電子回路論第 8 回

Electric Circuits for Physicists

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Outline

5.1 Transmission lines

TEM mode Lecher line

Micro-strip line

TE, TM mode Waveguide

Optical fiber

5.2 Propagation in transmission lines

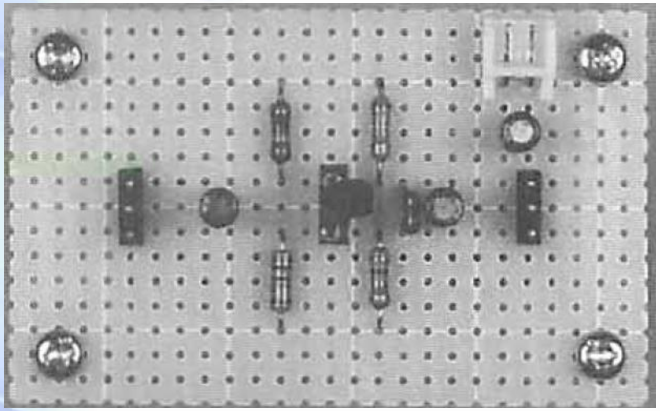
Termination and connection

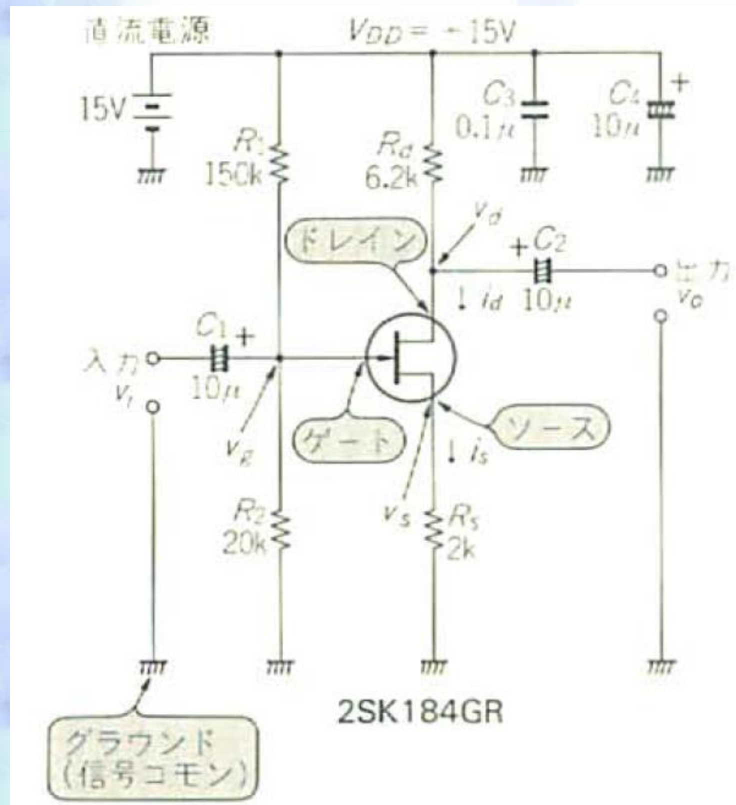
Smith chart

Scattering matrix

Impedance matching

Comment: bias + signal superposition





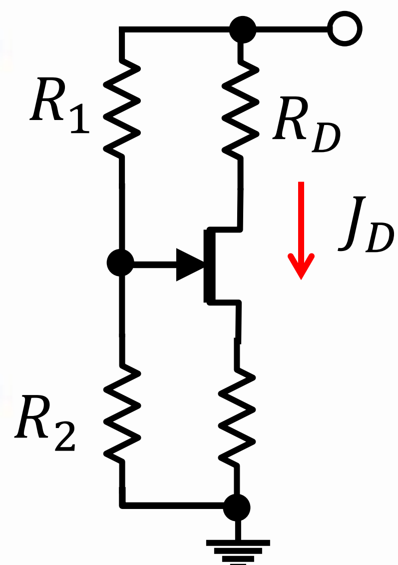
For bias (dc) circuits

All the capacitors can be viewed as break line.

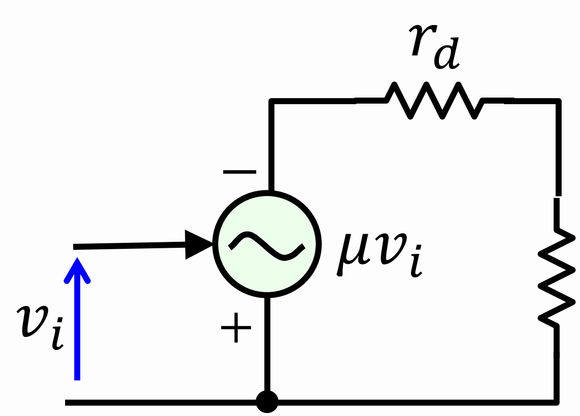
For small amplitude (high-frequency) circuits

All the capacitors can be viewed as short circuits.

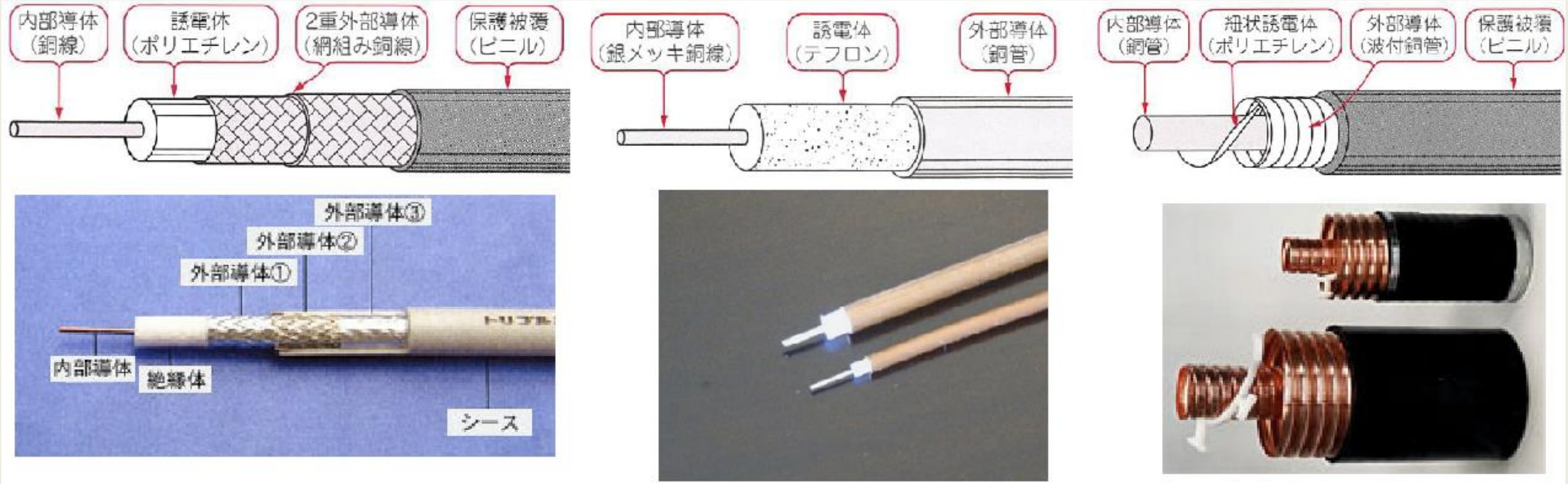
Self-bias



Source-grounded



Coaxial cable 2



(a) (b) (c)

図12 同軸ケーブルの型名(JIS C3501)

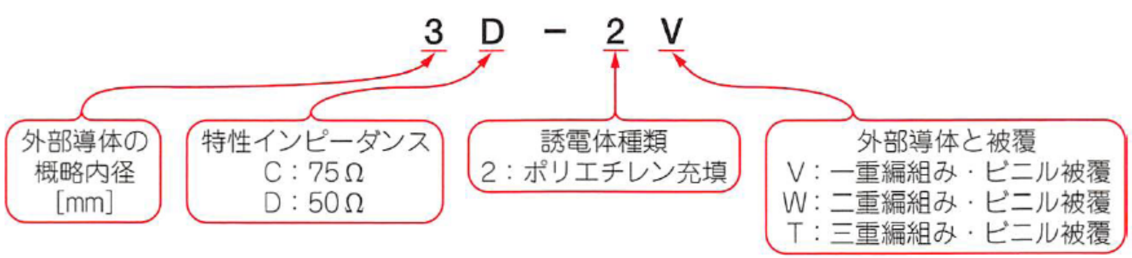
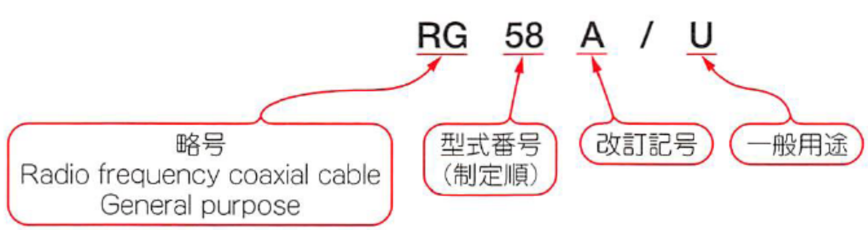
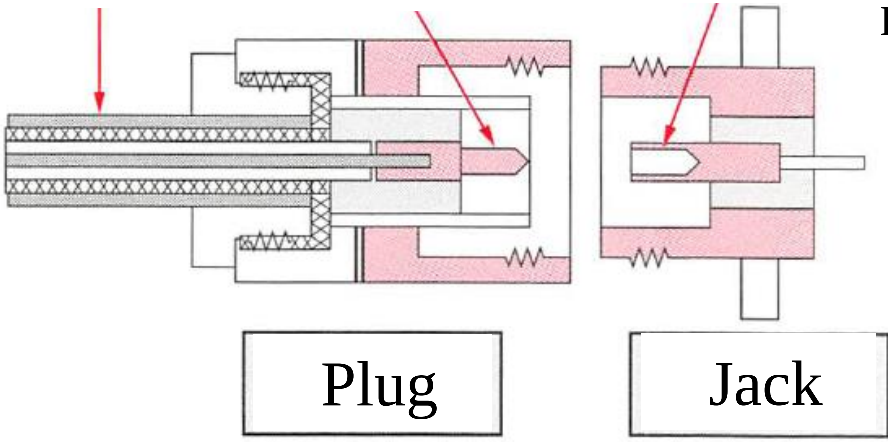


図13 MIL規格での同軸ケーブル型名の例



Coaxial connectors

図22 coaxial connector (schematic)

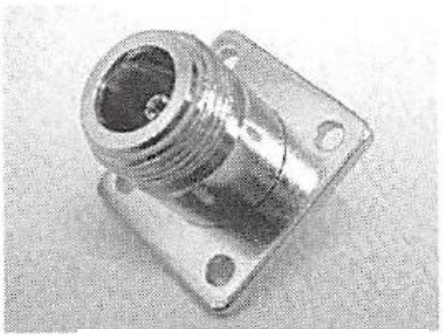


Highest available frequencies for coaxial connectors

type	outer diam.	highest freq.
BNC	約 7 mm	2 ～ 4 GHz
N	約 7 mm	10 ～ 18 GHz
7 mm	7 mm	～ 18 GHz
SMA	4.15 mm	18 GHz
3.5 mm	3.5 mm	26.5 GHz
K	2.92 mm	40 GHz
2.4 mm	2.4 mm	50 GHz
V	1.85 mm	65 GHz
W	1.1 mm	110 GHz
1.0 mm	1.0 mm	110 GHz

Coaxial connectors

N-type connectors

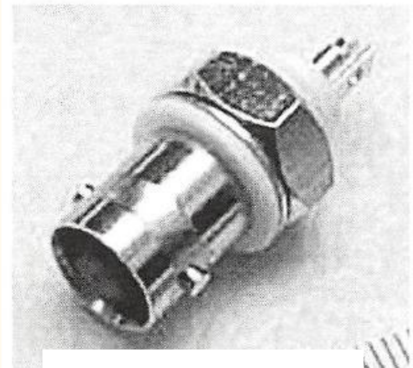


(a) jack with flange

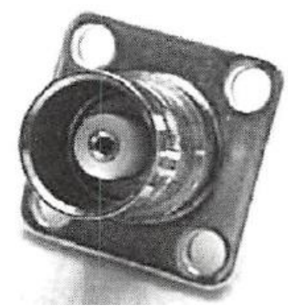
(b) plug

(c) plug [disassembled (b)]

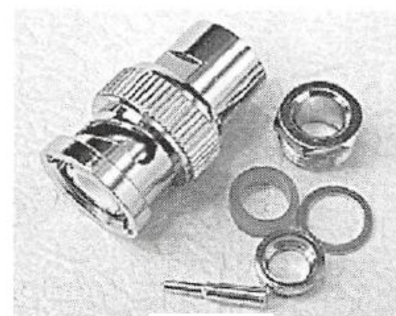
BNC-type connectors



isolated jack
(not for high freq.)



