mu+\gamma^2)\begin{pmatrix}E_x\\E_y\end{pmatrix} = \begin{pmatrix}-\gamma\partial_x & -i\omega\mu\partial_y\\ -\gamma\partial_y & i\omega\mu\partial_x\end{pmatrix}\begin{pmatrix}E_z\\H_z\end{pmatrix},$$

$$(\omega^2\epsilon\mu+\gamma^2)\begin{pmatrix}H_x\\H_y\end{pmatrix} = \begin{pmatrix}i\omega\mu\partial_y & -\gamma\partial_x\\ -i\omega\mu\partial_x & -\gamma\partial_y\end{pmatrix}\begin{pmatrix}E_z\\H_z\end{pmatrix}.$$

However in TEM (transverse electric and magnetic) mode:

$E_z = H_z = 0$ *i.e.*, the RHSs are zero.

For the fields along *x* and *y* to survive, $\omega^2\epsilon\mu+\gamma^2 = 0 \quad \therefore \gamma = \pm i\omega\sqrt{\epsilon\mu}$

Propagation velocity $v = \dfrac{\omega}{\omega\sqrt{\epsilon\mu}} = \dfrac{1}{\sqrt{\epsilon\mu}}$

In such a case, from Maxwell equations: $\mathrm{rot}_{xy}\boldsymbol{H} = 0, \quad \mathrm{rot}_{xy}\boldsymbol{E} = 0$

→ Potentials are conceivable for ***H*** and ***E***.

Maxwell theory

$$\boldsymbol{E} = \nabla_{xy}\mathcal{U}/\sqrt{\epsilon}, \quad \boldsymbol{H} = \nabla_{xy}\mathcal{V}/\sqrt{\mu}$$

$$\frac{\partial \mathcal{U}}{\partial x} = \frac{\partial \mathcal{V}}{\partial y}, \quad \frac{\partial \mathcal{U}}{\partial y} = -\frac{\partial \mathcal{V}}{\partial x}$$ Cauchy-Riemann theorem

Characteristic impedance: $Z_0 = \dfrac{\mathcal{U}_a - \mathcal{U}_b}{J\sqrt{\epsilon}}$

If we can express V and J in the form of distributed constant circuit model (L and C model), the equivalence is certified.

Capacitance part

$$V = \frac{q}{\epsilon}\int_a^b \frac{dr}{2\pi r} = \frac{q}{2\pi\epsilon}\log\frac{b}{a} = \frac{q}{C}$$

$$\therefore C = \frac{2\pi\epsilon}{\log(b/a)}$$

Maxwell theory

Inductance part

Core current J, shield current $-J$

$$H(r) = \frac{J}{2\pi r}, \quad B(r) = \frac{\mu J}{2\pi r}$$

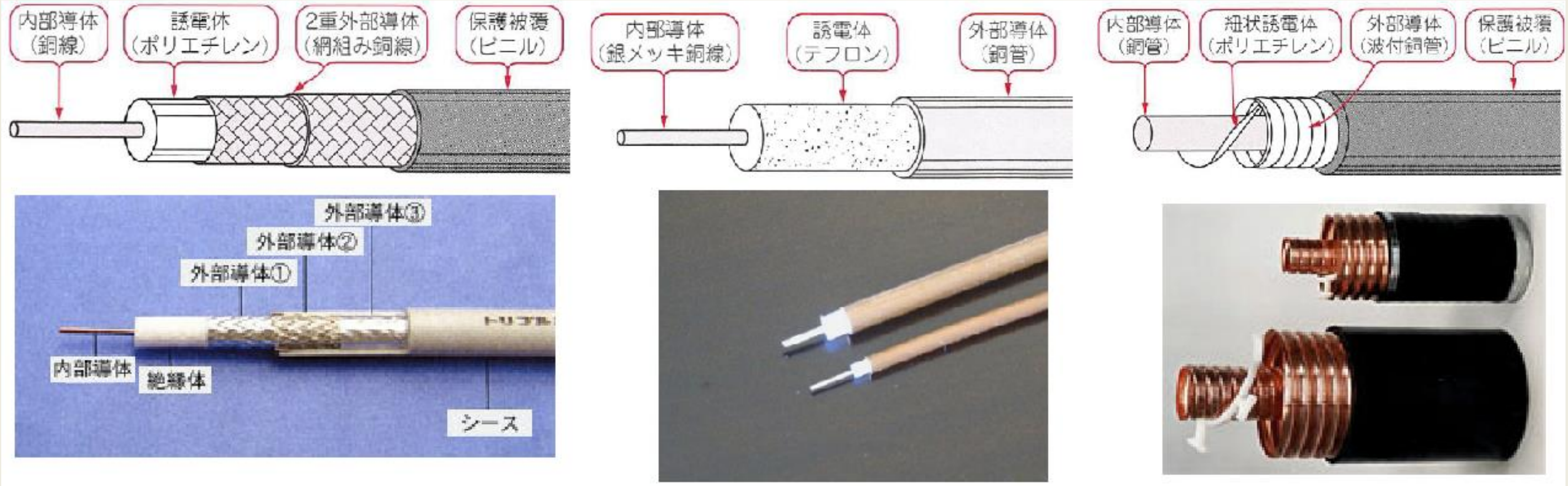
Flux per length: $\Phi = \int_a^b dr B(r) = \frac{\mu J}{2\pi} \log \frac{b}{a}$

