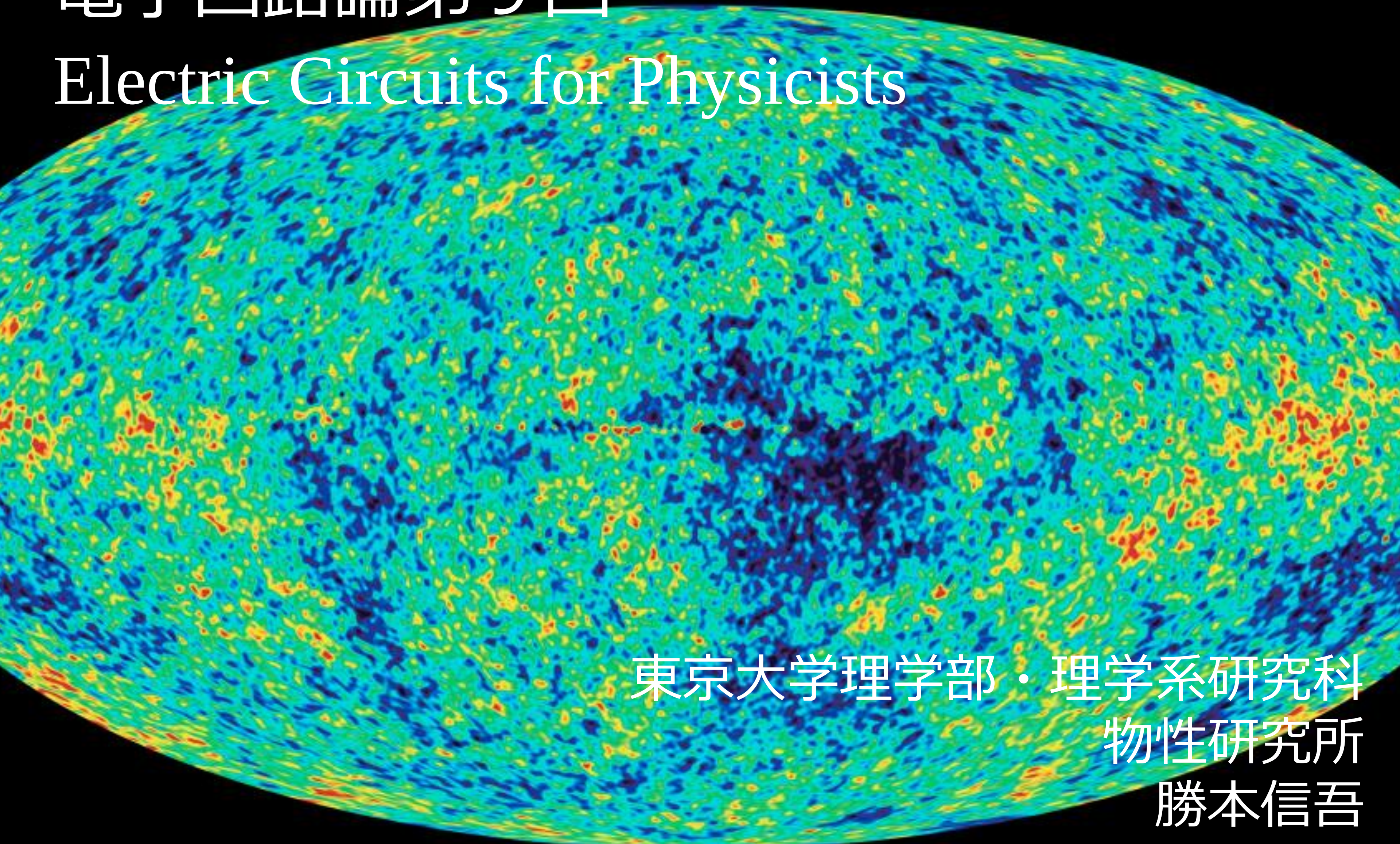
電子回路論第９回

Electric Circuits for Physicists



東京大学理学部・理学系研究科
物性研究所
勝本信吾
Shingo Katsumoto


GSFC

Outline

5.3 S-parameter representation of devices

Impedance matching with Z, S parameters

Impedance matching with immittance chart

5.4 Non-TEM mode transmission lines

5.5 Non-linear elements and Toda lattice

Ch.6 Noises and Signals

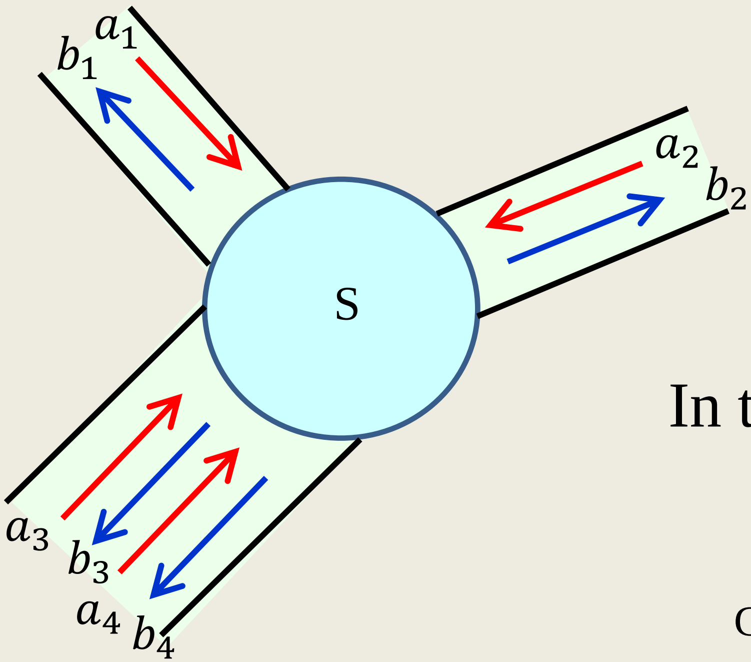
6.1 Fluctuations

6.2 Fluctuation-dissipation theorem

Review: Scattering (S) matrix (S parameters)

Transmission lines: wave propagating modes → Channels

Take $|a_i|^2$, $|b_i|^2$ to be powers (energy flow).

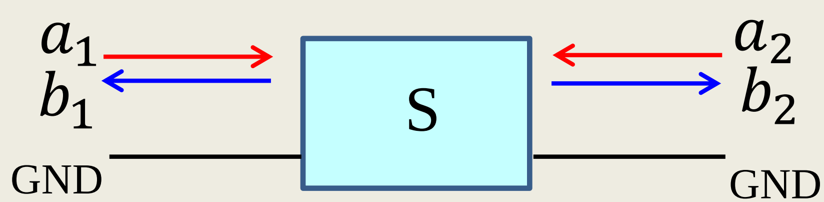


output S-matrix input

$$\begin{pmatrix} b_1 \\ \vdots \\ b_i \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} S_{11} & \cdots & S_{1i} & \cdots & S_{1n} \\ \vdots & \ddots & & & \vdots \\ S_{i1} & & S_{ii} & & S_{in} \\ \vdots & & & \ddots & \vdots \\ S_{n1} & \cdots & S_{ni} & \cdots & S_{nn} \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{pmatrix}$$

$$\boldsymbol{b} = \mathrm{S}\boldsymbol{a}$$

In the case of two-terminal pair circuit



$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} r_\mathrm{l} & t_\mathrm{r} \\ t_\mathrm{l} & r_\mathrm{r} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

