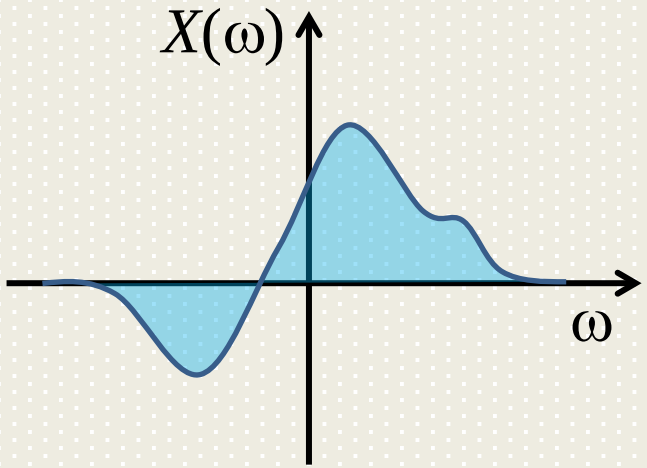
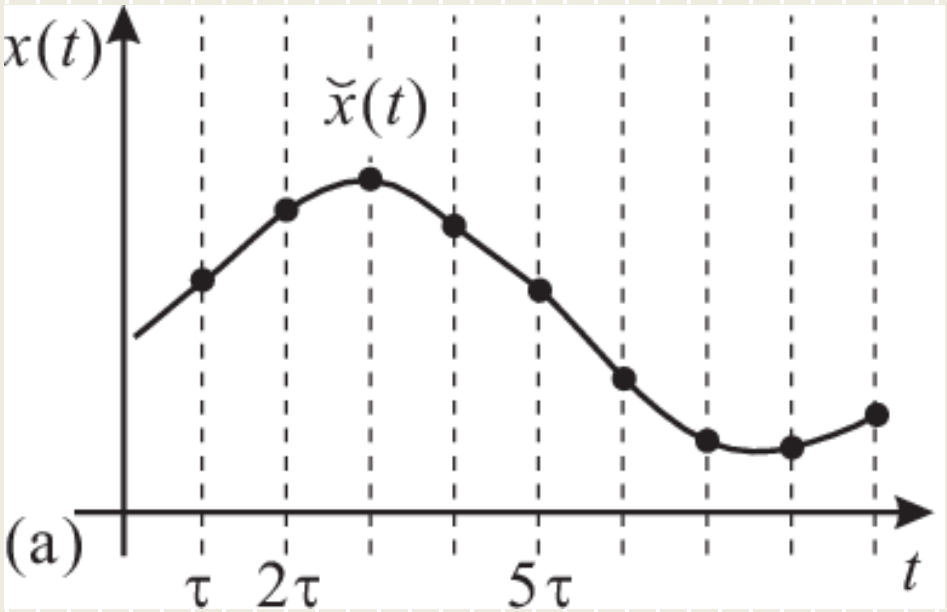
電子回路論13

