電子回路論第11回
Electric Circuits for Physicists #11

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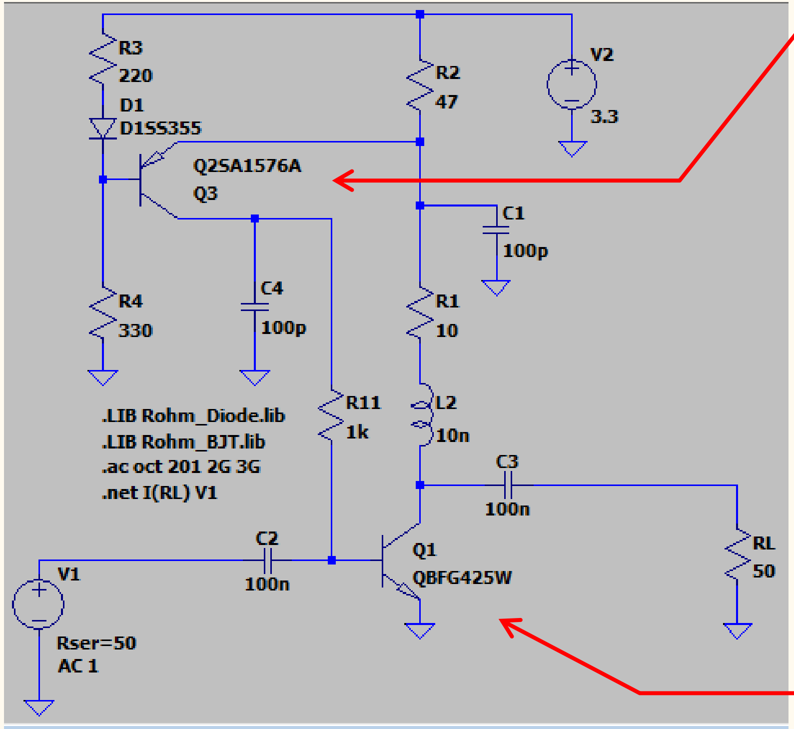
Noise matching circuit simulation (LTSpice)



Transistor for dc bias

Input-output parameter measurement circuit

Result:

$$Z_{\text{in}} = 25.0\ \omega + 8.2i\ \Omega$$
$$Z_{\text{out}} = 49.6\ \Omega - 19.8i\ \Omega$$
$$S_{21} = 2.92 + 6.17i$$
$$S_{12} = 0.027 + 0.047i$$

Transistor for hf amplification
(device under test)
spice model is provided by Rohm corp.

Transistor gijutsu 2015 No.1- No.3 A. Kawada

Noise matching circuit simulation (LTSpice)

QBFG425W noise data:
(at 2.45 GHz)

F_{min} [dB]	= 2.3 (Noise figure)
Γ_{mag}	= 0.2 (reflection amplitude)
Γ_{angle} [°]	= -153 (reflection angle)
r_n [Ω]	= 0.21 (normalized to 50 Ω)

Noise figure circles

$$C_{\rm nf} = \frac{r_{\rm opt}}{N+1} \quad \text{(center)}, \qquad R_{\rm nf} = \frac{\sqrt{N(N+1-|r_{\rm opt}|^2)}}{N+1} \quad \text{(radius)} \qquad (1)$$

$$\text{where} \quad N \equiv \frac{F - F_{\rm min}}{4R_{\rm n}Y_0}|1 + r_{\rm opt}|^2. \qquad (2)$$

R_n: noise equivalent resistance, Y_0: characteristic admittance

Noise matching circuit simulation (LTSpice)

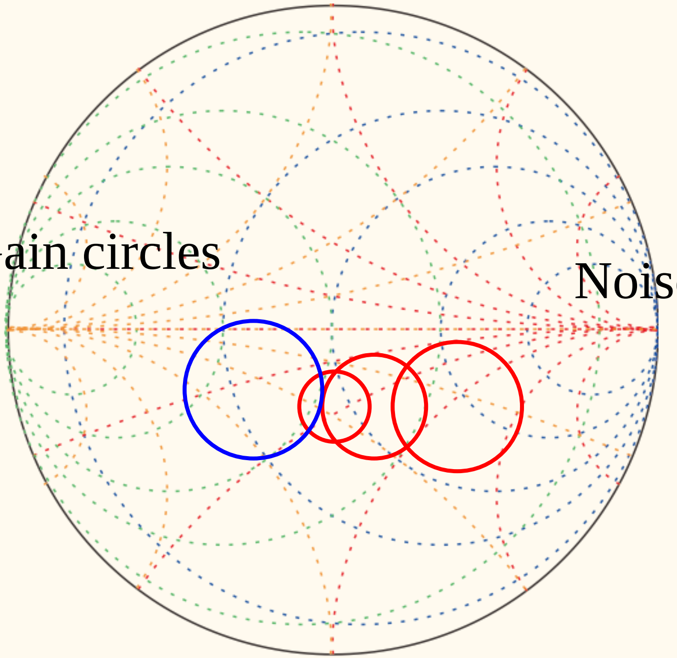
Constant gain also gives a circle on the Smith chart

$$\text{(center) } C_\text{s} = \frac{g_s S_{11}^*}{1-(1-g_s)|S_{11}|^2}, \quad \text{(radius) } R_s = \frac{\sqrt{1-g_s}(1-|S_{11}|^2)}{1-(1-g_s)|S_{11}|^2} \tag{3}$$

