

# 電子回路論第 9 回

## Electric Circuits for Physicists

東京大学理学部・理学系研究科  
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# Outline

## 5.3 S-parameter representation of devices

Impedance matching with  $Z$ ,  $S$  parameters

Impedance matching with immittance chart

## 5.4 Non-TEM mode transmission lines

## 5.5 Non-linear elements and Toda lattice

## Ch.6 Noises and Signals

### 6.1 Fluctuations

### 6.2 Fluctuation-dissipation theorem

# Review: Scattering (S) matrix (S parameters)

Transmission lines: wave propagating modes  $\rightarrow$  Channels

Take  $|a_i|^2$ ,  $|b_i|^2$  to be powers (energy flow).

output

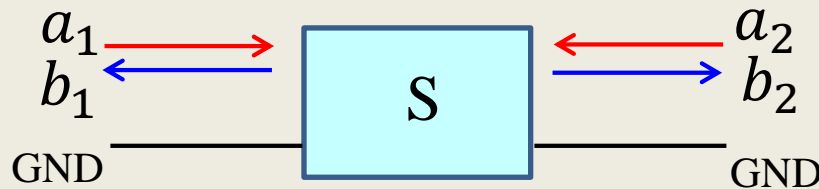
S-matrix

input

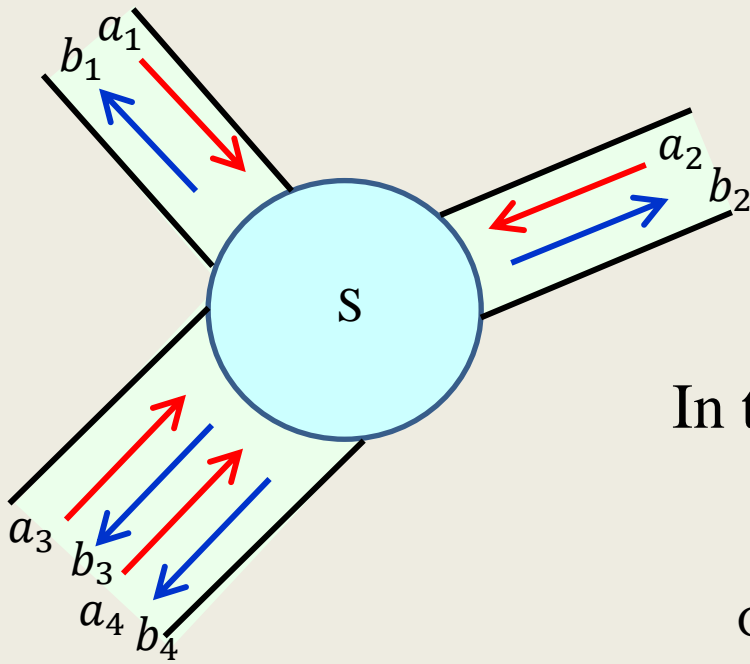
$$\begin{pmatrix} b_1 \\ \vdots \\ b_i \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} S_{11} & \cdots & S_{1i} & \cdots & S_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ S_{i1} & & S_{ii} & & S_{in} \\ \vdots & & \vdots & \ddots & \vdots \\ S_{n1} & \cdots & S_{ni} & \cdots & S_{nn} \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{pmatrix}$$

$$\mathbf{b} = \mathbf{S}\mathbf{a}$$

In the case of two-terminal pair circuit



$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} r_1 & t_r \\ t_1 & r_r \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$





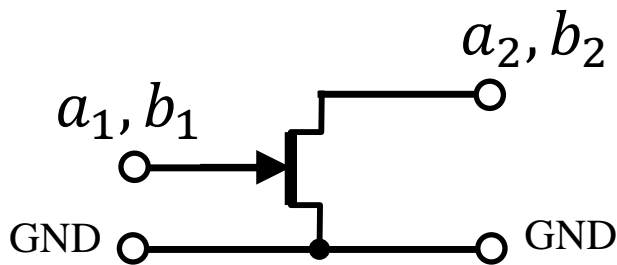
# S-parameter representation of high-frequency devices



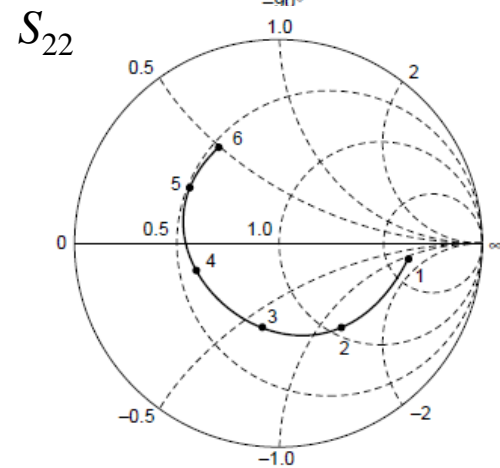
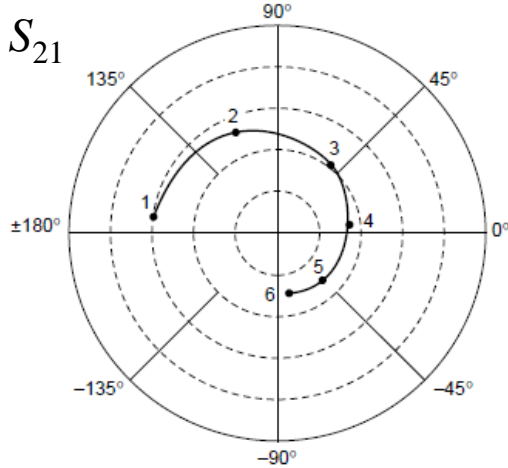
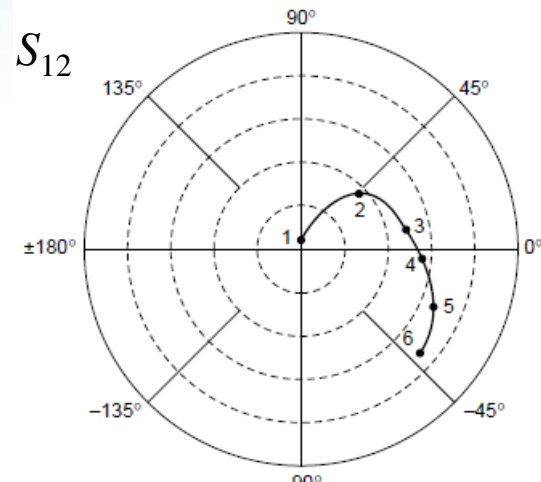
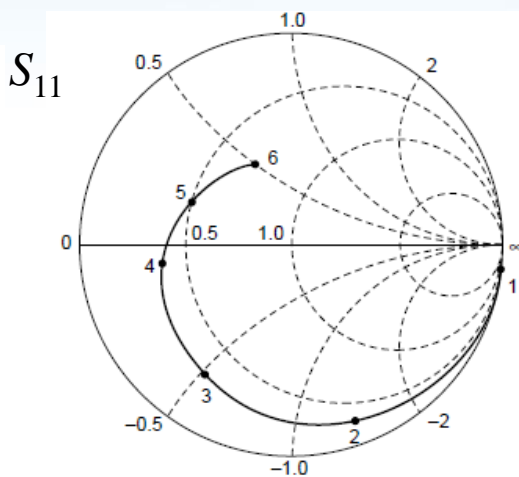
<b>NEC</b>	<b>DATA SHEET</b>
<b>GaAs MES FET</b> <b>NE76084</b>	
<b>C to Ku BAND LOW NOISE AMPLIFIER</b> <b>N-CHANNEL GaAs MES FET</b>	

## S-PARAMETERS

$V_{DS} = 3\text{ V}$ ,  $I_D = 10\text{ mA}$   
 START 500 MHz, STOP 18 GHz, STEP 500 MHz