S-parameter representation of high-frequency devices

DATA SHEET

NEC

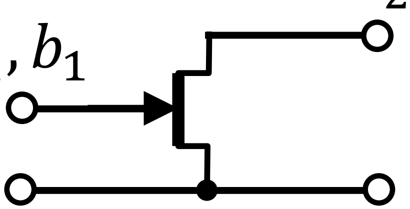
GaAs MES FET

NE76084

C to Ku BAND LOW NOISE AMPLIFIER
N-CHANNEL GaAs MES FET

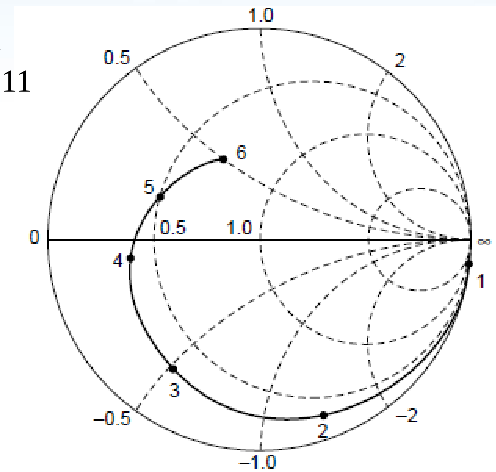
S-PARAMETERS

V_{DS} = 3 V, I_D = 10 mA
START 500 MHz, STOP 18 GHz, STEP 500 MHz

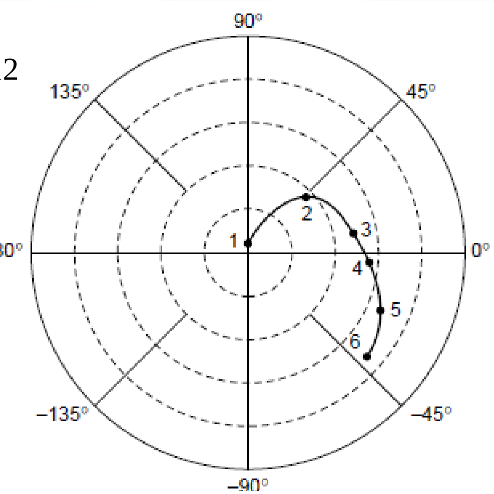


$$S_{11} = r_l, S_{22} = r_r$$

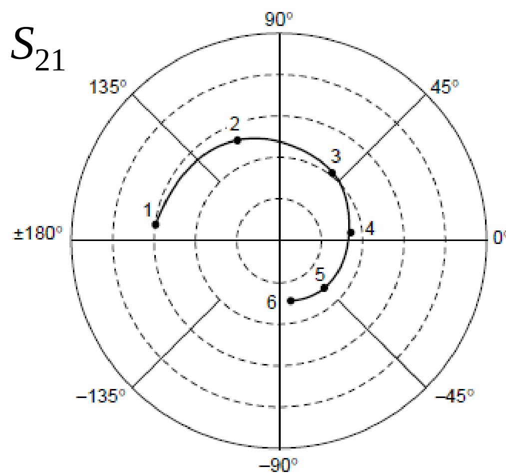
S_{11}



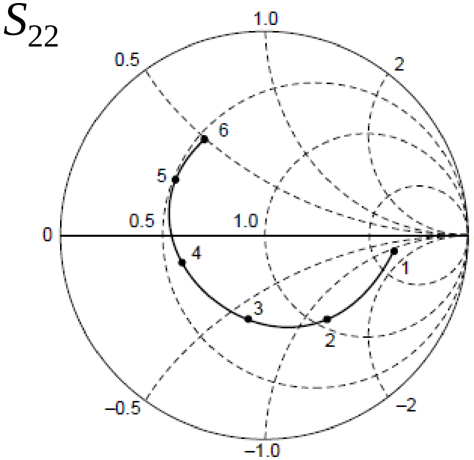
S_{12}



S_{21}



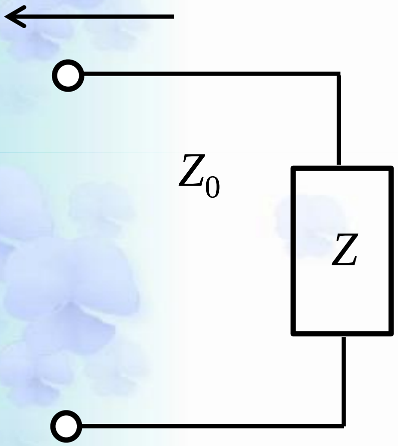
S_{22}



Review: Smith Chart

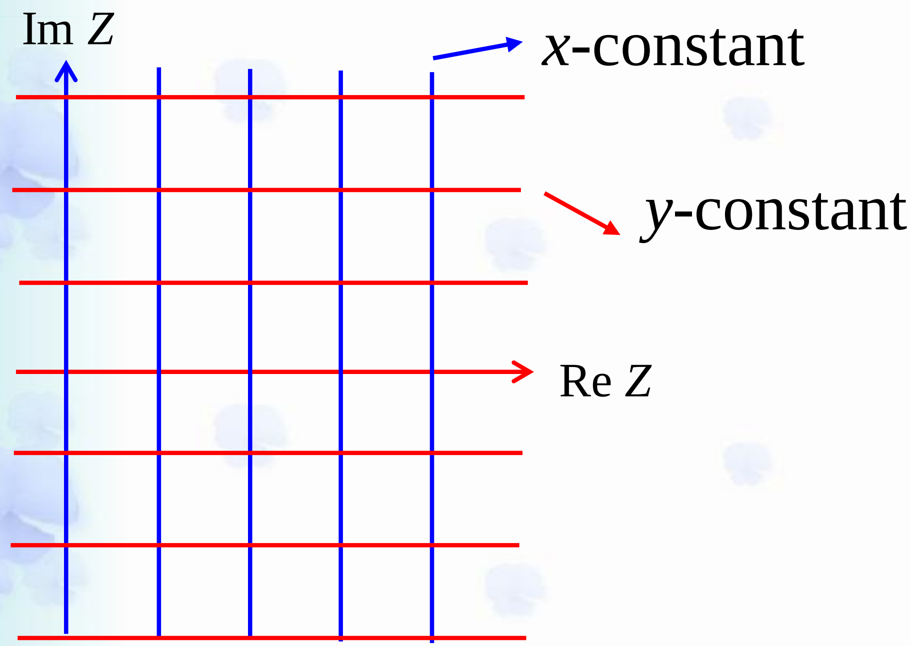
P.H. Smith
1905-1987

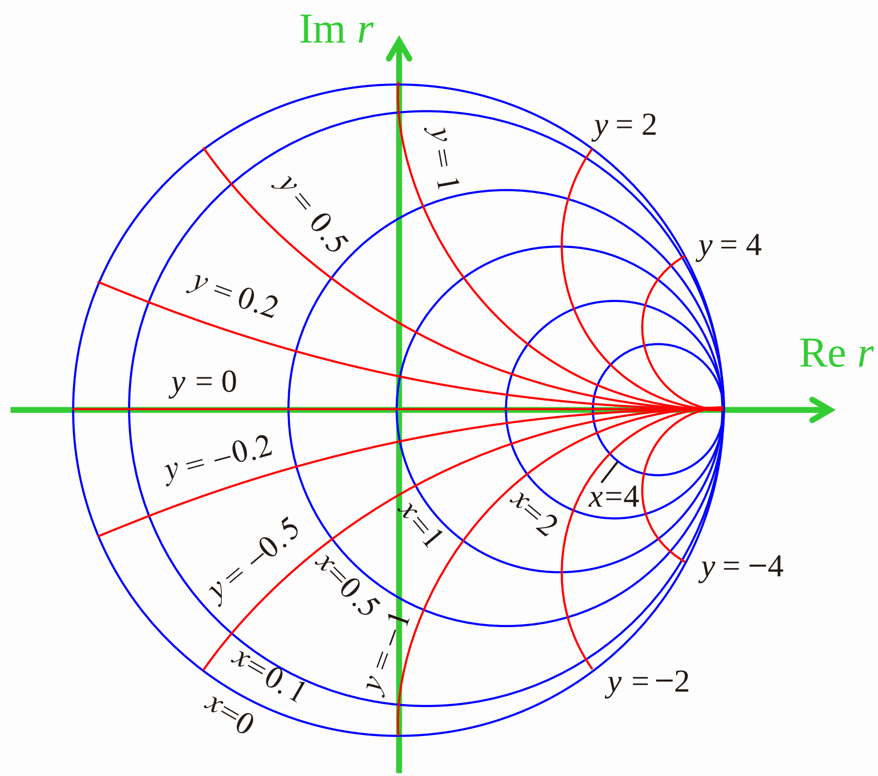
r : reflection coefficient



$$r = \frac{V_-}{V_+} = \frac{Z - Z_0}{Z + Z_0} = \frac{z-1}{z+1}$$

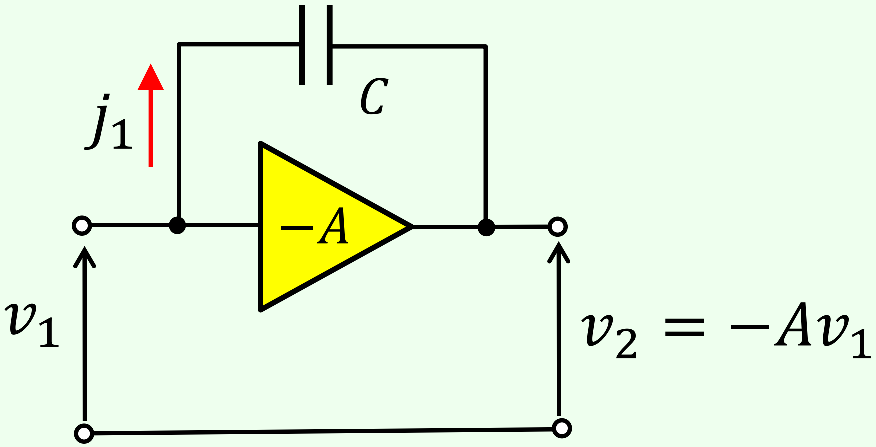
$$z = \frac{Z}{Z_0} = x + iy$$



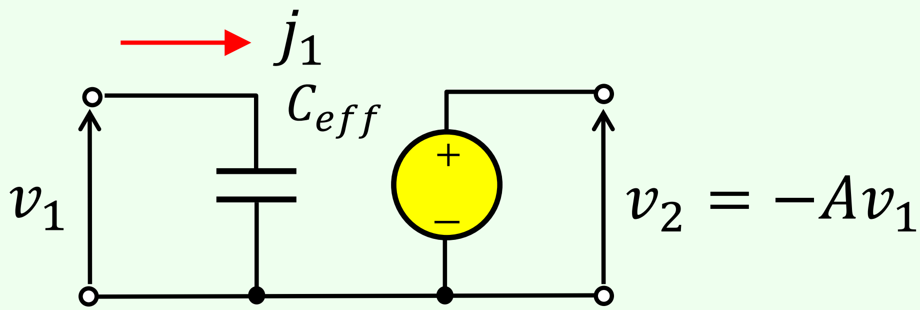


Comment: Mirror effect

An amplifier may change the effective impedance of passive elements.