2015.1.8 勝本信吾

fppt.com

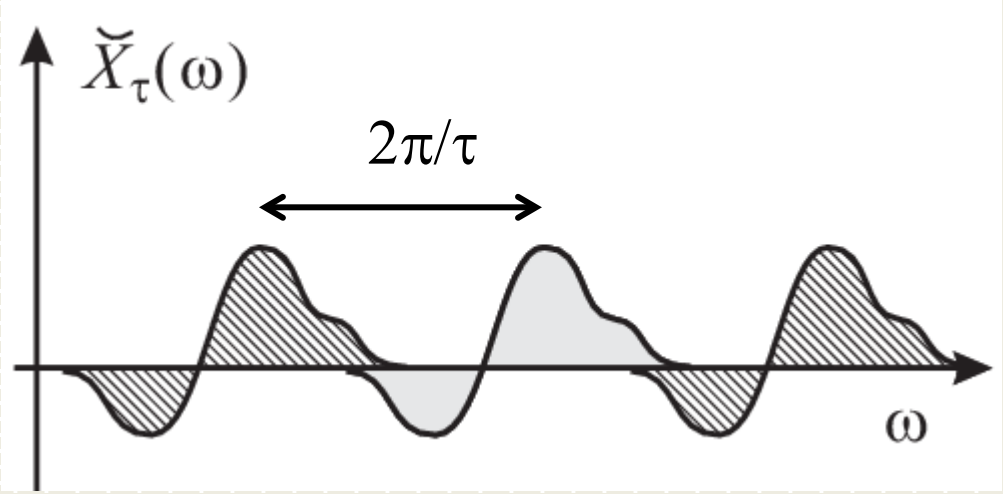
標本化定理

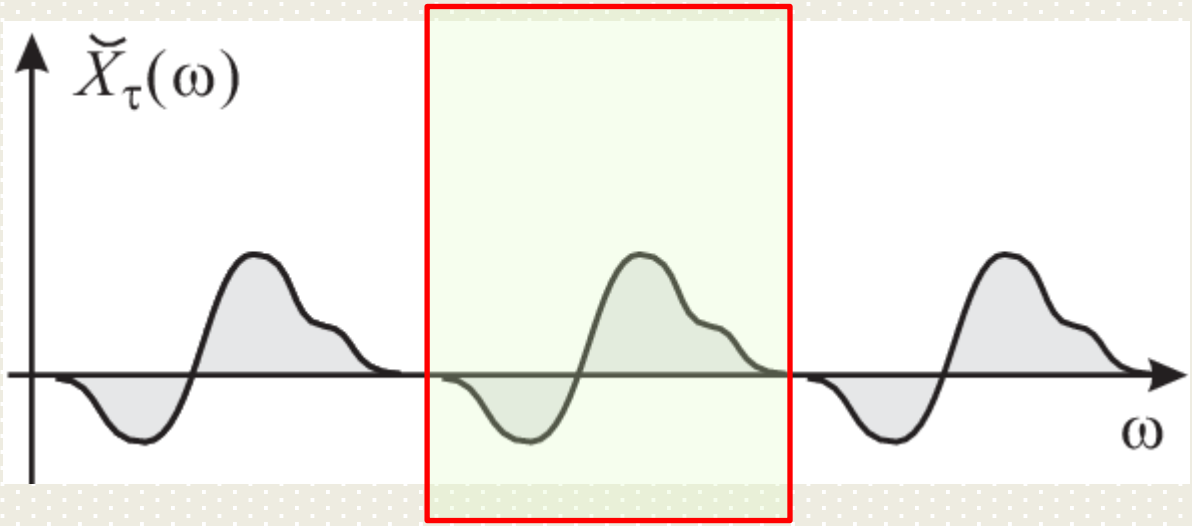




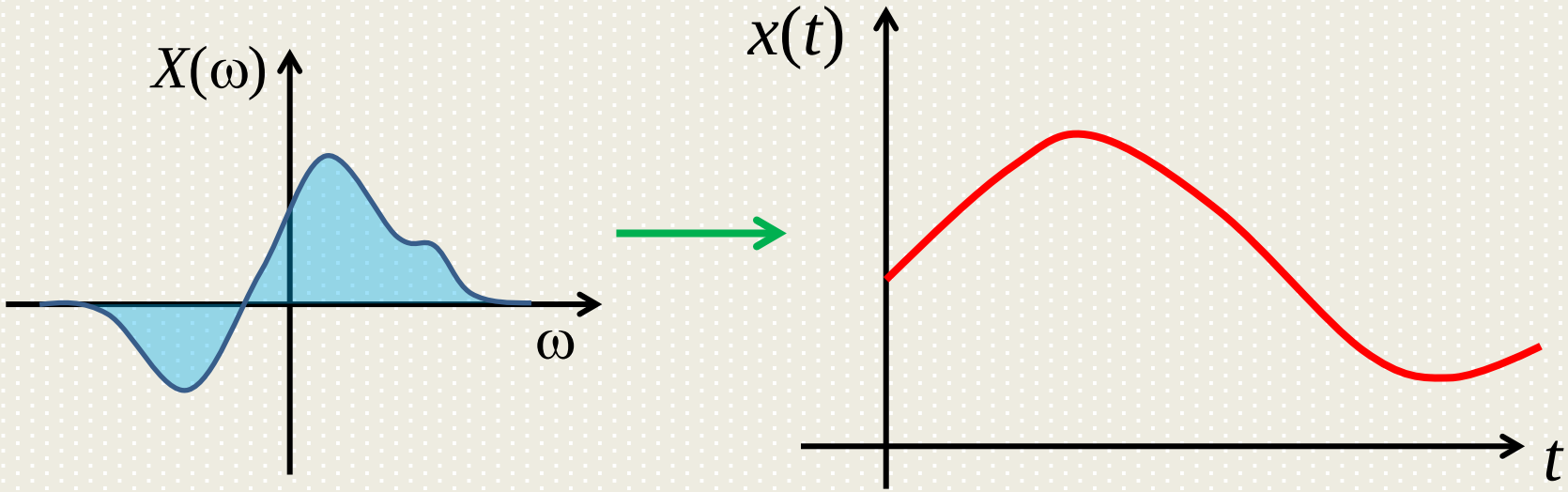
1928 H. Nyquist
1949 C. Shannon
染谷勲

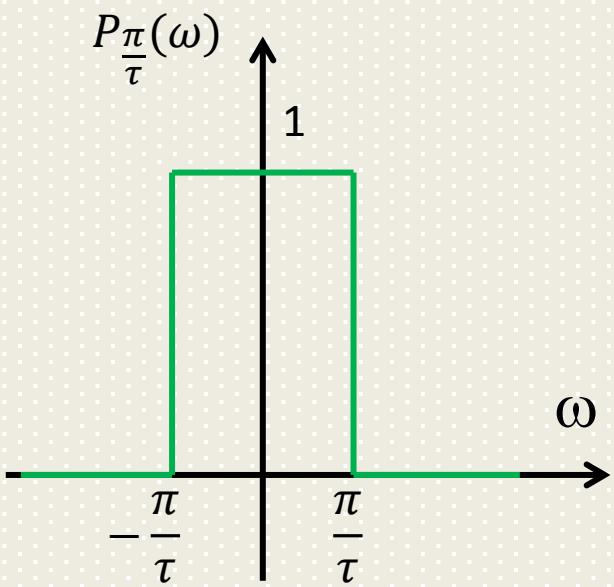
周波数スペクトルの「切り出し」

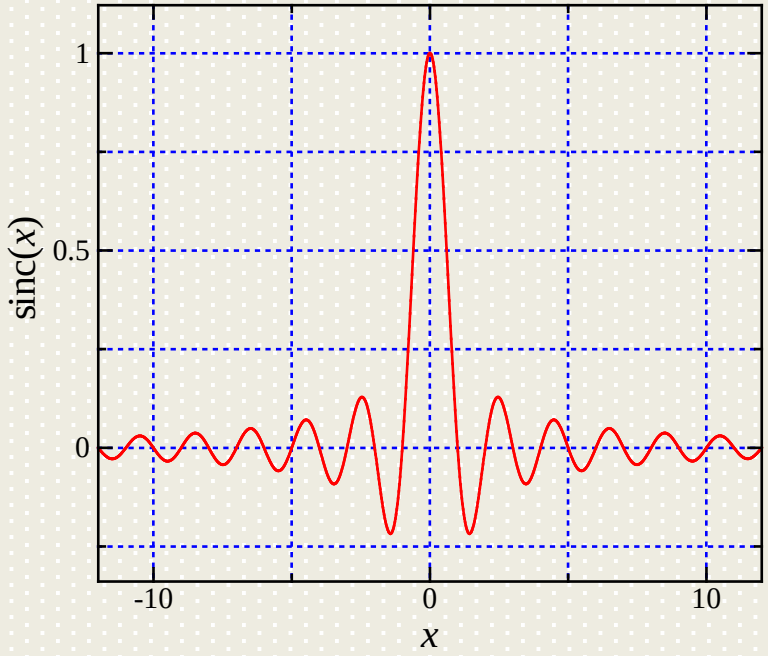




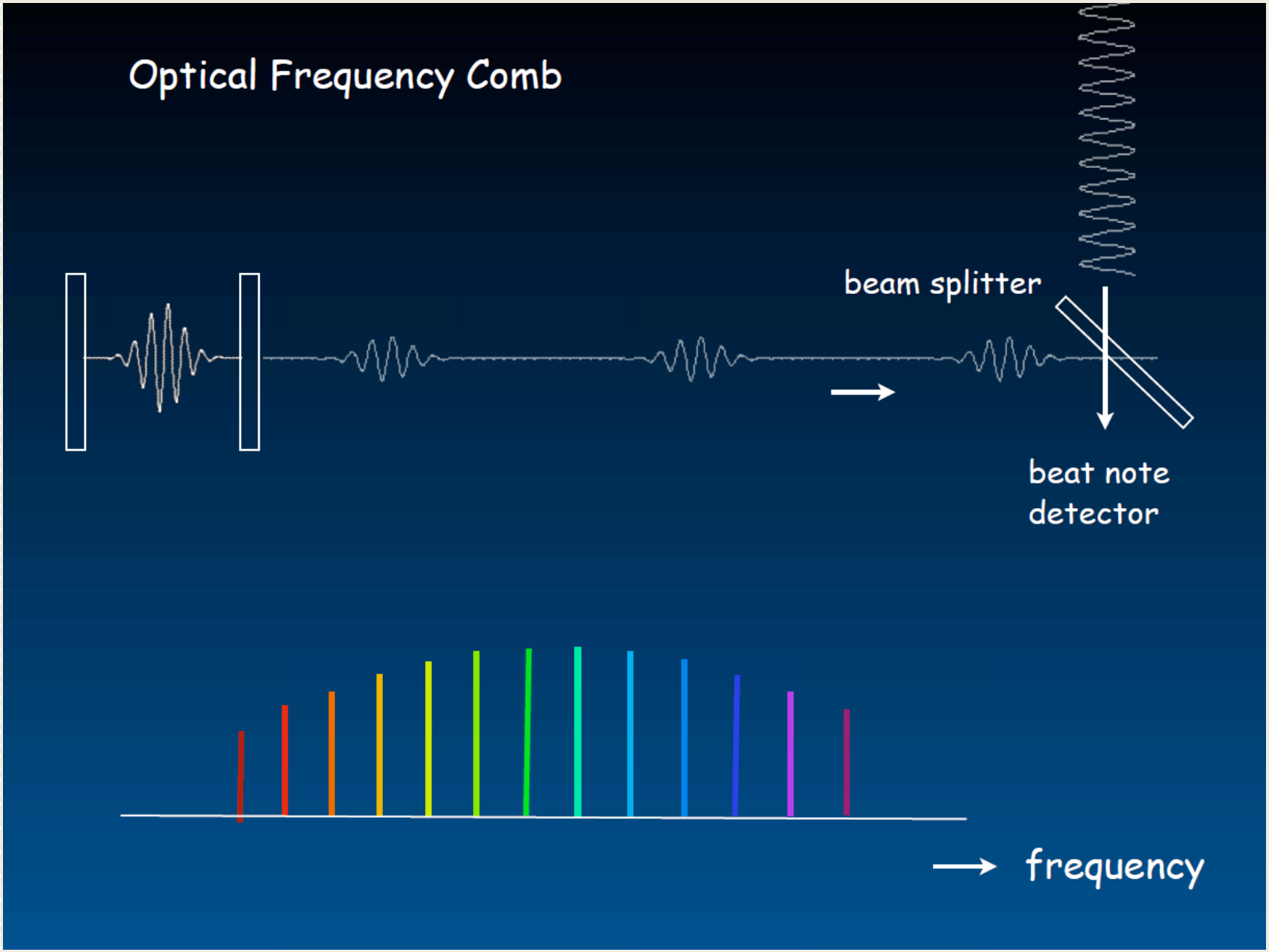
信号の復元


$X(\omega)$
ω
$x(t)$
t

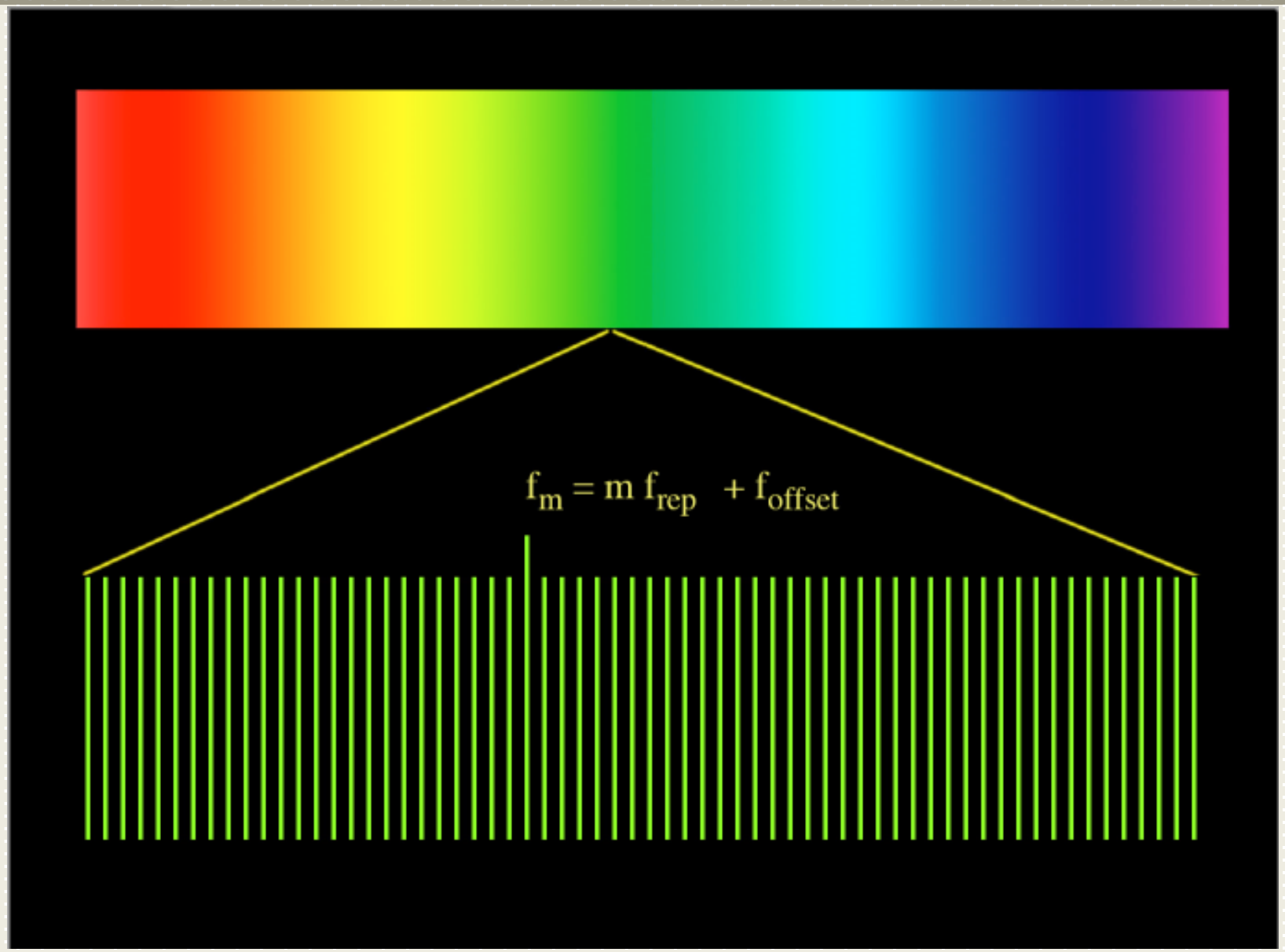

$P_{\frac{\pi}{\tau}}(\omega)$
1
ω
$-\frac{\pi}{\tau}$
$\frac{\pi}{\tau}$


1
0.5
0
sinc(x)
-10
0
10
x

Optical Frequency Comb

