Problems for Final Report (physics of semiconductors)

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Problem1

Let us consider a pn-junction of Si at the temperature 300 K. In the p-layer tthe acceptor (boron, B) concentration is 10^{21} m^{-3} and in the n-layer the donor (phosphorous, P) concentration is 10^{20} m^{-3}. The doping profile is abrupt.

1. Obtain the built-in potential.
2. Calculate the depletion layer widths for p- and n-layers at reverse bias voltage -5 V.
3. Calculate the differential capacitance at reverse bias voltage -5 V for the area 1 mm$\times$1 mm.

Let put another p-layer and make a pnp transistor (gedankenexperiment). The hole diffusion length in the base is 10 μm.

4. Calculate $h_{\rm FE}$ for base widths 0.5 μm and 0.1 μm. (Ignore depletion layer widths, other non-ideal factors. Calculate under the simplest approximation.)

Problem2

Figure 1 (in the next page) shows the Shubnikov-de Haas oscillation and the quantum Hall effect in two-dimensional electrons.

1. Calculate the electron concentration from the low ($\leq$ 0.5 T) field data.
2. Something happened around 0.65T. What is it?

Problem3

Consider a double barrier resonant diode with GaAs as the well material and $\rm Al_{0.4}Ga_{0.6}As$ as the barrier material. Let us adopt $E_{\rm g}$=1.424 eV for GaAs and $E_{\rm g}$=1.424+1.265x+0.265x^2 (eV) for $\rm Al_{\it x}Ga_{1-\it x}As$ and $\Delta E_{\rm c}$: $\Delta E_{\rm v}$ =6:4. The electron effective mass in GaAs is 0.067m_0 and ignore the change in AlxGa$_{1-x}$As. Consider n-type electrodes (note that in the lecture we considered p-type).

1. Obtain the transfer matrix of 5 nm thickness GaAs-$\rm Al_{0.4}Ga_{0.6}As$.
2. Calculate the transmission probability of resonant diode with two 5 nm barriers and a 5 nm well region as a function of incident energy (from 0 to the top of the barrier with an appropriate interval) and plot in a figure.

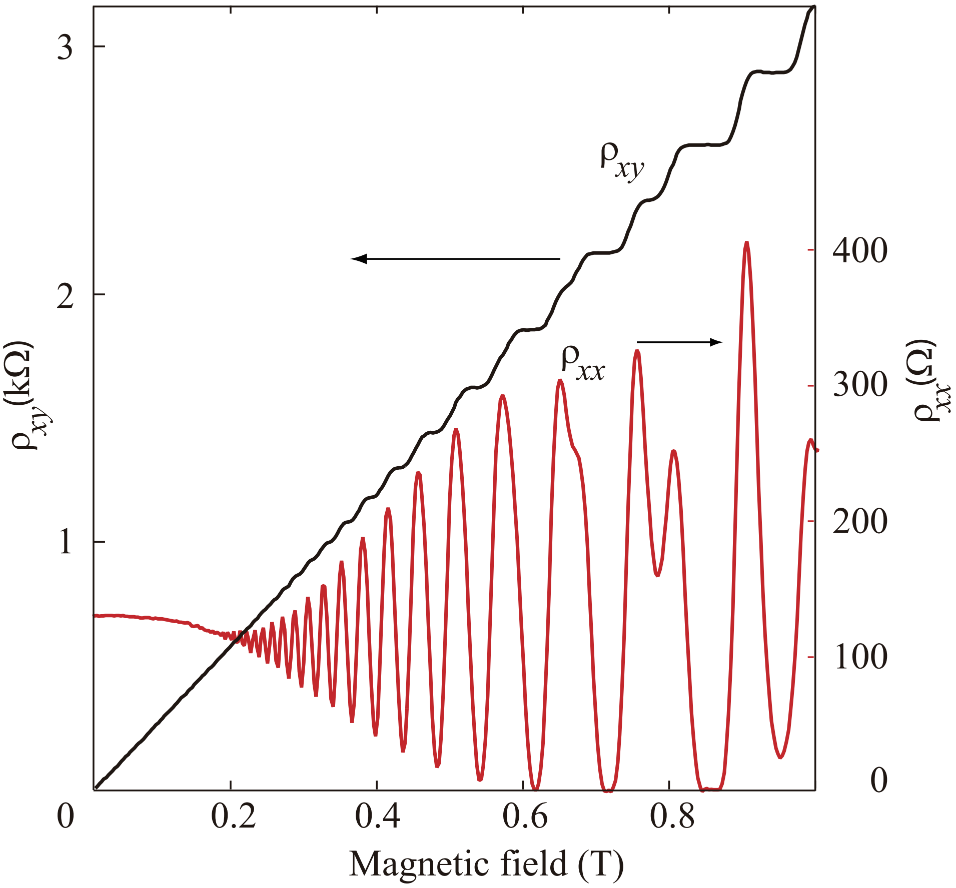
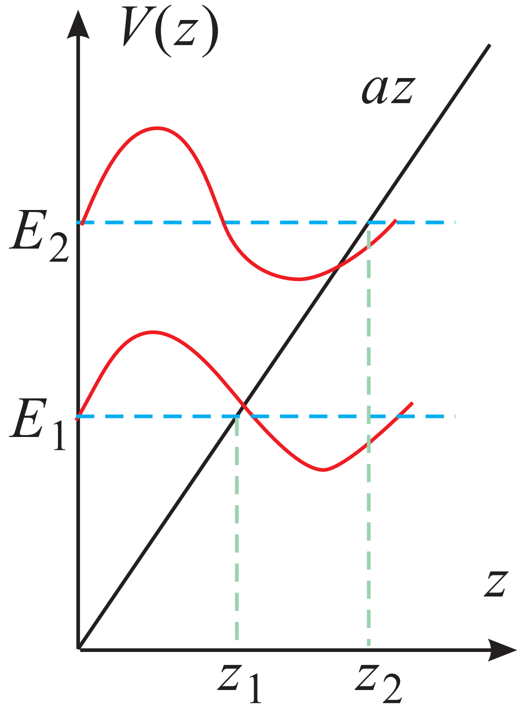