jack with flange

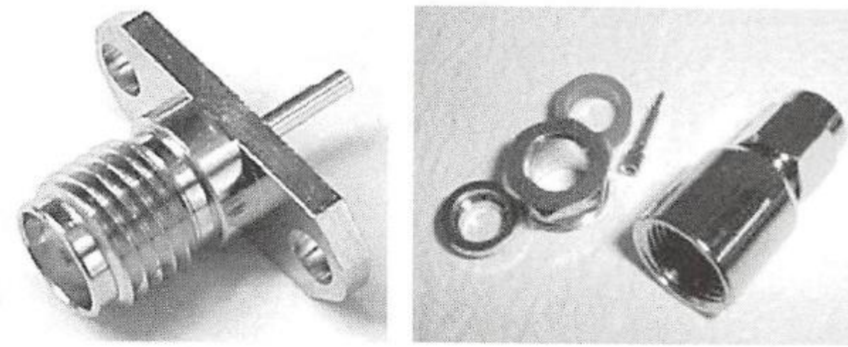


plug

Coaxial connectors 2

SMA-type

写真4 SMA型コネクタ

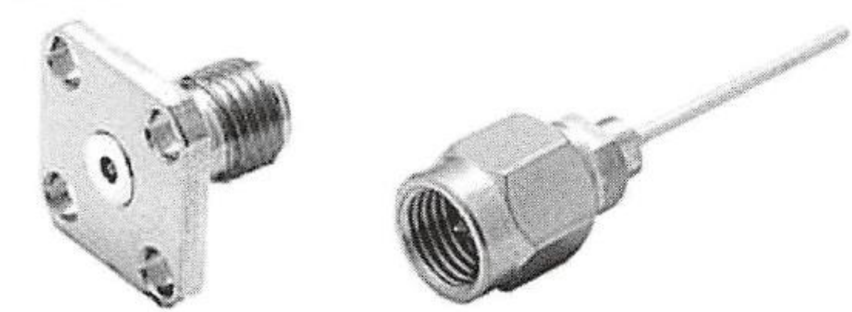


(a) ジャック　(b) プラグ

jack　plug

K-type

写真6 K型コネクタ

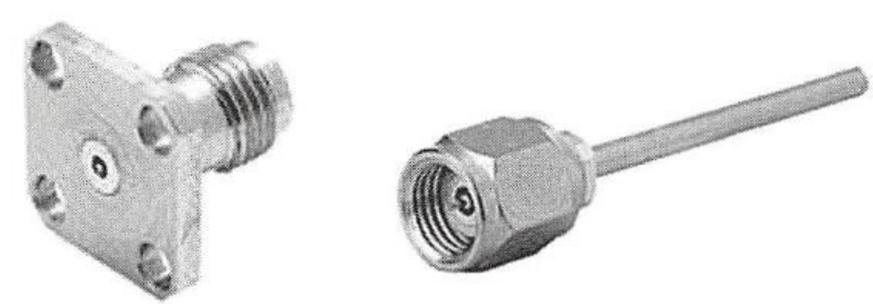


(a) ジャック　(b) プラグ

jack　plug

V-type

写真7 V型コネクタ



(a) ジャック　(b) プラグ

jack　plug

LEMO cables and connectors

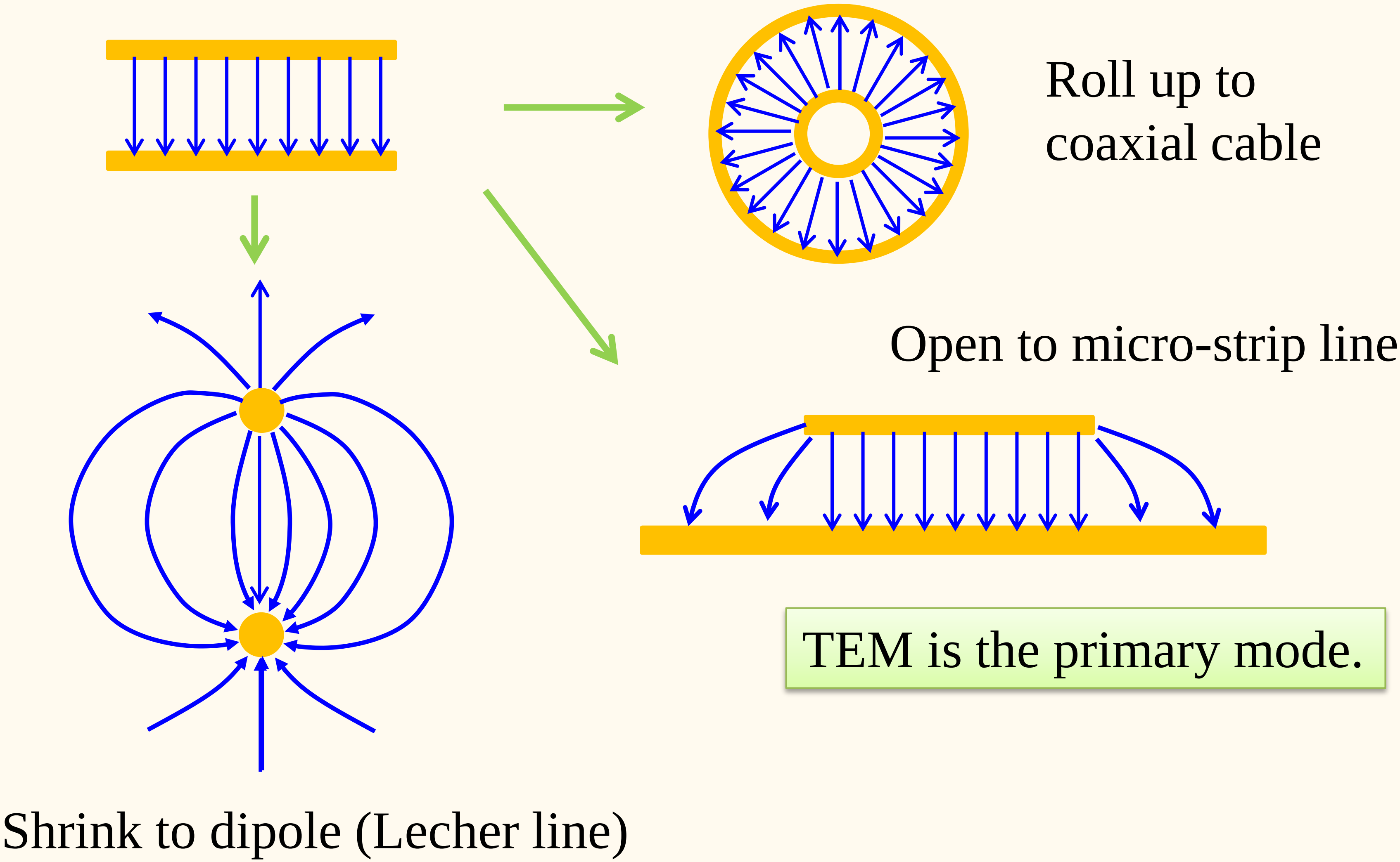
http://www.lemo.com/

High-energy physics experiment, etc.

Transmission lines with TEM mode

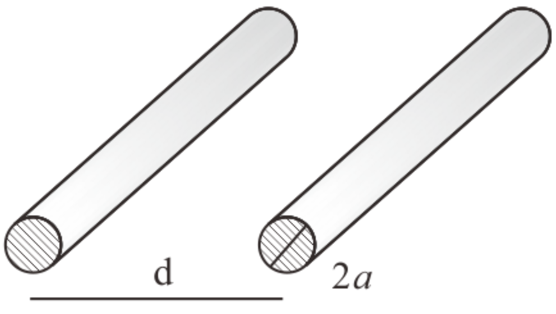
Transmission lines with two conductors are “families”.

Electromagnetic field confinement with parallel-plate capacitor

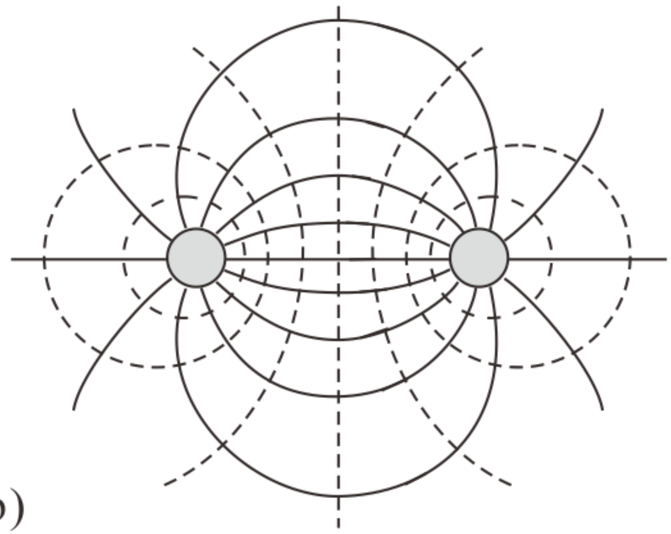


TEM is the primary mode.

Lecher line



(a)

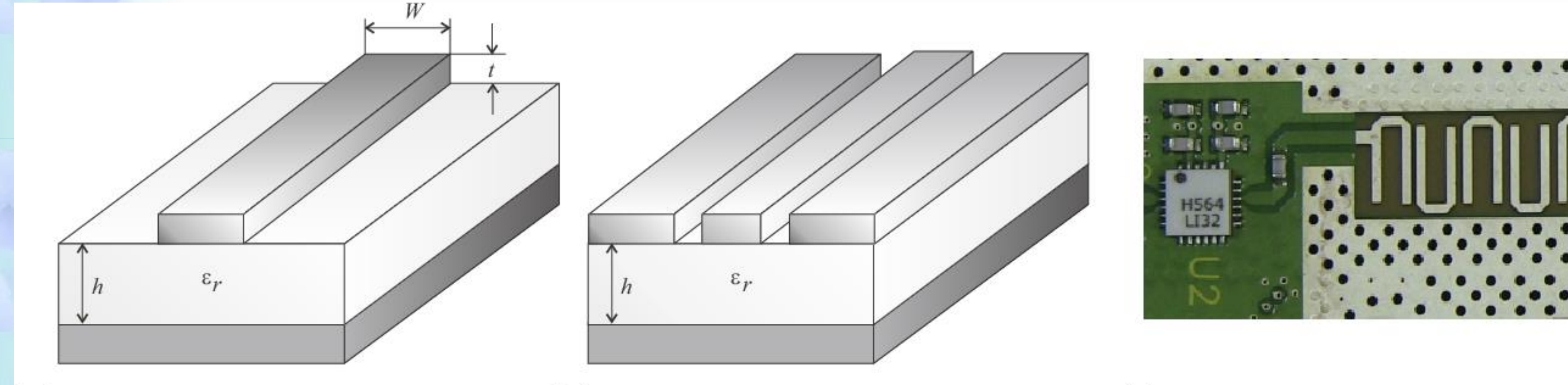


(b)

(c)

$$\phi_1 = -\phi_2 = \frac{J\sqrt{\mu}}{2\pi} \log \frac{d}{a} \qquad Z_0 = \sqrt{\frac{\mu}{\epsilon}} \frac{1}{\pi} \log \frac{d}{a}$$

Micro strip line



(a) (b) (c)

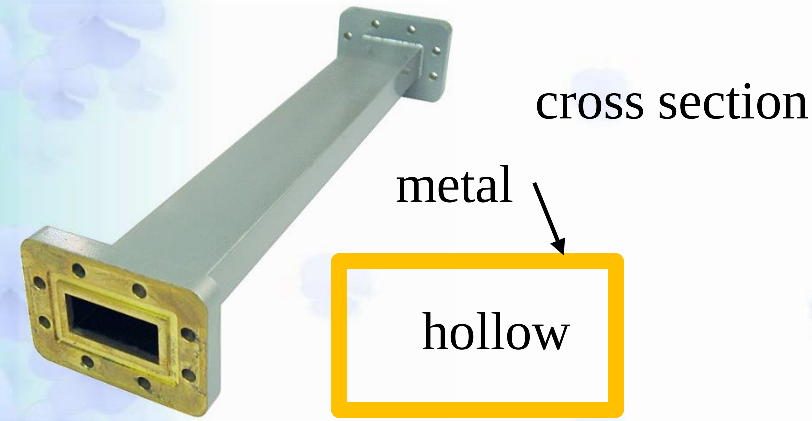
Wide (W/h>3.3) strip

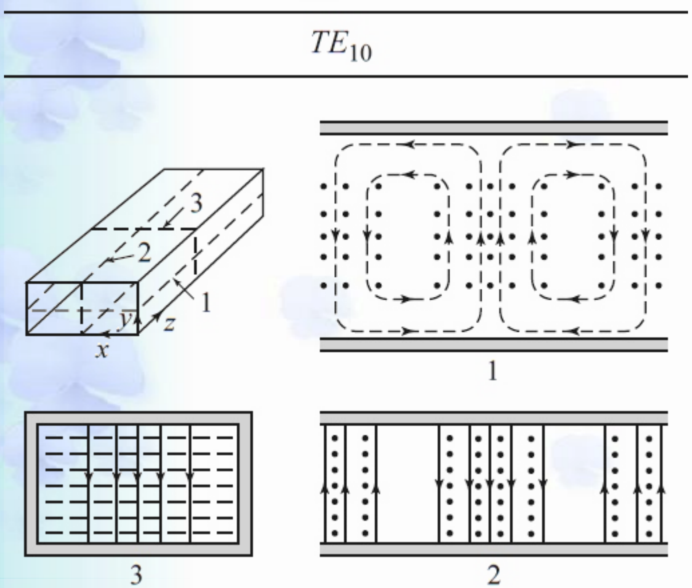
$$Z(W,h,\epsilon_r) = \frac{Z_{F0}}{2\sqrt{\epsilon_r}}\left\{\frac{W}{2h} + \frac{1}{\pi}\log 4 + \frac{\epsilon_r+1}{2\pi\epsilon_r}\log\left[\frac{\pi e}{2}\left(\frac{W}{2h}+0.94\right)\right]\frac{\epsilon_r-1}{2\pi\epsilon_r^2}\log\frac{e\pi^2}{16}\right\}^{-1}$$

Narrow (W/h<3.3) strip

$$Z(W,h,\epsilon_r) = \frac{Z_{F0}}{\pi\sqrt{2(\epsilon_r+1)}}\left\{\log\left[\frac{4h}{W}+\sqrt{\left(\frac{4h}{W}\right)^2+2}\right] - \frac{1}{2}\frac{\epsilon_r-1}{\epsilon_r+1}\left(\log\frac{\pi}{2}+\frac{1}{\epsilon_r}\log\frac{4}{\pi}\right)\right\}$$

Waveguide





Electromagnetic field is confined into a simply-connected space.

TEM mode cannot exist.