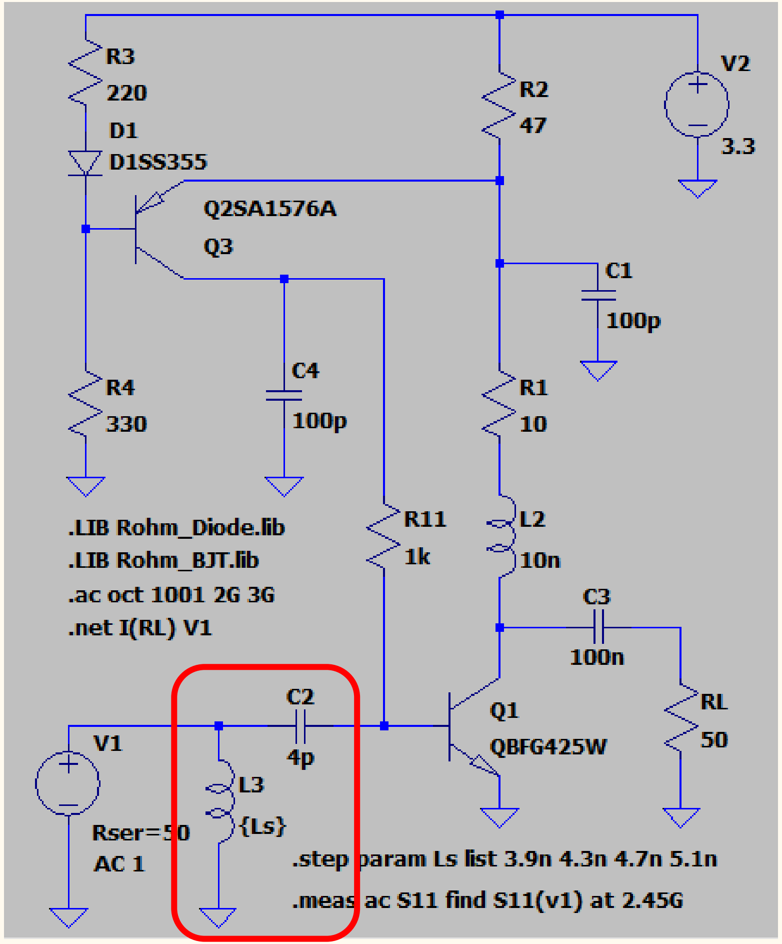
$$g_s = G_s(1-|S_{11}|^2)$$

Find smallest NF touching point

Tune to 50 Ω

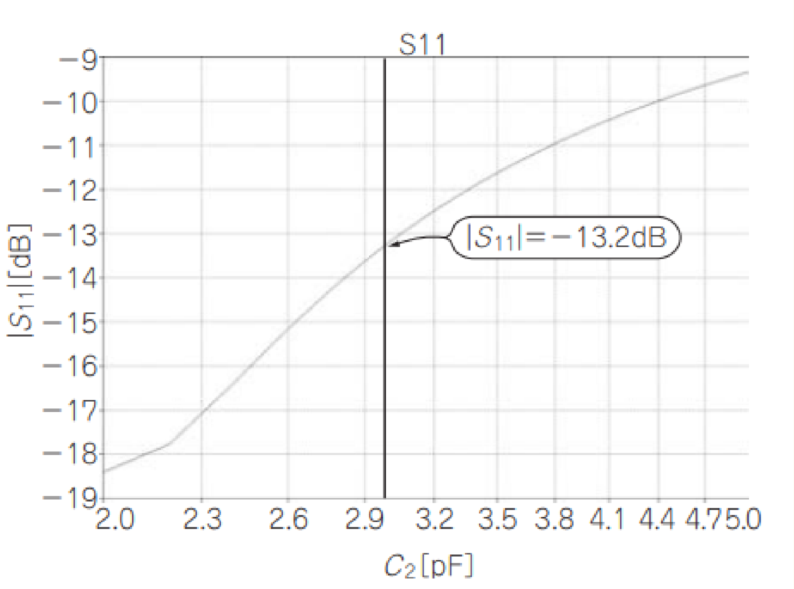


Impedance matching circuit simulation (LTSpice)