Self inductance per length: $L = \frac{\mu}{2\pi} \log(b/a)$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \log\left(\frac{b}{a}\right)$$

cf. Characteristic impedance of vacuum $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 376\Omega$

Coaxial cable 2



(a) (b) (c)

図12 同軸ケーブルの型名(JIS C3501)

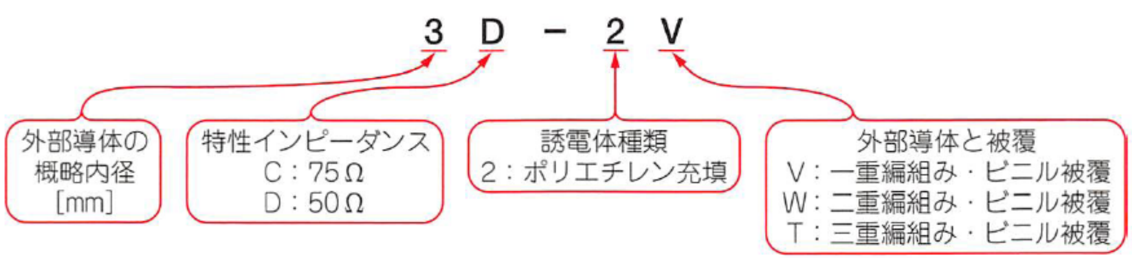
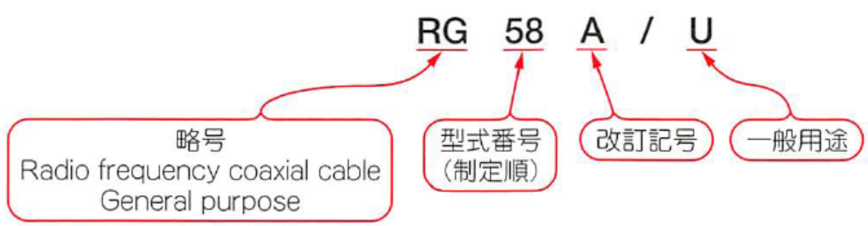
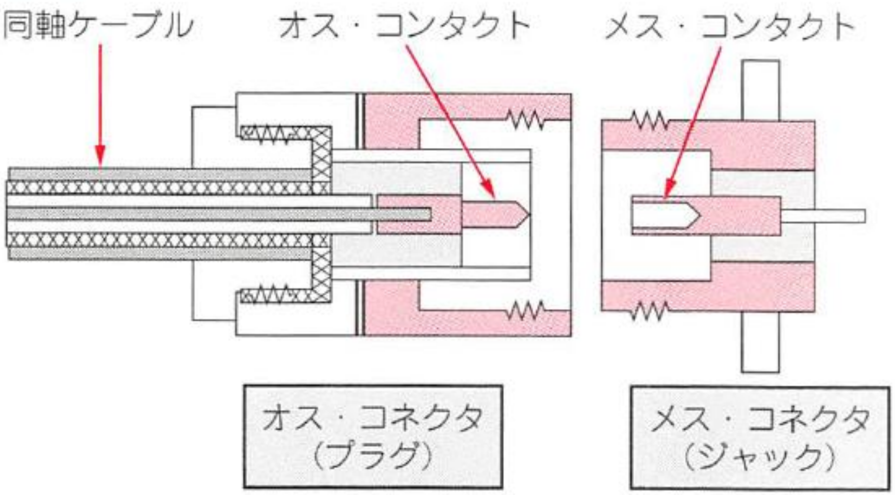