Maxwell equations give

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] E_z = -(\omega^2 \epsilon\mu + \gamma^2) E_z,$$

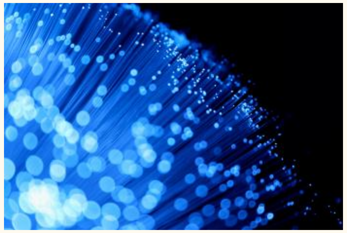
$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] H_z = -(\omega^2 \epsilon\mu + \gamma^2) H_z.$$

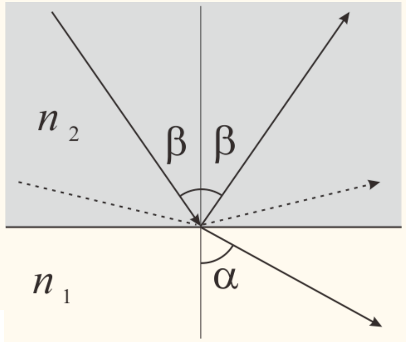
Helmholtz equation

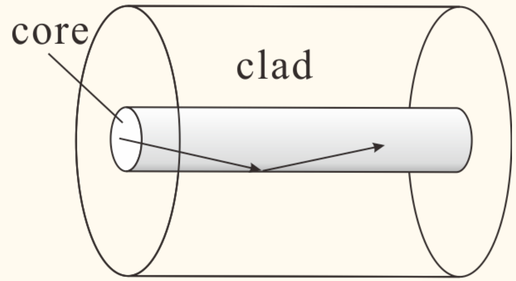
$E_z = 0$: TE mode,

$H_z = 0$: TM mode

Optical fiber

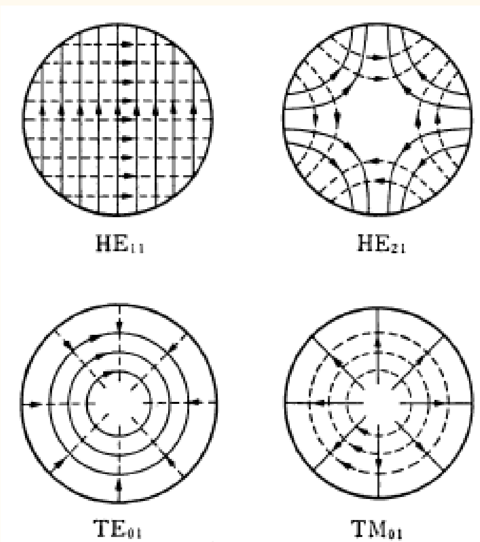




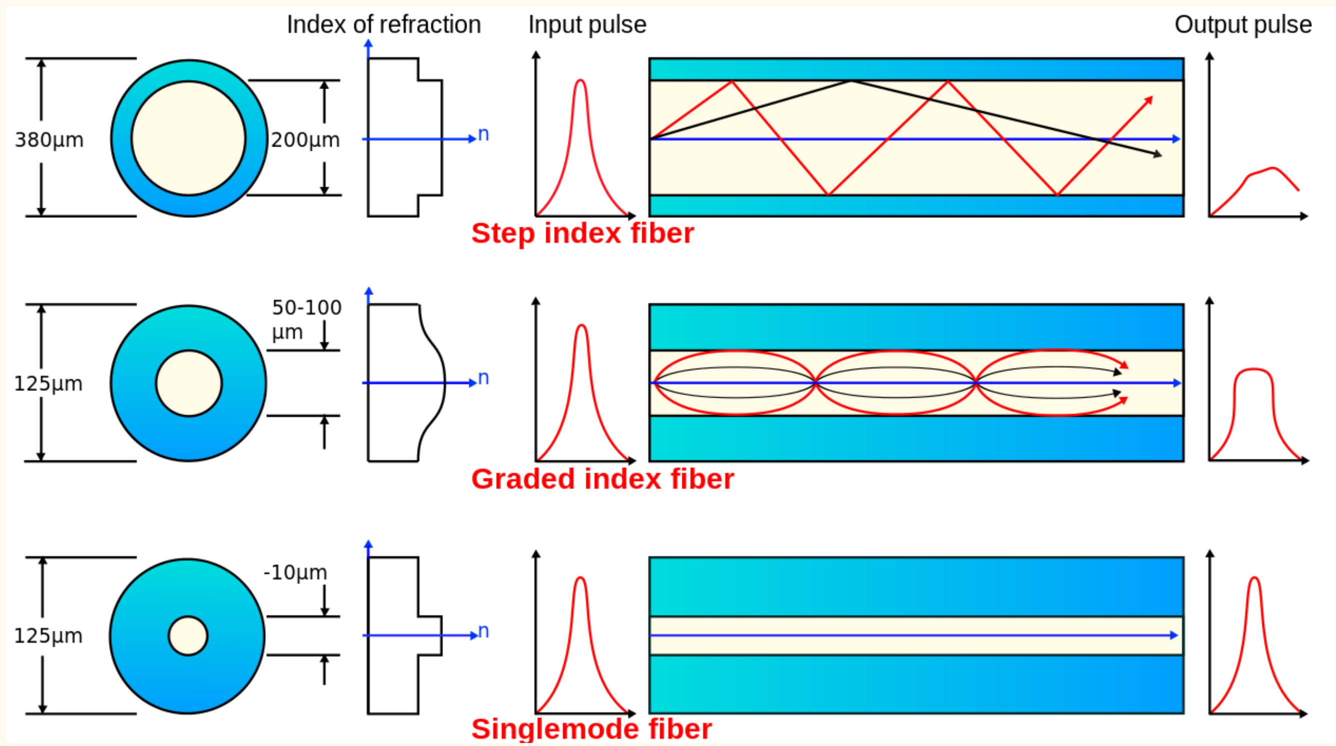


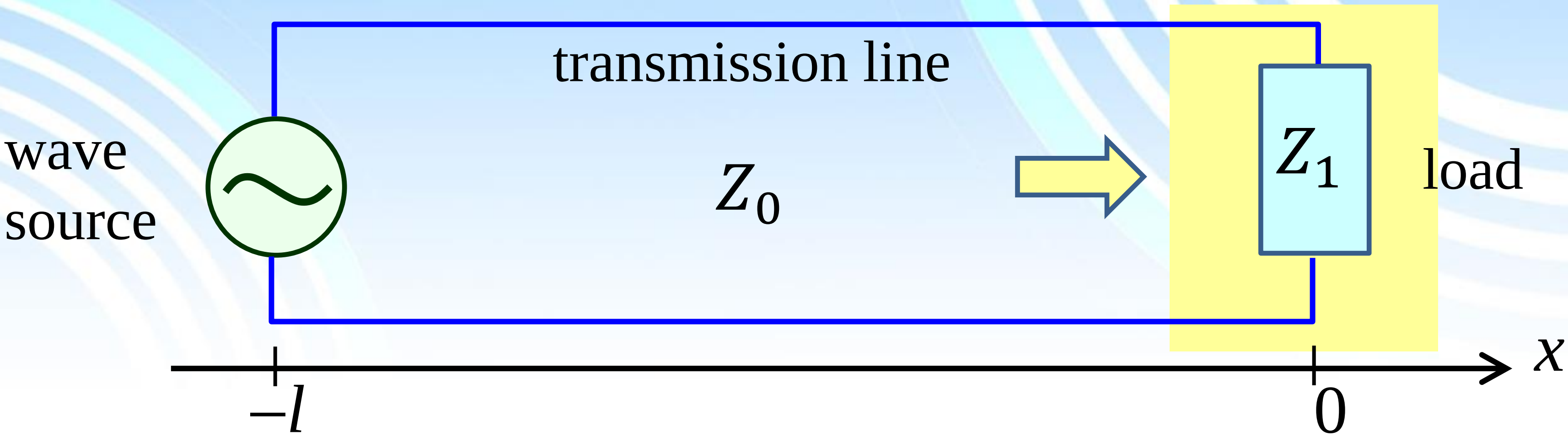
step-type optical fiber

Difference in dielectric constant



no dispersion





Termination of a transmission line with length l and characteristic impedance Z_0 at $x = 0$ with a resistor Z_1.

At $x = 0$:

$$J = J_+ + J_- \quad \text{(definition right positive)}$$

(J_+: progressive, J_-: retrograde)

$$V = V_+ + V_- = Z_0(J_+ - J_-)$$

Comment: Sign of Ohm's law in transmission lines reflects direction of waves (depends on the definitions).

Termination of transmission line

$$\pm 2Z_0 J_\pm = 2V_\pm = J \pm Z_0 V$$

synthesized impedance: $Z_1 = \dfrac{V}{J} = \dfrac{J_+ - J_-}{J_+ + J_-} Z_0$

reflection coefficient: $r = \dfrac{V_-}{V_+} = -\dfrac{J_-}{J_+} = \dfrac{Z_1 - Z_0}{Z_1 + Z_0}$

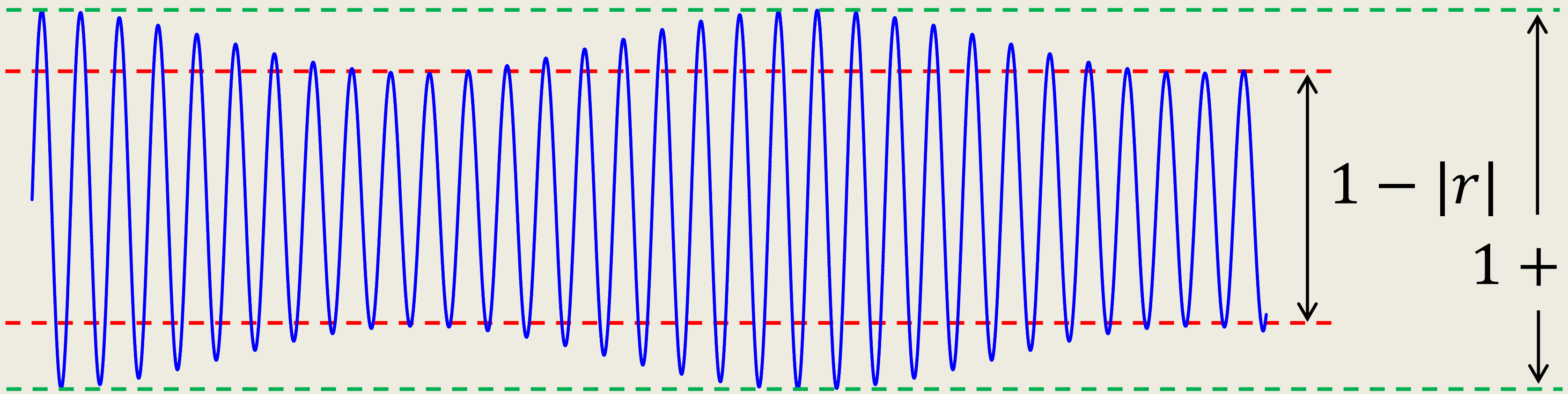
$Z_1 = Z_0$: no reflection, i.e., impedance matching

$Z_1 = +\infty$ (open circuit end) : $r = 1$, i.e., free end

$Z_1 = 0$ (short circuit end) : $r = -1$, i.e., fixed end

Connection and termination

Finite reflection → Standing wave



Voltage-Standing Wave Ratio (VSWR): $= \dfrac{1+|r|}{1-|r|}$

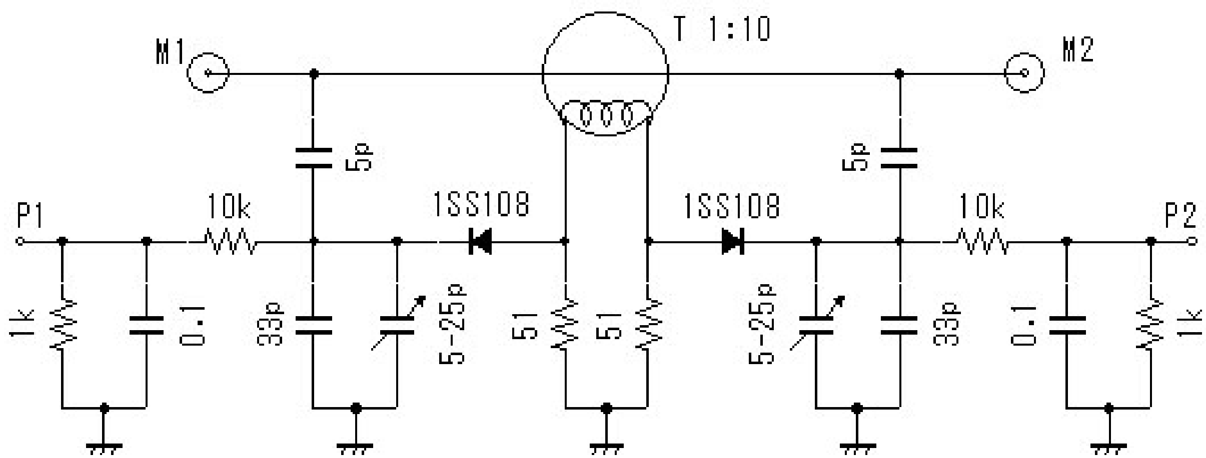
SWR measurement

SWR Meters:

desktop types

cross-meter

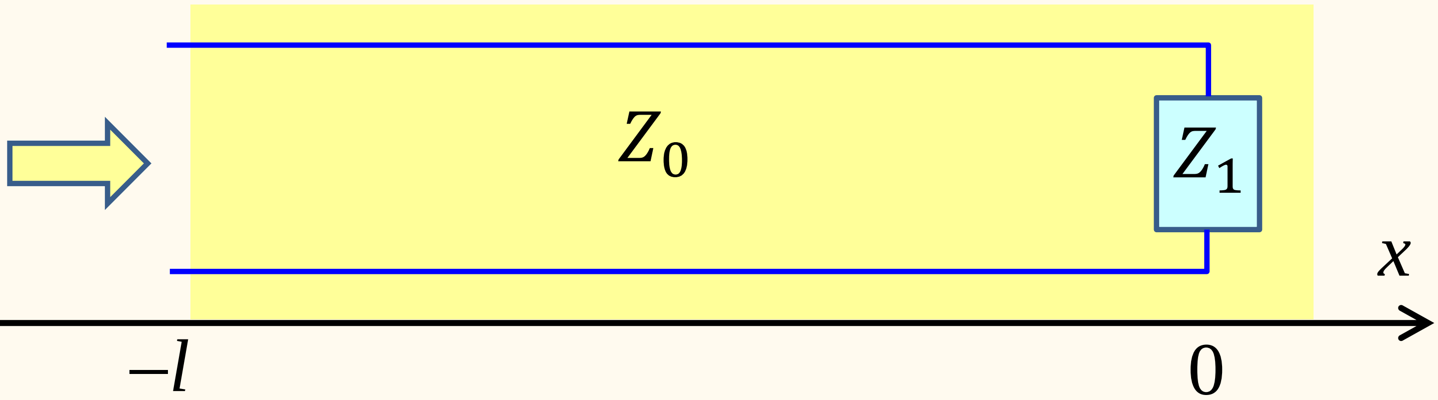
handy type



directional coupler

Synthesized impedance

total impedance
from this side



At $x = -l$

$$\left.\begin{aligned} V &= V_{+0}\exp(\kappa l) + V_{-0}\exp(-\kappa l) = [J_{+0}\exp(\kappa l) - J_{-0}\exp(-\kappa l)]Z_0 \\ J &= J_{+0}\exp(\kappa l) + J_{-0}\exp(-\kappa l) \end{aligned}\right\}$$

$$Z_l = \frac{V}{J} = \frac{J_{+0}e^{\kappa l} - J_{-0}e^{-\kappa l}}{J_{+0}e^{\kappa l} + J_{-0}e^{-\kappa l}} Z_0$$

Reflection coefficient: $r_l = \dfrac{V_-}{V_+} = \dfrac{V_{-0}e^{-\kappa l}}{V_{+0}e^{\kappa l}} = r\exp(-2\kappa l)$

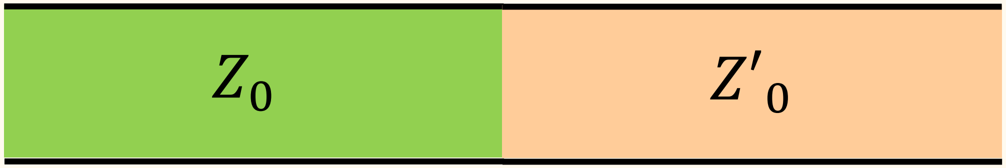
Connection and termination

Transmission line connection.
Characteristic impedance Z_0, ${Z_0}'$

At the connection point, only the local relation between V and J affects the reflection coefficient.