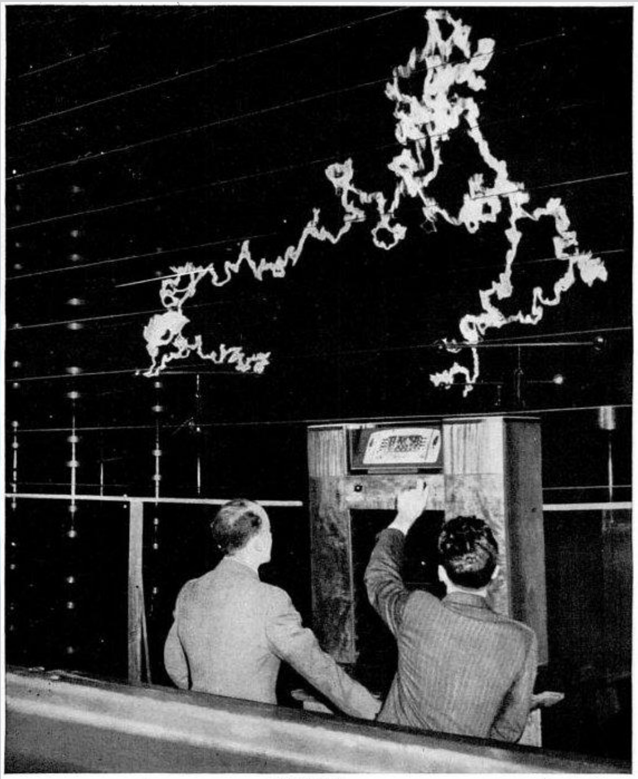


Tuning of input matching circuit

The lowest nf condition gives C_2= 4.06 pF, L_3 = 4.51 nH



Outline

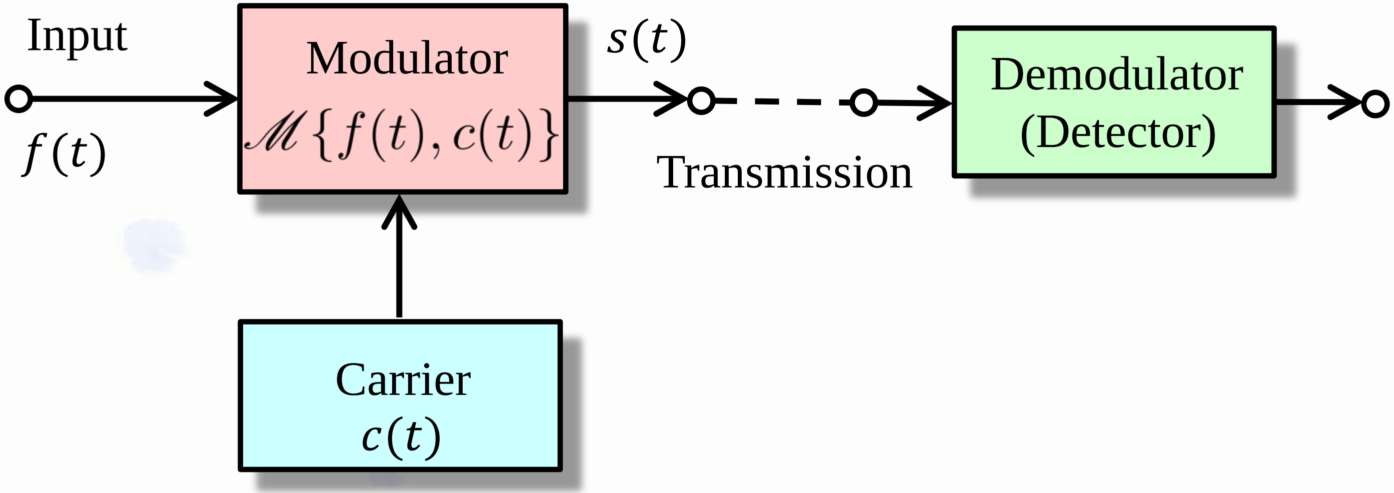


FM broadcast test

6.3 Signal transmission

Electric communication
- Baseband communication
- Carrier communication

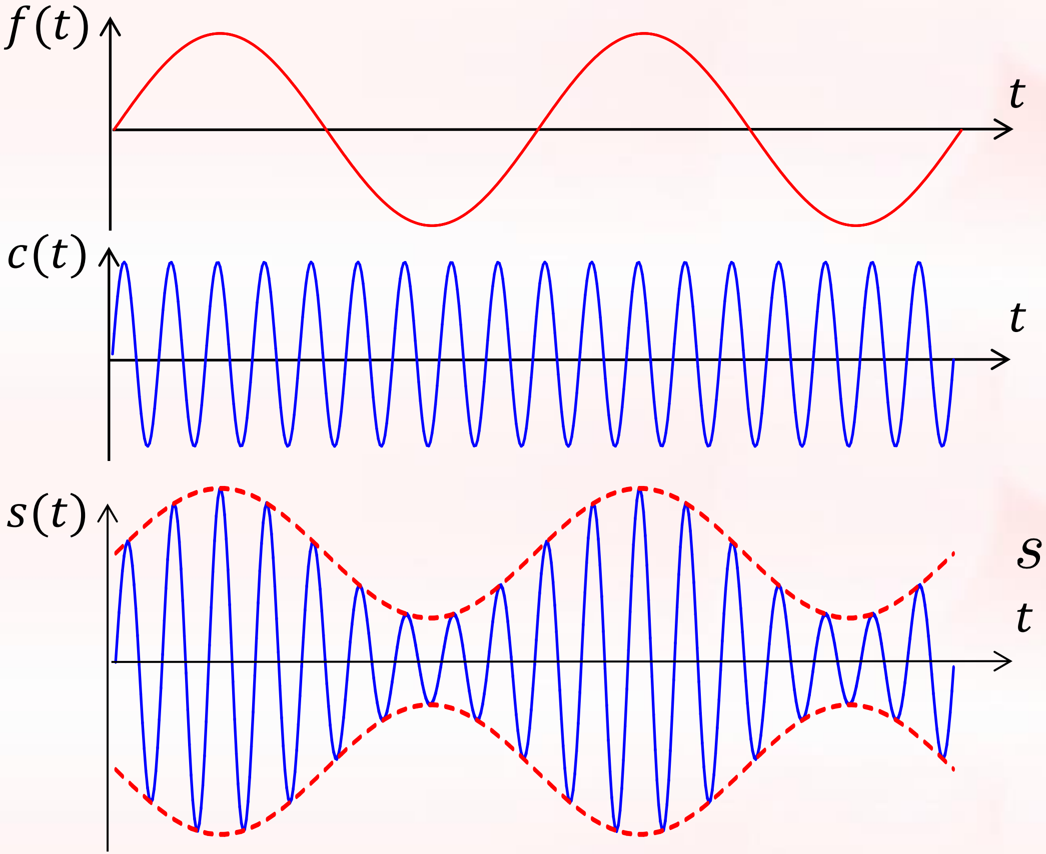
Carrier communication



Modulation
- Amplitude modulation
- Frequency (Phase) modulation

Analog
Pulse

6.3.2 Amplitude modulation



$$c(t) = A\cos\omega_c t$$

$$s(t) = A[1 + mf(t)]\cos\omega_c t \quad (4)$$

m: Modulation index

$$0 < m \leq 1$$

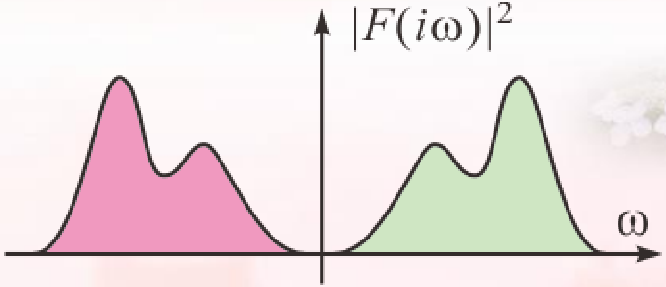
6.3.2 Amplitude modulation

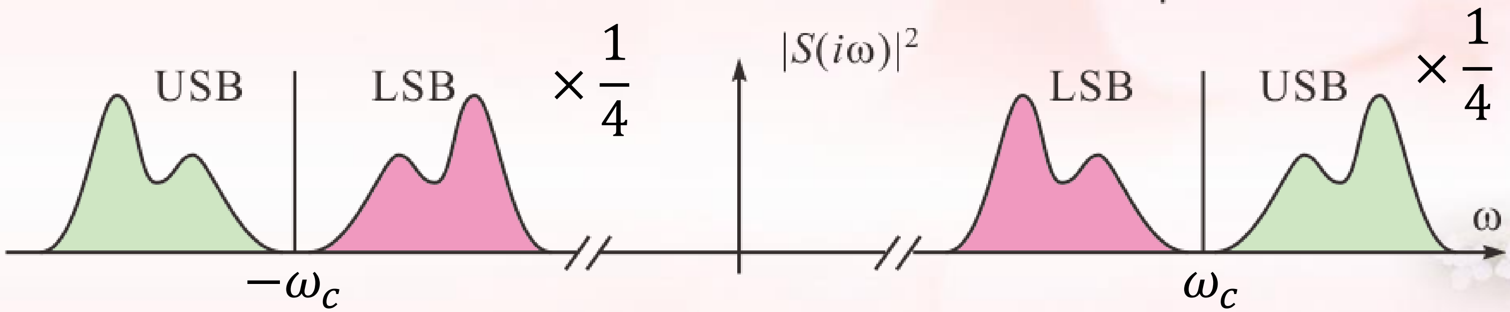
$$S(i\omega) = \int_{-\infty}^{\infty} s(t)e^{i\omega t}dt = \int_{-\infty}^{\infty} A[1 + mf(t)]\cos(\omega_c t)e^{i\omega t}dt$$