$$S_{11} = r_l, S_{22} = r_r$$



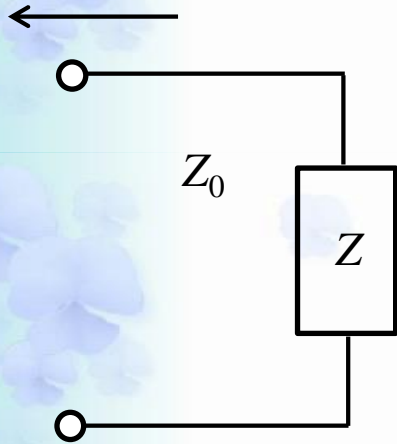
# Review: Smith Chart

P.H. Smith

1905-1987

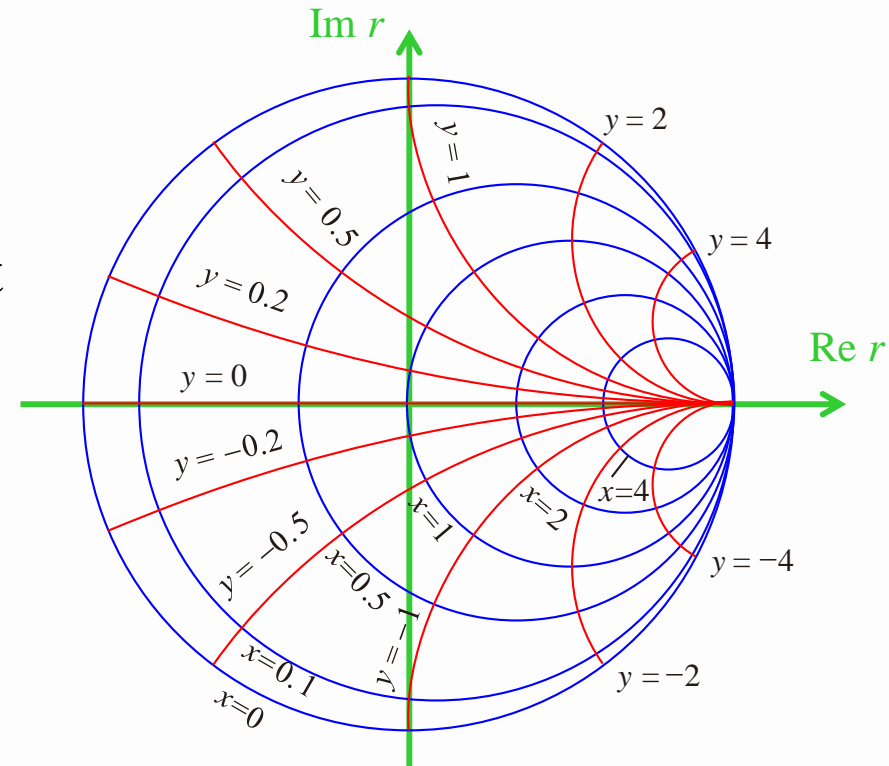
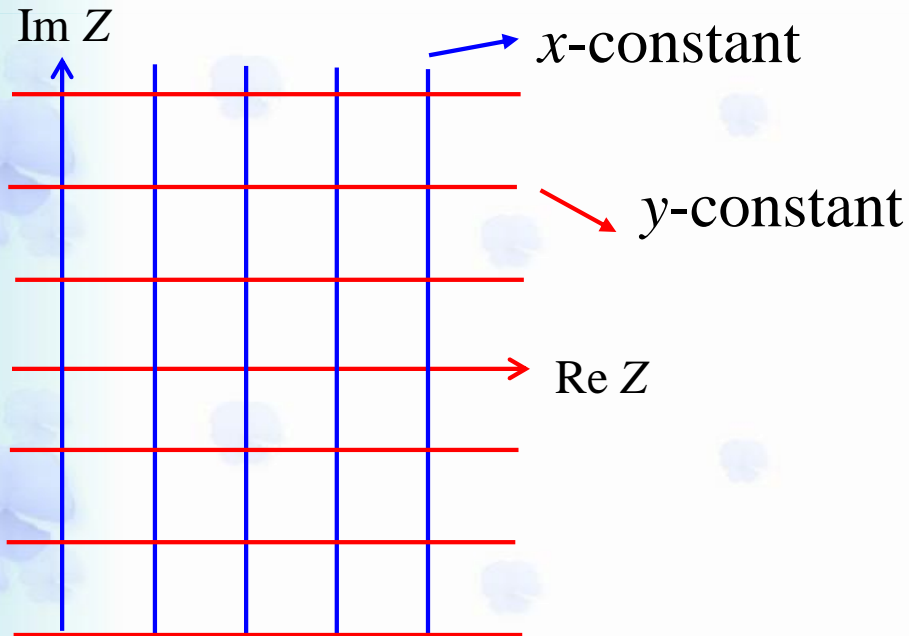


$r$  : reflection coefficient



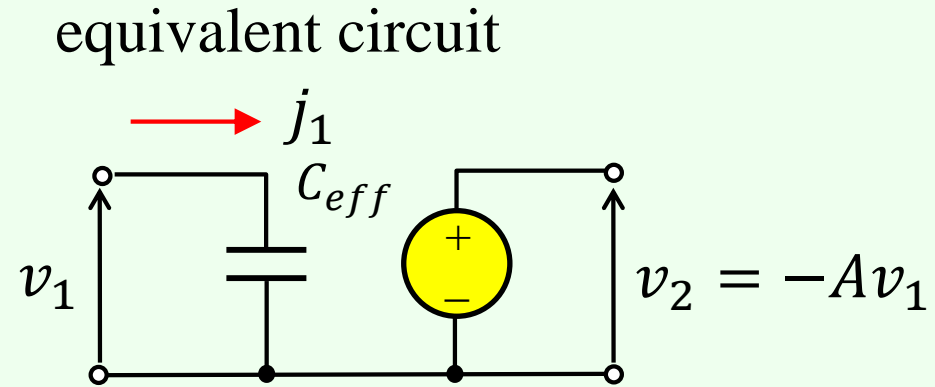
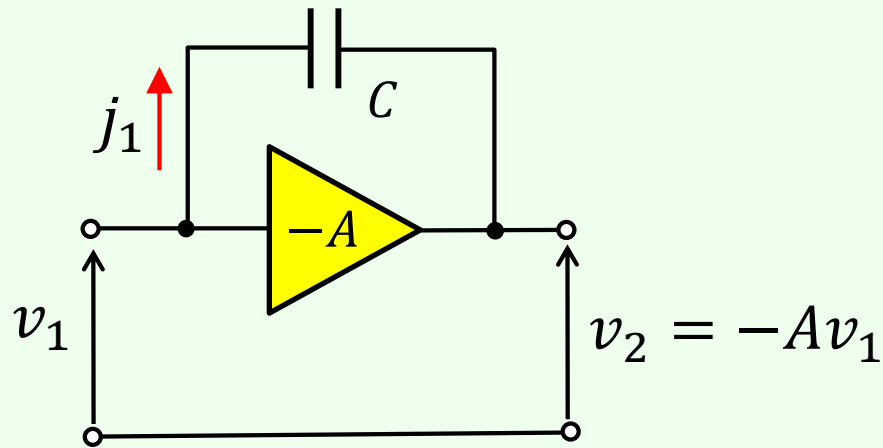
$$r = \frac{V_-}{V_+} = \frac{Z - Z_0}{Z + Z_0} = \frac{z - 1}{z + 1}$$

$$z = \frac{Z}{Z_0} = x + iy$$



# Comment: Mirror effect

An amplifier may change the effective impedance of passive elements.



$$j_1 = sC(v_1 - v_2) = sC(1 + A)v_1$$

$$s = \frac{j_1}{(1 + A)Cv_1}$$

$$C_{eff} \frac{dv_1}{dt} = sC_{eff}v_1 = j_1$$

$$s = \frac{1}{C_{eff}} \frac{j_1}{v_1}$$

$$C_{eff} = (1 + A)C$$

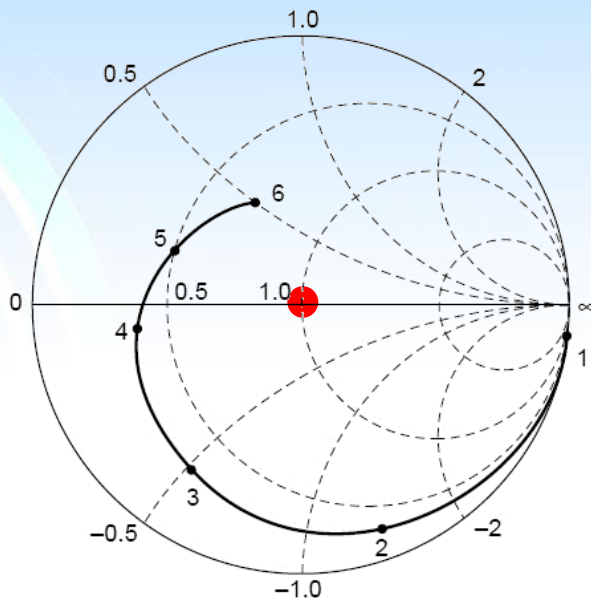
: mirror effect

# S-parameter representation of high-frequency devices

Ex) NE76084 MES FET 0.5 ~ 18GHz

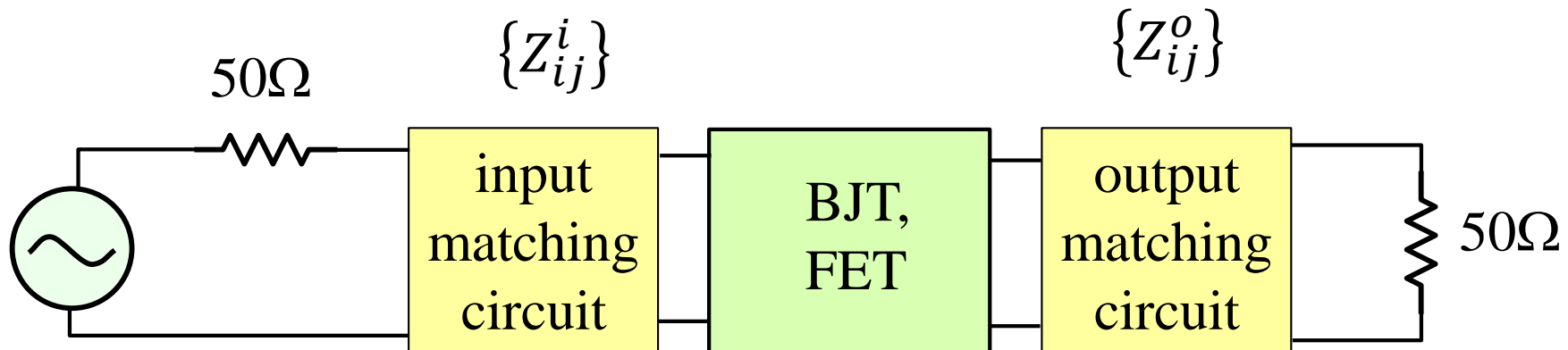


$S_{11}$



The datasheet tells that we need impedance matching circuits with transmission lines with  $Z_0 = 50 \Omega$ .