図13 MIL規格での同軸ケーブル型名の例



Coaxial connectors

図22 同軸コネクタの構造(概念図)

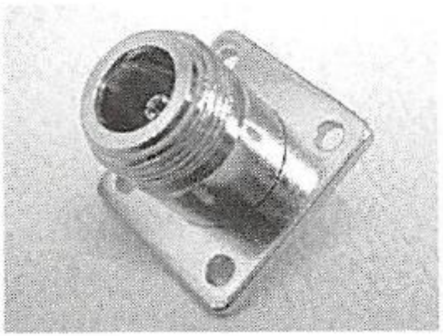


代表的な同軸コネクタの最高使用周波数例

形　式	外部導体内径	最高使用周波数
BNC	約 7 mm	2 ~ 4 GHz
N	約 7 mm	10 ~ 18 GHz
7 mm	7 mm	~ 18 GHz
SMA	4.15 mm	18 GHz
3.5 mm	3.5 mm	26.5 GHz
K	2.92 mm	40 GHz
2.4 mm	2.4 mm	50 GHz
V	1.85 mm	65 GHz
W	1.1 mm	110 GHz
1.0 mm	1.0 mm	110 GHz

Coaxial connectors

写真2 N型コネクタ

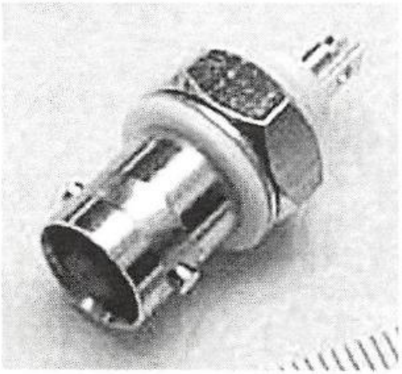


(a) フランジ付きジャック

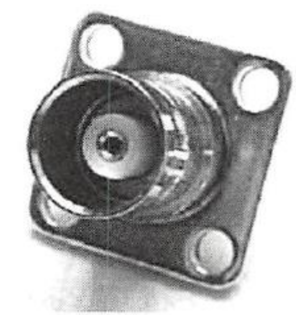
(b) プラグ

(c) プラグ[(b)を分解]

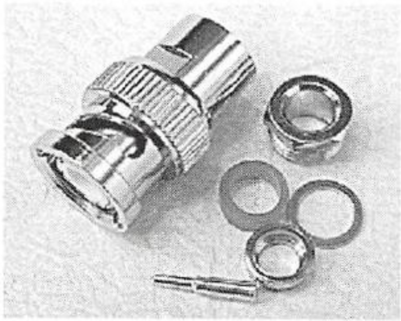
写真3 BNC型コネクタ



(a) 絶縁型ジャック
(高周波に向かない)