$$= A\left\{\pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] + \frac{m}{2}[F(i(\omega - \omega_c)) + F(i(\omega + \omega_c))]\right\} \quad (5)$$

$$F(i\omega) = \mathscr{F}\{f(t)\}$$

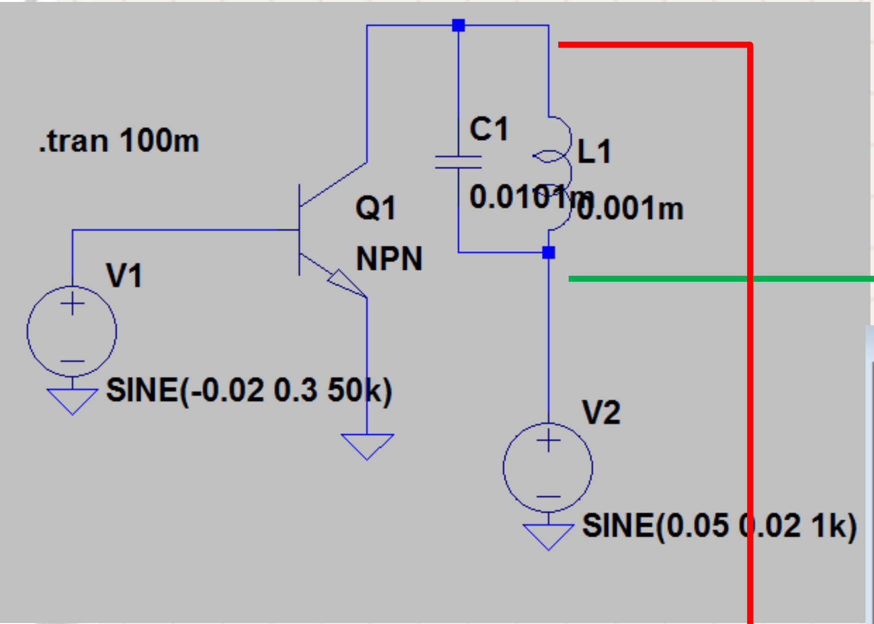
$f(t)$: Real $\quad F(i\omega) = F^*(-i\omega)$





Upper side band (USB), Lower side band (LSB)

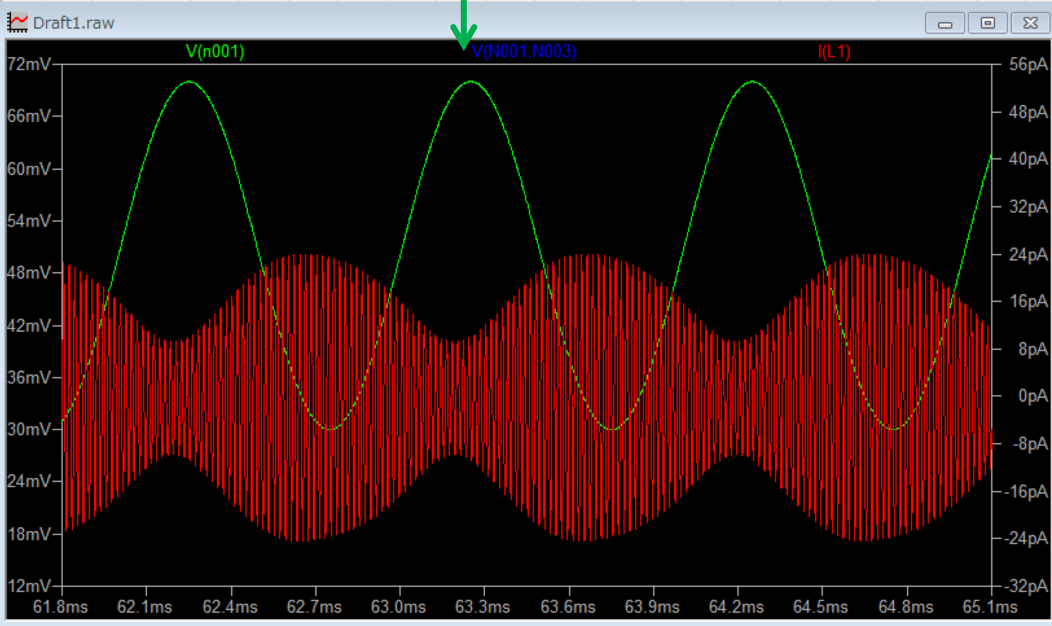
6.3.2 Amplitude modulation (circuit example)