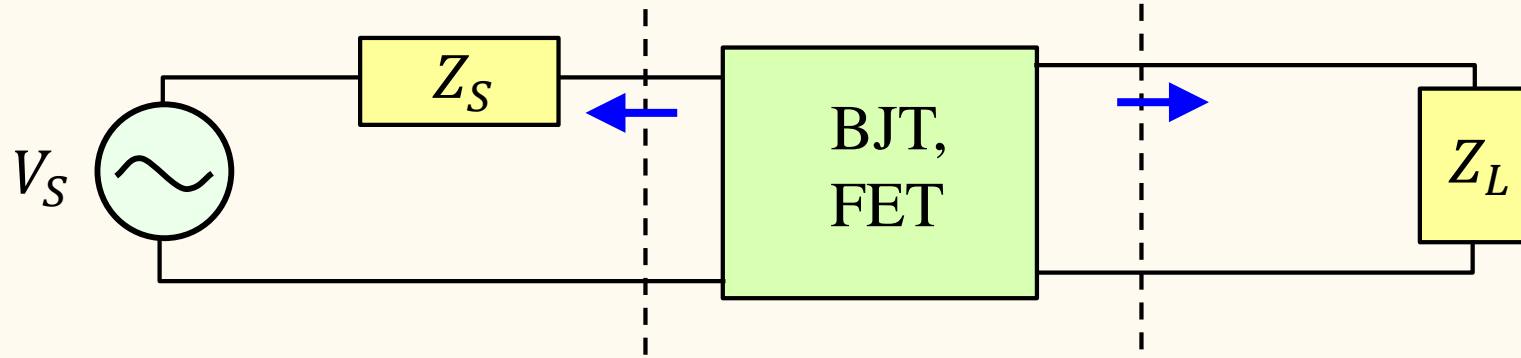
Insert input and output matching circuits to kill reflections.





# Impedance matching circuits

The circuit is summarized at the boundaries as

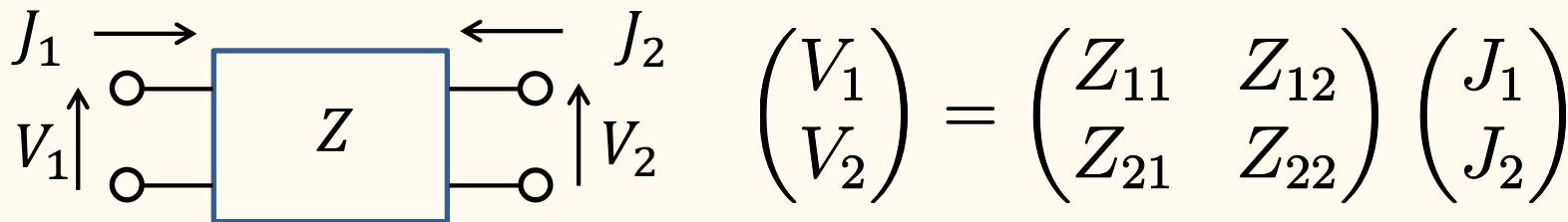


If we know Z-parameters of the input/output matching circuits, from Ho-Thevenin's theorem

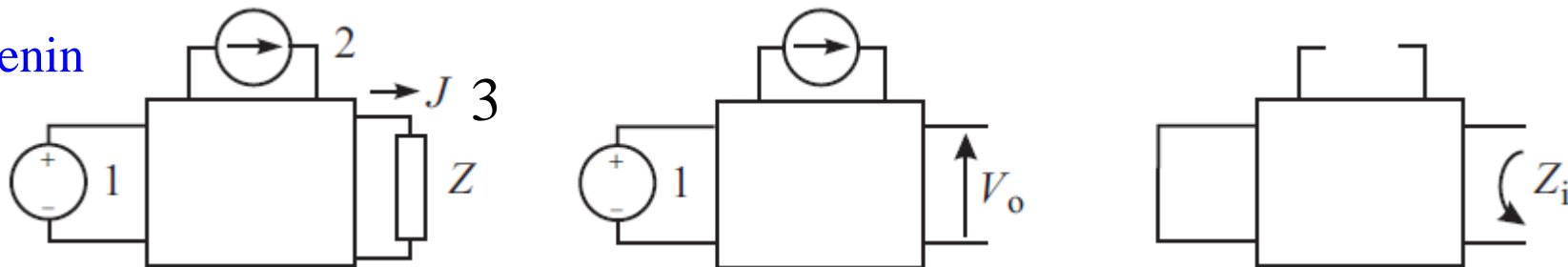
$$Z_S = Z_{22}^i - \frac{Z_{12}^i Z_{21}^i}{50 + Z_{11}^i}, \quad Z_L = Z_{11}^o - \frac{Z_{12}^o Z_{21}^o}{50 + Z_{22}^o}.$$

# Z-matrix, Ho-Thevenin's theorem

Z-matrix



Ho-Thevenin



1. Measure the open terminal voltage  $V_0$ .
2. Turn off all the power sources (voltage sources: short, current sources: open). Measure the open circuit impedance  $Z_i$ .

Then

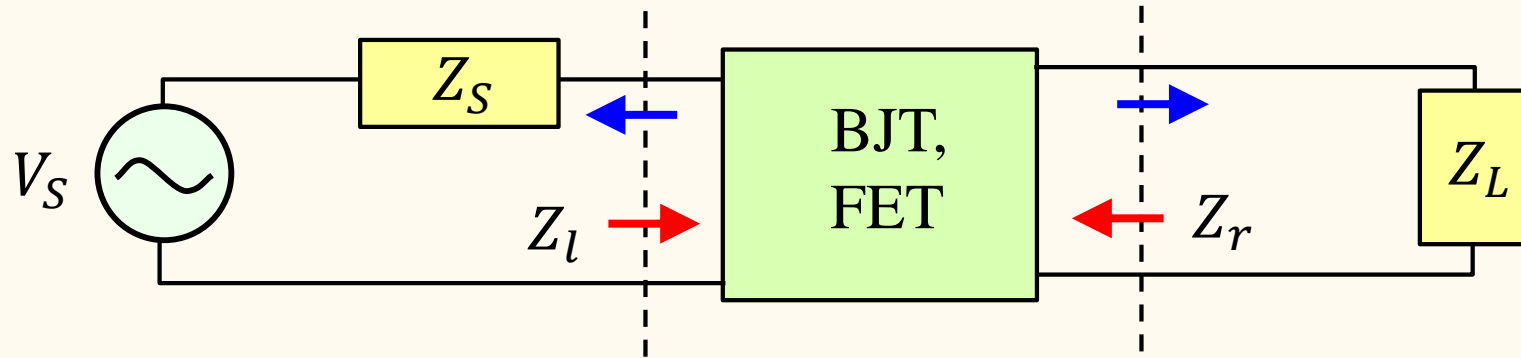
$$J = \frac{V_0}{Z + Z_i}$$

$$V_s = 0 \quad \therefore V_1 = -50J_1 = Z_{11}J_1 + Z_{12}J_2 \quad \therefore J_1 = -\frac{Z_{12}}{50 + Z_{11}}J_2$$

$$V_2 = Z_{21}J_1 + Z_{22}J_2 = \left( Z_{22} - \frac{Z_{21}Z_{12}}{50 + Z_{11}} \right) J_2 \rightarrow Z_S$$

# Impedance matching circuits

The circuit is summarized at the boundaries as



If we know  $Z$ -parameters of the input/output matching circuits, from Ho-Thevenin's theorem

$$Z_S = Z_{22}^i - \frac{Z_{12}^i Z_{21}^i}{50 + Z_{11}^i}, \quad Z_L = Z_{11}^o - \frac{Z_{12}^o Z_{21}^o}{50 + Z_{22}^o}.$$