Figure 1: Shubnikov-de Haas oscillation of a two-dimensional electron system

Problem4

Let us consider the rectangular potential illustrate in the right.



1. First consider the most coarse approximation. Choosing a kinetic energy E determines the effective potential with E/a. Now let us approximate the potential with a rectangular potential of width E/a, bottom $V(0)$, infinite barrier height. Let m^* be the effective mass and obtain the eigen energies from lower level with index $n = 1, 2, \cdots$.

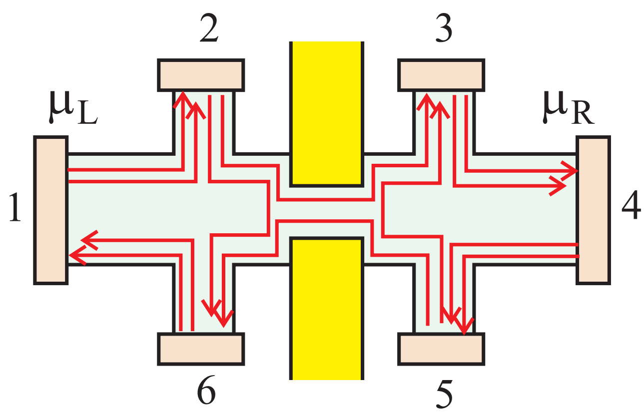
2. Compare the above result with more accurate one on Airy functions.

3. Also try comparison with Wenzel-Kramers-Brillouin (WKB) approximation for wavefunction penetration into the barrier.

Problem5

In the left figure the green region indicates 2DEG, 1 to 6 are the electric contacts, the yellow regions are metallic gates. The structure has a quantum point contact in the middle. In the integer quantum Hall state with filling factor ν, the sample has ν edge modes. With applying gate voltage, we can tune the number of modes which transmit through the QPC, to χ. Other modes are completely reflected by the QPC. The current is through 1 and 4.

1. Obtain the longitudinal resistance $R_{\rm L}$, which is measured from the voltage between 2 and 3 V_{23} or 6 and 5 V_{65}.
2. Obtain the Hall resistance $R_{\rm H}$, measured from V_{26} or V_{35}.



Problem6

Consider a 2DEG under IQHE with ν =1. The edge modes can bring finite current without energy dissipation and the resistance is zero. The conductance of one-dimensional edge mode is then the inverse of the resistance and infinity. Let write the quantum resistance h/e^2 as $R_{\rm q}$.

Two dimensional resistivity tensor is written as

$$\boldsymbol{\rho} = \begin{pmatrix} 0 & R_{\rm q} \\ -R_{\rm q} & 0 \end{pmatrix}.$$

Then the two dimensional conductivity tensor defined by the inverse of resistivity tensor is

$$\boldsymbol{\sigma} = \boldsymbol{\rho}^{-1} = \frac{1}{R_{\rm q}^2}\begin{pmatrix} 0 & -R_{\rm q} \\ R_{\rm q} & 0 \end{pmatrix}.$$

That is, σ_{xx}=0! Does the calculation contain an error? If it does, what is the error? Or can you solve the puzzle?