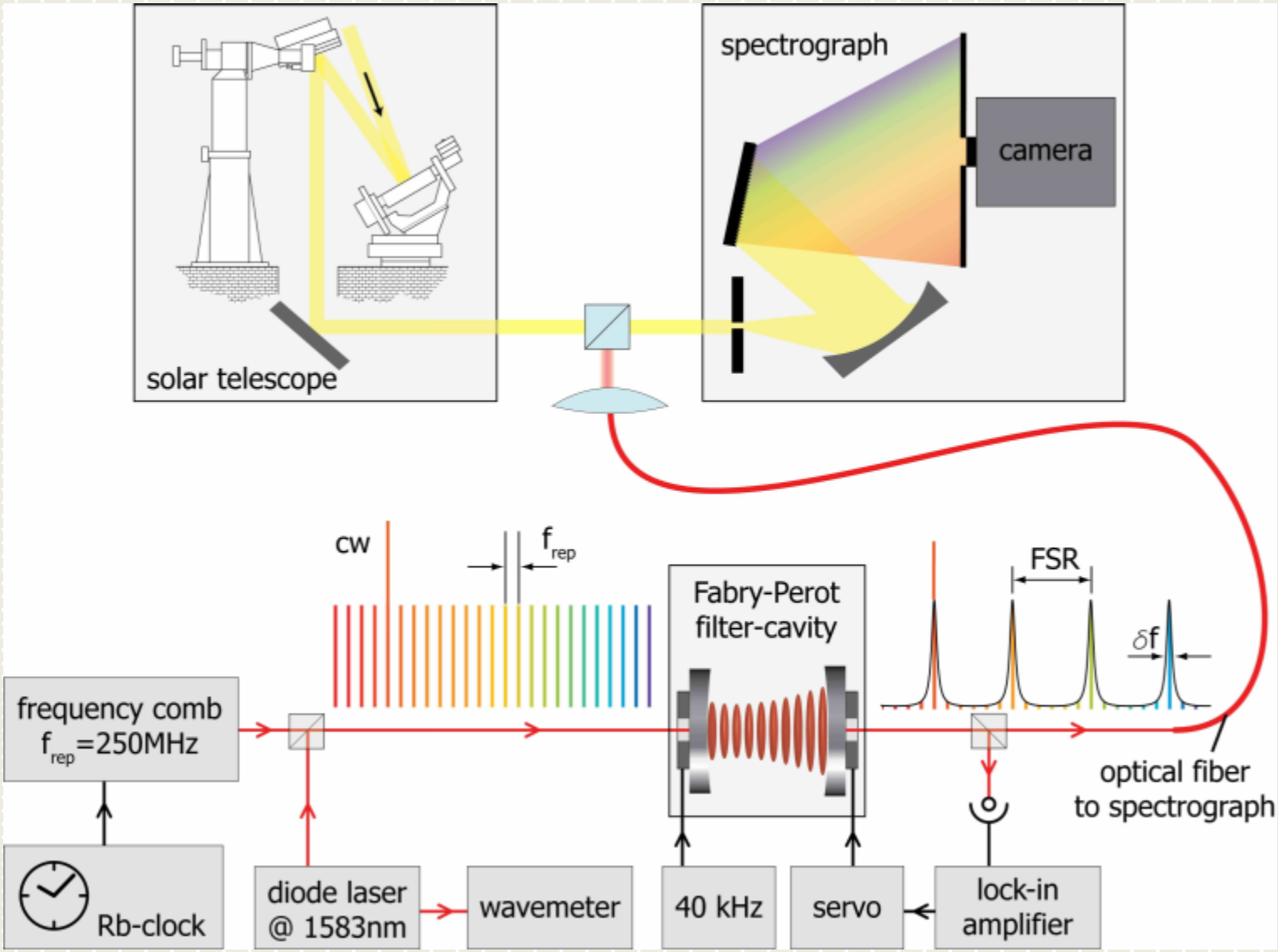


Frequency Comb

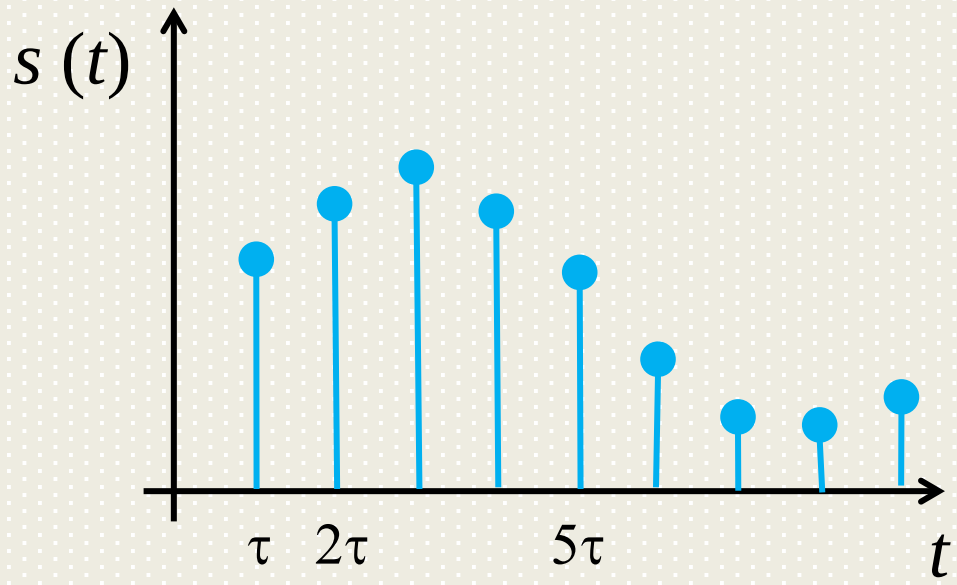


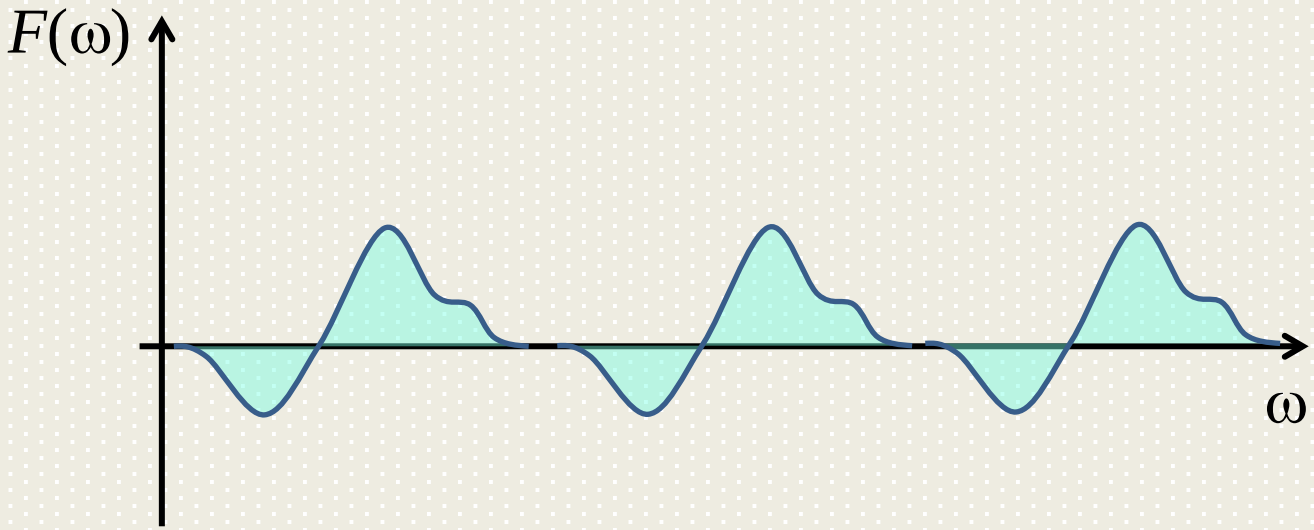
Theodor Hänsch, Max-Planck Institute, Science 2008

宇宙膨張ドップラー効果の測定



パルス振幅変調





Fast Fourier Transform FFT

$$F_k = \sum_{n=0}^{N-1} f_n W_N^{kn}$$

Discrete Fourier Transform, DFT

N : assumed to be even

$$= \sum_{n=0}^{N/2-1} f_{2n} W_N^{2nk} + \sum_{n=0}^{N/2-1} f_{2n+1} W_N^{(2n+1)k}$$

$$= \sum_{n=0}^{N/2-1} f_{2n} W_{N/2}^{nk} + W_N^k \sum_{n=0}^{N/2-1} f_{2n+1} W_{N/2}^{nk}. \quad \because W_N^{2nk} = W_{N/2}^{nk}$$