$\{Z_{ij}\}$  : BJT (FET)  $Z$ -parameters, again Ho-Thevenin says

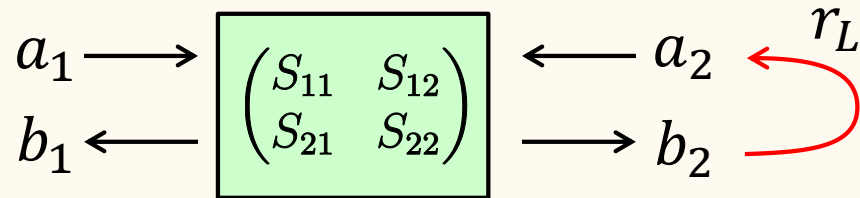
$$Z_l = Z_{11} - \frac{Z_{12} Z_{21}}{Z_L + Z_{22}}, \quad Z_r = Z_{22} - \frac{Z_{12} Z_{21}}{Z_S + Z_{11}}$$

matching condition:  $Z_l = Z_S^*$ ,  $Z_r = Z_L^*$

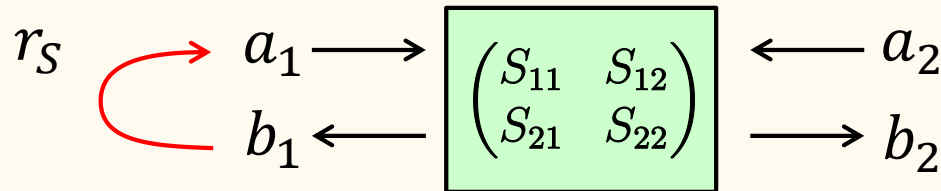


# Impedance matching with S-parameters

In S-parameter treatment, we use complex reflection coefficients to express load, source etc.



$$r_{\text{in}} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}r_L}{1 - S_{22}r_L}$$



$$r_{\text{out}} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}r_S}{1 - S_{11}r_S}$$

Matching condition:  $r_L = r_{\text{out}}^*$ ,  $r_S = r_{\text{in}}^*$

Solution

$$r_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|M|^2}}{2M}, \quad r_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|N|^2}}{2N}$$

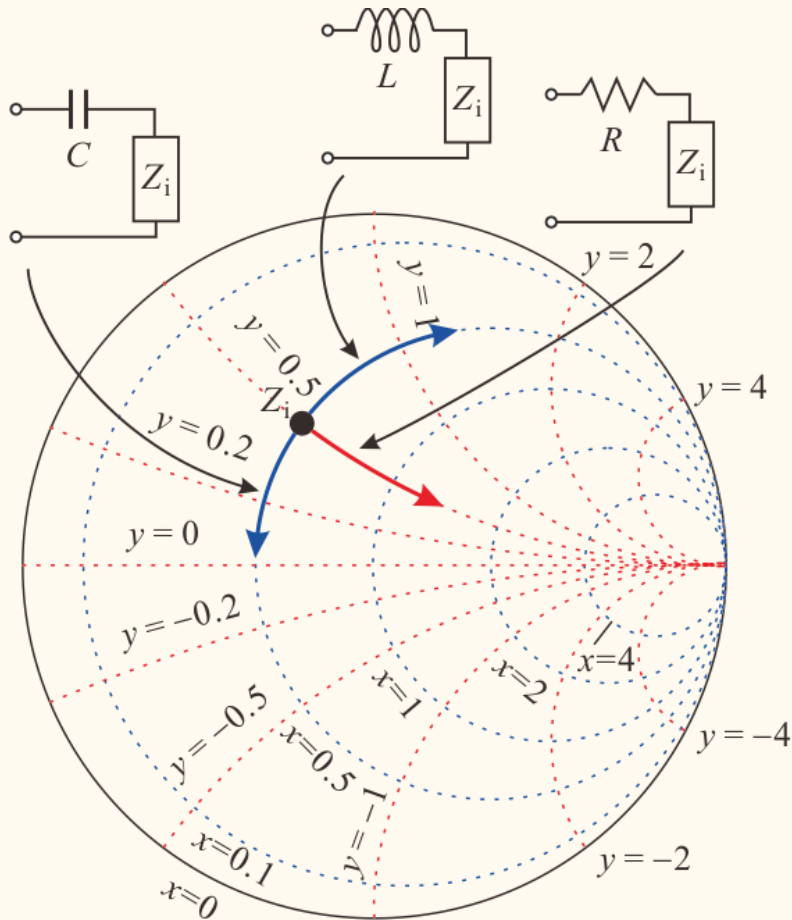
with

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\det S|^2, \quad B_2 = 1 - |S_{11}|^2 + |S_{22}|^2 - |\det S|^2,$$

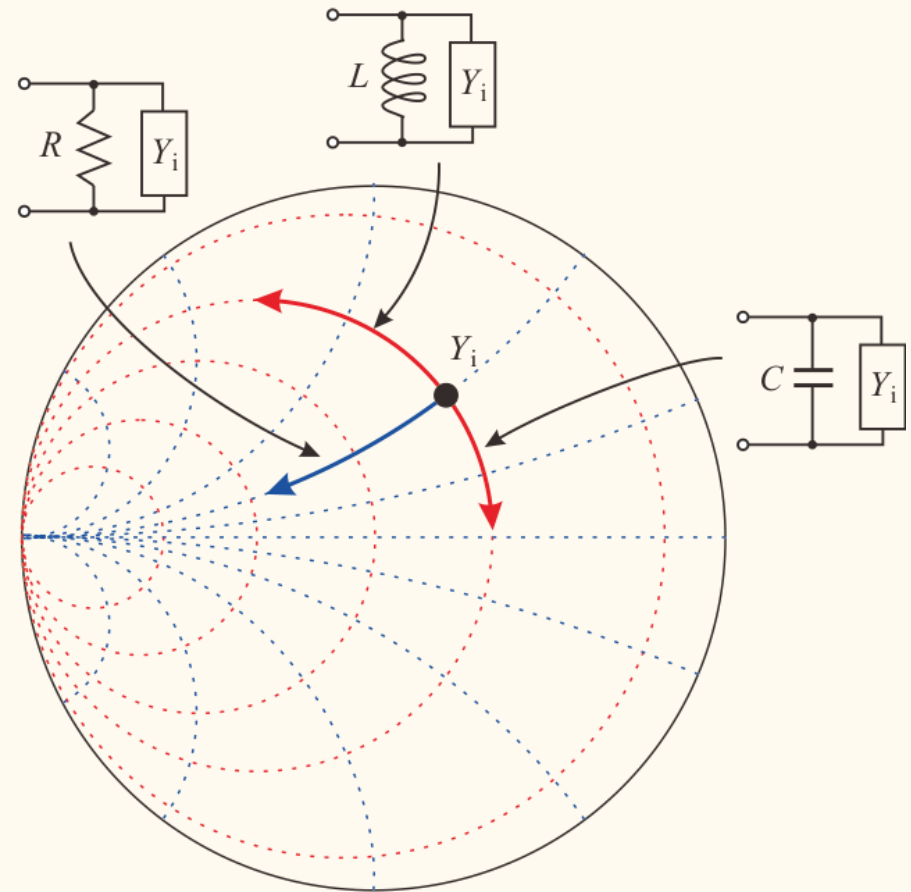
$$N = S_{22} - S_{11}^* \det S, \quad M = S_{11} - S_{22}^* \det S$$

