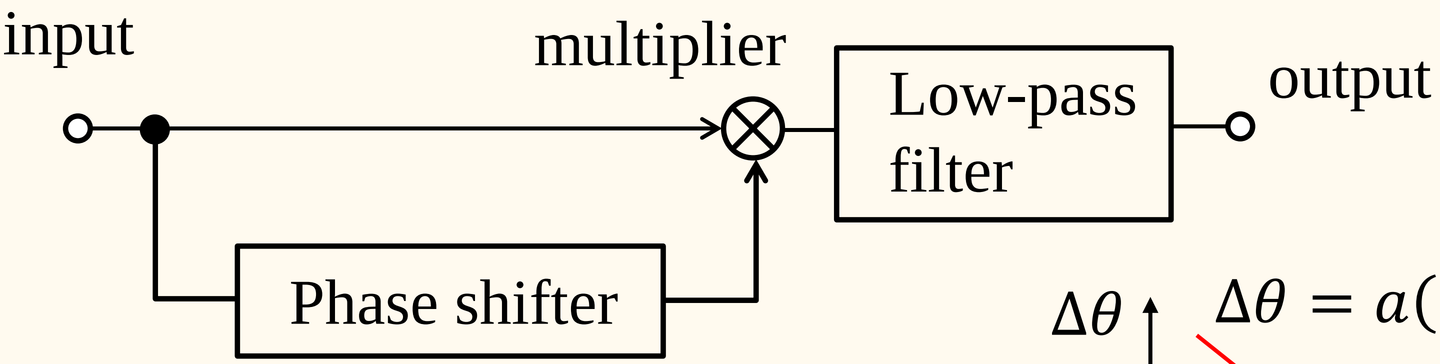
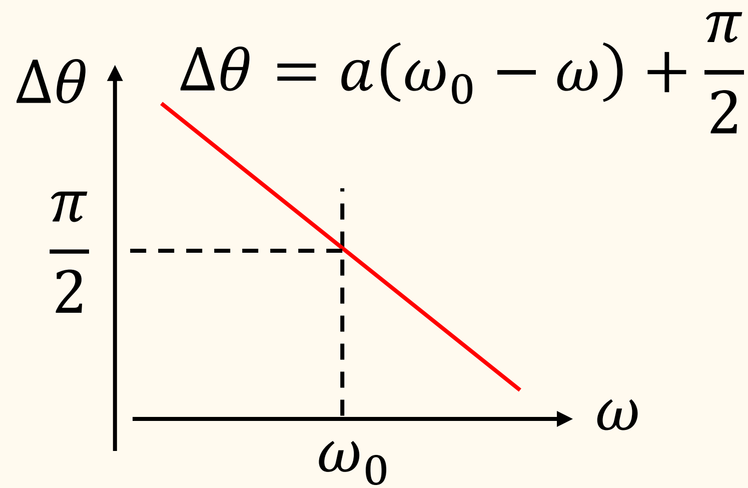
Exercise F-1

Show that the following circuit works as a demodulator of frequency modulation (FM) signal (quadrature demodulator).



Here the phase shifter gives the shift proportional to the frequency difference between input and the career frequency ω_0. The shift at ω_0 is $\pi/2$ as shown in the right (this can be achieved with resonant circuits). The low-pass filter cuts components with frequencies as high as ω_0.



Exercise F-1

(hint) Assume the original signal $f(t)$ is much slower than the carrier $A\cos(\omega_0 t)$. Then the input can be approximated as

$$s(t) = A\cos\{[\omega_0 + k_f f(t)]t\}.$$

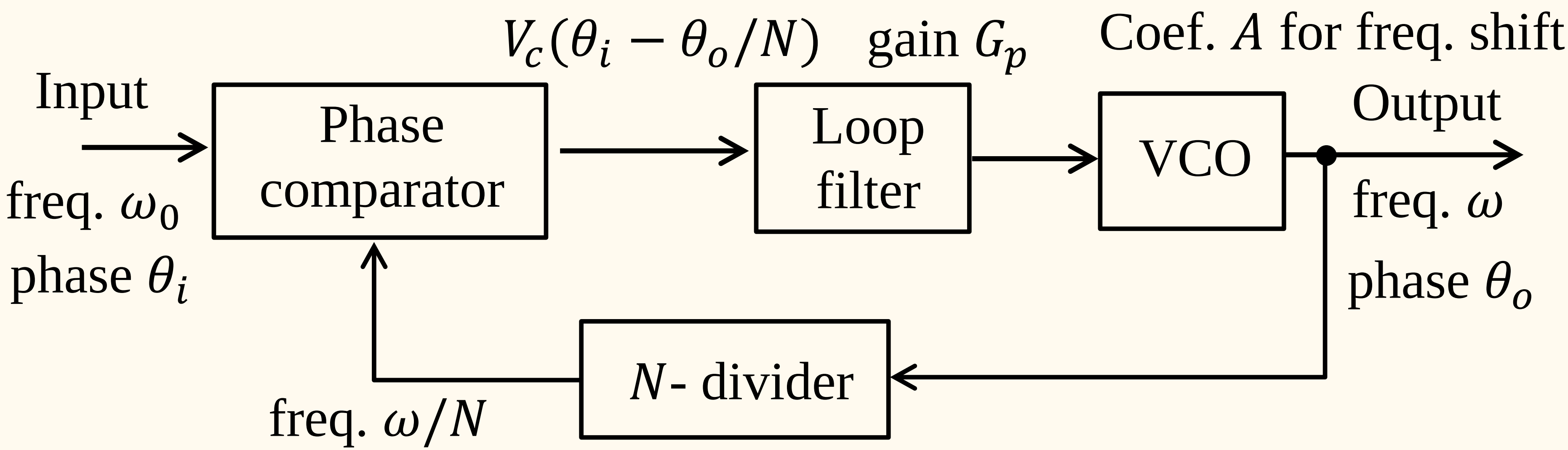
Then the phase shifter output is

$$q_{\mathrm{ps}}(t) = A\sin\{[\omega_0 + k_f f(t)]t - ak_f f(t)\}.$$

Taking product and high-frequency filtering gives …
(use $ak_f f(t) \ll 1$).

Exercise F-2

In the following phase lock loop (PLL) circuit, the initial ($t = 0$) oscillation frequency of voltage-controlled oscillator (VCO) ω deviates from $N\omega_0$ by $\Delta\omega$. Obtain the relaxation time of ω to $N\omega_0$.



(hint) Here we can put $\theta_i = 0$ hence input $= V_i \sin \omega_0 t$ without loosing generality. Similarly output $= V_o \sin[N\omega_0 t + \theta_o(t)]$. Now $\omega = N\omega_0 + d\theta_o/dt$ and it is easy to write $d\theta_o/dt$ with A, G_p, V_c, $\theta_o(t)$ and a constant.

Exercise F-3

Solve the difference equation below with z-transform.

$$\begin{cases} x(n) - 2x(n-1) = n & (n \geq 0) \\ x(n) = 0 & (n < 0) \end{cases}$$

(hint) z-transform of n is $\dfrac{z}{(z-1)^2}$ as in the table (slide no.14).

Then z-transform of $x(n)$: $X(z)$ is easily obtained. Inverse z-transform gives $x(n)$.

Answer sheet submission deadline: 11^{th} Jan. 2017.