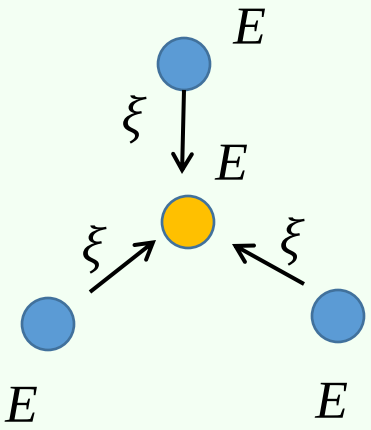
Exercise C-6-20

Energy gap opening in one-dimensional lattice can be easily understood by solving 2x2 Schrodinger equation:

$$i\hbar\frac{\partial}{\partial t}\boldsymbol{a} = \begin{pmatrix} E & \xi \\ \xi & E \end{pmatrix}\boldsymbol{a}$$

which gives eigenvalues $E \pm \xi$



For systematic treatment, the space group theory is the best method to consider this kind of symmetry.
But in the case of graphene, a simple consideration similar to the above is enough to understand why we the off-diagonal terms in Hamiltonian leave degeneracy.
Consider the case illustrated in the left figure and calculate the eigenvalues. Write a brief comment why the degeneracy is not lifted.