# Practical impedance matching with Smith chart

Series and parallel connection of passive elements and traces on charts



Smith chart



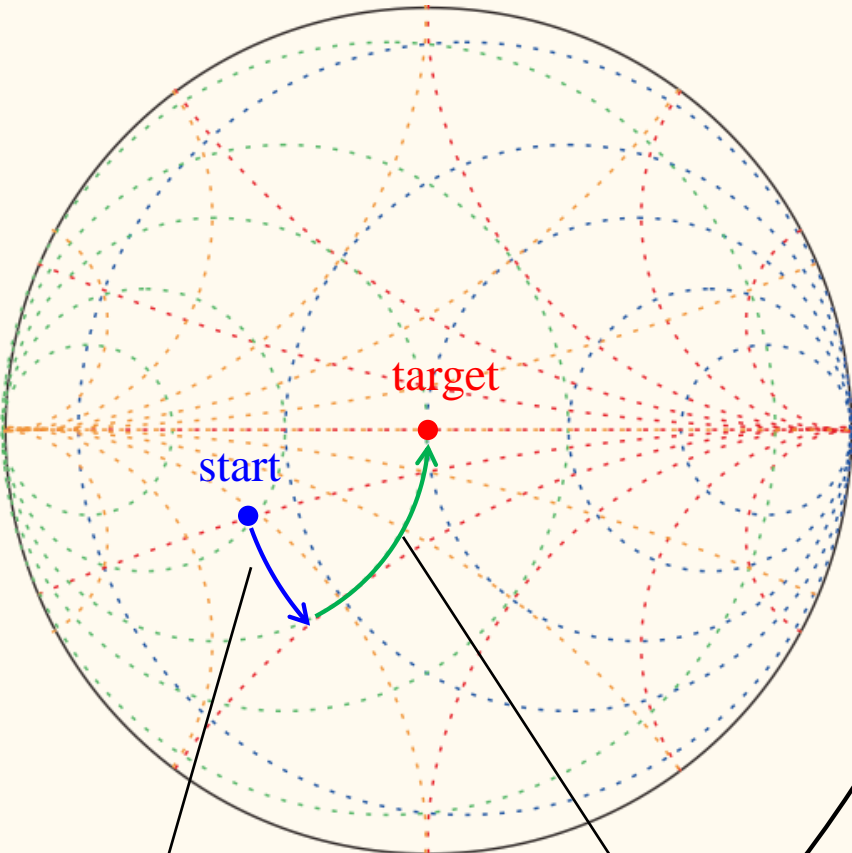
Admittance chart

# An example of impedance matching

frequency 100 MHz  $\approx$  628 Mrad/s

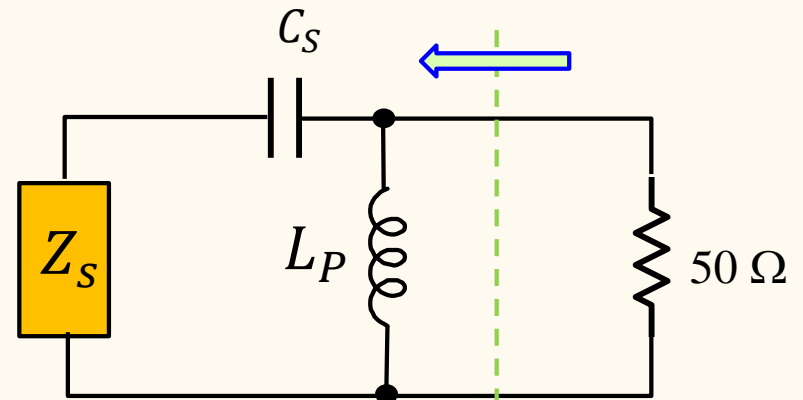
immittance chart

Im  $r$   $\uparrow$



$\text{Re}[Z]=0.4$

$\text{Re}[Y]=1$



$$Z_s = 20 - 10i \quad (\Omega)$$

$$= 0.4 - 0.2i \quad (Z_0)$$

$$\text{equalize: } 0.4 + iy = \frac{1}{1 + iq}$$

$$y = -\sqrt{0.24} \approx -0.49$$

$$-0.49 = -0.2 - \frac{1}{\omega C_S Z_0}$$

$$C_S \approx \frac{1}{2\pi \times 10^6 \times 50 \times 0.29} \approx 110 \text{ pF}$$

similarly

$$L_P \approx 65 \text{ nH}$$



# Impedance matching designer

<http://home.sandiego.edu/~ekim/e194rfs01/jwmatcher/matcher2.html>


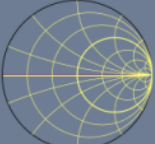
[http://leleivre.com/rf\\_lcmatch.html](http://leleivre.com/rf_lcmatch.html)

# Useful freeware: Smith v4.0

<http://fritz.dellsperger.net/smith.html>

← → ↻ fritz.dellsperger.net/smith.html ☆ W ↕

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 **Fritz Dellsperger** 

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## Smith-Chart Software and Related Documents

**NEW Software Smith V4.0**

[Smith V4.0](#) 6'664kB exe **Computer Smith-Chart Tool** and S-Parameter Plot, Setup Smith V4.0.exe 11.2016

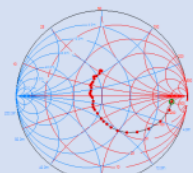
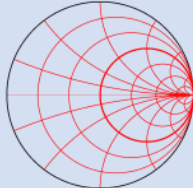
### 1. Smith-Chart Diagram

- Matching ladder networks with capacitors, inductors, resistors, serie and parallel RLC, transformers, serie lines and open or shorted stubs
- Free settable normalisation impedance for the Smith chart
- Circles and contours for stability, noise figure, gain, VSWR and Q
- Edit element values after insertion
- Tune element values using sliders (Tuning Cockpit) **NEW**
- Sweep versus frequency or datapoints
- Serial transmission line with loss
- Export datapoint and circle info to ASCII-file for post-processing in spreadsheets or math software
- Import datapoints from S-parameter files (Touchstone, CITI, EZNEC)
- Undo- und Redo-Function
- Save and load designs (licensed version only)
- Save netlist (licensed version only)
- Print Smith-Chart, schematic, datapoints, circle info and S-Plot graphs
- Copy to clipboard for documentation purposes
- Settings for color and line widths for all graphs

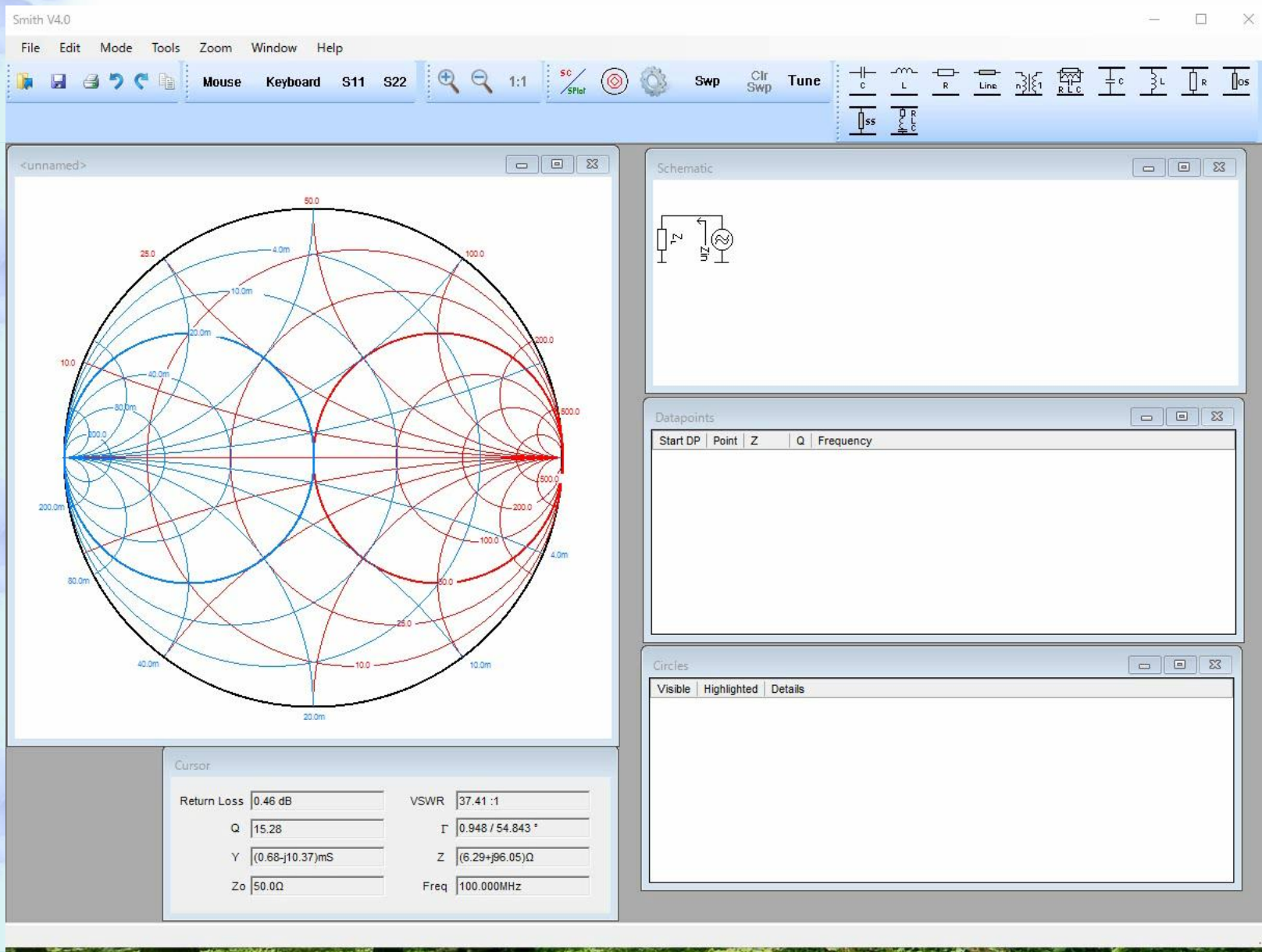
### 2. S-Plot

- Read S-Parameter - Files in Touchstone@-, CITI- and EZNEC-Format
- Graphical display of s11, s12, s21 and s22
- Graphical display and listing of MAG (maximum operating power gain), MSG (maximum stable gain), stability factor k and u and returnloss
- Linear or logarithmic frequency axis