$$k = N/2, \cdots, N-1 \qquad k \to k + N/2$$

$$W_{N/2}^{nN/2} = 1, \quad W_N^{k+N/2} = -W_N^k$$ より

Fast Fourier Transform FFT

$$F_{k+N/2} = \sum_{n=0}^{N/2-1} f_{2n} W_{N/2}^{n(k+N/2)} + W_N^{k+N/2} \sum_{n=0}^{N/2-1} f_{2n+1} W_{N/2}^{n(k+N/2)}$$

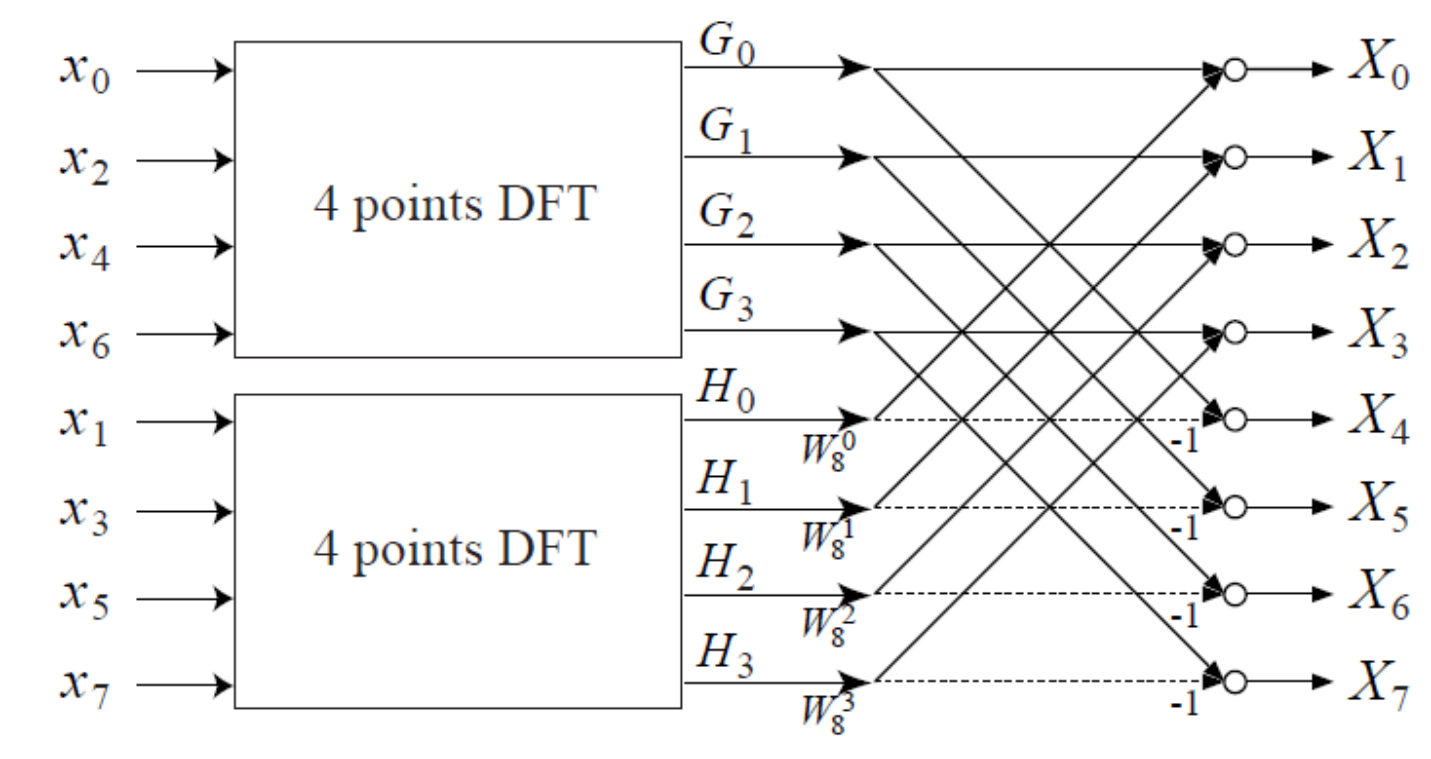
$$= \sum_{n=0}^{N/2-1} f_{2n} W_{N/2}^{nk} - W_N^k \sum_{n=0}^{N/2-1} W_{N/2}^{nk}$$

$$\begin{cases} X_k = \displaystyle\sum_{n=0}^{N/2-1} f_{2n} W_{N/2}^{nk}, \\ Y_k = \displaystyle\sum_{n=0}^{N/2-1} f_{2n+1} W_{N/2}^{nk} \quad (k = 0, \cdots, N/2-1) \end{cases}$$