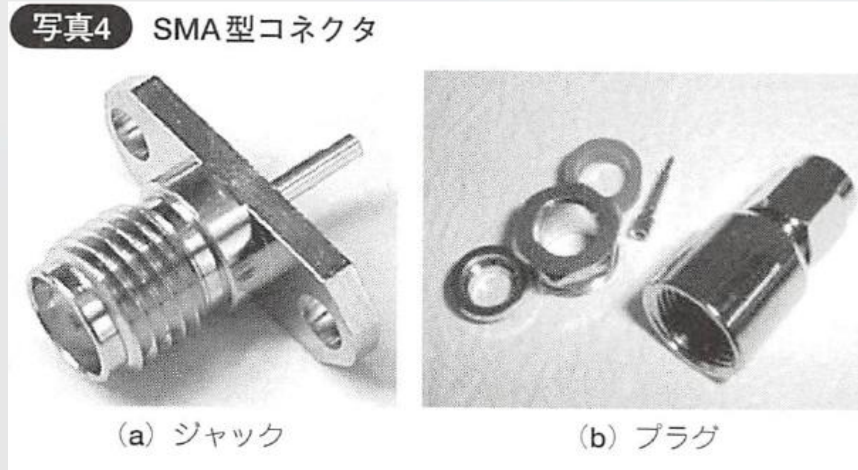
(b) フランジ付きジャック



(c) プラグ

Coaxial connectors 2

SMA-type



jack plug

K-type

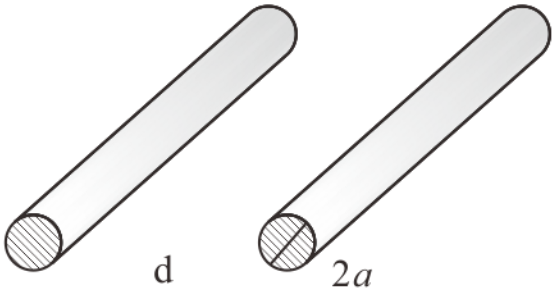
V-type

LEMO cables and connectors

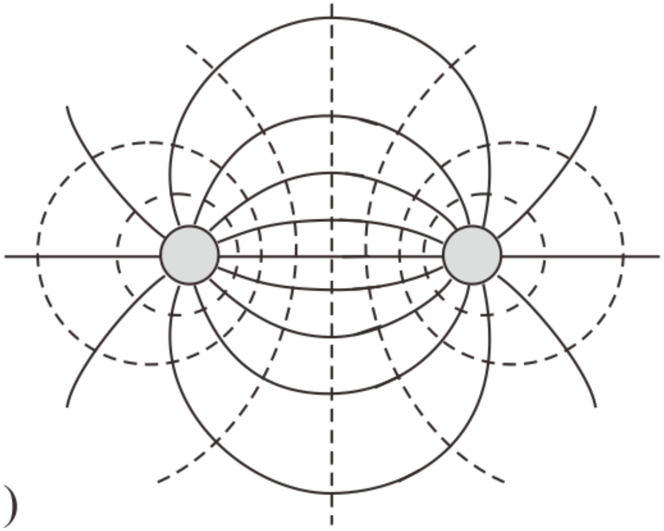
http://www.lemo.com/

High-energy physics experiment, etc.

Lecher line



(a)

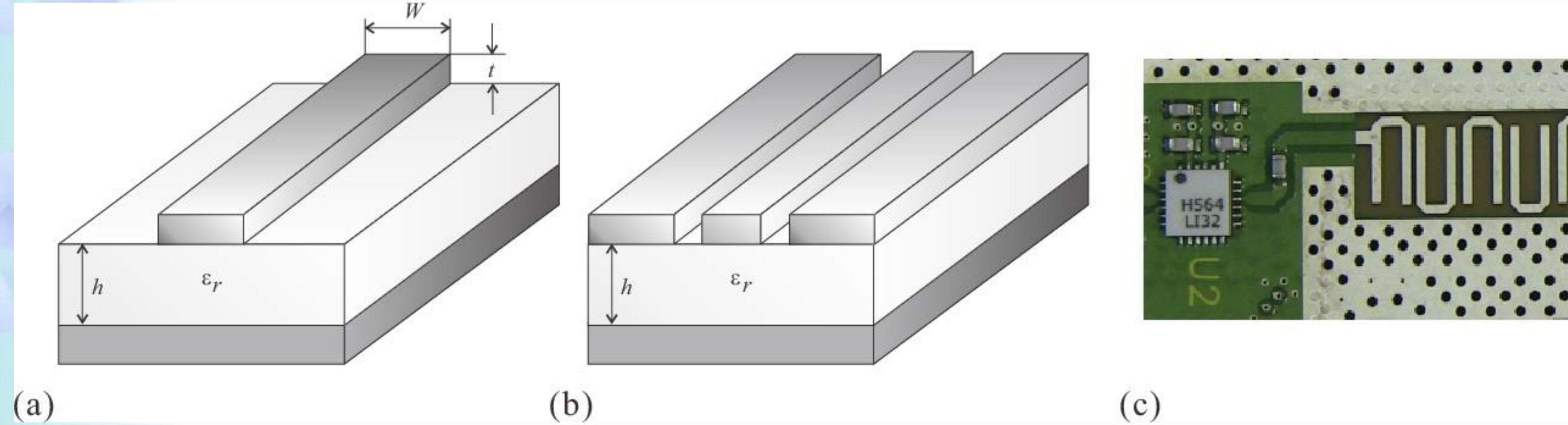


(b)