**Download  
New Version 4.0  
Octobre 2016**

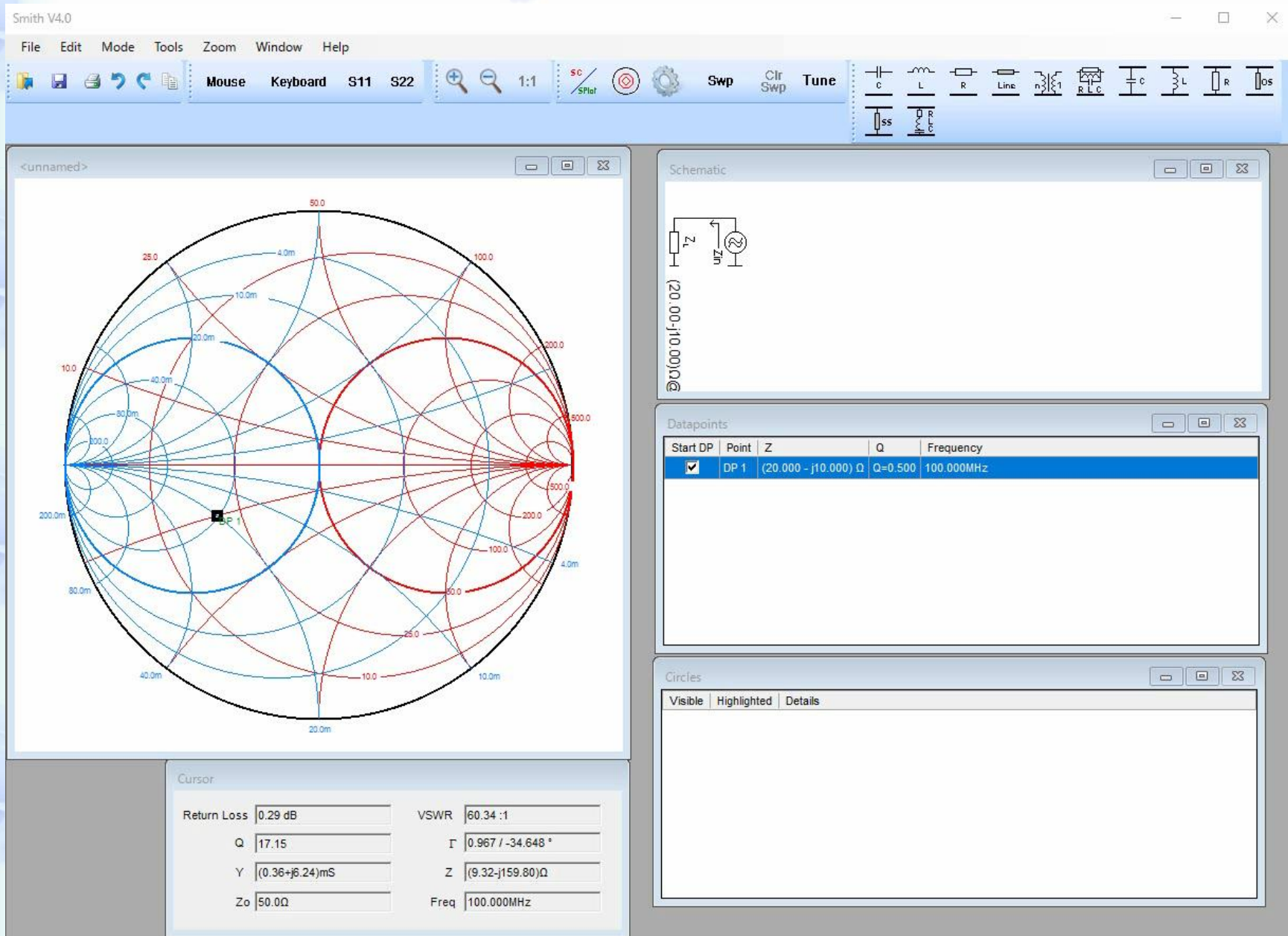


# Impedance matching with Smith V4 (1)

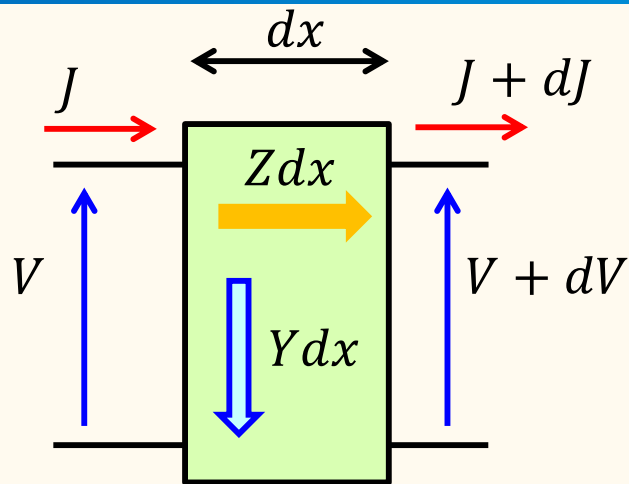




# Impedance matching with Smith V4 (2)



## 5.4 Non-TEM mode transmission line



$$\frac{V}{J} = \mp \frac{Z}{\kappa} = \mp \sqrt{\frac{Z}{Y}}$$

Characteristic impedance

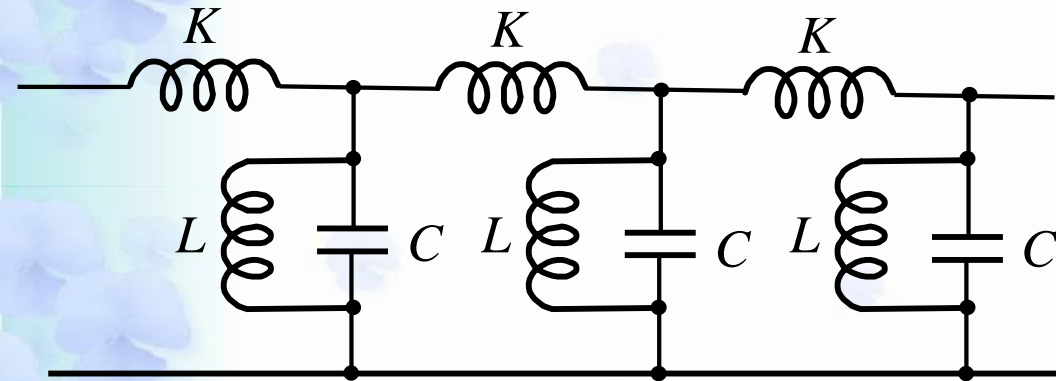
$$\text{LC model: } Z = i\omega L, \quad Y = i\omega C$$

The inductance represents magnetic fields circulating the core and the capacitance electric fields directing from the core to the shield.

$$Z_0 = \sqrt{\frac{L}{C}} \quad : \text{ real, dispersionless (no } \omega\text{-}k \text{ relation)}$$

Non-linear  $\omega$ -term in  $Z$  or  $Y \rightarrow$  dispersion (longitudinal components)

## 5.4 Non-TEM mode gives mass in transmission line



$C$ : capacitance per unit length  
 $L$ : inductance per inverse unit length  
 $K$ : inductance per unit length

$$Y = i\omega C + \frac{1}{i\omega L}$$

$$-k^2 = YZ = \left( i\omega C + \frac{1}{i\omega L} \right) i\omega K = -CK\omega^2 + \frac{K}{L}$$

Constant finite mass:  $E = \hbar\omega \propto k^2$

(Schrodinger eq.: Parabolic partial differential equation)

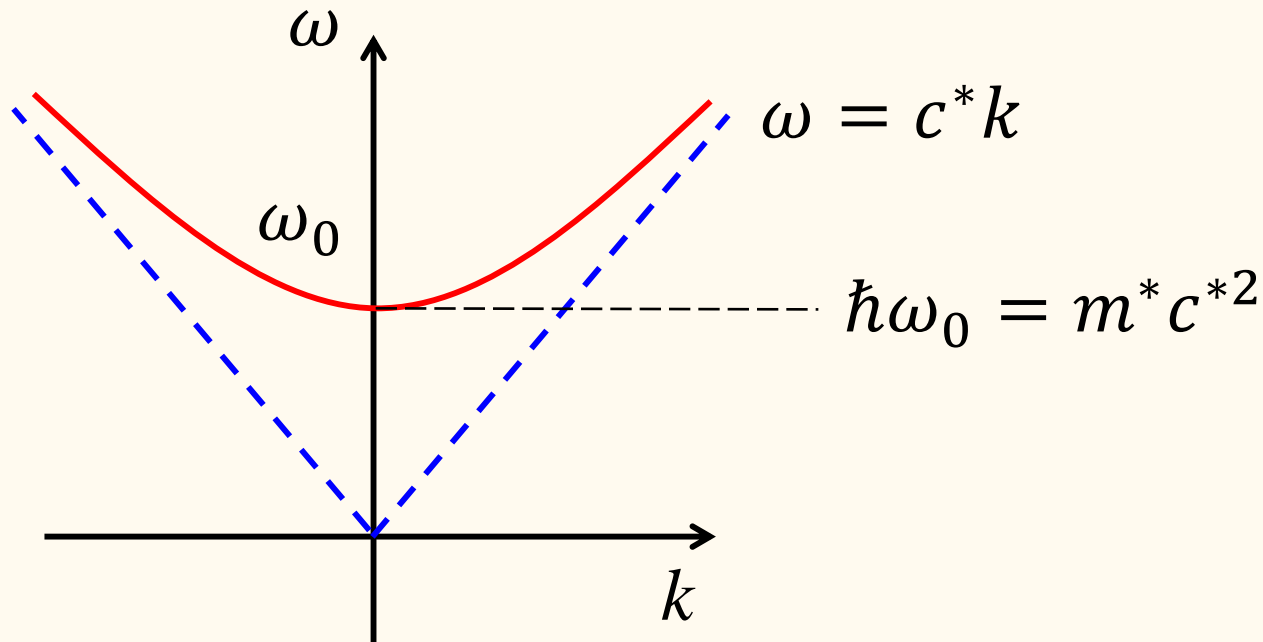
Coupling between linear dispersions: mass mechanism *cf.* Higgs