The local impedance from the left hand side is ${Z_0}'$.

$$r = \frac{Z_0' - Z_0}{Z_0' + Z_0}$$



5.2.3 Smith chart, Immittance chart

End impedance Z_1: Normalized end impedance $Z_n \equiv Z_1/Z_0$

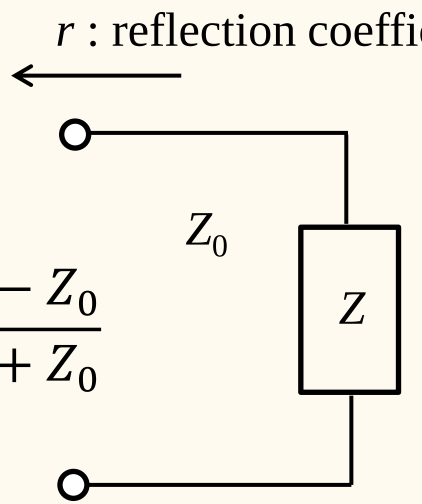
$$Z_n = x + iy, \quad r = u + iw \quad (x, y, u, w \in \mathbb{R})$$

$$u + iw = r = \frac{Z_n - 1}{Z_n + 1} = \frac{(x-1) + iy}{(x+1) + iy}$$

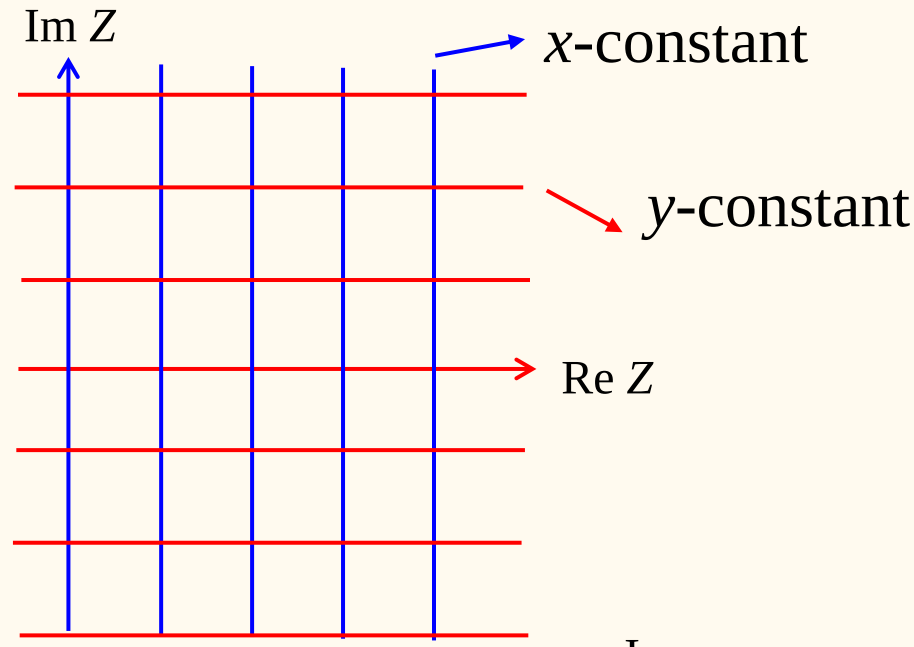
$$\left.\begin{aligned} &\text{real:} & x - 1 &= (x+1)u - yw \\ &\text{imaginary:} & y &= yu + w(x+1) \end{aligned}\right\}$$

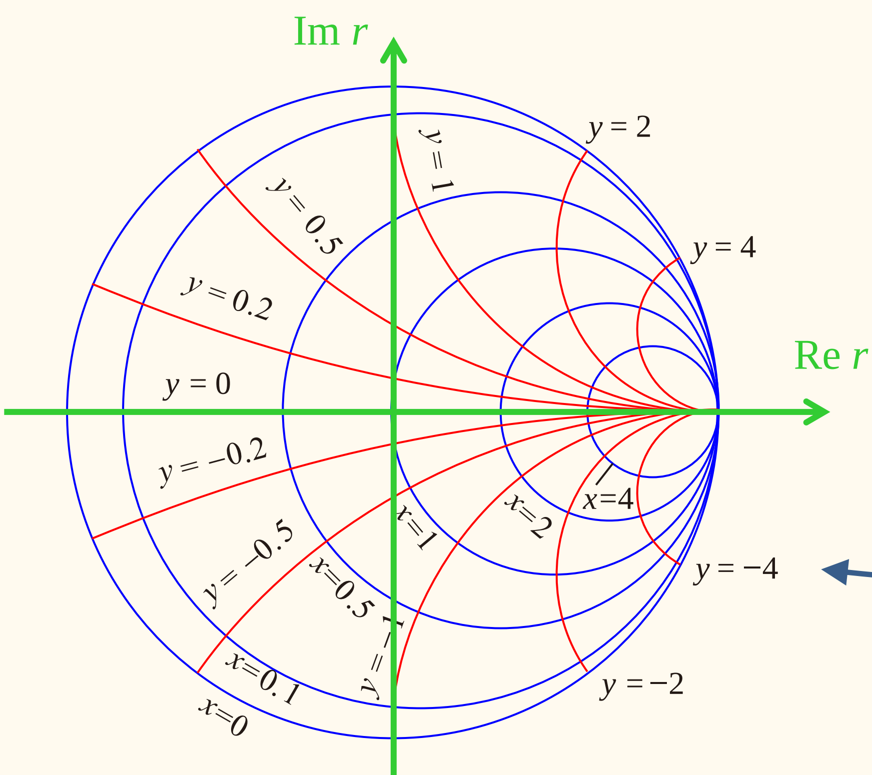
x: constant → $\left(u - \frac{x}{x+1}\right)^2 + w^2 = \frac{1}{(x+1)^2}$ constant resistance circle

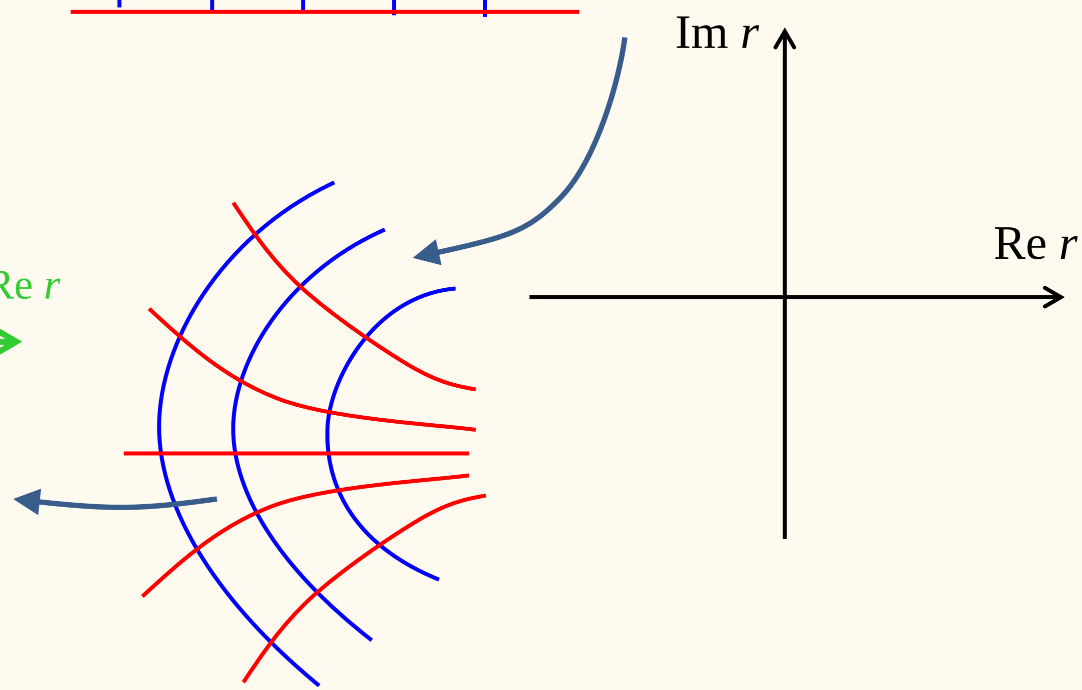
y: constant → $(u-1)^2 + \left(w - \frac{1}{y}\right)^2 = \frac{1}{y^2}$ constant reactance circle



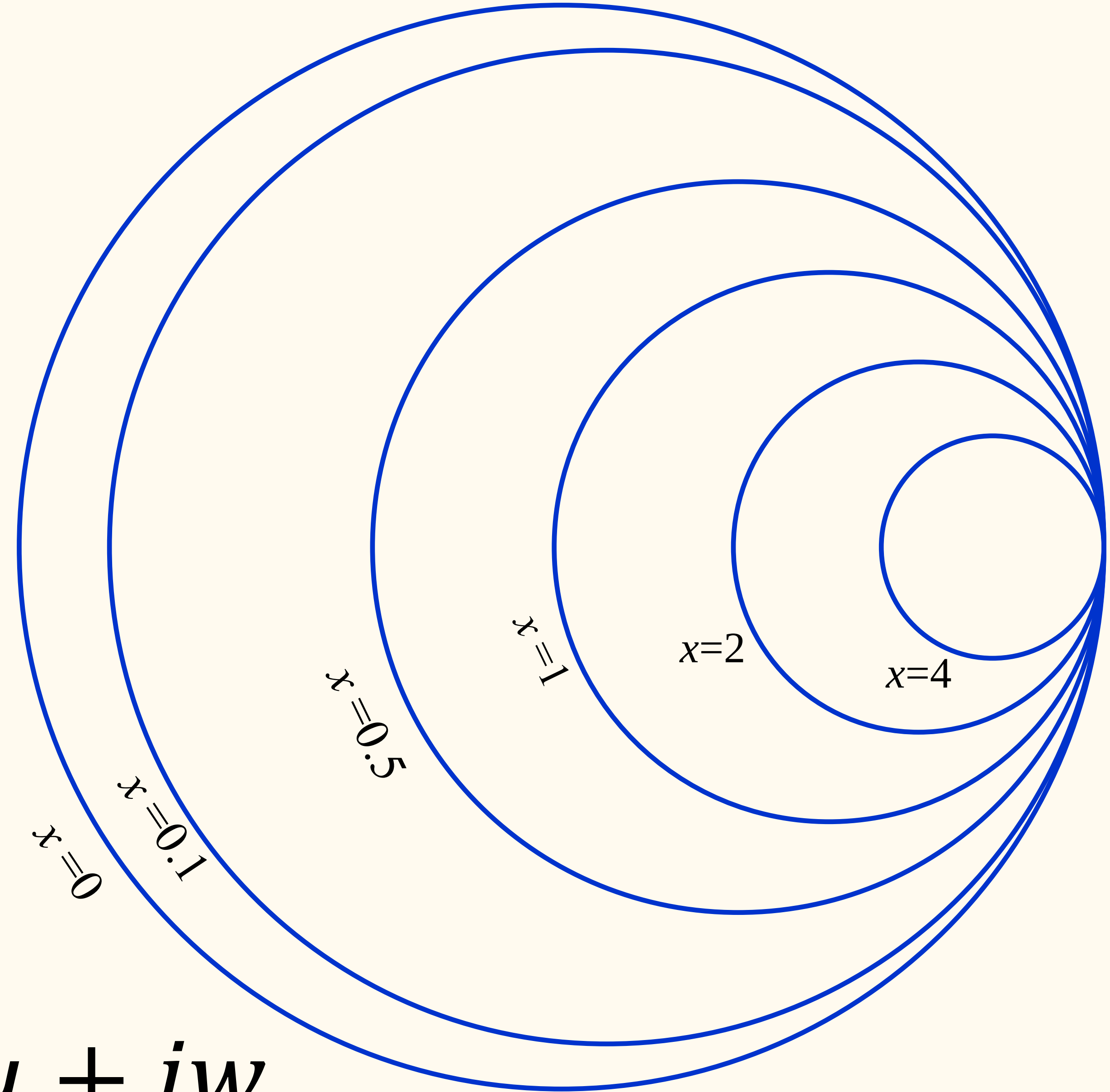
$$r = \frac{V_-}{V_+} = \frac{Z - Z_0}{Z + Z_0}$$