equivalent circuit



$$j_1 = sC(v_1 - v_2) = sC(1 + A)v_1$$

$$C_{\text{eff}} \frac{dv_1}{dt} = sC_{\text{eff}} v_1 = j_1$$

$$s = \frac{j_1}{(1 + A)Cv_1}$$

$$s = \frac{1}{C_{\text{eff}}} \frac{j_1}{v_1}$$

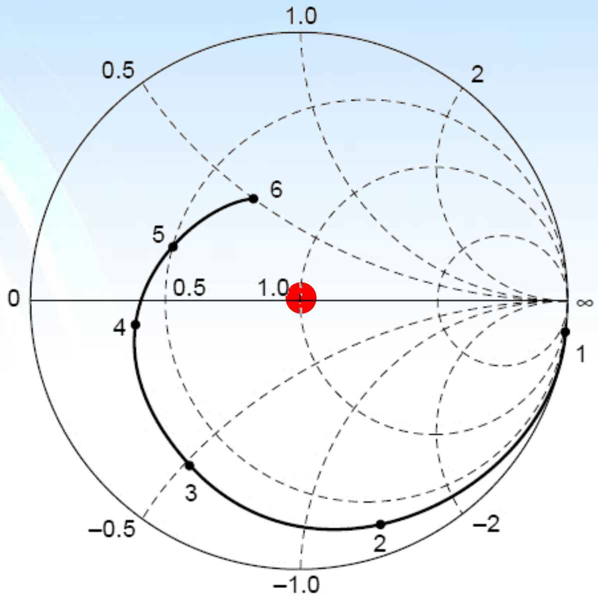
$$C_{\text{eff}} = (1 + A)C$$

: mirror effect

S-parameter representation of high-frequency devices

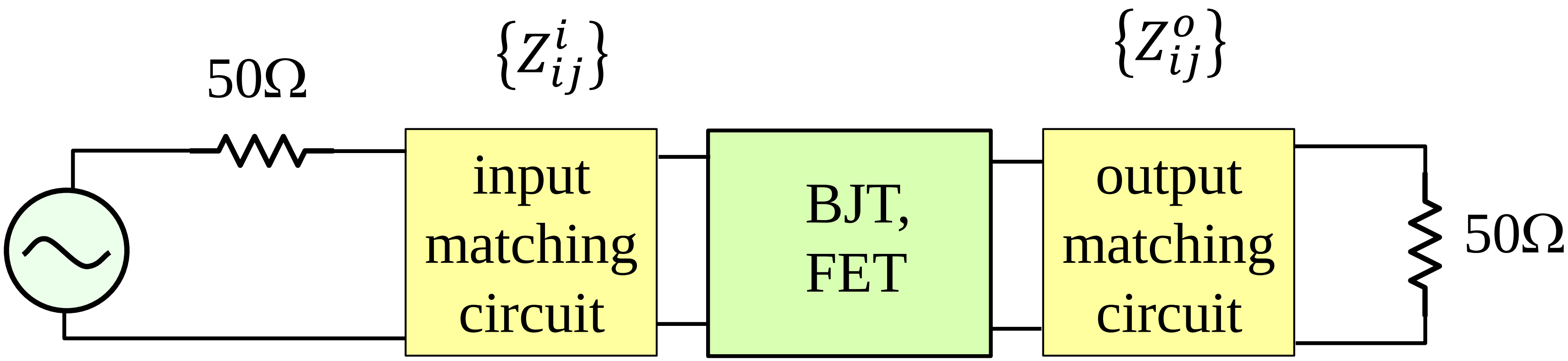
Ex) NE76084 MES FET 0.5～18GHz

S_{11}



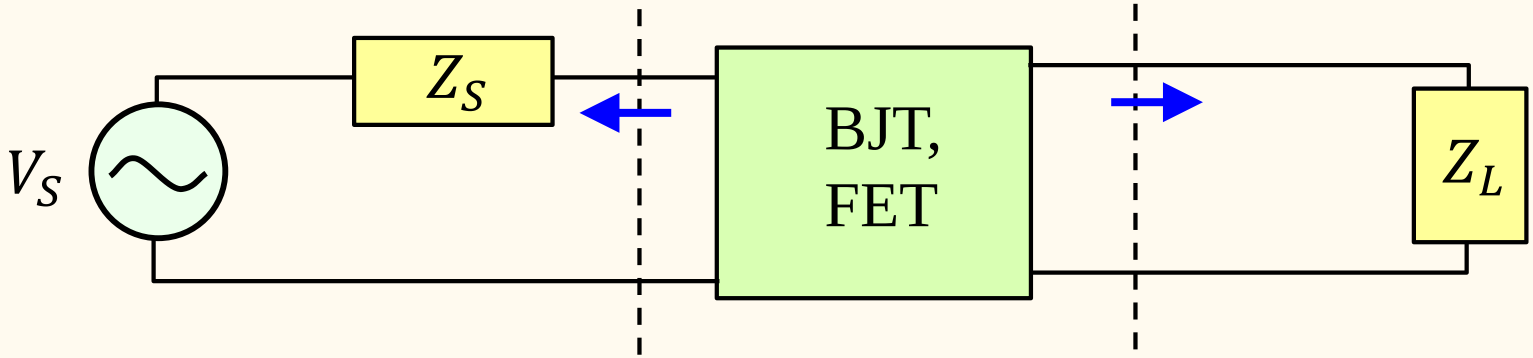
The datasheet tells that we need impedance matching circuits with transmission lines with $Z_0 = 50\ \Omega$.

Insert input and output matching circuits to kill reflections.



Impedance matching circuits

The circuit is summarized at the boundaries as

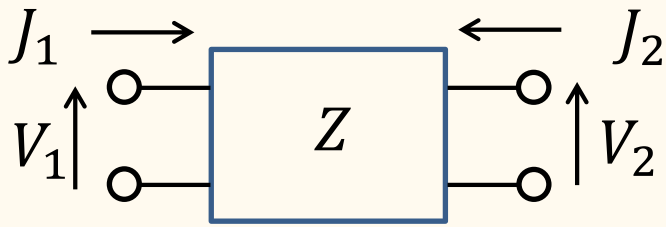


If we know Z-parameters of the input/output matching circuits, from Ho-Thevenin's theorem

$$Z_{\mathrm{S}} = Z^{\mathrm{i}}_{22} - \frac{Z^{\mathrm{i}}_{12} Z^{\mathrm{i}}_{21}}{50 + Z^{\mathrm{i}}_{11}}, \quad Z_{\mathrm{L}} = Z^{\mathrm{o}}_{11} - \frac{Z^{\mathrm{o}}_{12} Z^{\mathrm{o}}_{21}}{50 + Z^{\mathrm{o}}_{22}}.$$

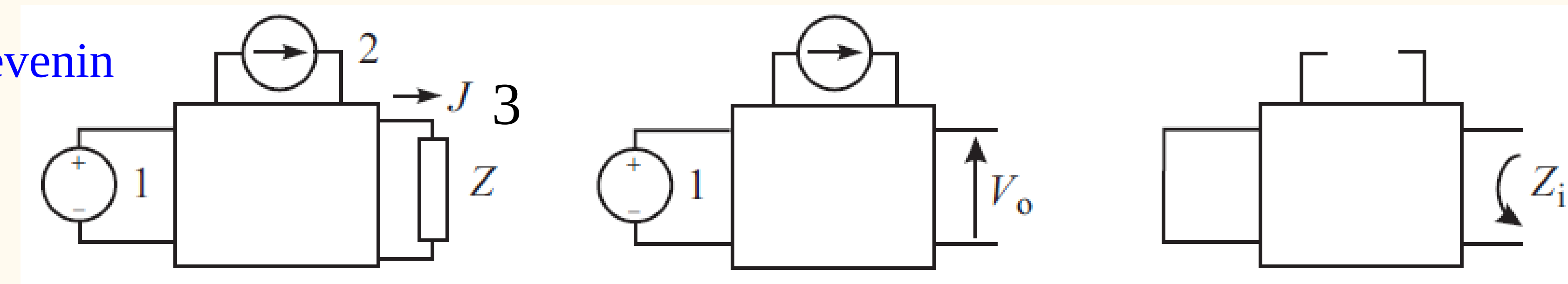
Z-matrix, Ho-Thevenin's theorem

Z-matrix



$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} J_1 \\ J_2 \end{pmatrix}$$

Ho-Thevenin



1. Measure the open terminal voltage V_0.
2. Turn off all the power sources (voltage sources: short, current sources: open). Measure the open circuit impedance Z_i.

Then

$$J = \frac{V_0}{Z + Z_i}$$

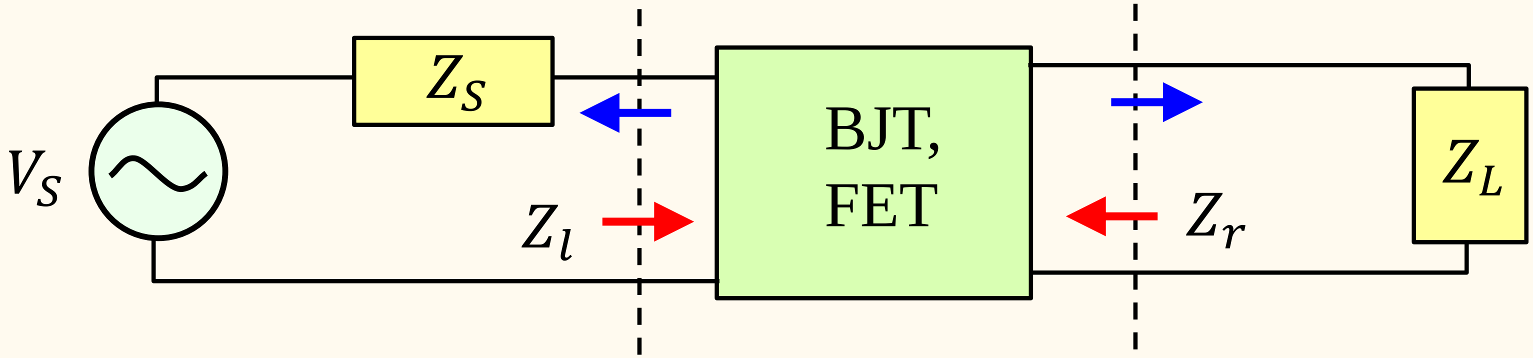
$$V_s = 0 \ \therefore V_1 = -50 J_1 = Z_{11} J_1 + Z_{12} J_2 \quad \therefore J_1 = -\frac{Z_2}{50 + Z_{11}} J_2$$

$$V_2 = Z_{21} J_1 + Z_{22} J_2 = \left(Z_{22} - \frac{Z_{21} Z_{12}}{50 + Z_{11}} \right) J_2$$

$\rightarrow Z_S$

Impedance matching circuits

The circuit is summarized at the boundaries as



If we know Z-parameters of the input/output matching circuits, from Ho-Thevenin's theorem

$$Z_{\rm S} = Z^{\rm i}_{22} - \frac{Z^{\rm i}_{12} Z^{\rm i}_{21}}{50 + Z^{\rm i}_{11}}, \quad Z_{\rm L} = Z^{\rm o}_{11} - \frac{Z^{\rm o}_{12} Z^{\rm o}_{21}}{50 + Z^{\rm o}_{22}}.$$