(c)

$$\phi_1 = -\phi_2 = \frac{J\sqrt{\mu}}{2\pi}\log\frac{d}{a} \qquad Z_0 = \sqrt{\frac{\mu}{\epsilon}}\frac{1}{\pi}\log\frac{d}{a}$$

Micro strip line



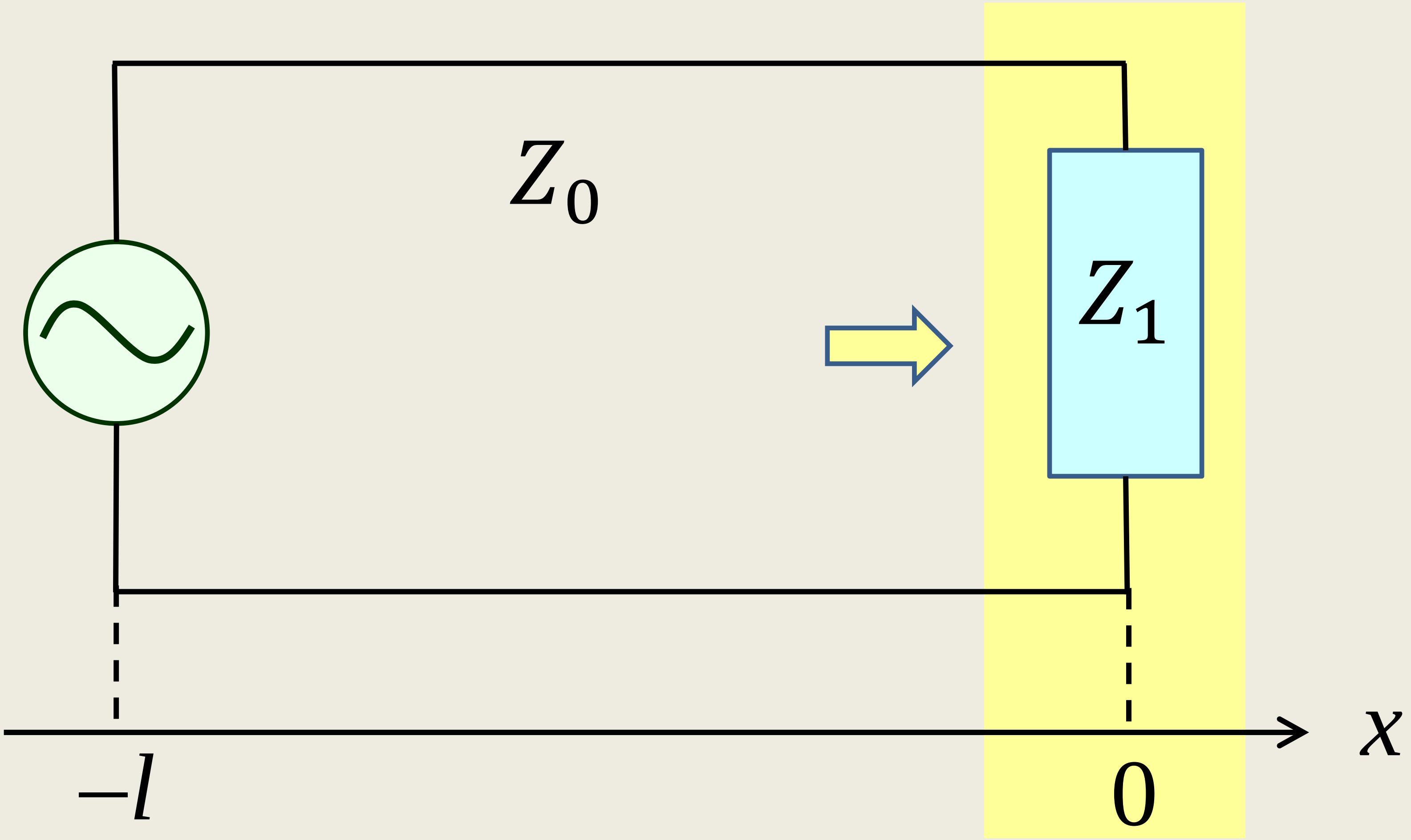
Wide (W/h>3.3) strip

$$Z(W,h,\epsilon_r) = \frac{Z_{F0}}{2\sqrt{\epsilon_r}} \left\{ \frac{W}{2h} + \frac{1}{\pi}\log 4 + \frac{\epsilon_r + 1}{2\pi\epsilon_r} \log\left[\frac{\pi e}{2}\left(\frac{W}{2h} + 0.94\right)\right] \frac{\epsilon_r - 1}{2\pi\epsilon_r^2} \log\frac{e\pi^2}{16} \right\}^{-1}$$