Fast Fourier Transform FFT

$$F_k = X_k + W_N^k Y_k, \qquad k = 0, \cdots, N/2 - 1$$

$$F_{k+N/2} = X_k - W_N^k Y_k,$$



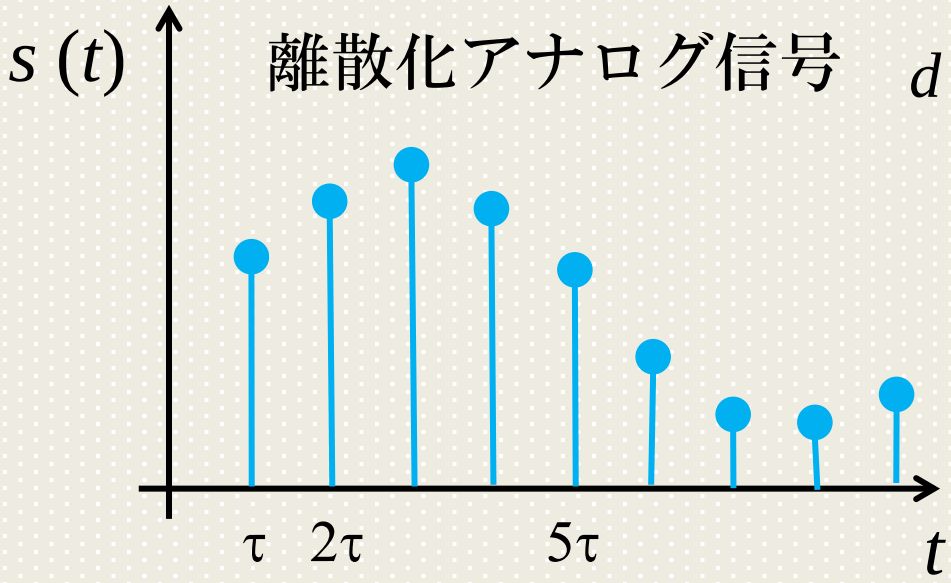
$N = 2^q$ とすると，N^2個の乗算を$(N/2)\log_2 N = qN/2$回の乗算に落とすことができたことになる．

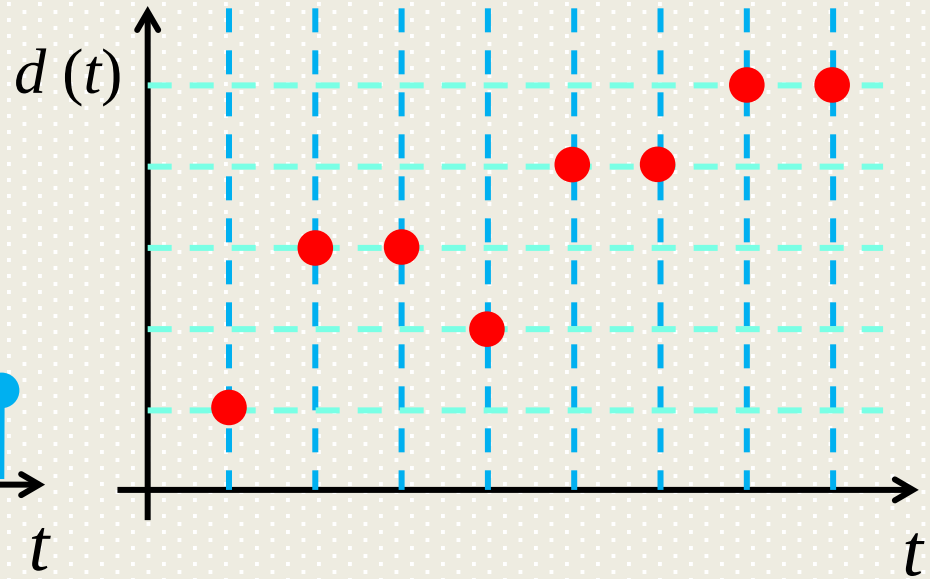
(片側) z 変換

f_n	$F(z)$	収束領域
$\delta(n)$	1	全平面
1	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
n	$\dfrac{z^{-1}}{(1-z^{-1})^2}$	$\lvert z\rvert > 1$
n^k	$\left(-z\dfrac{d}{dz}\right)^k \dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
a^n	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$\sin(n\omega\tau)$	$\dfrac{\sin(\omega\tau)z^{-1}}{1-2\cos(\omega\tau)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$e^{-n\alpha\tau}\cos(n\omega\tau)$	$\dfrac{1-e^{-\alpha\tau}\cos(\omega\tau)z^{-1}}{1-2e^{-\alpha\tau}\cos(\omega\tau)z^{-1}+e^{-2\alpha\tau}z^{-2}}$	$\lvert z\rvert > e^{-\alpha\tau}$

片側 z 変換の性質

性質名	信号	z 変換
線形性	$af_n + bg_n$	$aF(z) + bG(z)$
相似則	$f_{\alpha n}$	$F(z^{1/\alpha})$
時間移動	f_{n+k}	$z^k \left[F(z) - \sum_{l=0}^{k-1} f(l) z^l \right]$
時間移動 II	f_{n-k}	$z^{-k} F(z)$
スケール変換	$e^{\mp \alpha n} f_n$	$F(e^{\pm \alpha} z)$
スケール変換 II	$a^n x_n$	$F(a^{-1} z)$
インデックスとの積	$n f_n$	$-z \frac{d}{dz} F(z)$
微分	$n^k f_n$	$\left(-z \frac{d}{dz} \right)^n F(z)$
積分	$\frac{f_n}{n+a}$	$z^a \int_z^\infty \xi^{-a+1} F(\xi) d\xi$
畳み込み	$f_n * g_n$	$F(z) \cdot G(z)$
積	$f_n \cdot g_n$	$\frac{1}{2\pi i} \oint_c F(\xi) G\left(\frac{z}{\xi}\right) \xi^{-1} d\xi$





値も離散化 →ディジタル信号

信号単位： 0 or 1 bit (ビット)

論理演算： F or T

電圧レベル： L or H

多ビット→2進数演算→信号のパラレル化

論理ゲート

ディジタル信号→ 論理値 → 論理演算　をするのが
logic gatesである.

		input					output			
t		t_1	t_2	$\cdots$	t_m		t_1	t_2	$\cdots$	t_m
Ch.	1	0	1	$\cdots$	f_{1m}	1	1	1	$\cdots$	q_{1m}
	2	1	0	$\cdots$	f_{2m}	2	0	1	$\cdots$	q_{2m}
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	n	0	1	$\cdots$	f_{nm}	l	0	1	$\cdots$	f_{lm}

組み合わせ回路(combinational logic)

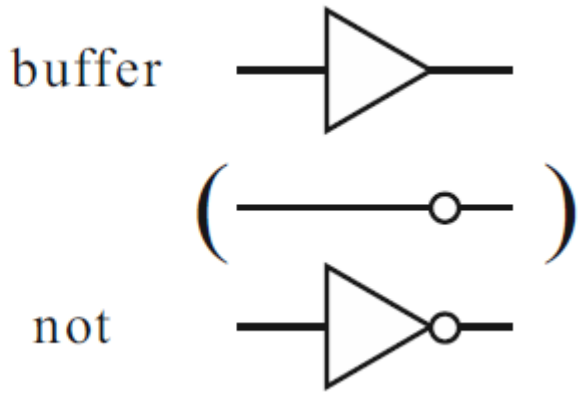
順序回路 (sequential logic)