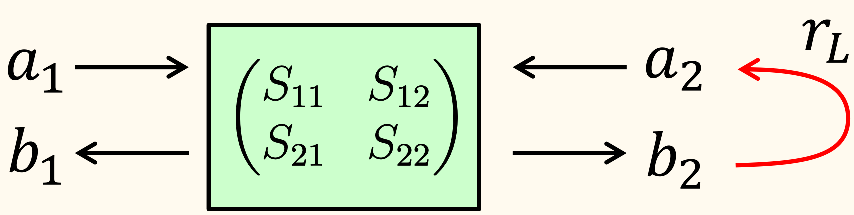
$\{Z_{ij}\}$: BJT (FET) Z-parameters, again Ho-Thevenin says

$$Z_l = Z_{11} - \frac{Z_{12} Z_{21}}{Z_{\rm L} + Z_{22}}, \quad Z_r = Z_{22} - \frac{Z_{12} Z_{21}}{Z_{\rm S} + Z_{11}}$$

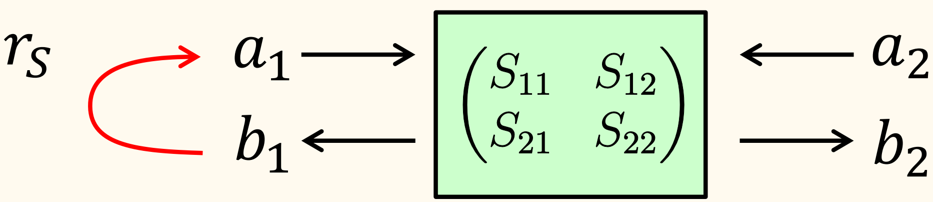
matching condition: $Z_l = Z_S^*,\ Z_r = Z_L^*$

Impedance matching with S-parameters

In S-parameter treatment, we use complex reflection coefficients to express load, source etc.



$$r_{\mathrm{in}} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}r_{\mathrm{L}}}{1 - S_{22}r_{\mathrm{L}}}$$



$$r_{\mathrm{out}} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}r_{\mathrm{S}}}{1 - S_{11}r_{\mathrm{S}}}$$

Matching condition: $r_{\mathrm{L}} = r_{\mathrm{out}}^*, \quad r_{\mathrm{S}} = r_{\mathrm{in}}^*$

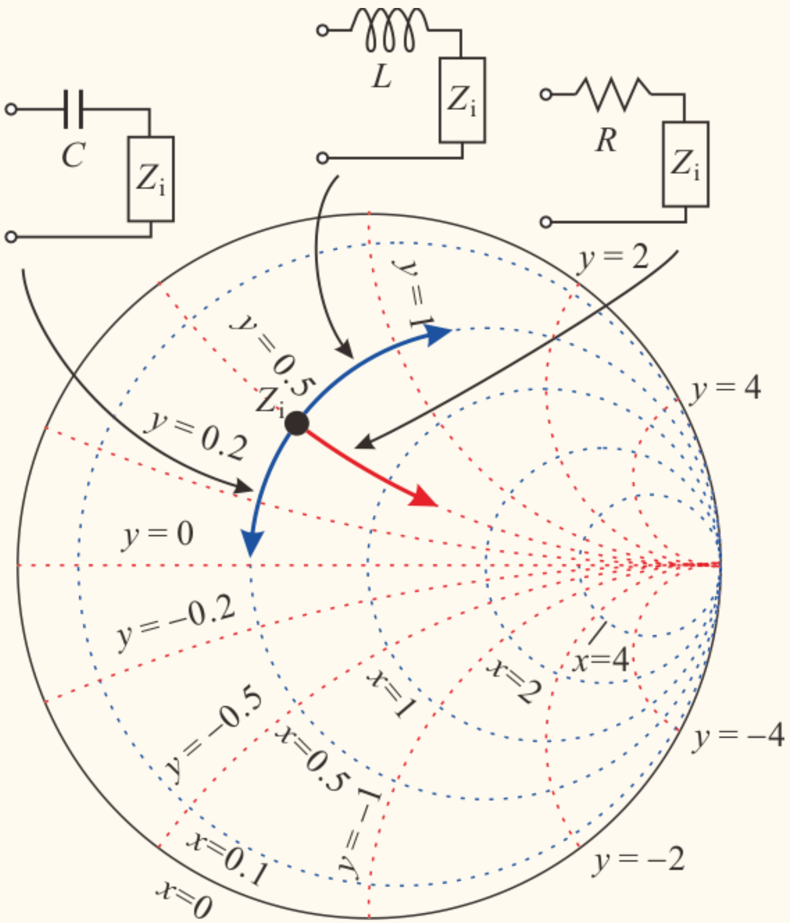
Solution
$$r_{\mathrm{S}} = \frac{B_1 \pm \sqrt{B_1^2 - 4|M|^2}}{2M}, \quad r_{\mathrm{L}} = \frac{B_2 \pm \sqrt{B_2^2 - 4|N|^2}}{2N}$$

with

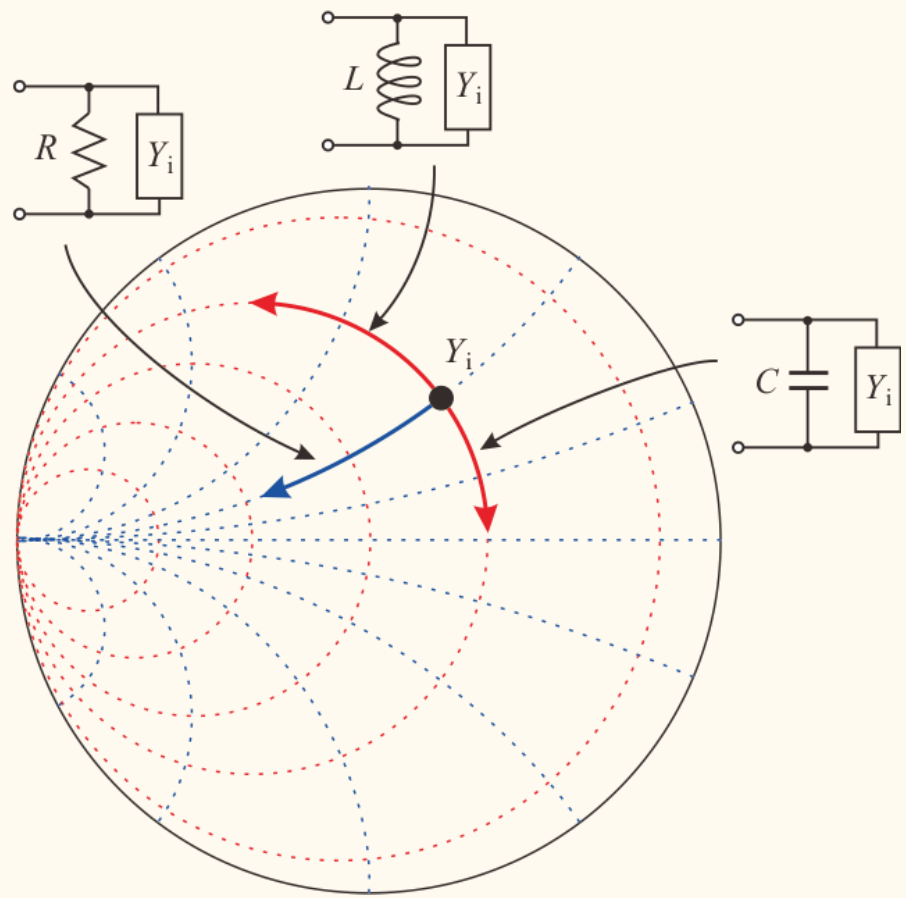
$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\mathrm{det}S|^2, \quad B_2 = 1 - |S_{11}|^2 + |S_{22}|^2 - |\mathrm{det}S|^2,$$
$$N = S_{22} - S_{11}^*\mathrm{det}S, \quad M = S_{11} - S_{22}^*\mathrm{det}S$$