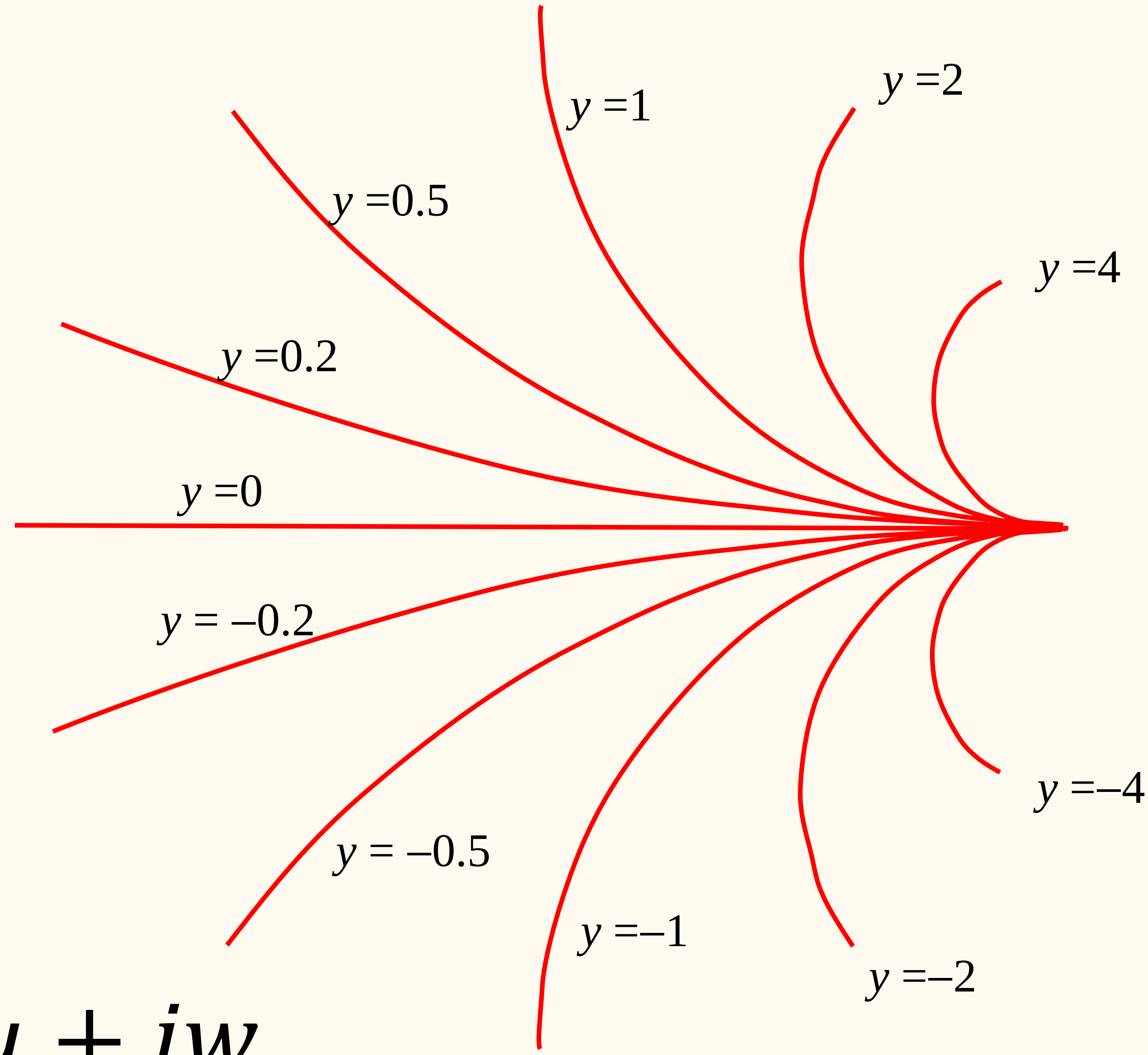


5.2.3 Smith chart, Immittance chart



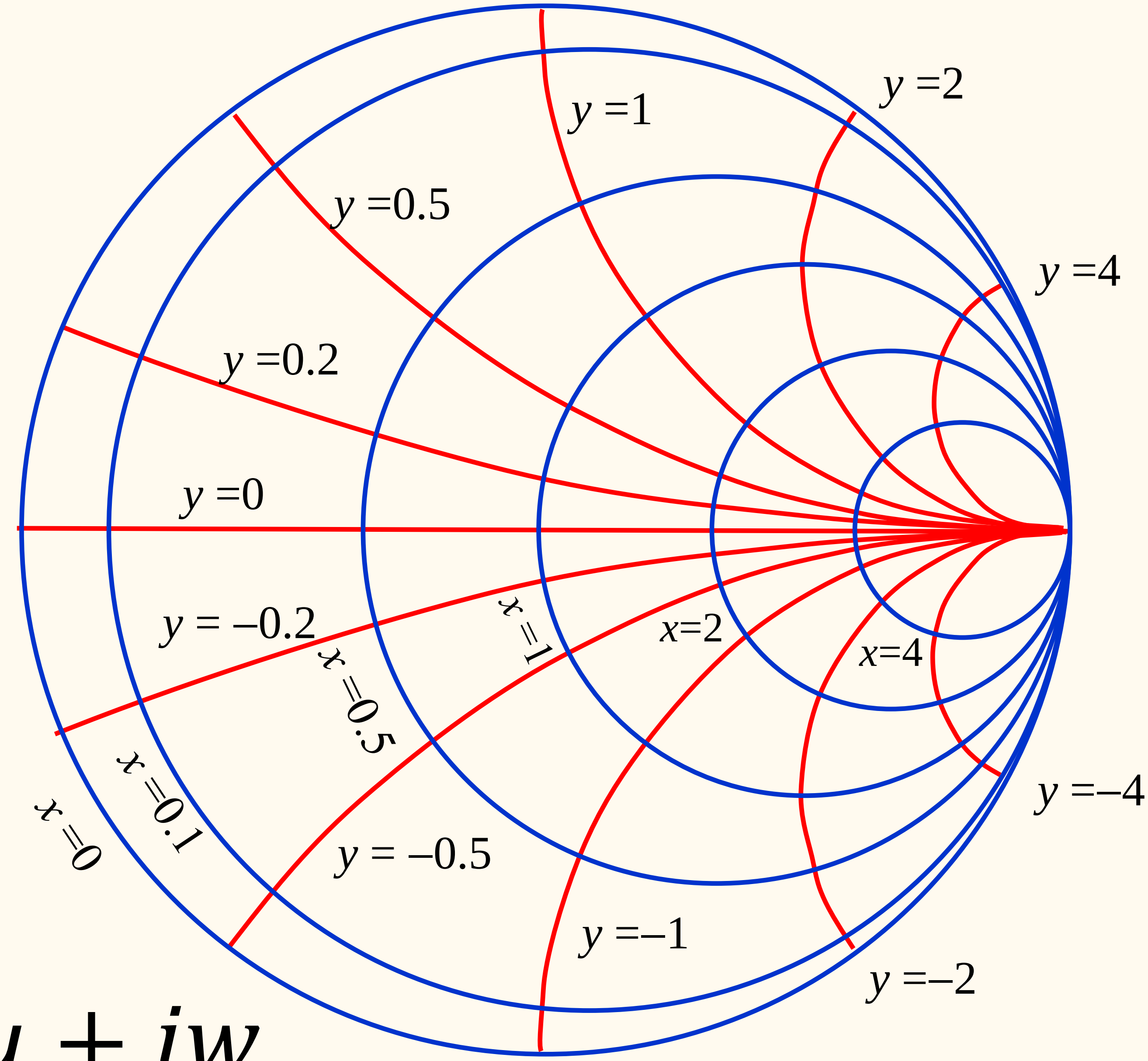
$$r = u + iw$$

5.2.3 Smith chart, Immittance chart



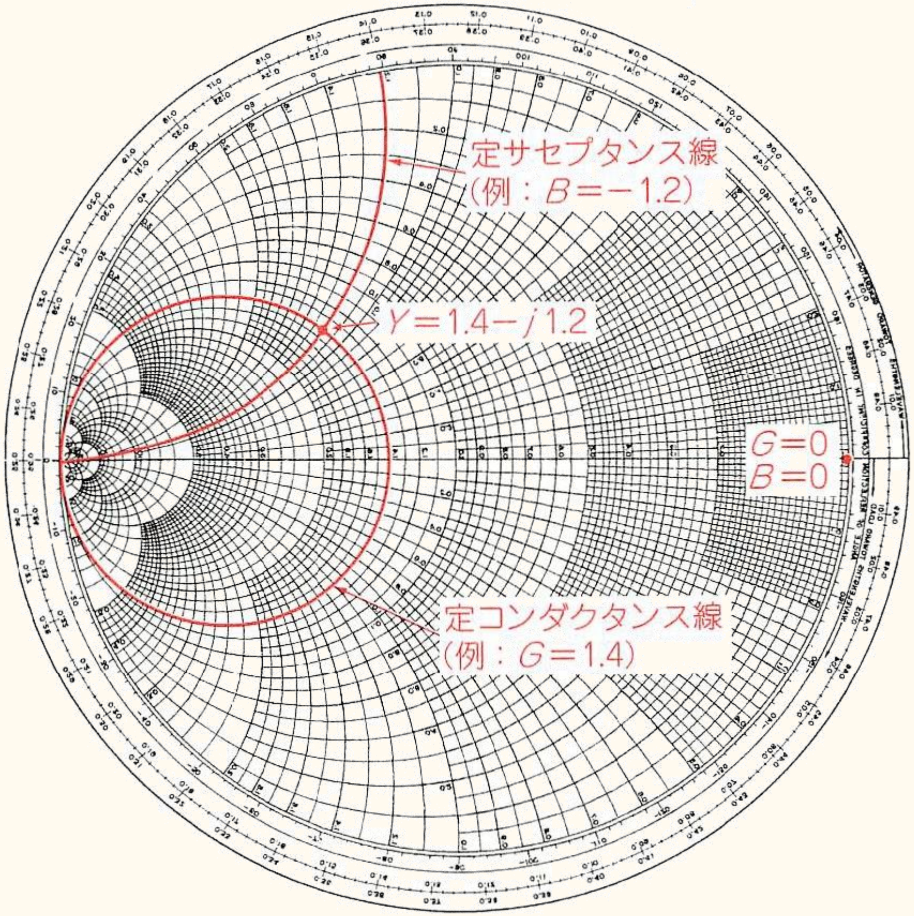
$$r = u + iw$$

5.2.3 Smith chart, Immittance chart



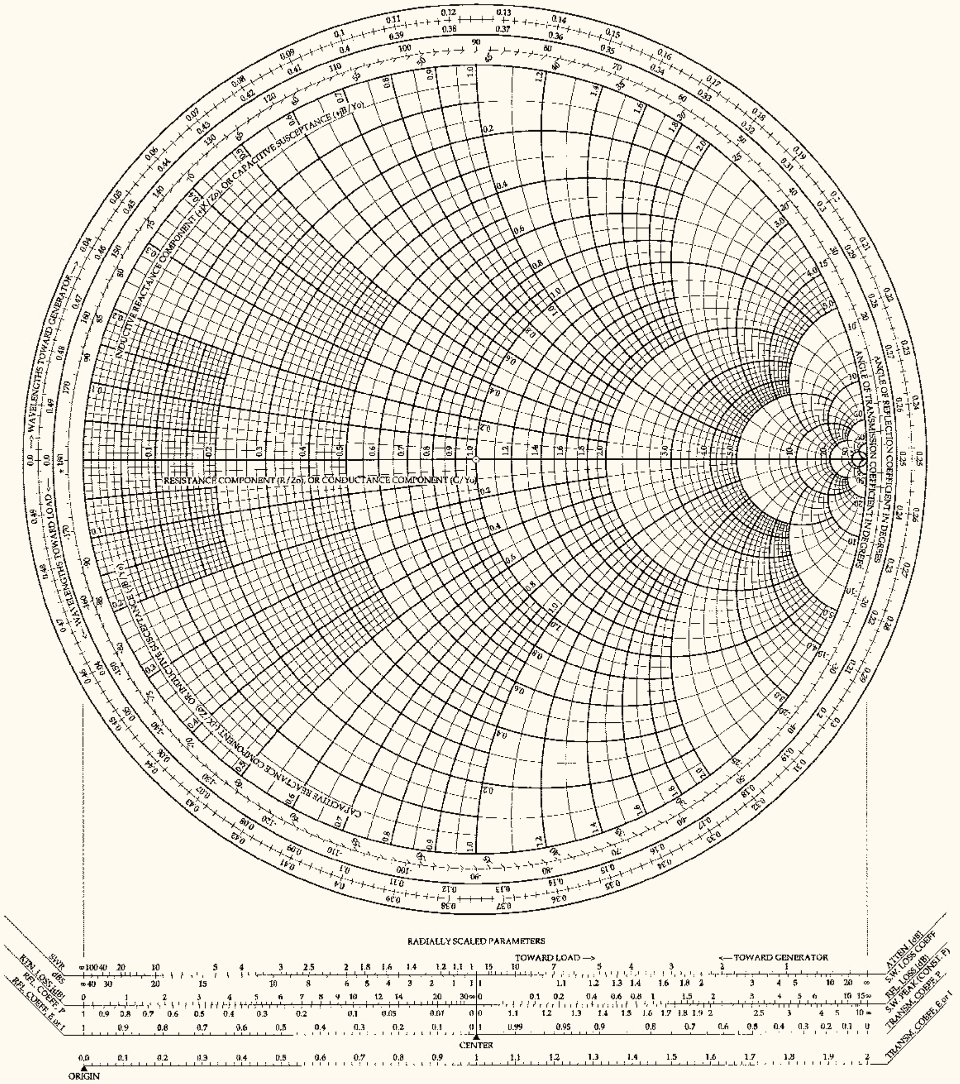
$$r = u + iw$$

Smith chart



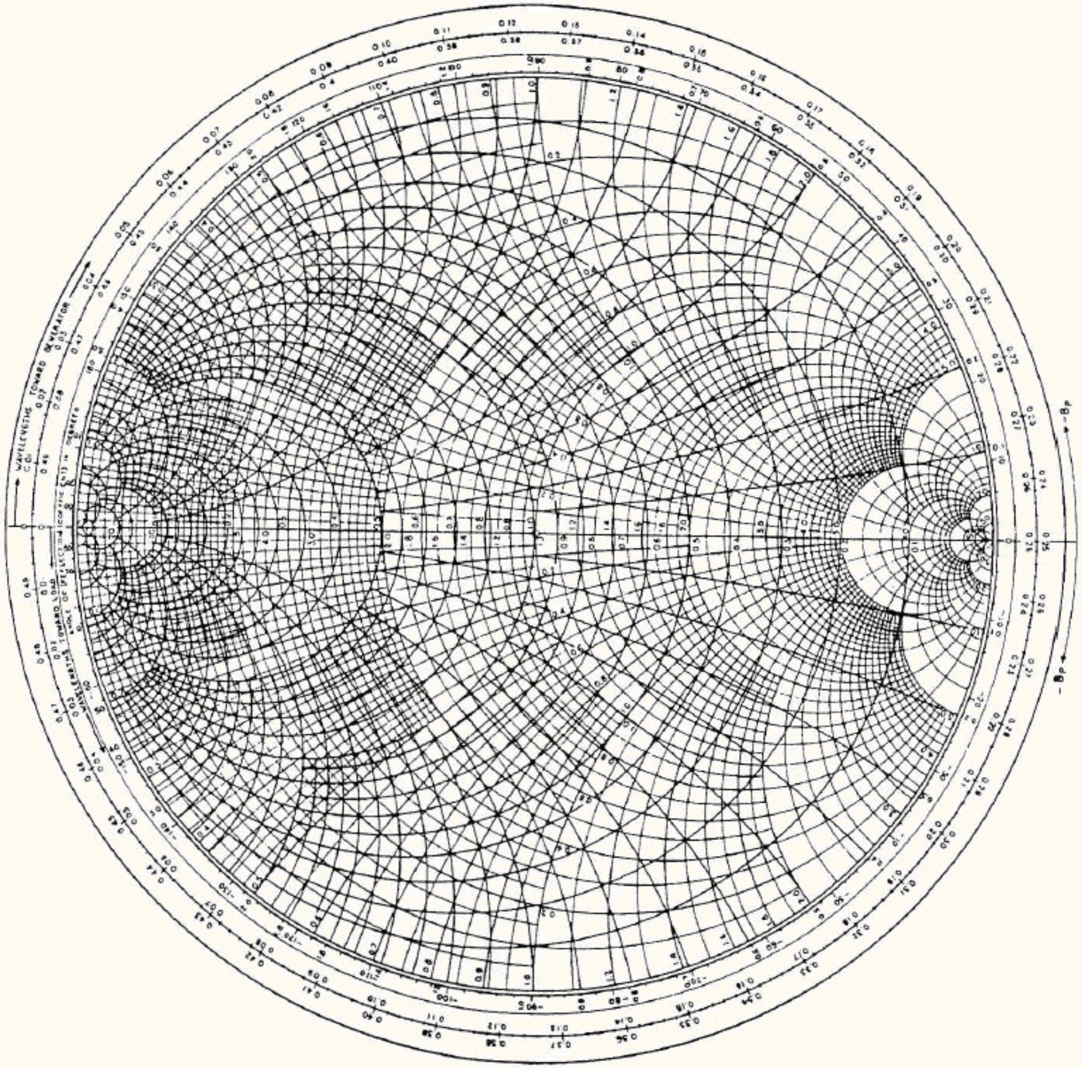
Admittance chart

$$r = u + iw$$



Smith chart

5.2.3 Smith chart, Immittance chart



Immittance chart

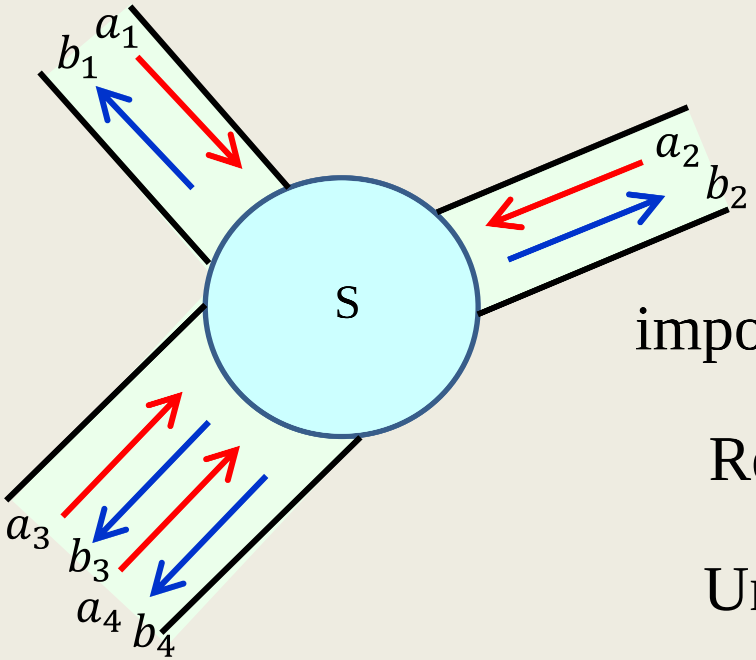
$$r = u + iw$$

5.3 Scattering (S) matrix (S parameters)

How to treat multipoint (crossing point) systematically?

Transmission lines: wave propagating modes → Channels

Take $|a_i|^2$, $|b_i|^2$ to be powers (energy flow).



output S-matrix input

$$\begin{pmatrix} b_1 \\ \vdots \\ b_i \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} S_{11} & \cdots & S_{1i} & \cdots & S_{1n} \\ \vdots & \ddots & & & \vdots \\ S_{i1} & & S_{ii} & & S_{in} \\ \vdots & & & \ddots & \vdots \\ S_{n1} & \cdots & S_{ni} & \cdots & S_{nn} \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{pmatrix}$$

$$\boldsymbol{b} = \mathrm{S}\boldsymbol{a}$$

important properties:

Reciprocity $S_{ij} = S_{ji}$

Unitarity $\sum_j S_{ji} S_{jk}^* = \delta_{ik}$

(In case, no dissipation, no amplification)

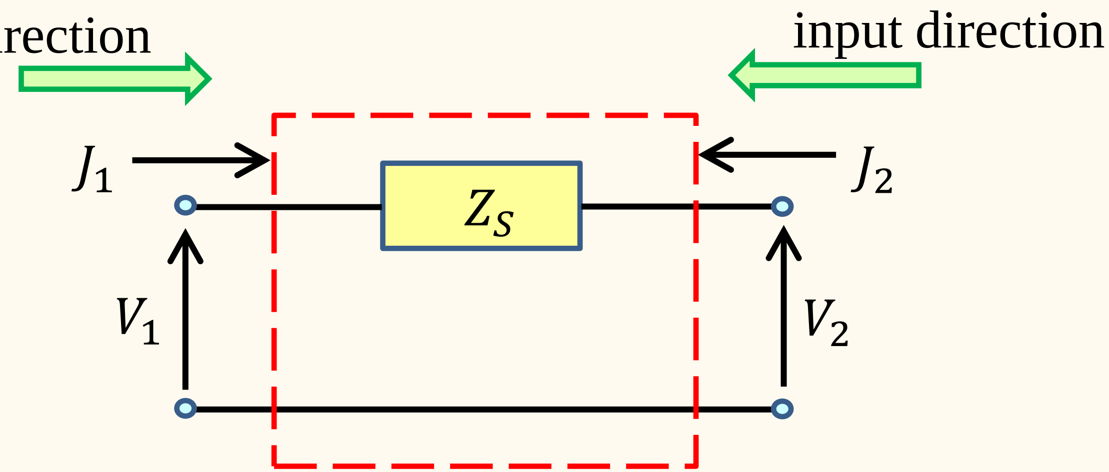
5.3 S matrix (S parameters)

Propagation with no dissipation

$$\begin{cases} a_n = \dfrac{V_{n+}}{\sqrt{Z_{0n}}} = J_{n+}\sqrt{Z_{0n}}, & \text{incident power wave} \\ b_n = \dfrac{V_{n-}}{\sqrt{Z_{0n}}} = J_{n-}\sqrt{Z_{0n}} & \text{reflected (transmitted) power wave} \end{cases}$$

$$|a_n|^2 = \frac{|V_{n+}|^2}{Z_{0n}} = |J_{n+}|^2 Z_{0n}$$

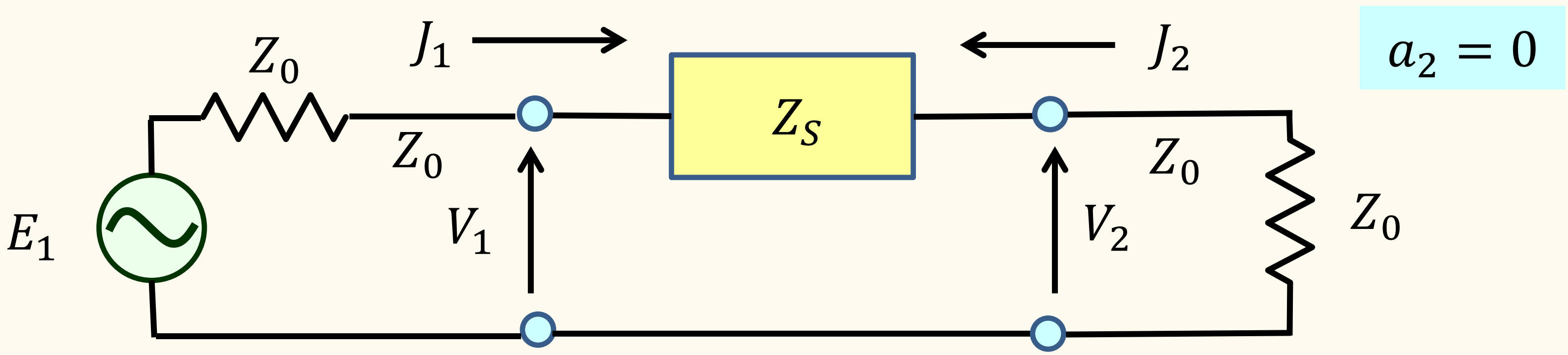