## 5.4 Non-TEM mode gives mass in transmission line

$$\frac{1}{\sqrt{LC}} = \omega_0 \text{ unchanged with } dx \rightarrow 0$$

$$Z = i\omega K, \quad Y = \frac{1 - (\omega/\omega_0)^2}{i\omega L}$$

$$ik = \kappa = \sqrt{YZ} = i \sqrt{\frac{K}{L} \left[ \left( \frac{\omega}{\omega_0} \right)^2 - 1 \right]} \quad \eta^2 \equiv \frac{K}{L}$$



## 5.4 Giving mass to LC transmission line

$$\omega \gg \omega_0 \rightarrow k \sim \eta \frac{\omega}{\omega_0} \quad \text{No dispersion}$$

$$\text{Velocity: } c^* = \frac{\omega}{k} = \frac{\omega_0}{\eta} = \frac{1}{\sqrt{KC}}$$

$$\omega \sim \omega_0 \quad \omega = \omega_0 + \delta\omega$$

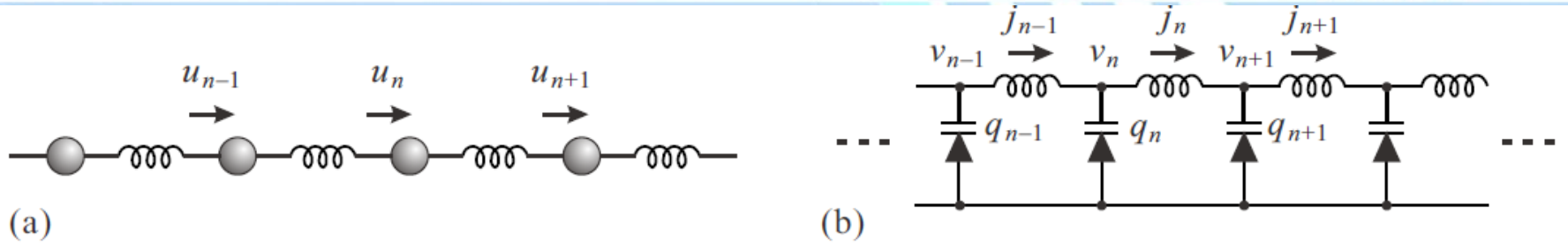
$$k^2 \approx 2\eta^2 \frac{\delta\omega}{\omega_0} \quad \therefore \epsilon \equiv \hbar\delta\omega = \frac{\hbar k^2}{2(\eta^2/\omega_0)} = \frac{\hbar^2 k^2}{2m^*}$$

$$m^* \equiv \frac{\hbar\eta^2}{\omega_0}$$

$$E_0 = \hbar\omega_0 = \frac{\hbar\eta^2}{\omega_0} \cdot \left(\frac{\omega_0}{\eta}\right)^2 = m^* c^{*2}$$



## 5.5 Non-linear LC transmission line and Toda lattice



Toda lattice is a typical non-linear system with exact (soliton) solutions. It is defined as follows:

The springs in (a) have Toda-potential:  $\phi(r) = \frac{a}{b}e^{-br} + ar \quad (ab > 0)$

Equation of motion:

$$m \frac{d^2 u_n}{dt^2} = -a \exp[-b(u_{n+1} - u_n)] + a \exp[-b(u_n - u_{n-1})]$$

For relative shift

$$r_n = u_{n+1} - u_n$$

$$m \frac{d^2 r_n}{dt^2} = a(2e^{-br_n} - e^{-br_{n+1}} - e^{-br_{n-1}})$$

Force of a spring:

$$f = -\phi'(r) = a(e^{-br} - 1)$$

# Solitons in Toda lattice

$$\frac{d^2}{dt^2} \log \left( 1 + \frac{f_n}{a} \right) = \frac{b}{m} (f_{n+1} + f_{n-1} - 2f_n)$$

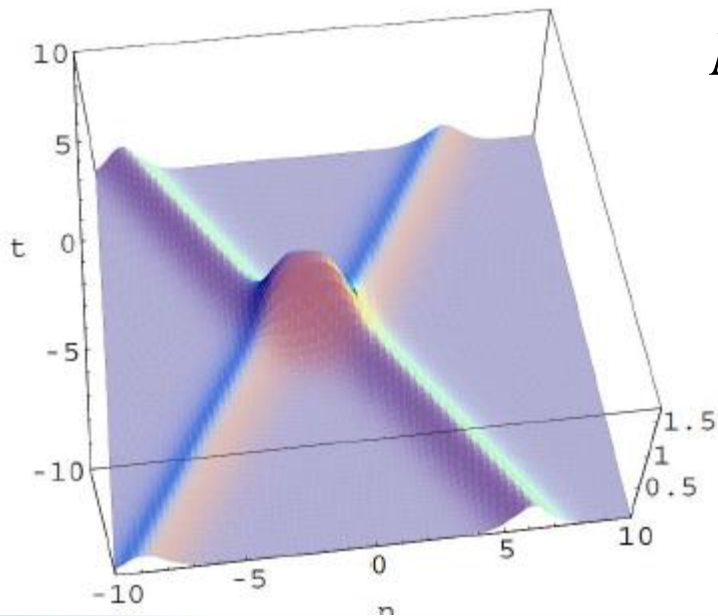
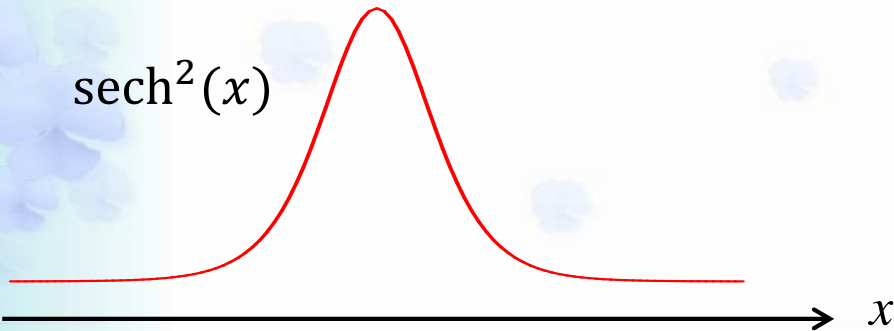
Soliton solution:

$$u_n = \omega^2 \operatorname{sech}^2(\kappa n + \sigma \omega t + \delta),$$

$$\sigma = \pm 1, \quad \omega = \sinh \kappa,$$

$\kappa, \delta$  : constants

$\operatorname{sech}^2(x)$



$N = 2$  soliton solution:

$$u_n = \frac{\tau_{n+1}\tau_{n-1}}{\tau_n^2} - 1,$$

$$\tau_n = 1 + e^{2\eta_1} + e^{2\eta_2} + A_{12}e^{2(\eta_1+\eta_2)},$$

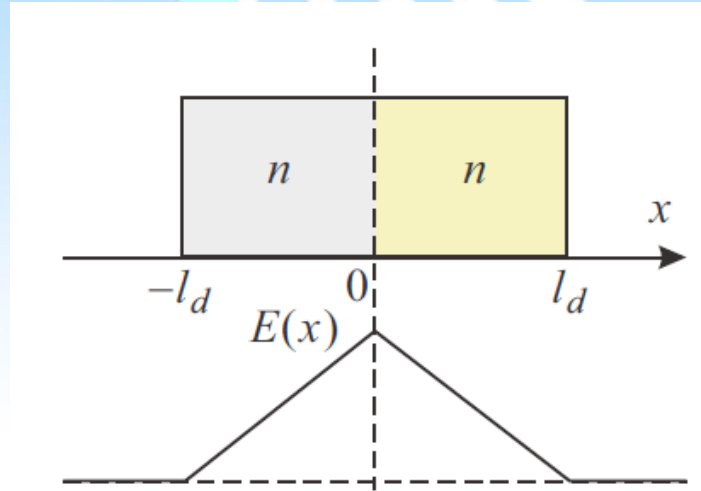
$$\eta_i = \kappa_i n + \sigma_i \omega_i t + \delta_i, \quad \sigma_i = \pm 1, \quad \omega_i = \sinh \kappa_i,$$

$$A_{12} = \frac{ab \sinh^2(\kappa_1 - \kappa_2) - m(\sigma_1 \omega_1 - \sigma_2 \omega_2)^2}{m(\sigma_1 \omega_1 + \sigma_2 \omega_2)^2 - ab \sinh^2(\kappa_1 + \kappa_2)}$$