組み合わせ回路：1入力ゲート

真理値表

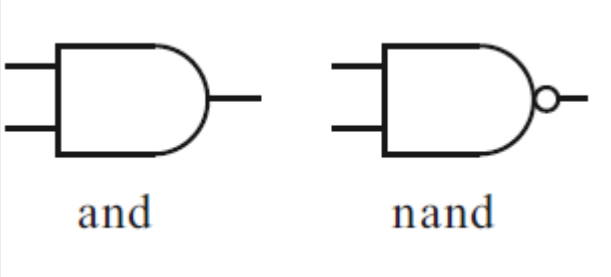
input	buffer	not
0	0	1
1	1	0

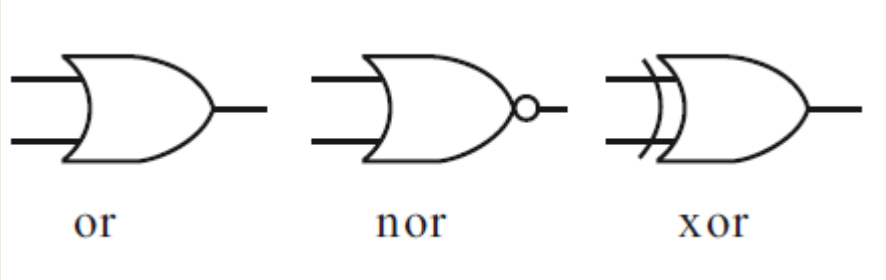
回路記号



組み合わせ回路：２入力ゲート

input1	input 2	and	or	xor	nand
0	0	0	0	0	1
1	0	0	1	1	1
0	1	0	1	1	1
1	1	1	1	0	0

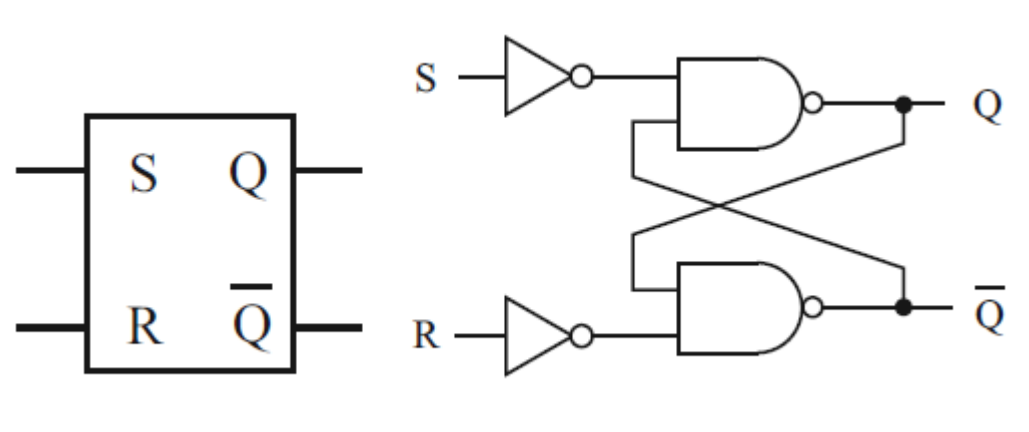




順序回路：フリップフロップ

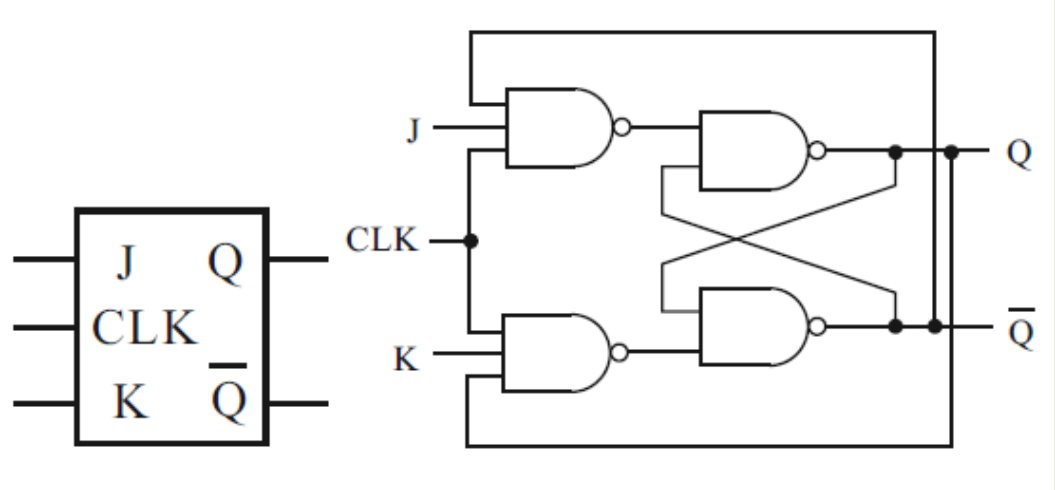
RS (reset-set) フリップフロップ (FF)

S	R	Q	$\bar{Q}$	動作
0	0	Q	$\bar{Q}$	不変
0	1	0	1	reset
1	0	1	0	set
1	1	不定		



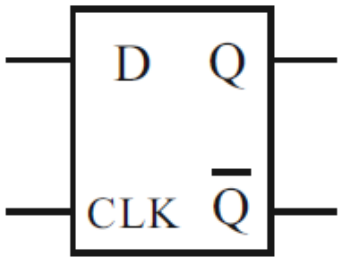
JK フリップフロップ

J	K	Q	次の CLK 入力時の Q
0	0	0	0
0	0	1	1
0	1	–	0
1	0	–	1
1	1	0	1
1	1	1	0



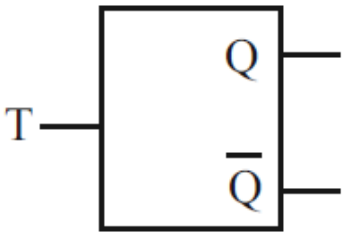
D-FF，T-FF

(a)



D	CLK	Q
0	↑	0
1	↑	1
–	↓	Q (hold)

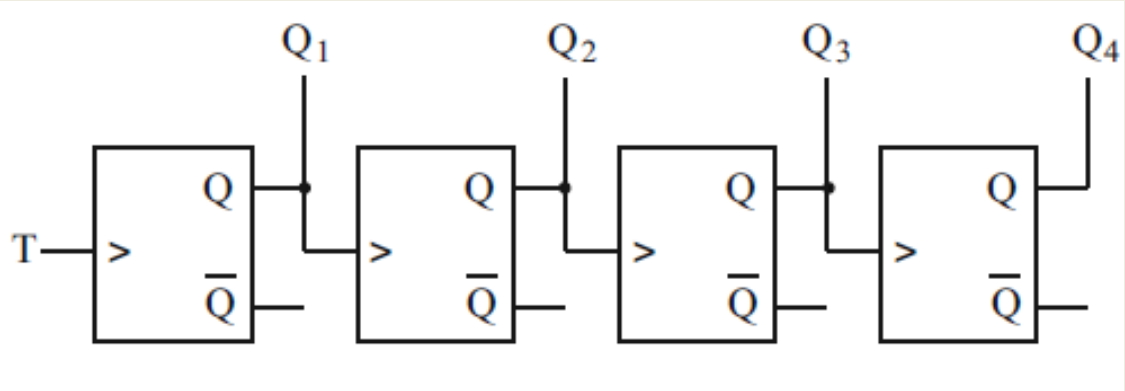
(b)