Practical impedance matching with Smith chart

Series and parallel connection of passive elements and traces on charts



Smith chart

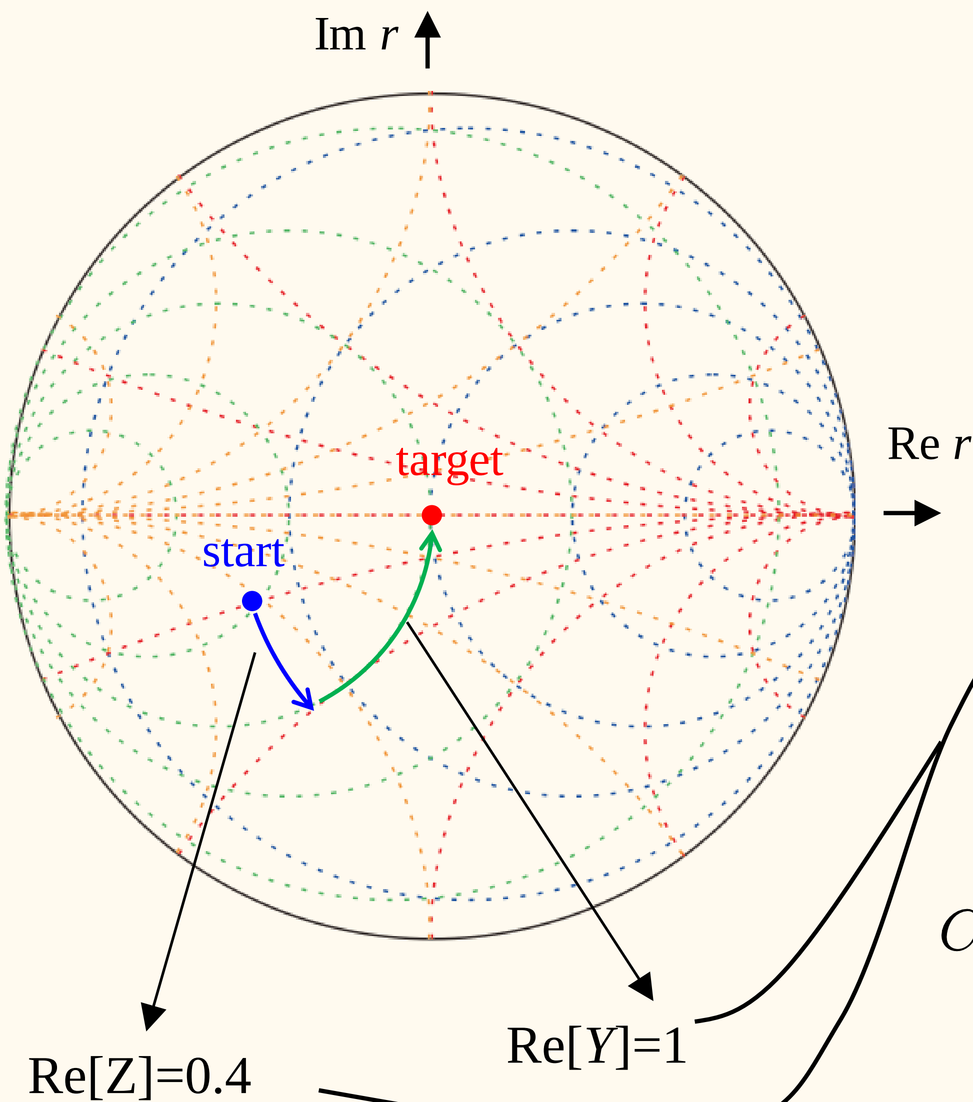


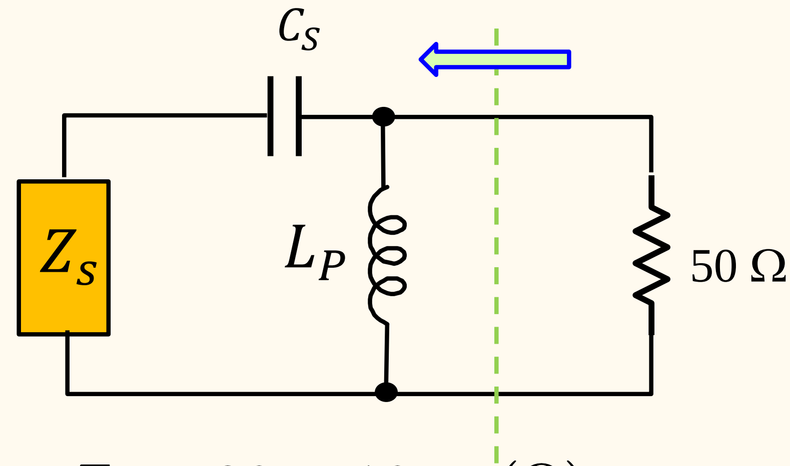
Admittance chart

An example of impedance matching

frequency 100 MHz ≈ 628 Mrad/s

immittance chart





$$Z_s = 20 - 10i \quad (\Omega)$$
$$= 0.4 - 0.2i \quad (Z_0)$$

equalize: $0.4 + iy = \dfrac{1}{1+iq}$

$$y = -\sqrt{0.24} \approx -0.49$$

$$-0.49 = -0.2 - \frac{1}{\omega C_S Z_0}$$

$$C_S \approx \frac{1}{2\pi \times 10^6 \times 50 \times 0.29} \approx 110 \text{ pF}$$