Collector modulation circuit

C-class (non-linear) amplification region

Modulation voltage

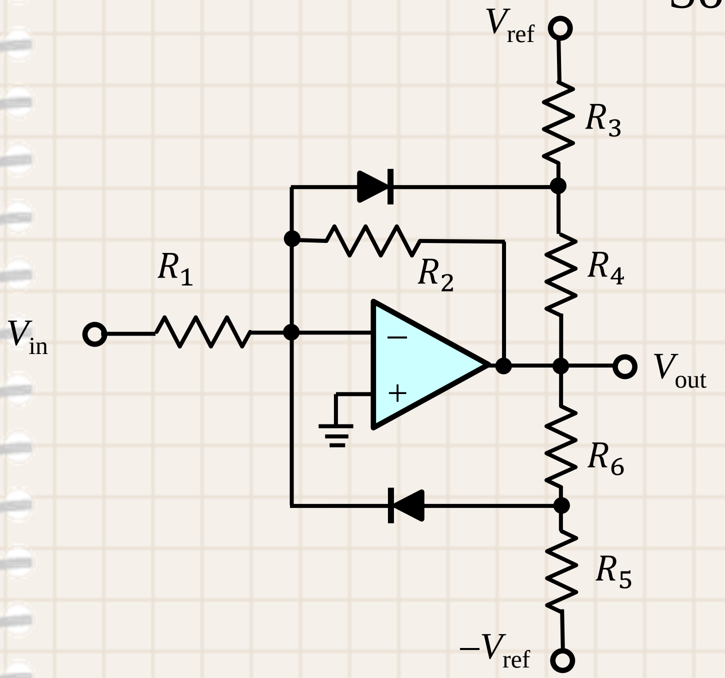


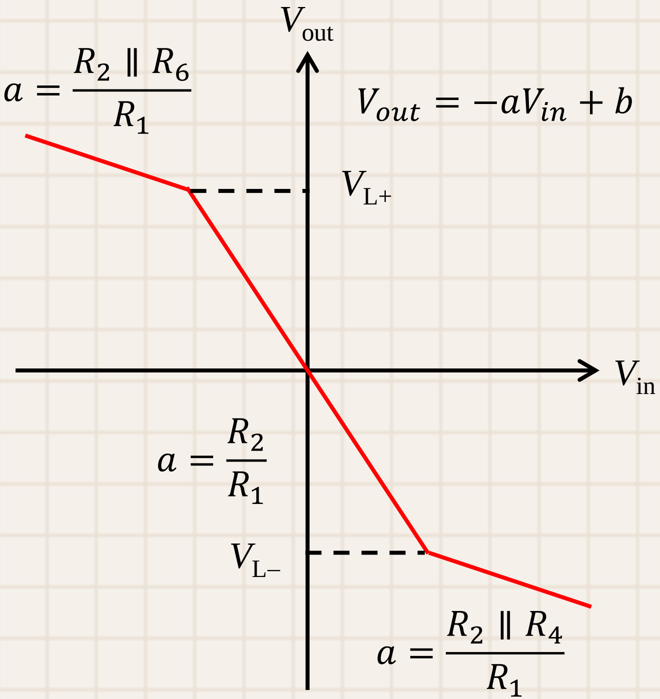
Current through the inductor

6.3.2 Amplitude modulation (circuit example 2)

Idea: Modulation of oscillator circuit

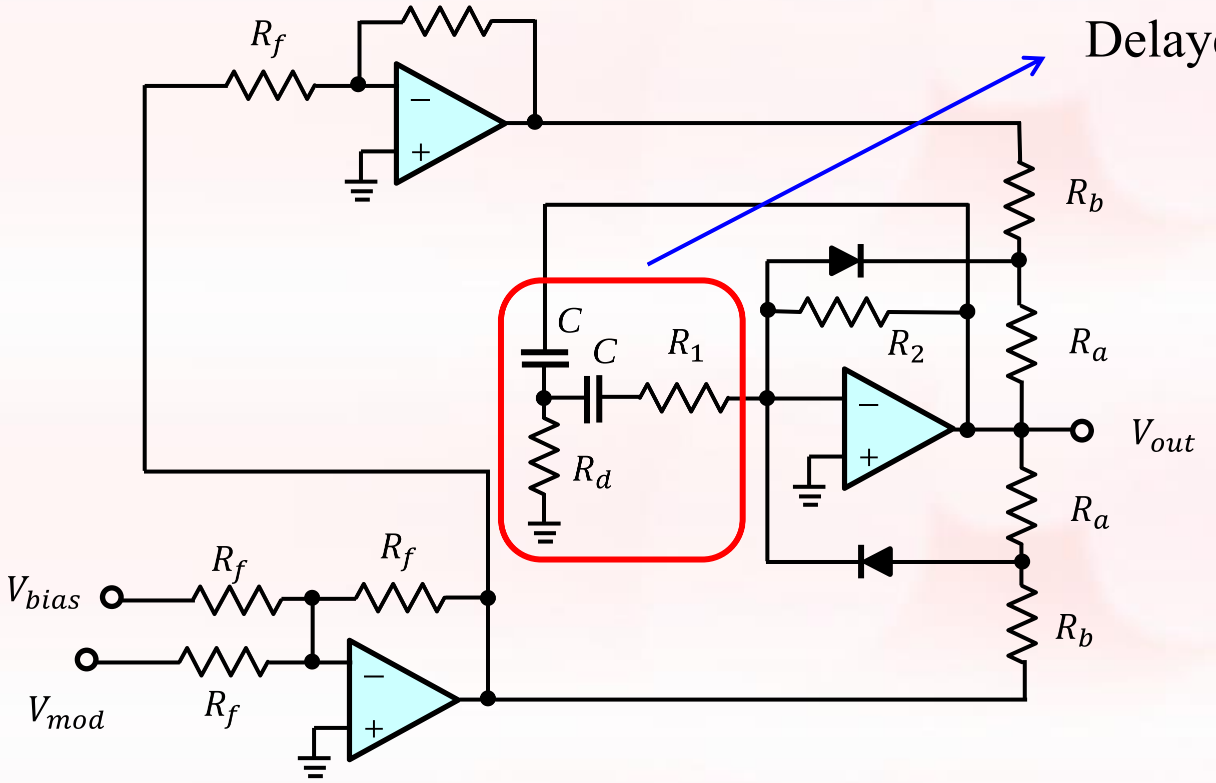
Soft limiter circuit





$$V_{L-} = -\frac{R_4}{R_3}V_{ref} - \left(1 + \frac{R_4}{R_3}\right)V_{th}$$: controllable with V_{ref}

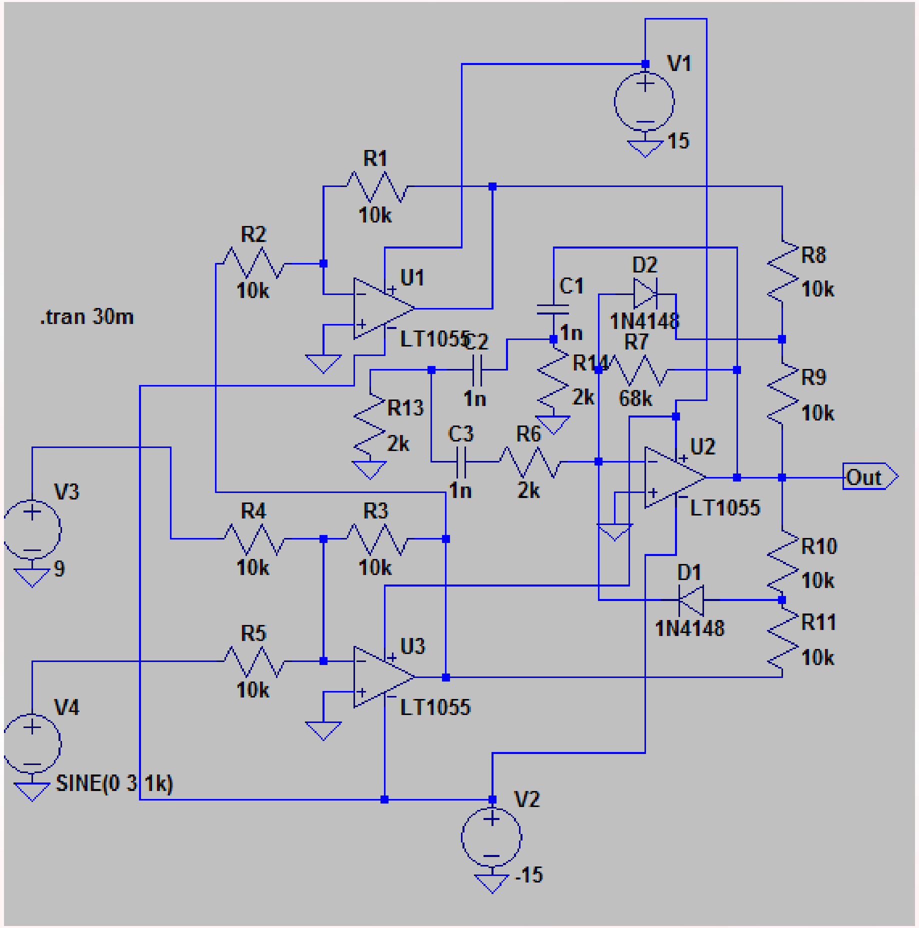
6.3.2 Amplitude modulation (circuit example 2)