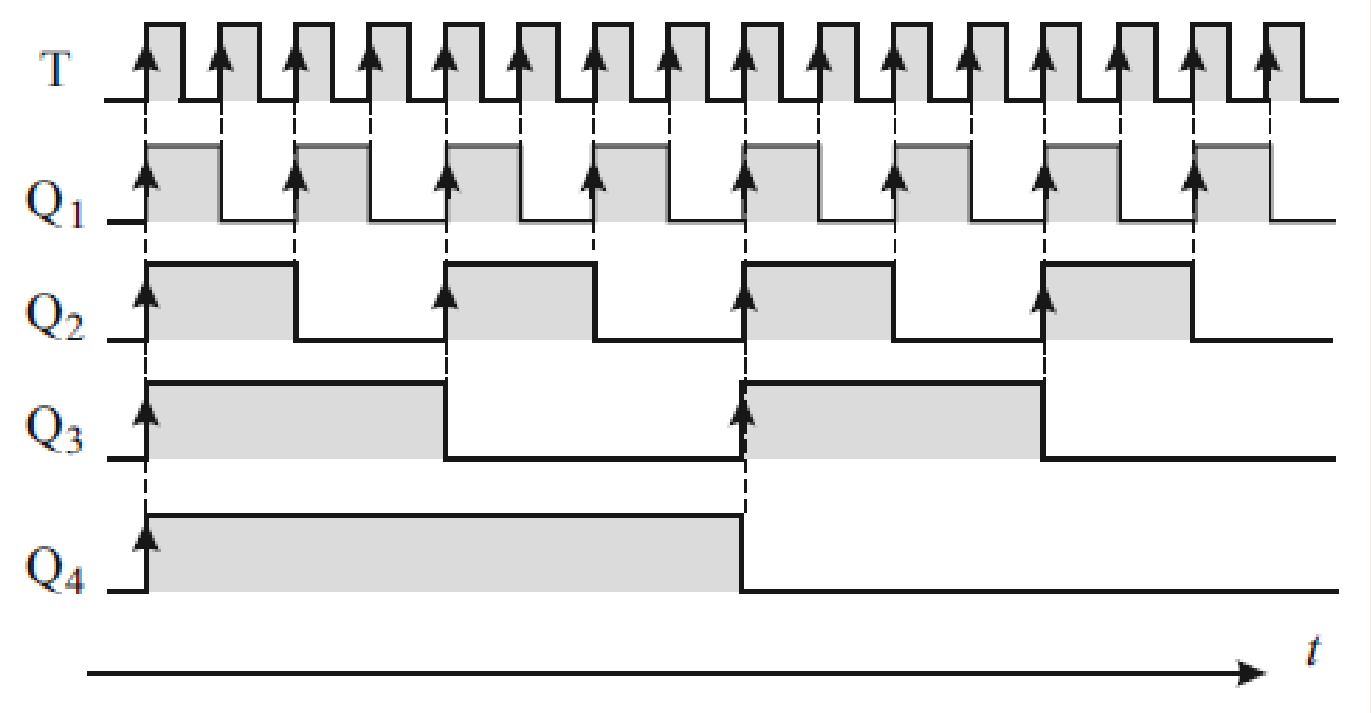
T	Q	Q
↓	0	0
↓	1	1
↑	0	1
↑	1	0

順序回路：カウンタ

非同期カウンタ
(リップルカウンタ)