Narrow (W/h<3.3) strip

$$Z(W,h,\epsilon_r) = \frac{Z_{F0}}{\pi\sqrt{2(\epsilon_r+1)}} \left\{ \log\left[\frac{4h}{W} + \sqrt{\left(\frac{4h}{W}\right)^2 + 2}\right] - \frac{1}{2}\frac{\epsilon_r - 1}{\epsilon_r + 1}\left(\log\frac{\pi}{2} + \frac{1}{\epsilon_r}\log\frac{4}{\pi}\right) \right\}$$

Connection and termination



Connection and termination

At $x = 0$: $\begin{cases} J = J_+ + J_- & \text{(definition right positive)} \\ \text{progressive} \quad \text{retrograde} & \\ V = V_+ + V_- = Z_0(J_+ - J_-) & \end{cases}$

$$Z_1 = \frac{V}{J} = \frac{J_+ - J_-}{J_+ + J_-} Z_0$$

Reflection coefficient: $$r = \frac{V_-}{V_+} = -\frac{J_-}{J_+} = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$Z_1 = Z_0$: no reflection, i.e., impedance matching

$Z_1 = +\infty$ (open circuit end) : $r = 1$, i.e., free end

$Z_1 = 0$ (short circuit end) : $r = -1$, i.e., fixed end

Connection and termination

Finite reflection → Standing wave

Voltage-Standing Wave Ratio (VSWR): $= \dfrac{1+|r|}{1-|r|}$

At $x = -l$

$$\left.\begin{aligned} V &= V_{+0}\exp(\kappa l) + V_{-0}\exp(-\kappa l) = [J_{+0}\exp(\kappa l) - J_{-0}\exp(-\kappa l)]Z_0 \\ J &= J_{+0}\exp(\kappa l) + J_{-0}\exp(-\kappa l) \end{aligned}\right\}$$

$$Z_l = \frac{V}{J} = \frac{J_{+0}e^{\kappa l} - J_{-0}e^{-\kappa l}}{J_{+0}e^{\kappa l} + J_{-0}e^{-\kappa l}} Z_0$$

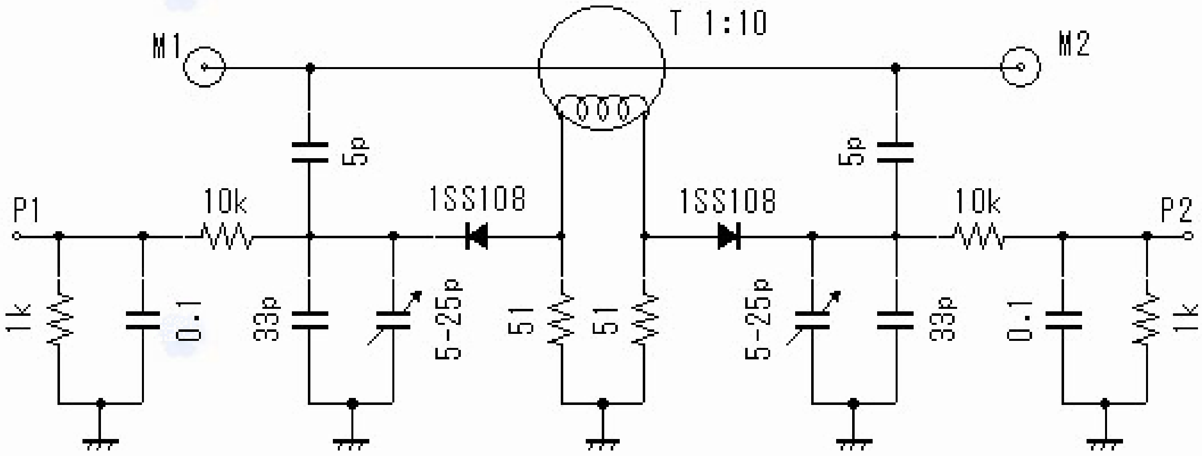
Reflection coefficient:

$$r_l = \frac{V_-}{V_+} = \frac{V_{-0}e^{-\kappa l}}{V_{+0}e^{\kappa l}} = r\exp(-2\kappa l)$$