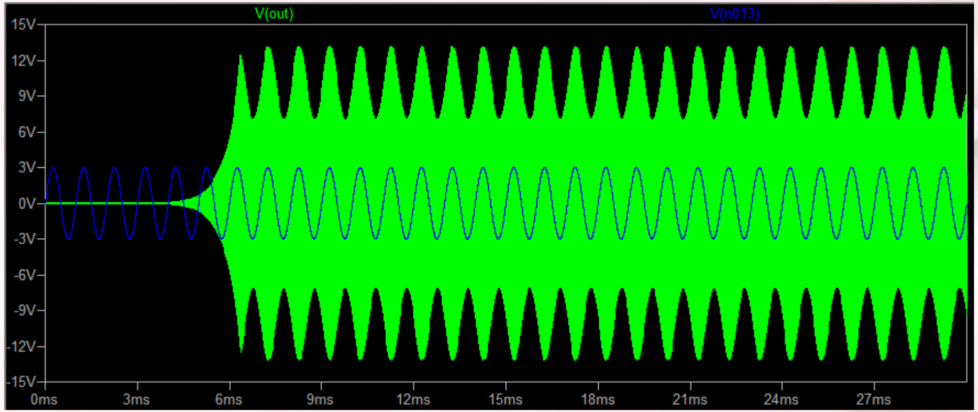


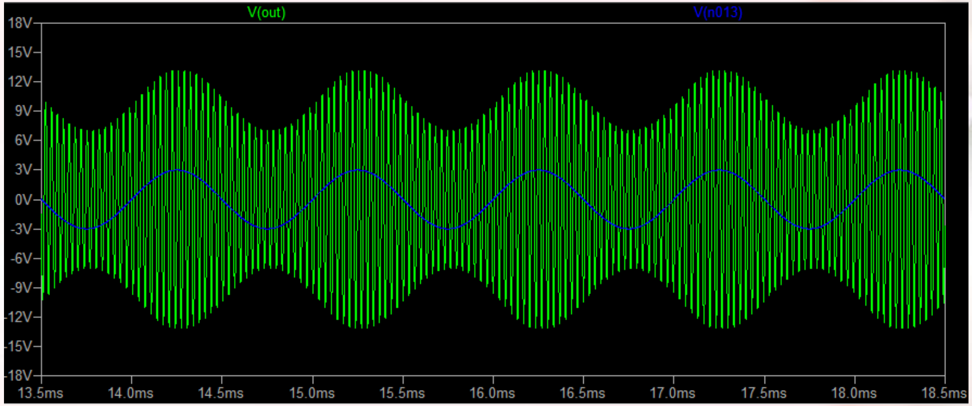
Delayed feedback → Oscillation

The amplitude is softly limited with the modulation voltage.

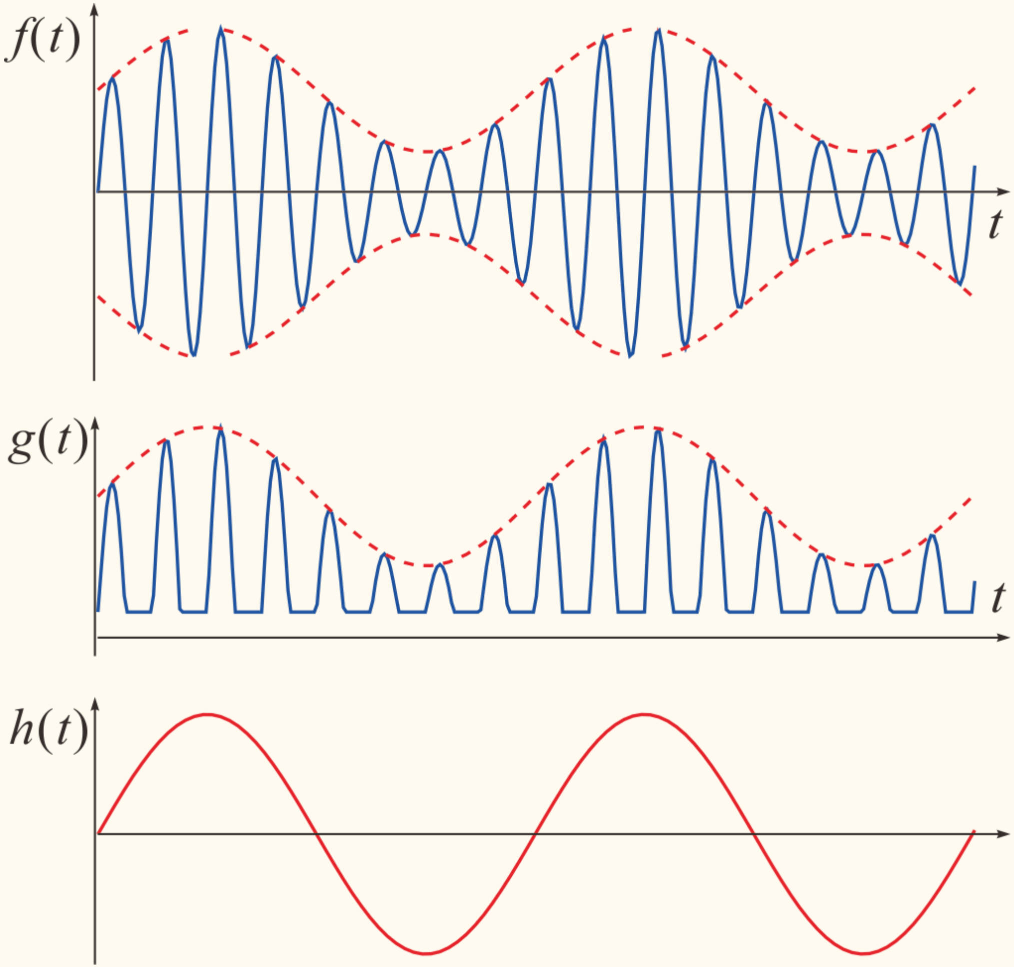
6.3.2 Amplitude modulation (circuit example 2)