similarly $L_P \approx 65$ nH

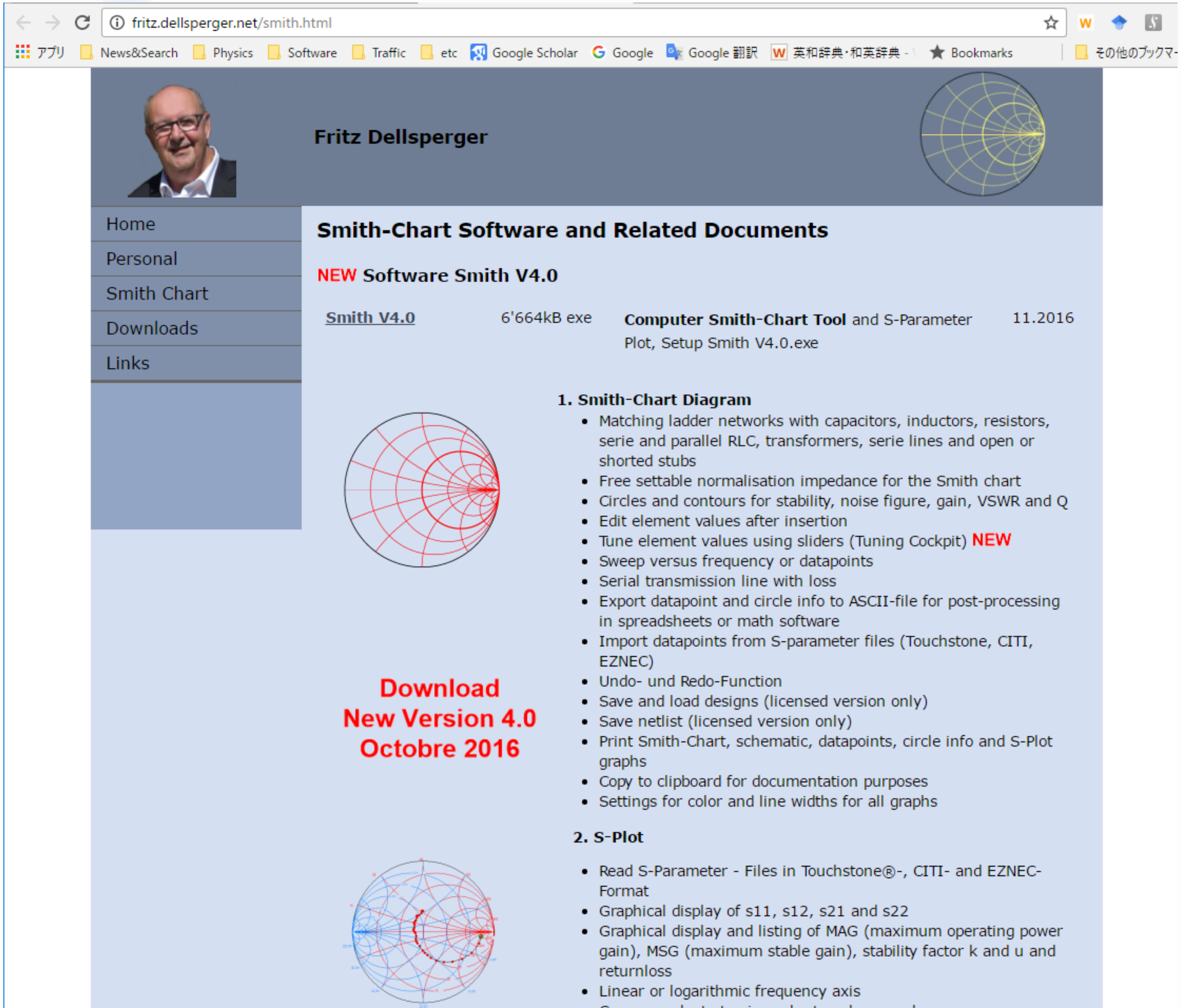
Impedance matching designer

http://home.sandiego.edu/~ekim/e194rfs01/jwmatcher/matcher2.html

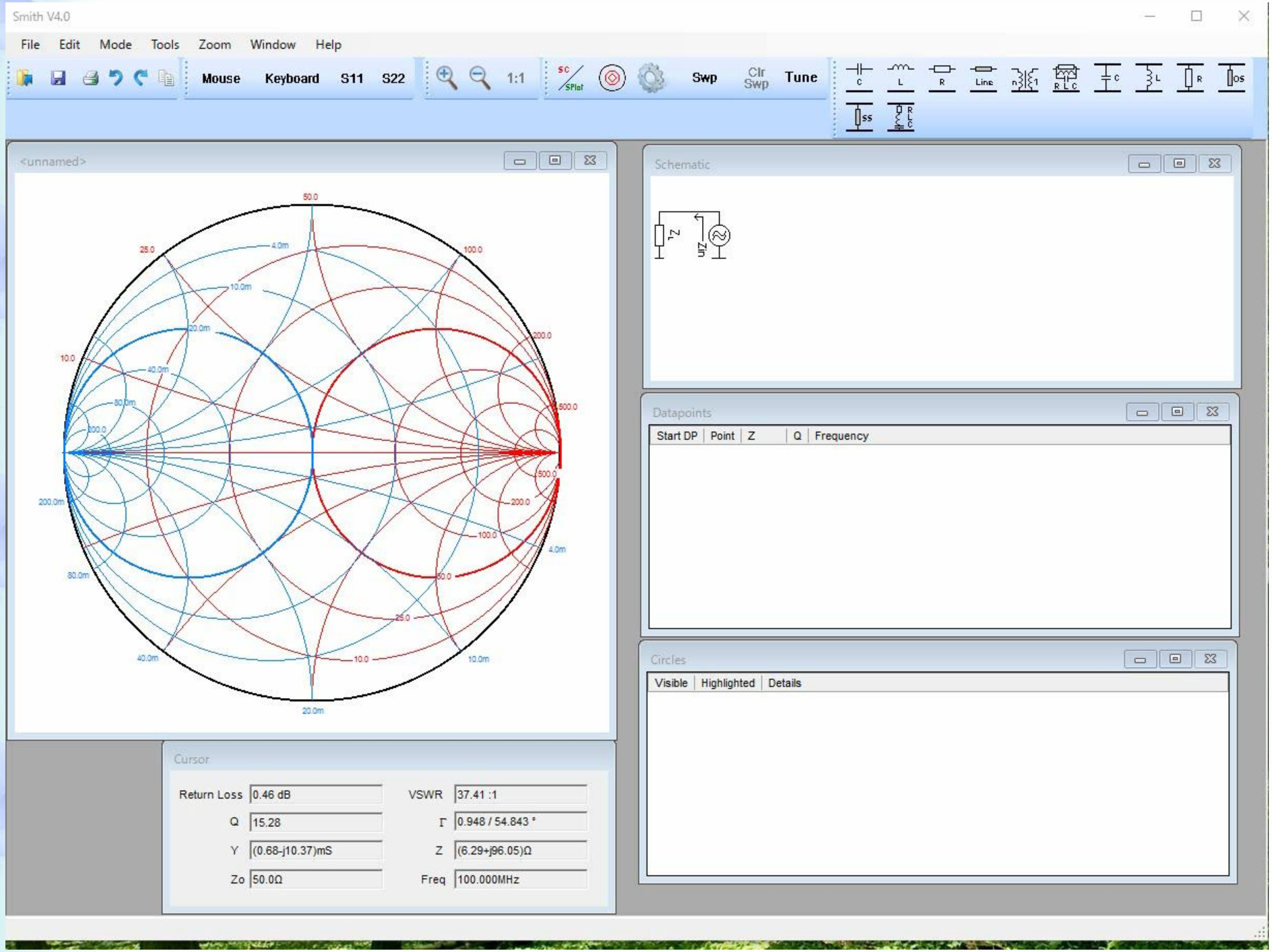
http://leleivre.com/rf_lcmatch.html

Useful freeware: Smith v4.0

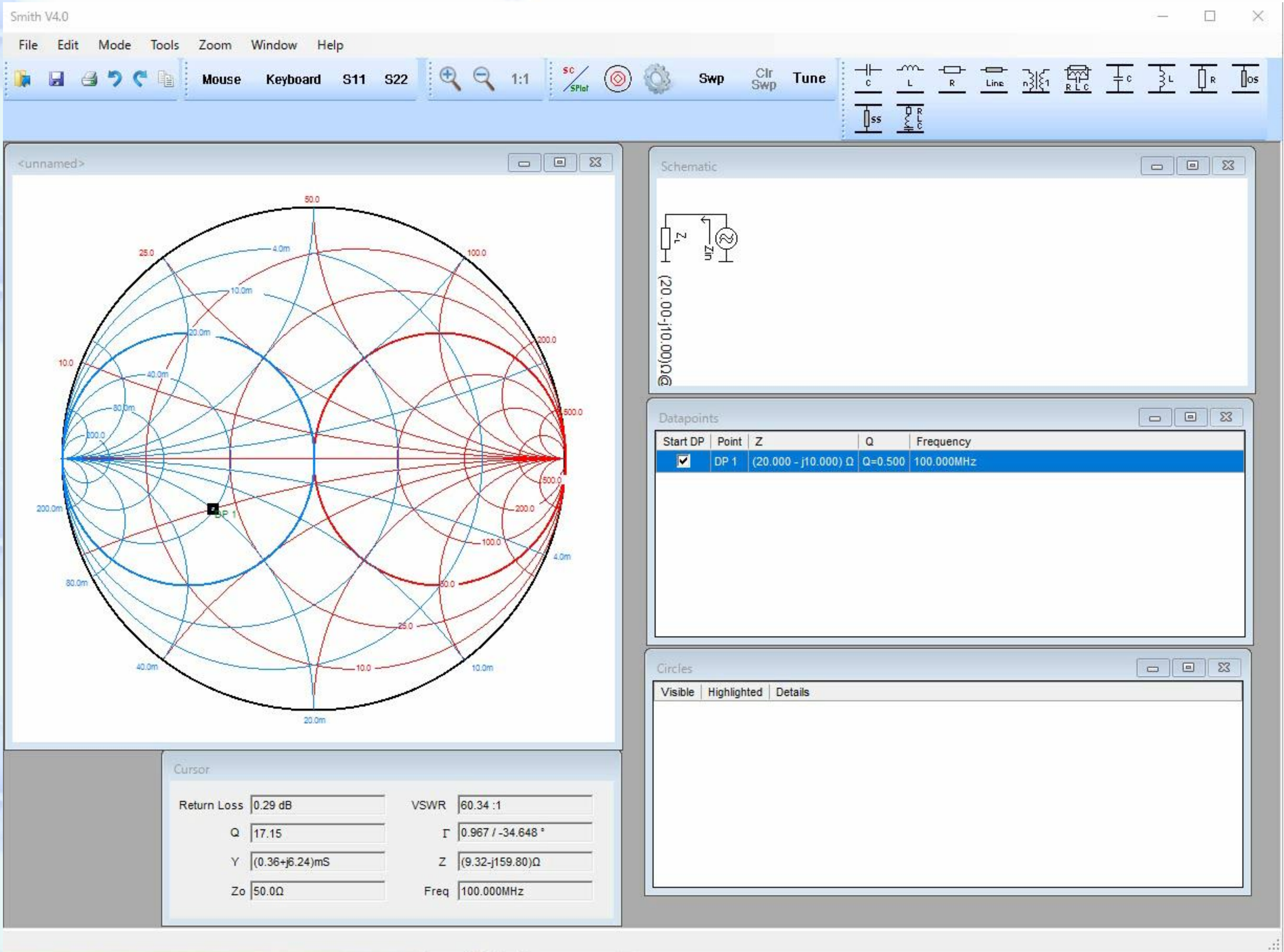
http://fritz.dellsperger.net/smith.html



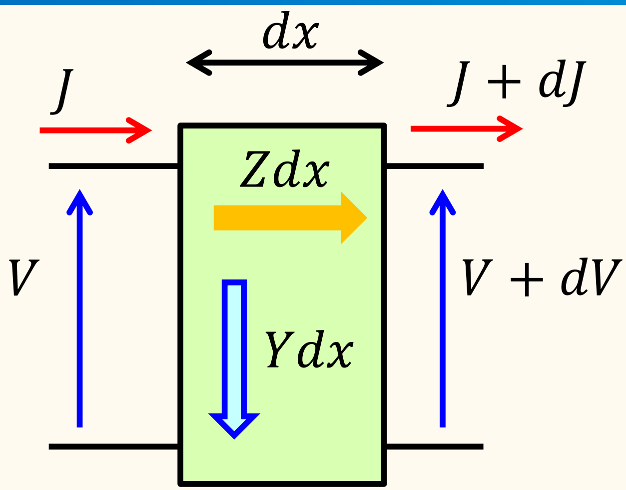
Impedance matching with Smith V4 (1)



Impedance matching with Smith V4 (2)



5.4 Non-TEM mode transmission line



$$\frac{V}{J} = \mp\frac{Z}{\kappa} = \mp\sqrt{\frac{Z}{Y}}$$

Characteristic impedance

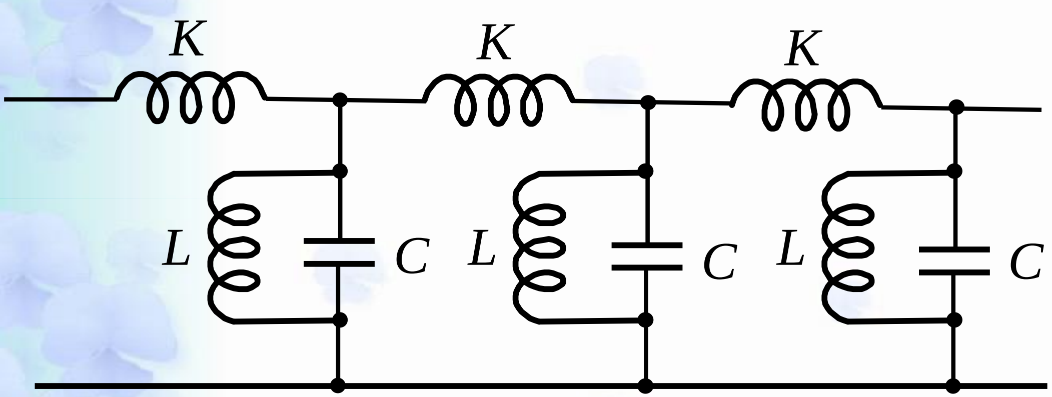
LC model: $Z = i\omega L,\ Y = i\omega C$

The inductance represents magnetic fields circulating the core and the capacitance electric fields directing from the core to the shield.

$$Z_0 = \sqrt{\frac{L}{C}}$$: real, dispersionless (no ω-k relation)

Non-linear ω-term in Z or Y → dispersion (longitudinal components)

5.4 Non-TEM mode gives mass in transmission line



C: capacitance per unit length
L: inductance per inverse unit length
K: inductance per unit length

$$Y = i\omega C + \frac{1}{i\omega L}$$

$$-k^2 = YZ = \left(i\omega C + \frac{1}{i\omega L} \right) i\omega K = -CK\omega^2 + \frac{K}{L}$$

Constant finite mass: $E = \hbar\omega \propto k^2$

(Schrodinger eq.: Parabolic partial differential equation)

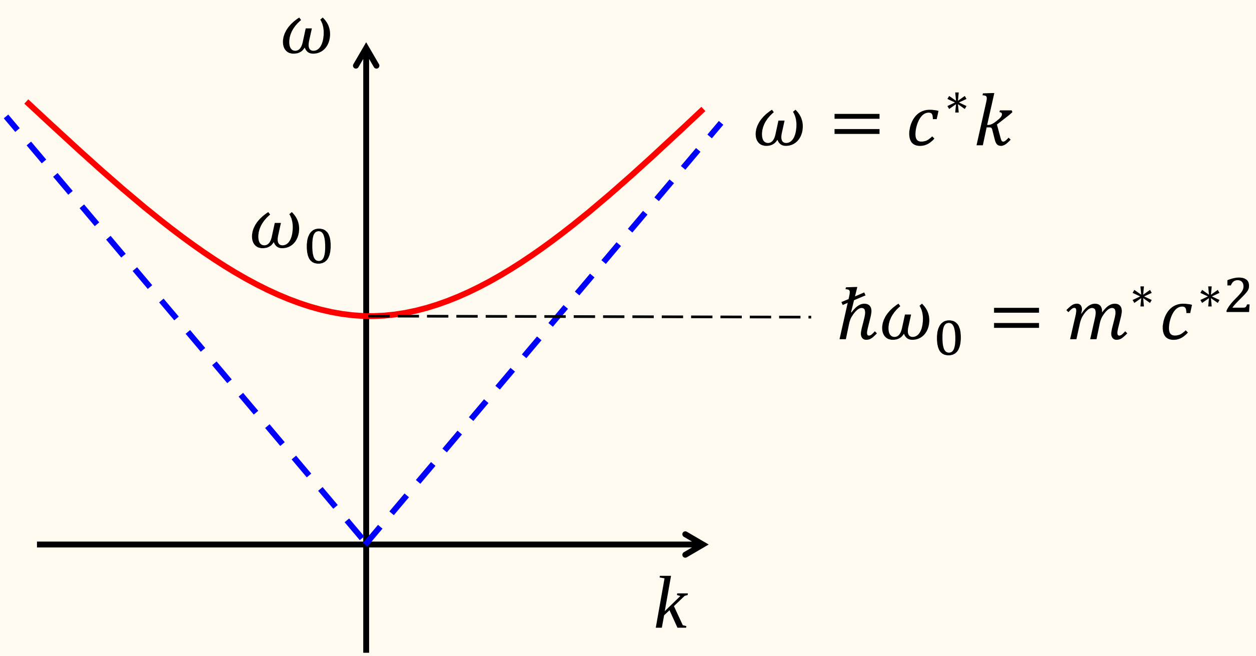
Coupling between linear dispersions: mass mechanism *cf.* Higgs