Simplest example: series impedance Z_S



Take voltage as the "flow" quantity.
(assume common characteristic impedance)

5.3 S-matrix (S-parameters)

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \qquad \begin{pmatrix} V_{1-} \\ V_{2-} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_{1+} \\ V_{2+} \end{pmatrix}$$



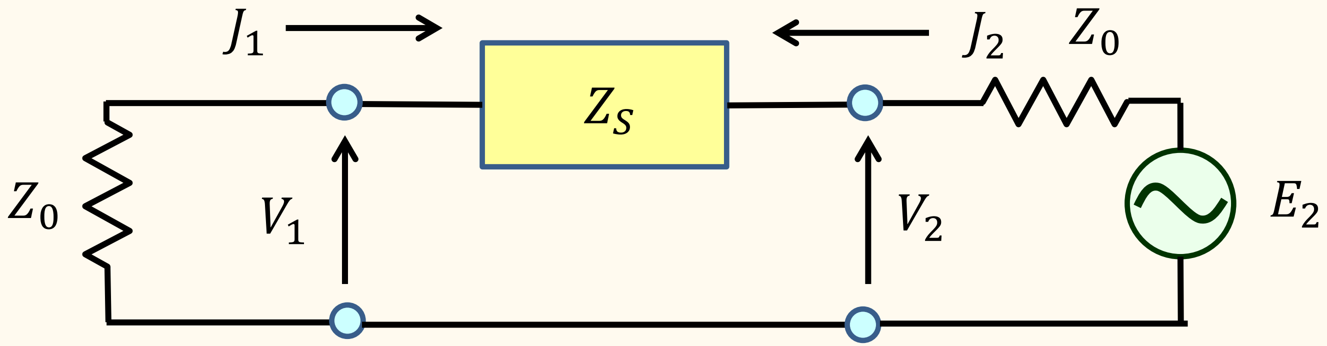
Terminate 2 with $Z_0 \rightarrow a_2 = 0$

$$S_{11} = \frac{V_{1-}}{V_{1+}} = \frac{V_1 - Z_0 J_1}{V_1 + Z_0 J_1} = \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{(Z_S + Z_0) - Z_0}{(Z_S + Z_0) + Z_0} = \frac{Z_S}{Z_S + 2Z_0}$$

$$S_{21} = \frac{V_{2-}}{V_{1+}} = \frac{V_2 - Z_0 J_2}{V_1 + Z_0 J_1} = \frac{Z_0 J_1 + Z_0 J_1}{(Z_S + Z_0) J_1 + Z_0 J_1} = \frac{2Z_0}{Z_S + 2Z_0} \quad (J_2 = -J_1)$$

5.3 S-matrix (S-parameters)

$a_1 = 0$



Terminate 1 with $Z_0 \rightarrow a_1 = 0$ (should be symmetric)

$$S_{12} = \frac{2V_1}{V_2 + Z_0 J_2} = \frac{2Z_0 J_2}{(Z_S + Z_0)J_2 + Z_0 J_2} = \frac{2Z_0}{Z_S + 2Z_0}$$

$$S_{22} = \frac{Z_S}{Z_S + 2Z_0}$$

Generally

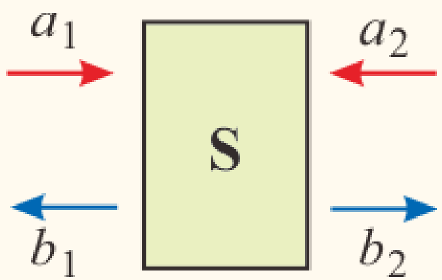
$$\mathrm{S} = \frac{1}{\det Z} \times \begin{pmatrix} (Z_{11} - Z_0)(Z_{22} + Z_0) - Z_{12}Z_{21} & 2Z_0 Z_{12} \\ 2Z_0 Z_{21} & (Z_{11} + Z_0)(Z_{22} - Z_0) - Z_{12}Z_{21} \end{pmatrix}$$

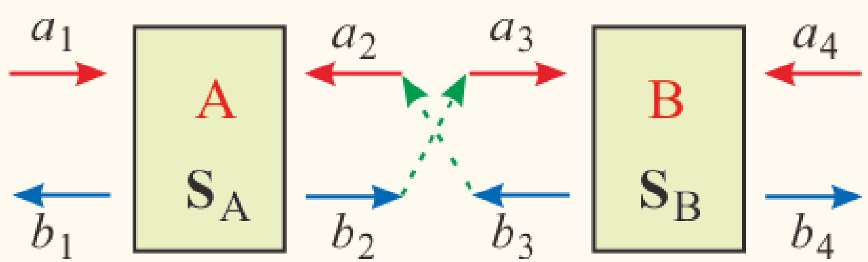
Cascade connection of S-matrices

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \mathbf{S} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} r_\mathrm{L} & t_\mathrm{R} \\ t_\mathrm{L} & r_\mathrm{R} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$r_{L,R}$, $t_{L,R}$: complex reflection, transmission coefficients satisfying

$$T_\mathrm{L,R} = |t_\mathrm{L,r}|^2 = 1 - R_\mathrm{L,R} = 1 - |r_\mathrm{L,R}|^2$$