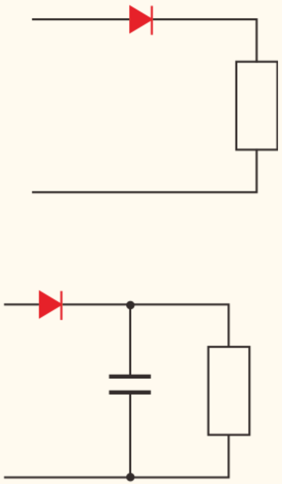




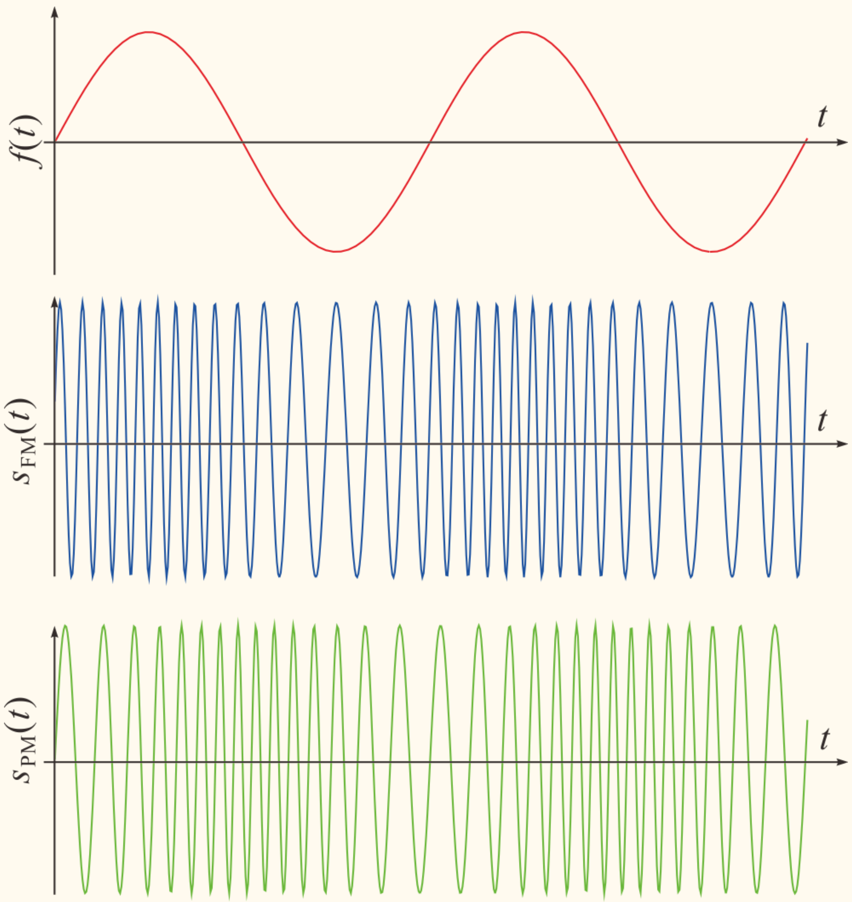


6.3.2 Amplitude modulation (demodulation)





6.3.3 Angle modulation



Signal

Frequency modulation (FM)

Phase modulation (PM)

6.3.3 Angle modulation

$$s(t) = A\cos\theta_{\mathrm{i}}(t), \quad \theta_{\mathrm{i}}(t) = \omega_{\mathrm{c}}t + \phi[t, f(t)] \tag{6}$$

Differential angular frequency: $\omega_{\mathrm{i}}(t) = \dfrac{d\theta_{\mathrm{i}}(t)}{dt} = \omega_{\mathrm{c}} + \dfrac{d\phi[t, f(t)]}{dt}$ (7)

$$\frac{d\phi[t, f(t)]}{dt} = k_f f(t) \quad (\mathrm{Frequency Modulation, FM}), \tag{8}$$

$$\phi[t, f(t)] = k_p f(t) \quad (\mathrm{Phase Modulation, PM}) \tag{9}$$

(k_f, k_p : depths of modulation)

$$s_{\mathrm{FM}}(t) = A\cos\left[\omega_{\mathrm{c}}t + k_f \int^t f(\tau)d\tau\right], \tag{10}$$

$$s_{\mathrm{PM}}(t) = A\cos[\omega_{\mathrm{c}}t + k_f f(t)] \tag{11}$$

Frequency ω component: only phase shift $\pi/2$:
No difference in signal outlook.

6.3.3 Angle modulation (frequency modulation)

$$f(t) = A_p \cos \omega_p t \qquad \text{: Signal wave} \qquad (12)$$

$$s_{\mathrm{FM}} = A \cos(\omega_c t + \beta \sin \omega_p t) = A \mathrm{Re}\left[\exp(i\omega_c t) \exp(i\beta \sin \omega_p t)\right] \qquad (13)$$

$$\left(\beta \equiv \frac{k_f A_p}{\omega_p} = \frac{\Delta f}{f_p}\right)$$

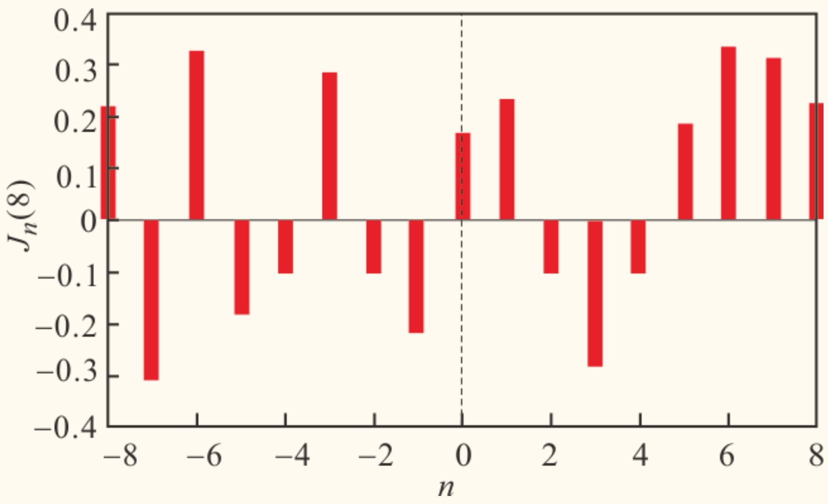
$\sin \omega_p t$: Periodic function with $T = 2\pi/\omega_p$, hence Fourier expansion is possible.