SWR Meters:

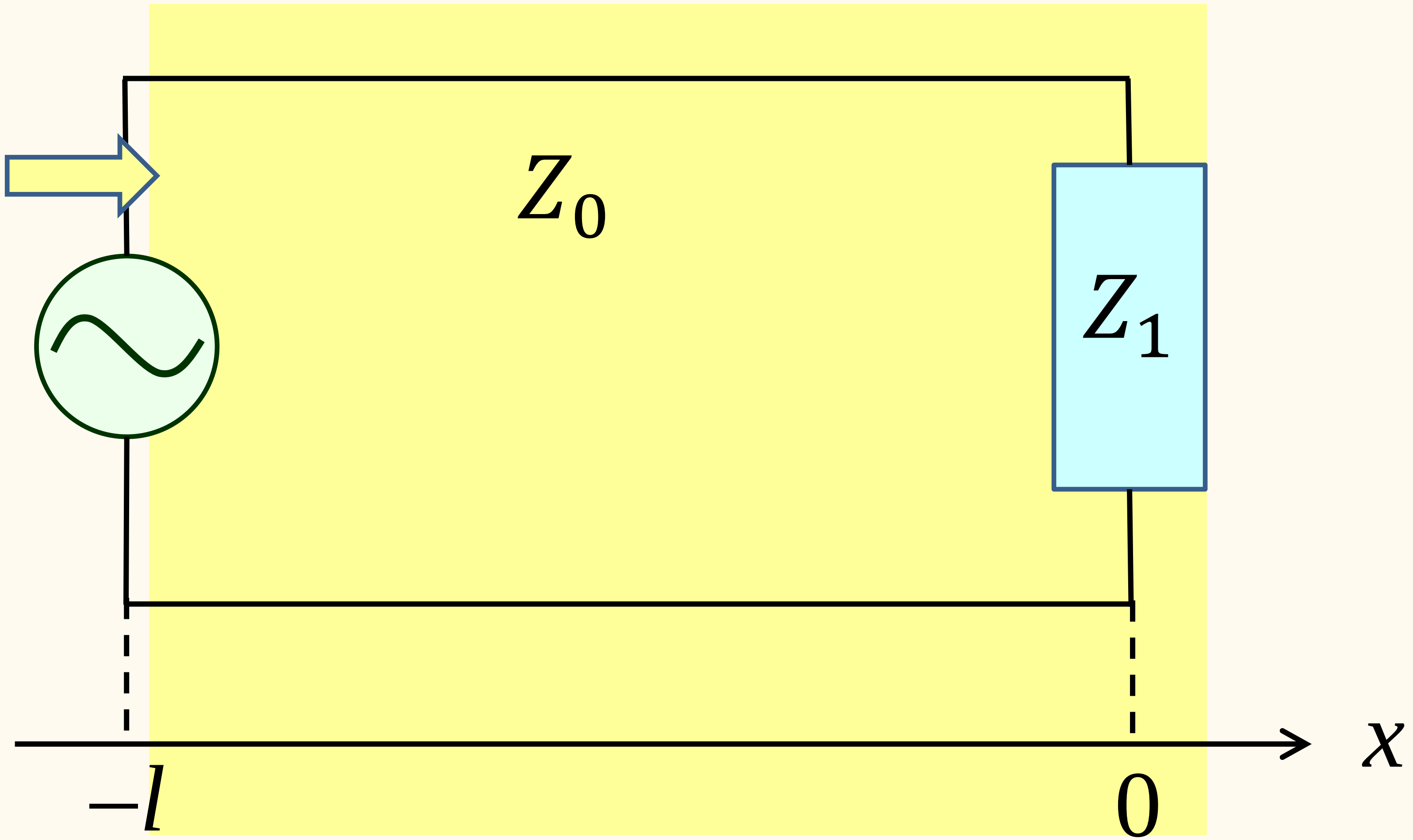
Desktop types

Cross-meter



Handy type

Connection and termination



Connection and termination

Transmission line connection.
Characteristic impedance Z_0, ${Z_0}'$

At the connection point, only the local relation between V and J affects the reflection coefficient.

The local impedance from the left hand side is ${Z_0}'$.

$$r = \frac{Z_0' - Z_0}{Z_0' + Z_0}$$