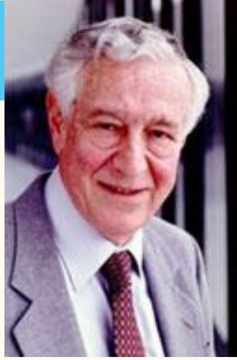




$$\mathbf{S}_\mathrm{AB} = \begin{pmatrix} r_\mathrm{L}^\mathrm{AB} & t_\mathrm{R}^\mathrm{AB} \\ t_\mathrm{L}^\mathrm{AB} & r_\mathrm{R}^\mathrm{AB} \end{pmatrix} = \begin{pmatrix} r_\mathrm{L}^\mathrm{A} + t_\mathrm{R}^\mathrm{A} r_\mathrm{L}^\mathrm{B} \left(I - r_\mathrm{R}^\mathrm{A} r_\mathrm{L}^\mathrm{B}\right)^{-1} t_\mathrm{L}^\mathrm{A} & t_\mathrm{R}^\mathrm{A} \left(I - r_\mathrm{L}^\mathrm{B} r_\mathrm{R}^\mathrm{A}\right)^{-1} t_\mathrm{R}^\mathrm{B} \\ t_\mathrm{L}^\mathrm{B} \left(I - r_\mathrm{R}^\mathrm{A} r_\mathrm{L}^\mathrm{B}\right)^{-1} t_\mathrm{L}^\mathrm{A} & r_\mathrm{R}^\mathrm{B} + t_\mathrm{L}^\mathrm{B} \left(I - r_\mathrm{R}^\mathrm{A} r_\mathrm{L}^\mathrm{B}\right)^{-1} r_\mathrm{R}^\mathrm{A} t_\mathrm{R}^\mathrm{B} \end{pmatrix}$$

$$(I - r_R^A r_L^B)^{-1} = I + r_R^A r_L^B + (r_R^A r_L^B)^2 + \cdots$$

Conduction channels in quantum transport



Rolf Landauer

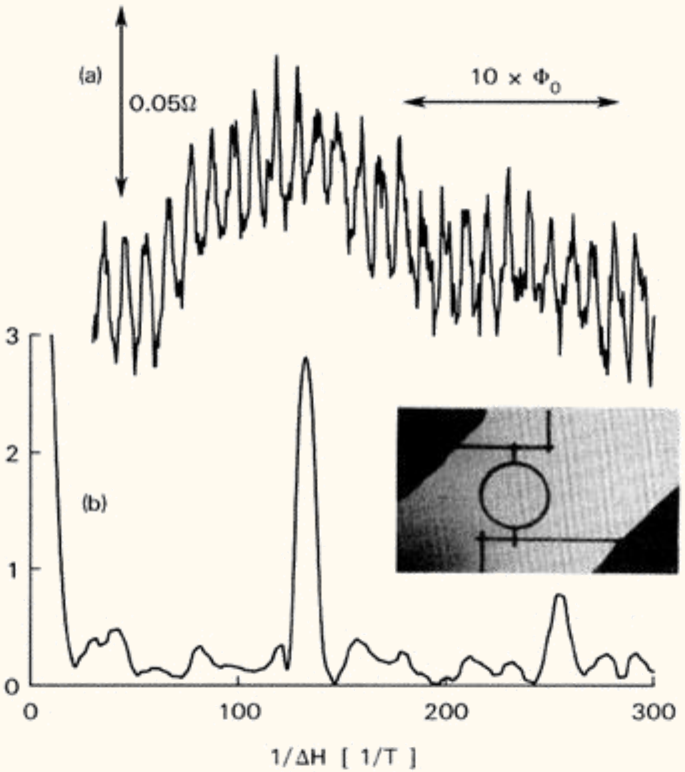
Electron (quantum mechanical) waves also have propagating modes in solids.

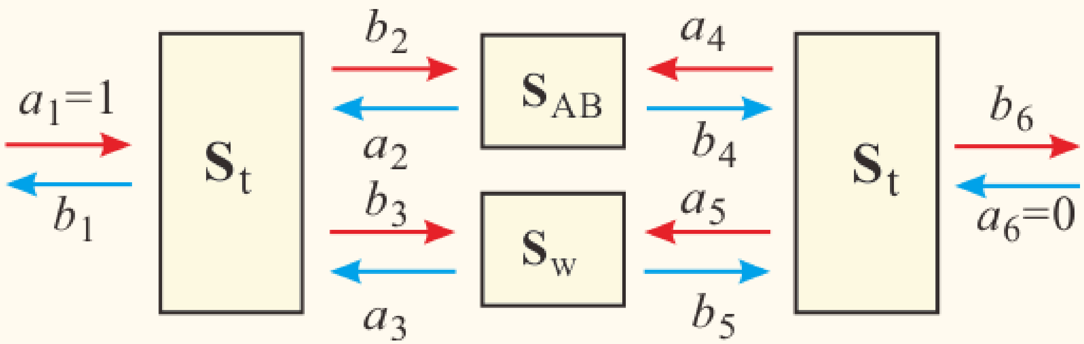
→ Conduction channel

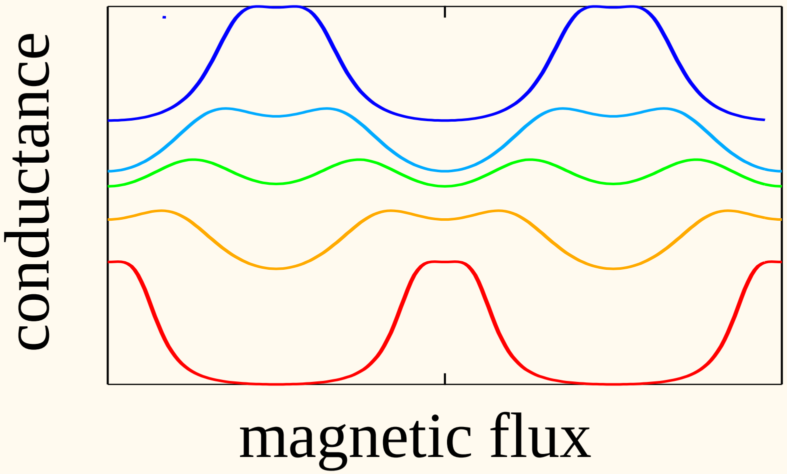
Landauer eq.:

the conductance of a single perfect quantum channel is $\dfrac{e^2}{h}$

AB ring S-matrix model







S-parameter representation of high-frequency devices

DATA SHEET

NEC

GaAs MES FET

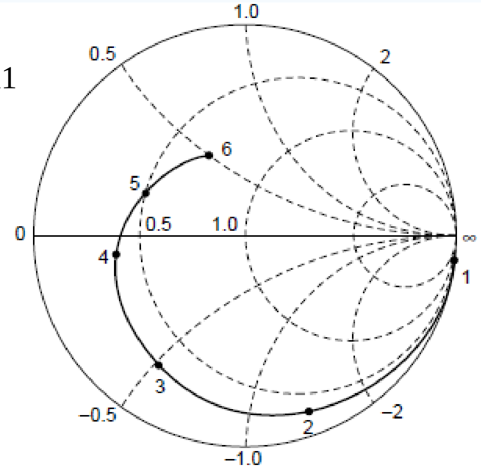
NE76084

C to Ku BAND LOW NOISE AMPLIFIER
N-CHANNEL GaAs MES FET

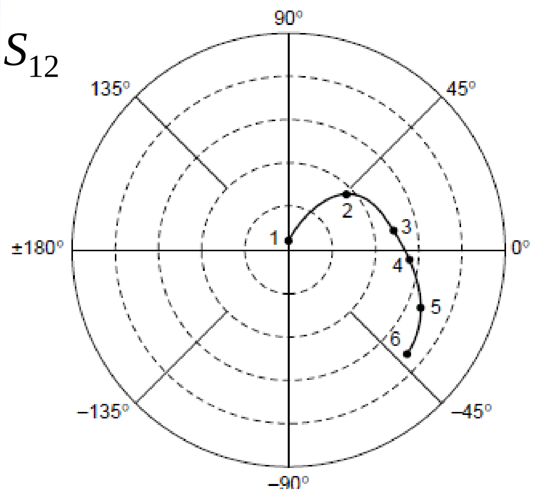
S-PARAMETERS

V_{DS} = 3 V, I_D = 10 mA
START 500 MHz, STOP 18 GHz, STEP 500 MHz

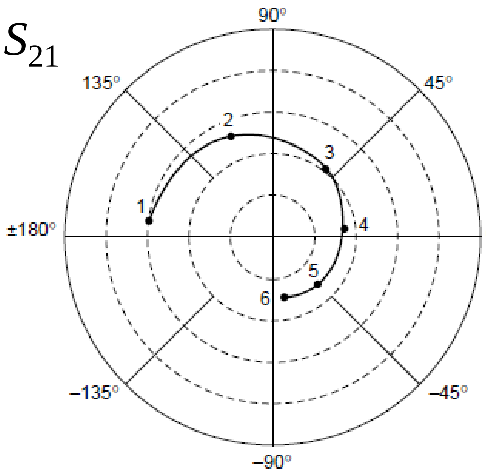
S_{11}



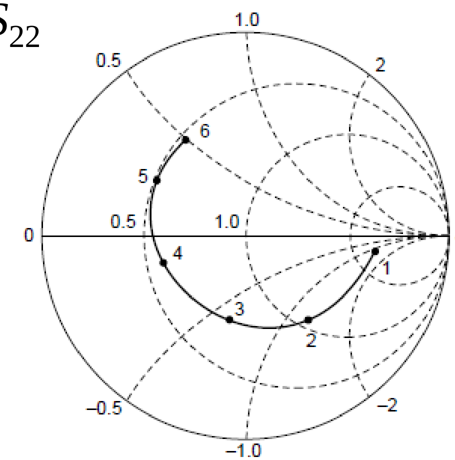
S_{12}



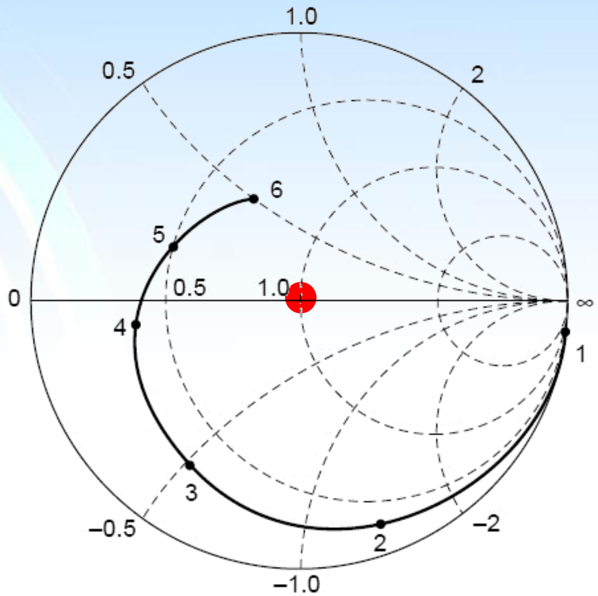
S_{21}



S_{22}



Ex) NE76084 MES FET 0.5～18GHz



S_{11}

The datasheet tells that we need impedance matching circuits with transmission lines.

If we know Z-parameters:

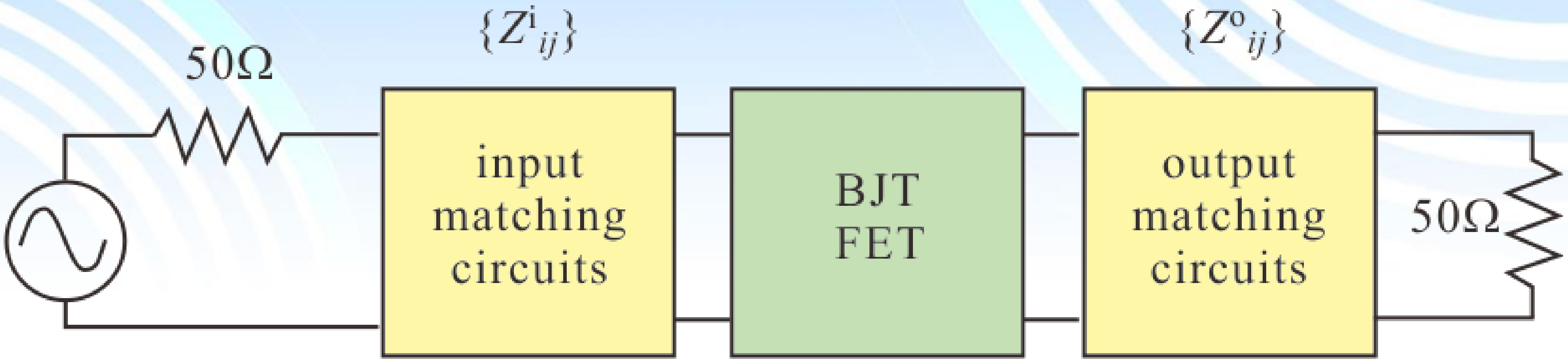
From Ho-Thevenin theorem

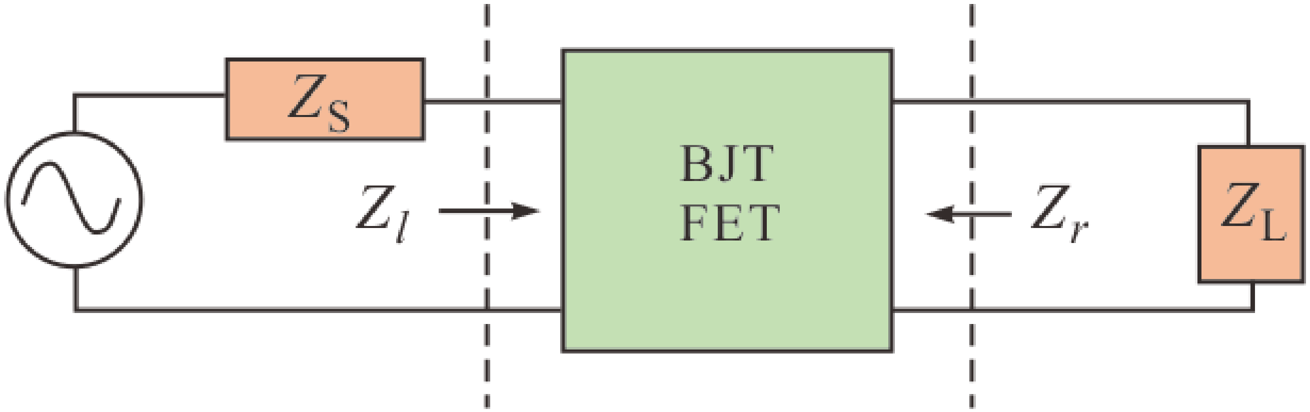
$$Z_\mathrm{S} = Z^\mathrm{i}_{22} - \frac{Z^\mathrm{i}_{12} Z^\mathrm{i}_{21}}{50 + Z^\mathrm{i}_{11}}, \quad Z_\mathrm{L} = Z^\mathrm{o}_{11} - \frac{Z^\mathrm{o}_{12} Z^\mathrm{o}_{21}}{50 + Z^\mathrm{o}_{22}}$$