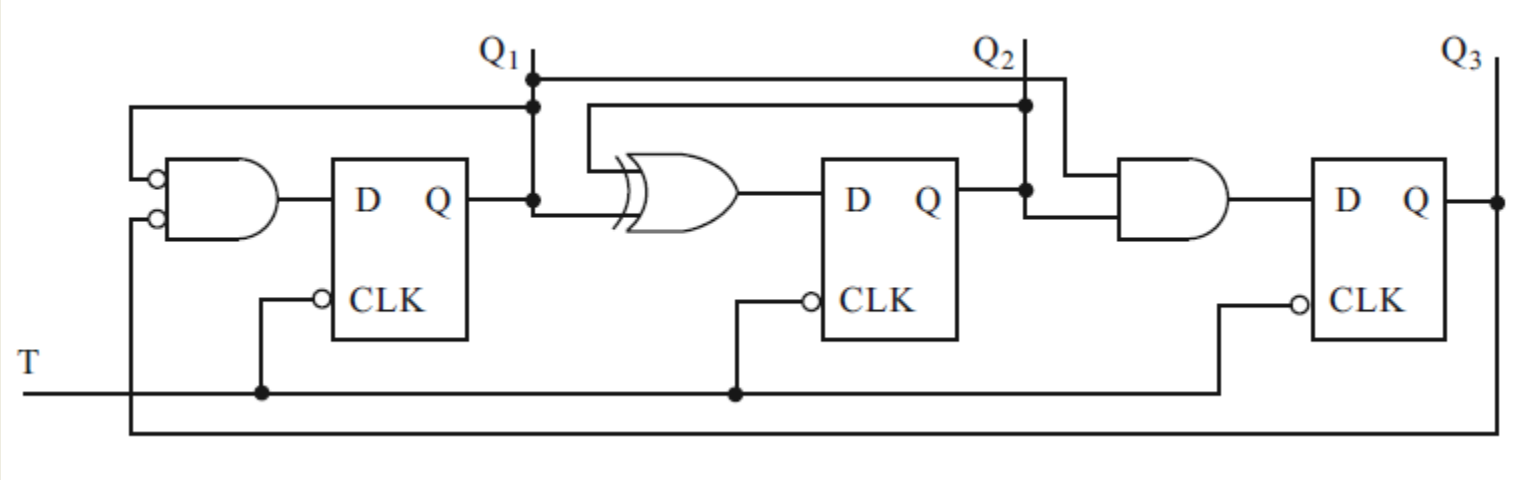
タイミング
チャート



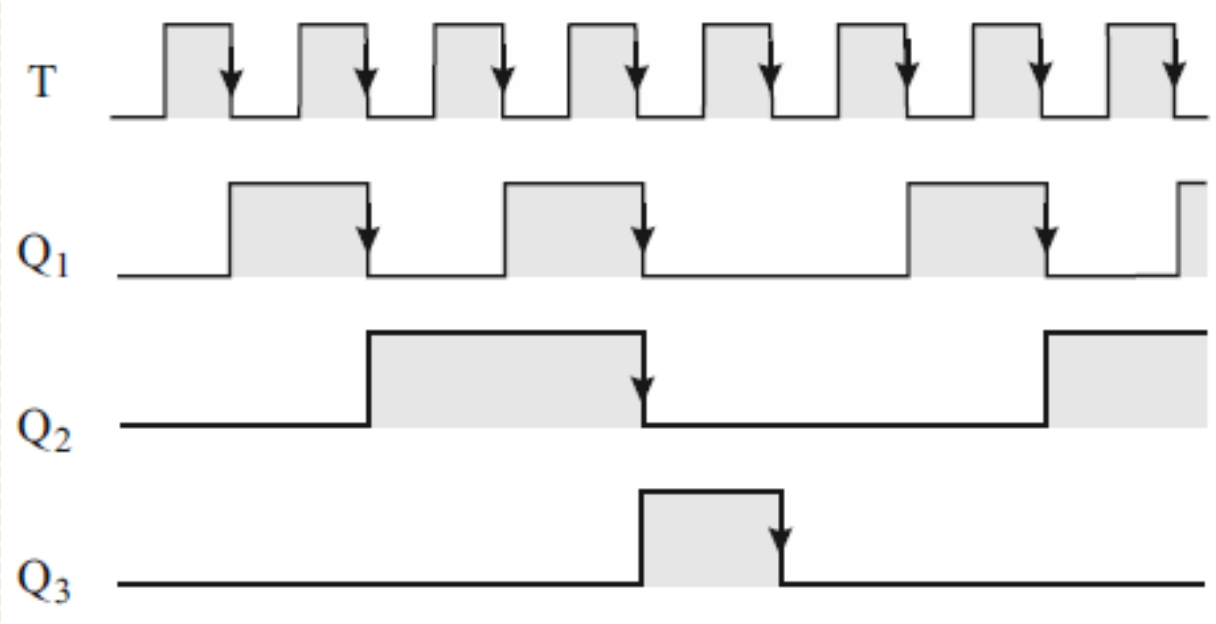
順序回路：カウンタ

同期カウンタ

回路図

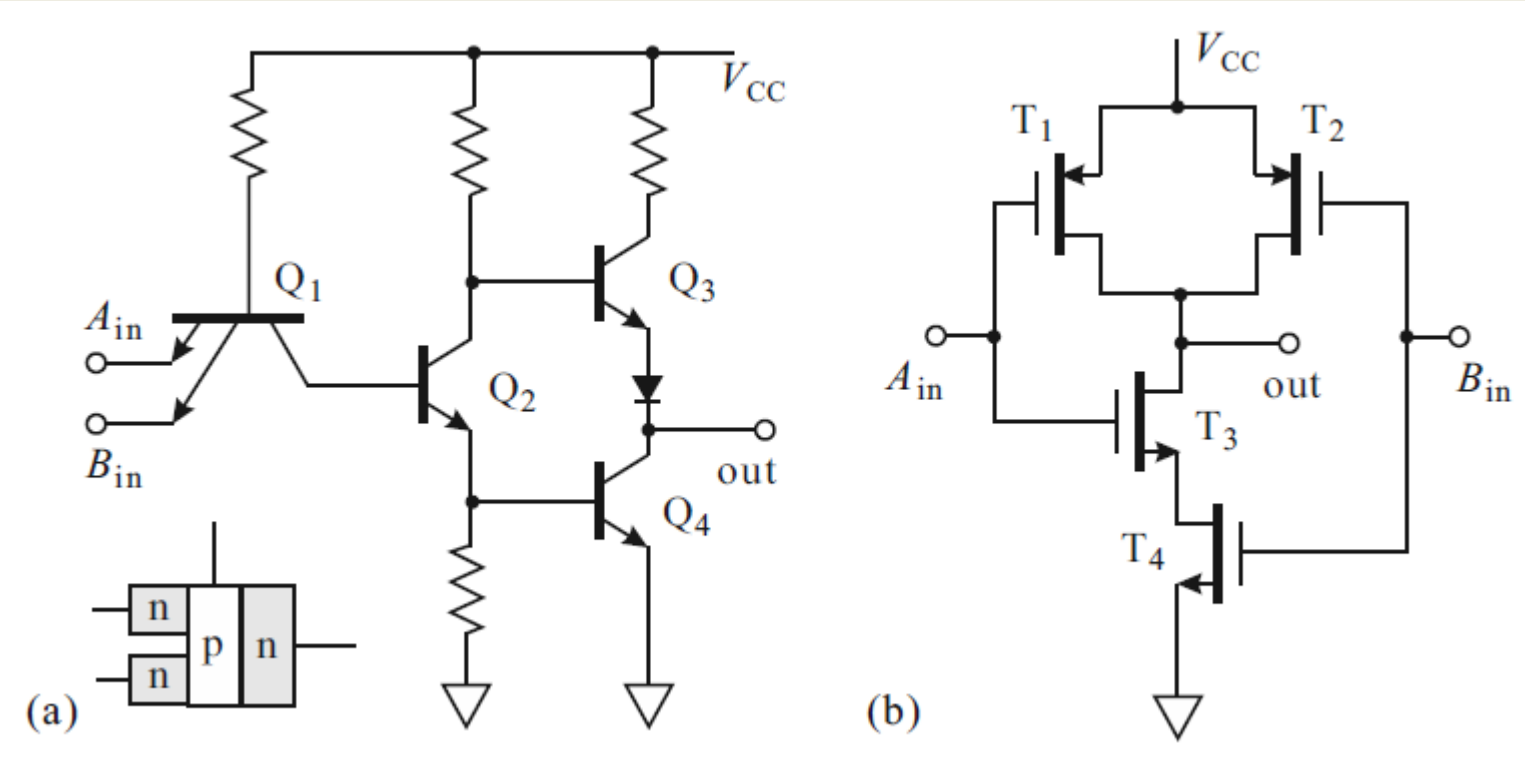


タイミングチャート



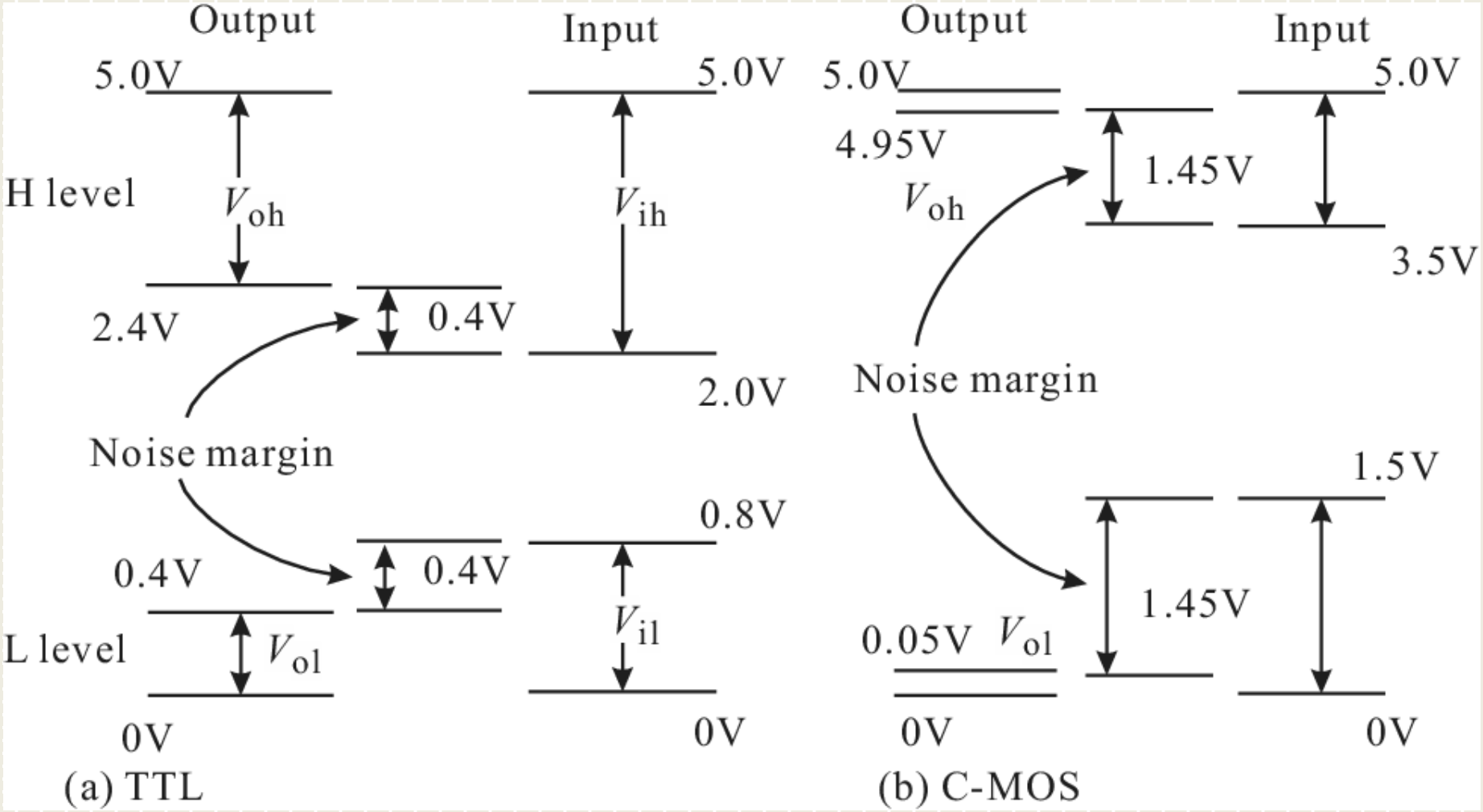
論理ゲートの等価回路

NAND 回路



TTL　　C-MOS

論理ゲートの動作電圧


Output
Input
5.0V
5.0V
H level
V_{oh}
V_{ih}
2.4V
0.4V
2.0V
Noise margin
0.8V
0.4V
0.4V
V_{il}
L level
V_{ol}
0V
0V
(a) TTL
Output
Input
5.0V
5.0V
4.95V
1.45V
V_{oh}
3.5V
Noise margin
1.5V
1.45V
0.05V
V_{ol}
0V
0V
(b) C-MOS