# Non-linear capacitance: Vari-cap



Varicap BB505

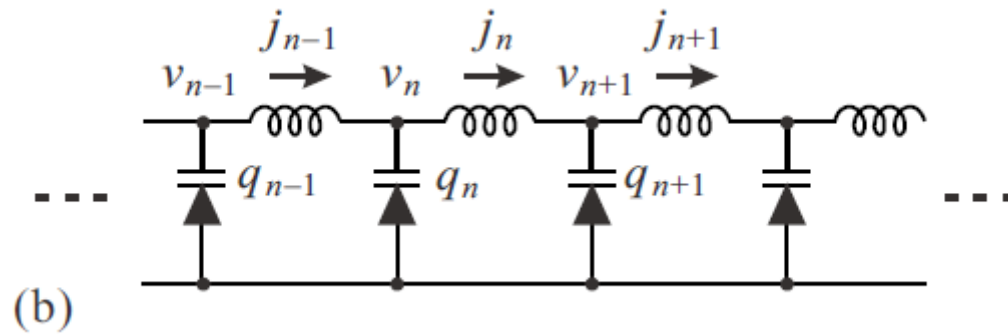


$$V_b = \frac{en}{\epsilon} \int_{-l_d}^0 2(x + l_d) dx + \frac{en}{\epsilon} \int_0^{l_d} 2(l_d - x) dx = \frac{2enl_d^2}{\epsilon}$$

$$V + V_b = \frac{2en}{\epsilon} \left( l_d + \frac{Q}{nS} \right)^2 \quad \therefore C = \frac{dQ}{dV} = \sqrt{\frac{\epsilon}{2en}} \frac{nS}{\sqrt{V + V_b}}$$

$$V + V_b = V_0 + \delta V \quad \delta V \rightarrow V$$

# L-Varicap transmission line



$$L \frac{dJ_n}{dt} = v_n - v_{n-1},$$

$$\frac{dq_n}{dt} = J_{n-1} - J_n,$$

$$q_n = \int_0^{v_n} C(V) dV, \quad C(V) = \frac{Q(V_0)}{F(V_0) + V - V_0}$$

$$q_n = Q(V_0) \log \left[ 1 + \frac{V_n}{F(V_0)} \right] + \text{const.}$$

$$\frac{d^2}{dt^2} \log \left[ 1 + \frac{V_n}{F(V_0)} \right] = \frac{1}{LQ(V_0)} (V_{n-1} + V_{n+1} - 2V_n)$$

# Solitons in non-linear circuit

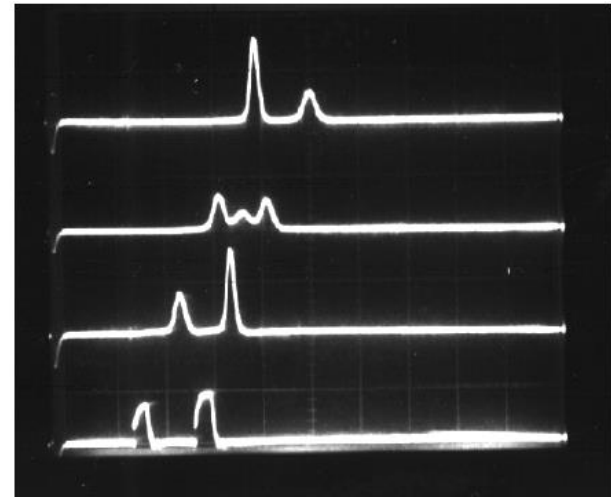
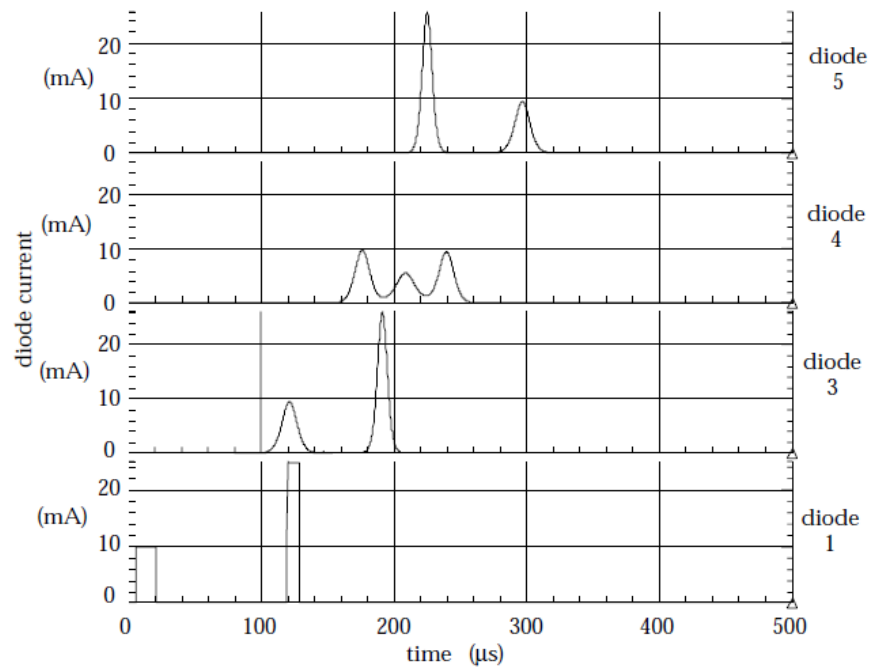
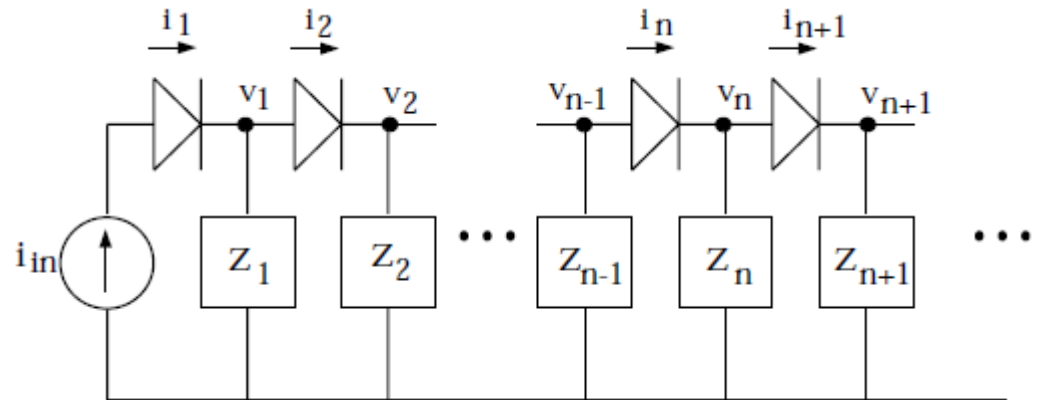
International Journal of Bifurcation and Chaos, Vol. 9, No. 4 (1999) 571-590  
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## CIRCUIT IMPLEMENTATIONS OF SOLITON SYSTEMS\*

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# Toda lattice circuit, Soliton circuit

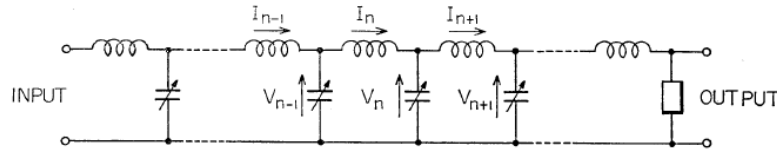
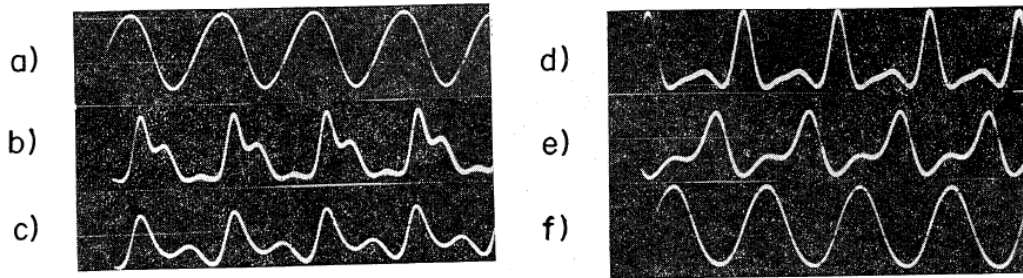


Fig. 1. A nonlinear network equivalent to a one-dimensional anharmonic lattice. The circuit element have an inductance  $L=22 \mu\text{H}$  or capacitance  $C(V)=27 V^{-0.48} \text{ pF}$ .



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## Studies on Lattice Solitons by Using Electrical Networks

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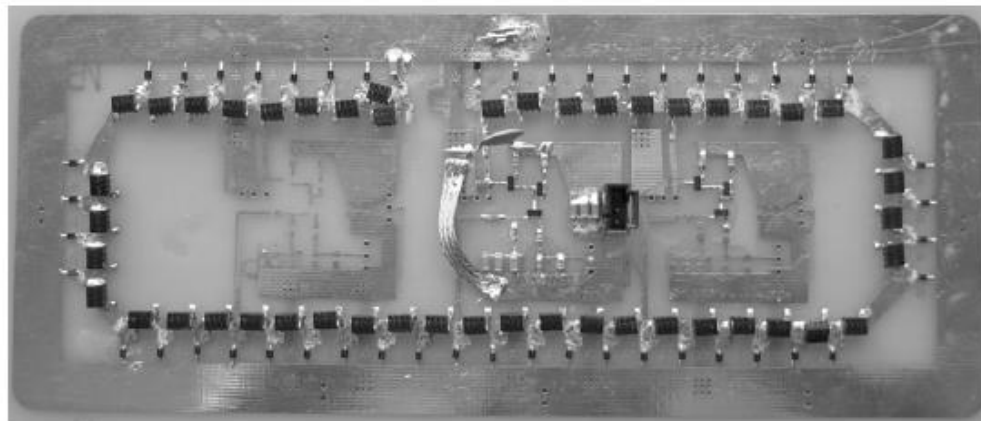


Fig. 16. Microwave soliton oscillator prototype.