5.4 Non-TEM mode gives mass in transmission line

$$\frac{1}{\sqrt{LC}} = \omega_0 \text{ unchanged with } dx \to 0$$

$$Z = i\omega K, \quad Y = \frac{1-(\omega/\omega_0)^2}{i\omega L}$$

$$ik = \kappa = \sqrt{YZ} = i\sqrt{\frac{K}{L}\left[\left(\frac{\omega}{\omega_0}\right)^2 - 1\right]} \qquad \eta^2 \equiv \frac{K}{L}$$



5.4 Giving mass to LC transmission line

$$\omega \gg \omega_0 \rightarrow k \sim \eta \frac{\omega}{\omega_0}$$ No dispersion

Velocity: $c^* = \frac{\omega}{k} = \frac{\omega_0}{\eta} = \frac{1}{\sqrt{KC}}$

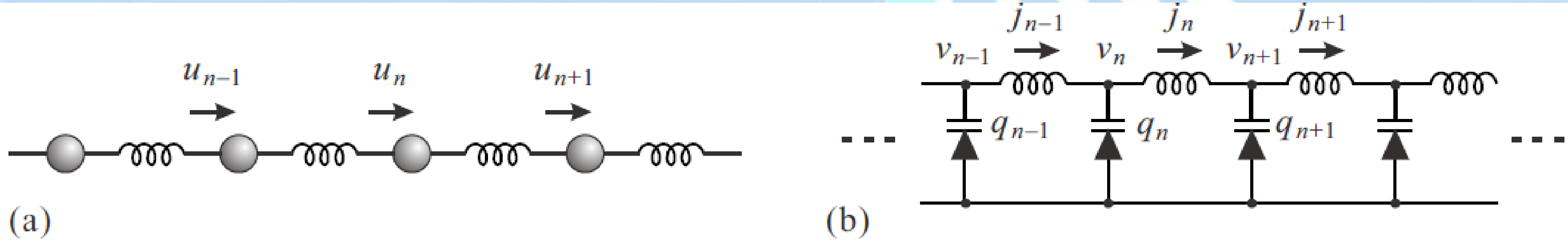
$$\omega \sim \omega_0 \quad \omega = \omega_0 + \delta\omega$$

$$k^2 \approx 2\eta^2 \frac{\delta\omega}{\omega_0} \quad \therefore \epsilon \equiv \hbar\delta\omega = \frac{\hbar k^2}{2(\eta^2/\omega_0)} = \frac{\hbar^2 k^2}{2m^*}$$

$$m^* \equiv \frac{\hbar\eta^2}{\omega_0}$$

$$E_0 = \hbar\omega_0 = \frac{\hbar\eta^2}{\omega_0} \cdot \left(\frac{\omega_0}{\eta}\right)^2 = m^* c^{*2}$$

5.5 Non-linear LC transmission line and Toda lattice



Toda lattice is a typical non-linear system with exact (soliton) solutions. It is defined as follows:

The springs in (a) have Toda-potential: $\phi(r) = \frac{a}{b}e^{-br} + ar \quad (ab > 0)$

Equation of motion:

$$m\frac{d^2u_n}{dt^2} = -a\exp[-b(u_{n+1} - u_n)] + a\exp[-b(u_n - u_{n-1})]$$

For relative shift $r_n = u_{n+1} - u_n$

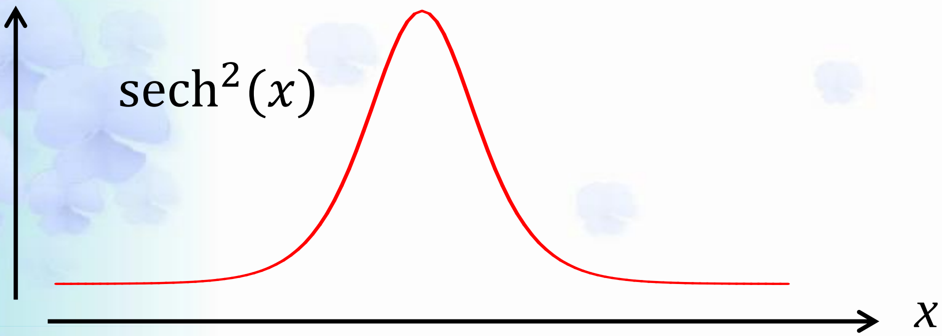
$$m\frac{d^2r_n}{dt^2} = a(2e^{-br_n} - e^{-br_{n+1}} - e^{-br_{n-1}})$$

Force of a spring: $f = -\phi'(r) = a(e^{-br} - 1)$

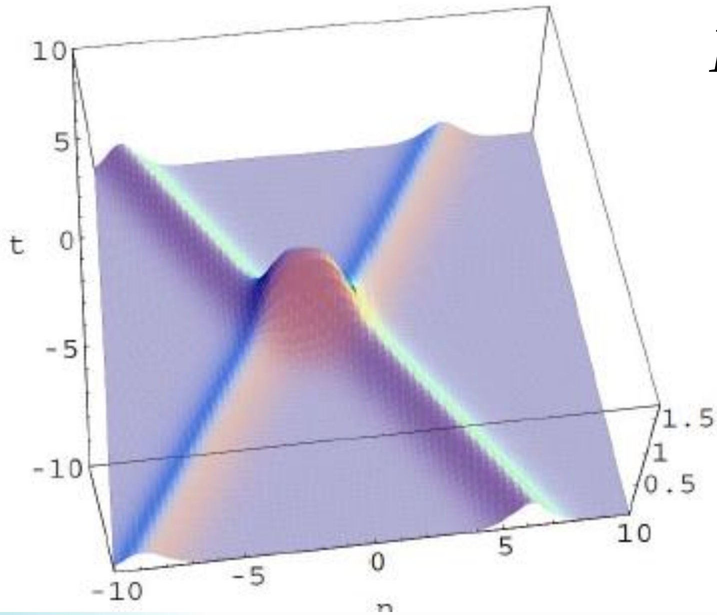
Solitons in Toda lattice

$$\frac{d^2}{dt^2}\log\left(1+\frac{f_n}{a}\right)=\frac{b}{m}(f_{n+1}+f_{n-1}-2f_n)$$

Soliton solution:



$$u_n=\omega^2\mathrm{sech}^2(\kappa n+\sigma\omega t+\delta),$$
$$\sigma=\pm 1,\quad \omega=\sinh\kappa,$$
$$\kappa,\quad \delta:\text{constants}$$

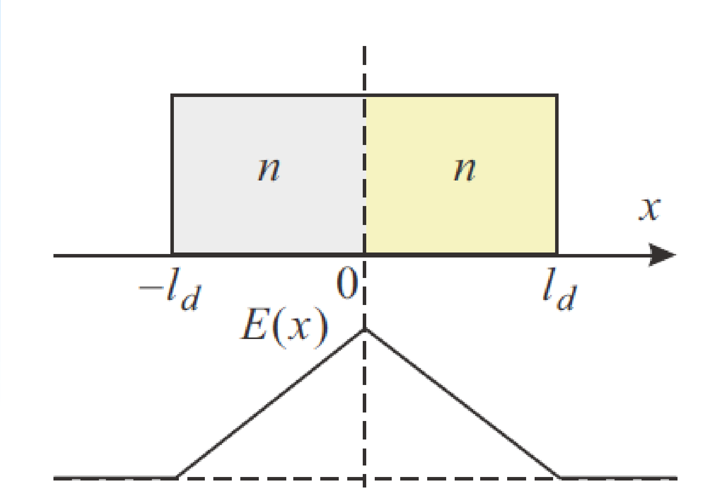


$N=2$ soliton solution:

$$u_n=\frac{\tau_{n+1}\tau_{n-1}}{\tau_n^2}-1,$$
$$\tau_n=1+e^{2\eta_1}+e^{2\eta_2}+A_{12}e^{2(\eta_1+\eta_2)},$$
$$\eta_i=\kappa_i n+\sigma_i\omega_i t+\delta_i,\quad \sigma_i=\pm 1,\ \omega_i=\sinh\kappa_i,$$
$$A_{12}=\frac{ab\sinh^2(\kappa_1-\kappa_2)-m(\sigma_1\omega_1-\sigma_2\omega_2)^2}{m(\sigma_1\omega_1+\sigma_2\omega_2)^2-ab\sinh^2(\kappa_1+\kappa_2)}$$