$\{Z_{ij}\}$: BJT (FET) Z-parameters, again Ho-Thevenin says

$$Z_l = Z_{11} - \frac{Z_{12} Z_{21}}{Z_\mathrm{L} + Z_{22}}, \quad Z_r = Z_{22} - \frac{Z_{12} Z_{21}}{Z_\mathrm{S} + Z_{11}}$$

S-parameter representation of high-frequency devices





Impedance matching with S-parameters

Generally the unitarity does not hold for amplification.

$$R_{\mathrm{in}} = S_{11} + \frac{S_{12}S_{21}R_{\mathrm{L}}}{1 - S_{22}R_{\mathrm{L}}} \qquad R_{\mathrm{out}} = S_{22} + \frac{S_{12}S_{21}R_{\mathrm{S}}}{1 - S_{11}R_{\mathrm{S}}}$$

Matching condition: $R_{\mathrm{L}} = R_{\mathrm{out}}^{*}, \quad R_{\mathrm{S}} = R_{\mathrm{in}}^{*}$

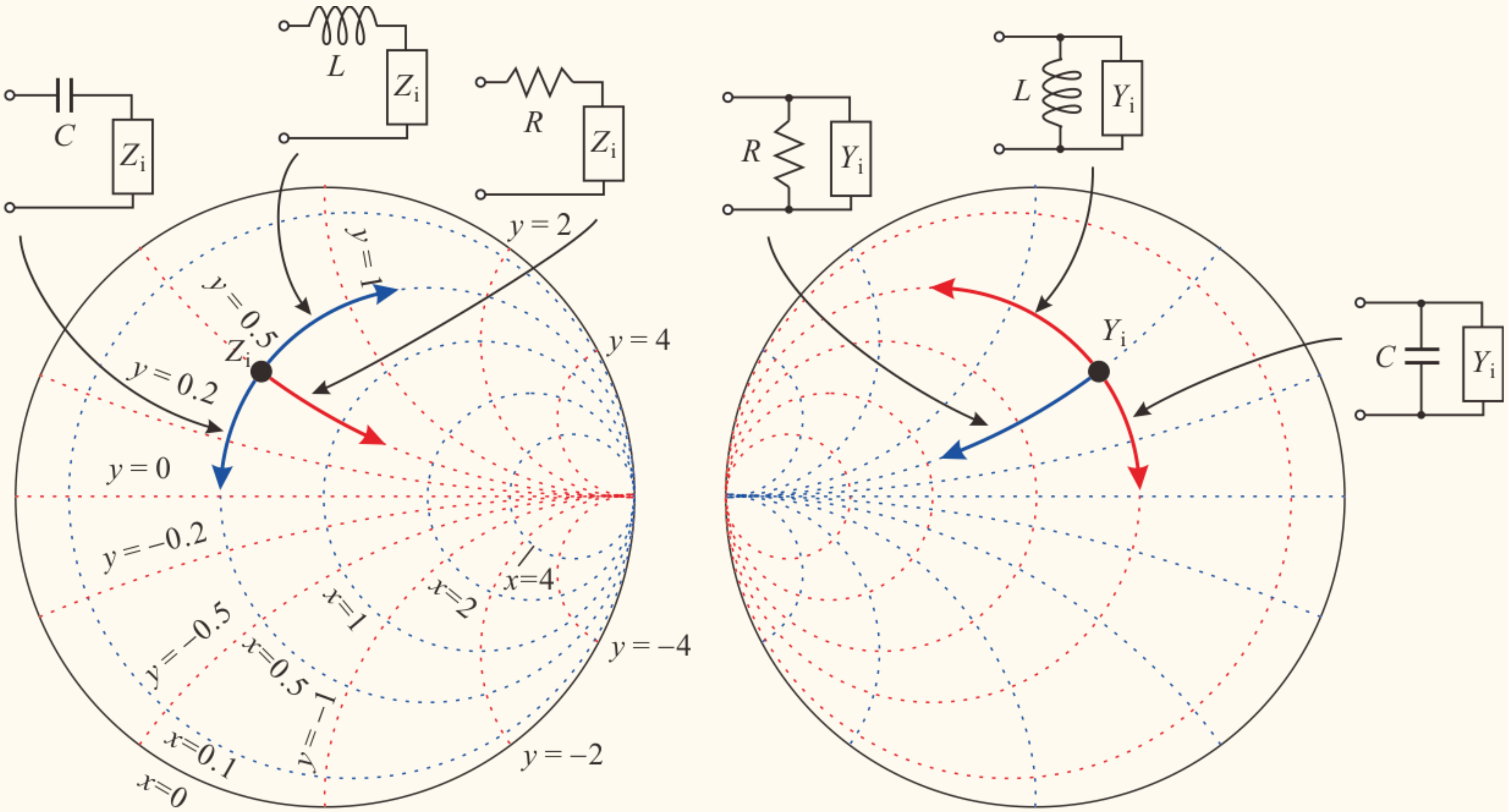
Solution $$R_{\mathrm{S}} = \frac{B_1 \pm \sqrt{B_1^2 - 4|M|^2}}{2M}, \quad R_{\mathrm{L}} = \frac{B_2 \pm \sqrt{B_2^2 - 4|N|^2}}{2N}$$ with

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\mathrm{det}S|^2, \quad B_2 = 1 - |S_{11}|^2 + |S_{22}|^2 - |\mathrm{det}S|^2,$$
$$N = S_{22} - S_{11}^{*}\mathrm{det}S, \quad M = S_{11} - S_{22}^{*}\mathrm{det}S$$

maximum available power gain $$G_{\mathrm{max}} = \left|\frac{S_{21}}{S_{12}}\right|(K - \sqrt{K^2 - 1})$$

$$K = \frac{1 + |\mathrm{det}S|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|}$$ stability factor

Practical impedance matching with Simth chart

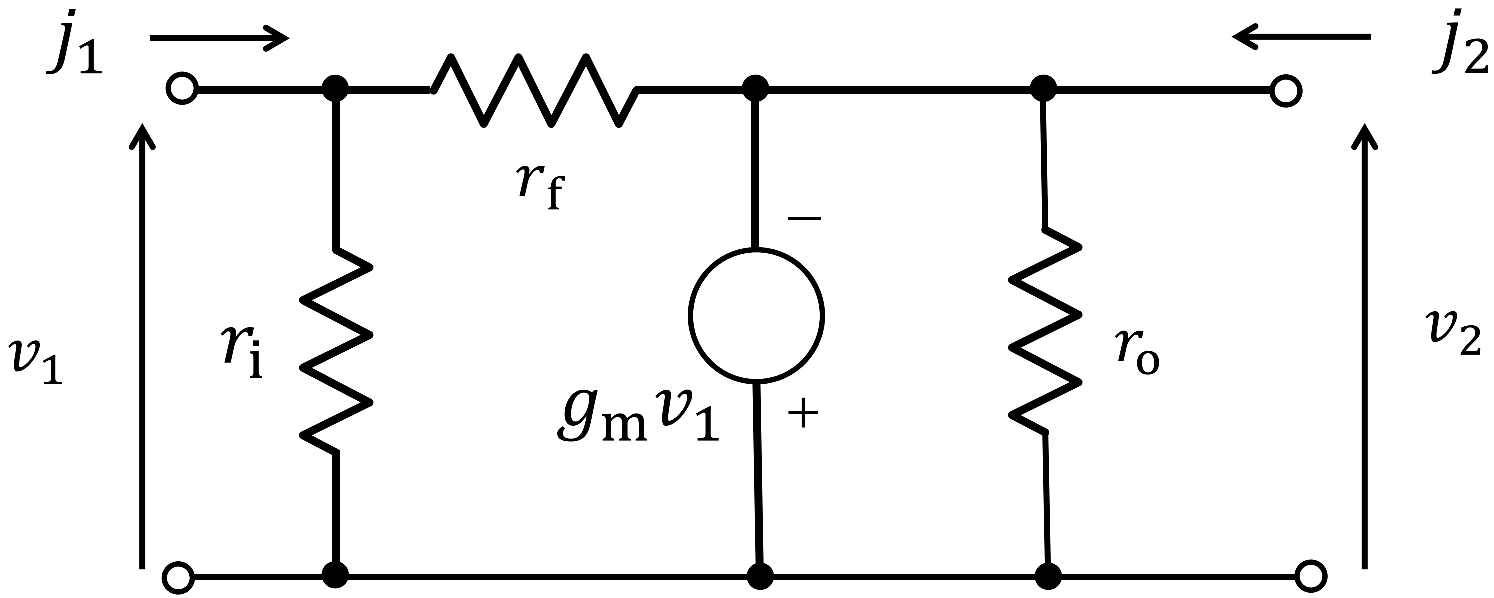


Impedance matching designer

http://home.sandiego.edu/~ekim/e194rfs01/jwmatcher/matcher2.html

http://leleivre.com/rf_lcmatch.html

Exercise D-1



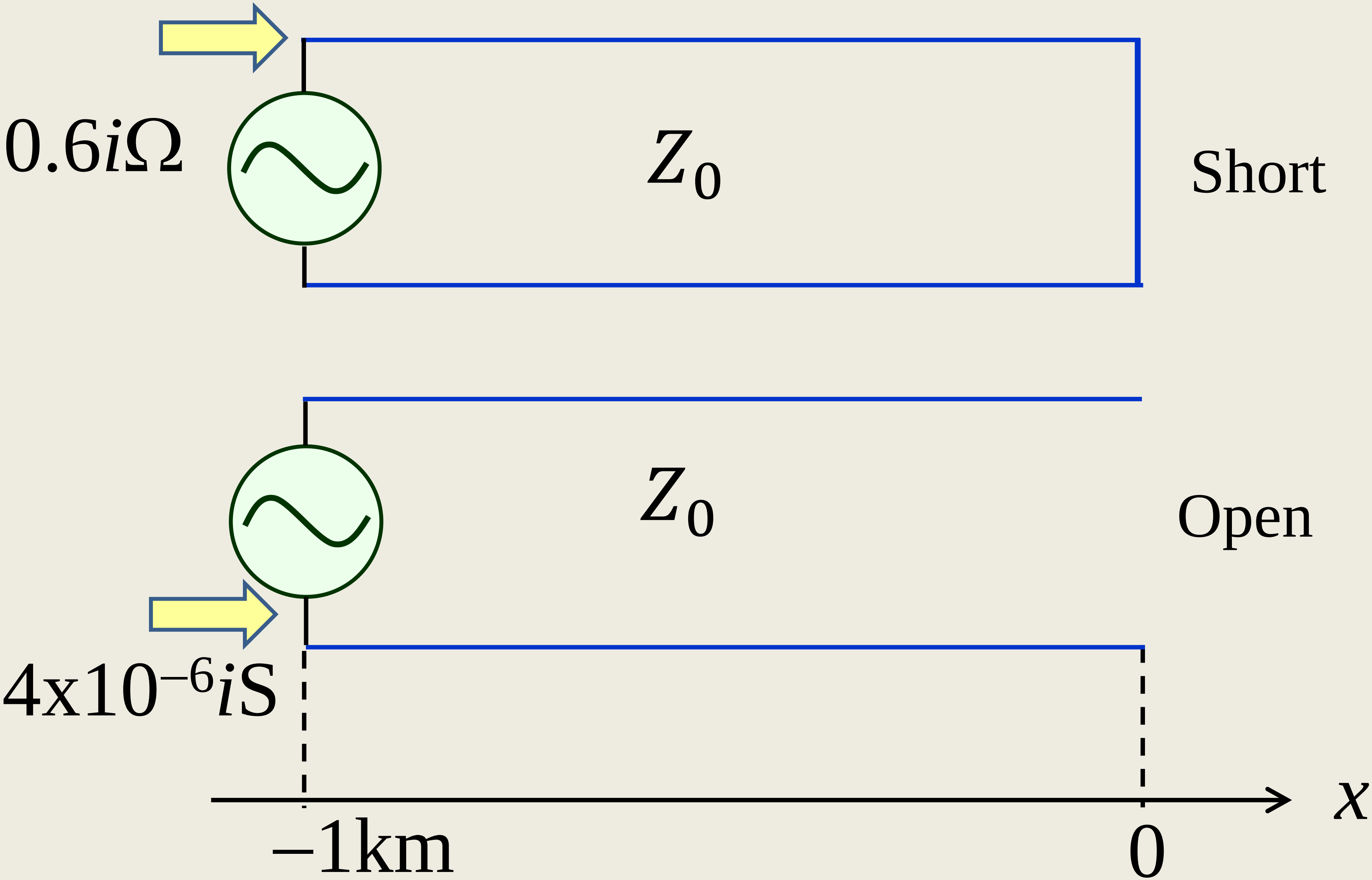
Obtain the Y matrix for the above equivalent circuit (π-shape circuit).

Exercise D-2

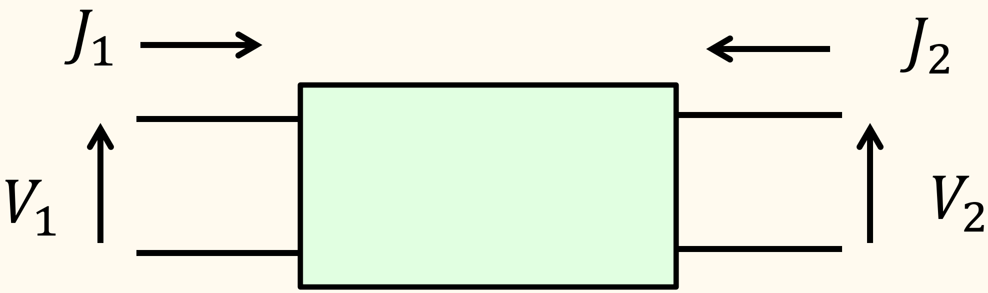
l =1km の伝送線路がある．終端側を短絡したところ，電源側から測定したインピダンスは0.6i Ωであった．一方，終端側を開放して電源側からアドミタンスを測定すると4x10^{-6}i Sであった．
この伝送線路の特性インピダンスを求めよ．

Consider a transmission line with the length l = 1km. First we short-circuited the end and measured the impedance from the signal source and obtained 0.6i Ω. Next we opened the end and measured the admittance from the signal source and obtained 4x10^{-6}i S.
What is the characteristic impedance of the transmission line?

Exercise D-2



Exercise D-3



$$\begin{pmatrix} V_1 \\ J_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ J_2 \end{pmatrix}$$

Remember F-matrix (cascade matrix) defined above.

Write down the F-matrix form of the transmission line shown below.