Non-linear capacitance: Vari-cap

Varicap BB505

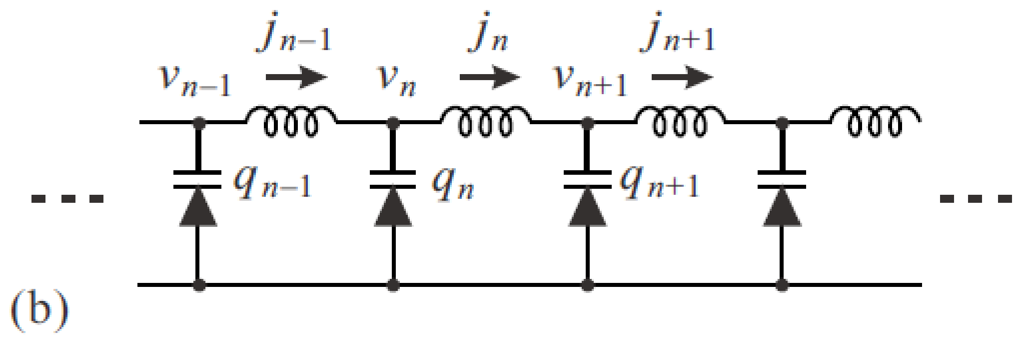


$$V_b = \frac{en}{\epsilon}\int_{-l_d}^{0} 2(x+l_d)dx + \frac{en}{\epsilon}\int_{0}^{l_d} 2(l_d - x)dx = \frac{2enl_d^2}{\epsilon}$$

$$V + V_b = \frac{2en}{\epsilon}\left(l_d + \frac{Q}{nS}\right)^2 \quad \therefore C = \frac{dQ}{dV} = \sqrt{\frac{\epsilon}{2en}}\frac{nS}{\sqrt{V+V_b}}$$

$$V + V_b = V_0 + \delta V \qquad \delta V \to V$$

L-Varicap transmission line



$$L\frac{dJ_n}{dt} = v_n - v_{n-1},$$

$$\frac{dq_n}{dt} = J_{n-1} - J_n,$$

$$q_n = \int_0^{v_n} C(V)dV, \quad C(V) = \frac{Q(V_0)}{F(V_0) + V - V_0}$$

$$q_n = Q(V_0)\log\left[1 + \frac{V_n}{F(V_0)}\right] + \text{const.}$$

$$\frac{d^2}{dt^2}\log\left[1 + \frac{V_n}{F(V_0)}\right] = \frac{1}{LQ(V_0)}(V_{n-1} + V_{n+1} - 2V_n)$$

Solitons in non-linear circuit

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CIRCUIT IMPLEMENTATIONS OF SOLITON SYSTEMS*

ANDREW C. SINGER
Department of Electrical and Computer Engineering,
University of Illinois, Urbana, IL 61801, USA

ALAN V. OPPENHEIM
Department of Electrical Engineering, MIT, Cambridge, MA 02139, USA

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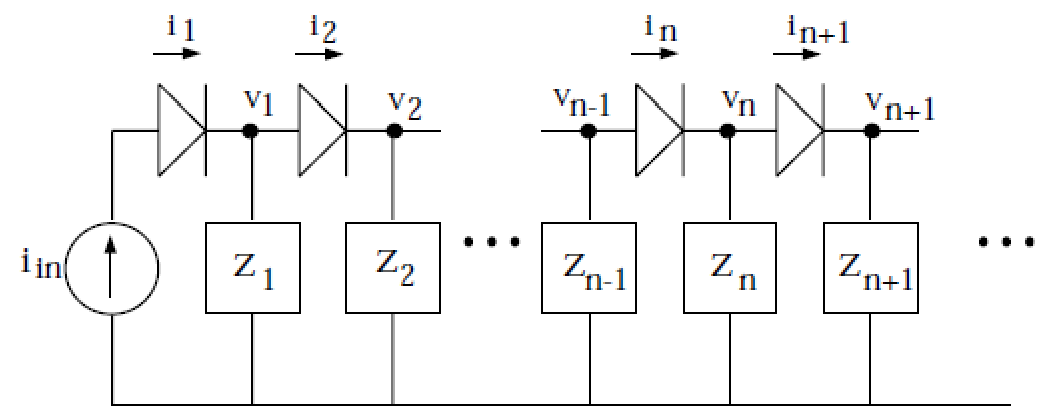
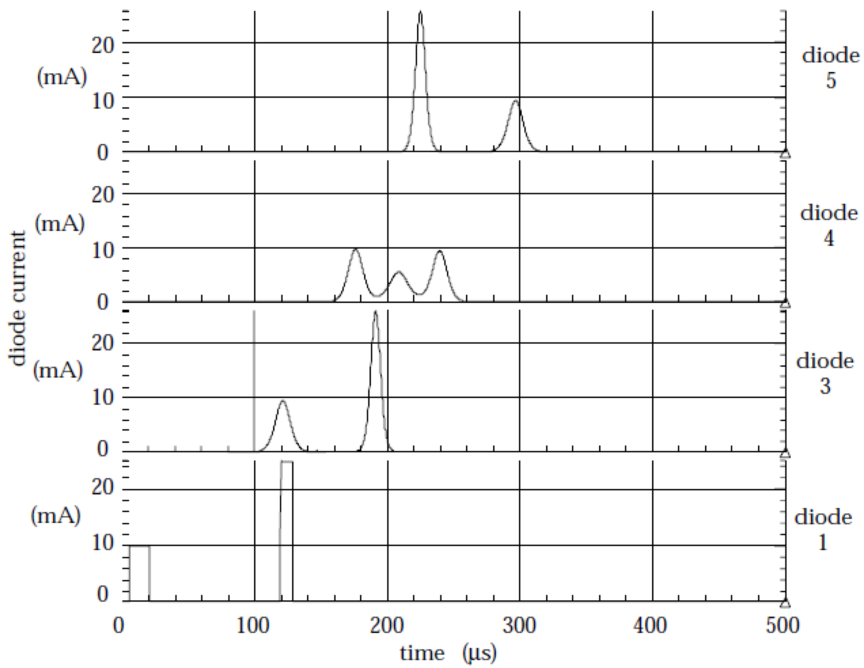
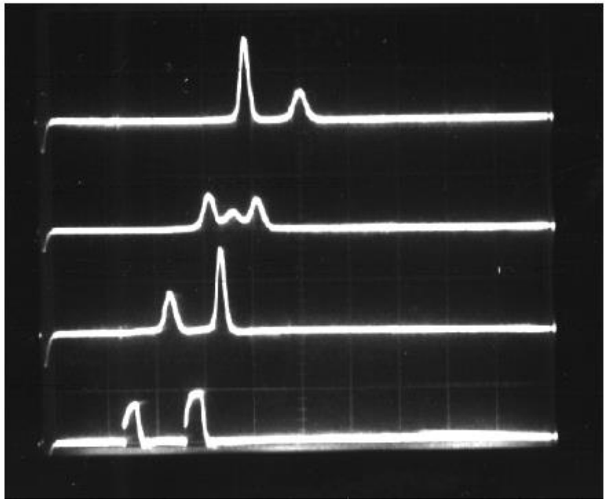






Toda lattice circuit, Soliton circuit

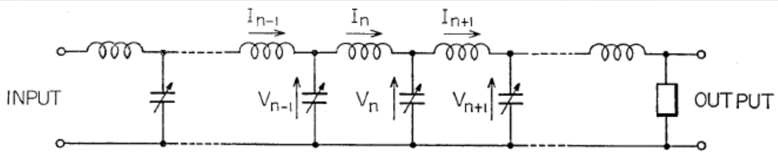
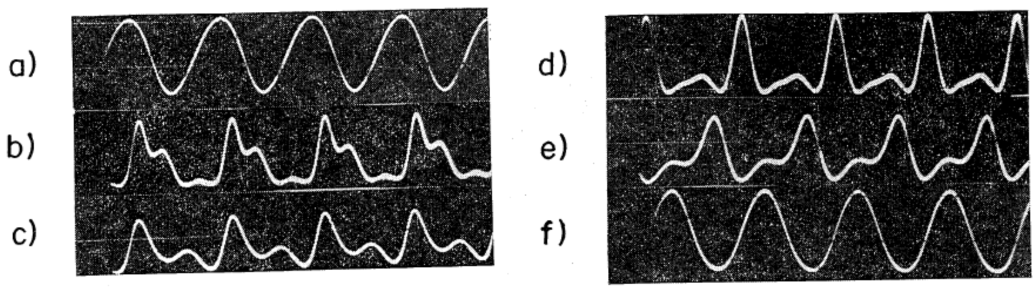


Fig. 1. A nonlinear network equivalent to a one-dimensional anharmonic lattice. The circuit element have an inductance $L=22\ \mu$H or capacitance $C(V)=27\ V^{-0.48}$ pF.



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Studies on Lattice Solitons by Using Electrical Networks

Ryogo HIROTA and Kimio SUZUKI

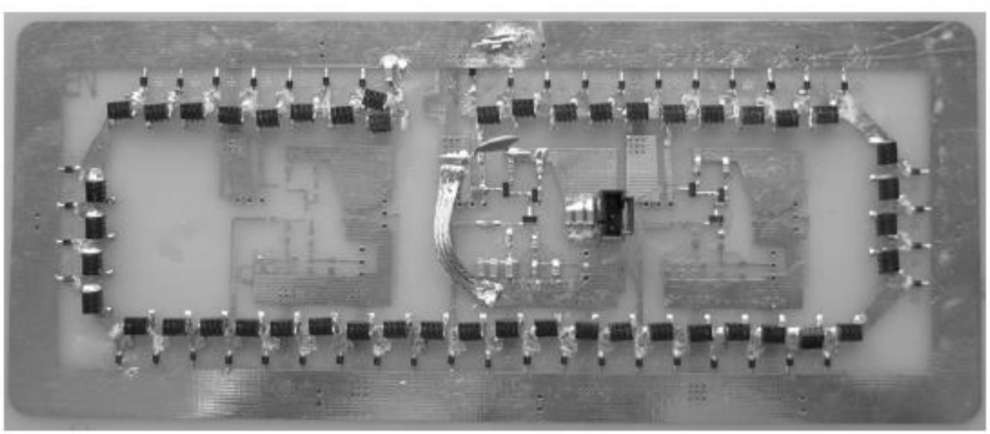

Fig. 16. Microwave soliton oscillator prototype.