$$\exp(i\beta \sin \omega_p t) = \sum_{n=-\infty}^{\infty} c_n \exp(in\omega_p t), \qquad (14)$$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} \exp(i\beta \sin \omega_p t') \exp(-in\omega_p t') dt'$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp[i(\beta \sin \theta - n\theta] d\theta = J_n(\beta) \qquad \text{First kind Bessel function} \qquad (15)$$

6.3.3 Angle modulation (frequency modulation)



$$s_{\mathrm{FM}}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[(\omega_c + n\omega_p)t]$$

Fourier transform:

$$S_{\mathrm{FM}}(i\omega) = \pi A \sum_{n=-\infty}^{\infty} J_n(\beta)\{\delta[\omega - (\omega_c + n\omega_p)] + \delta[\omega + (\omega_c + n\omega_p)]\}$$

Though the expansion series does not have clear cut-off, the actual band width falls

$$\omega_{\mathrm{bw}} = 2(\omega_{\mathrm{f}} + \xi\omega_{\mathrm{w}}) \quad (1 \le \xi \le 2)$$

Let us see the reasoning both for phase modulation (PM) and frequency modulation (FM).

6.3.3 Angle modulation band width

remember: $s_{\mathrm{FM}}(t) = A\cos\left[\omega_{\mathrm{c}}t + k_f \int^t f(\tau)d\tau\right], \quad s_{\mathrm{PM}}(t) = A\cos[\omega_{\mathrm{c}}t + k_f f(t)]$

Maximum frequency or phase shift:

$$\Delta\omega = k_f \left|f(t)|_{\max}\right| \equiv k_f f_{\max}, \quad \Delta\phi = k_f \left|\int^t f(\tau)d\tau\right|_{\max} \quad \text{for FM},$$

$$\Delta\omega = k_p |f'(t)|_{\max}, \quad \Delta\phi = k_p |f(t)|_{\max} \quad \text{for PM}$$

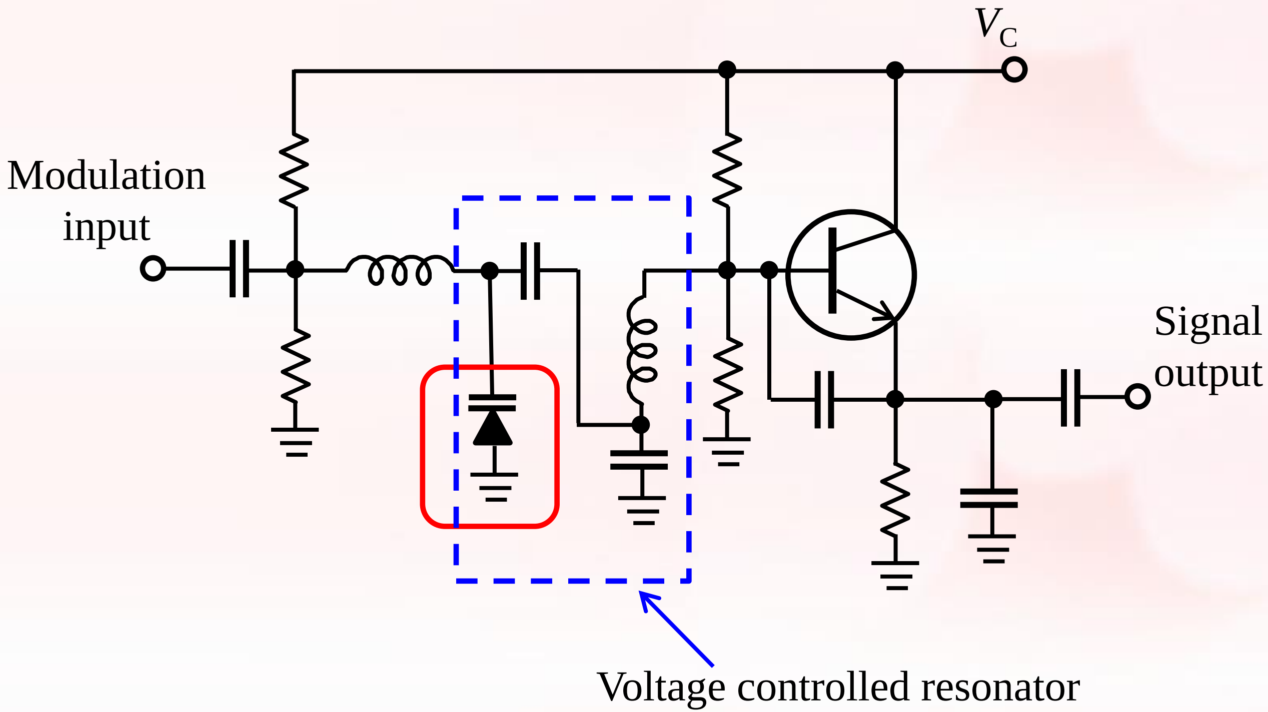
$f(t)$ band width: ω_W gives width $2\omega_W$ $\quad \omega_{\mathrm{bw1}} = 2(k_f f_{\max} + 2\omega_{\mathrm{w}}) = 2(\omega_{\mathrm{f}} + 2\omega_{\mathrm{w}})$ (16)

98% of total power in the whole frequency region $\quad \omega_{\mathrm{bw2}} = 2(\omega_{\mathrm{f}} + \omega_{\mathrm{w}})$ (17)

Actually, some value between (16) and (17) is taken for the band width.

$$2(\omega_f + \xi\omega_{\mathrm{W}}) \quad 1 \le \xi \le 2$$

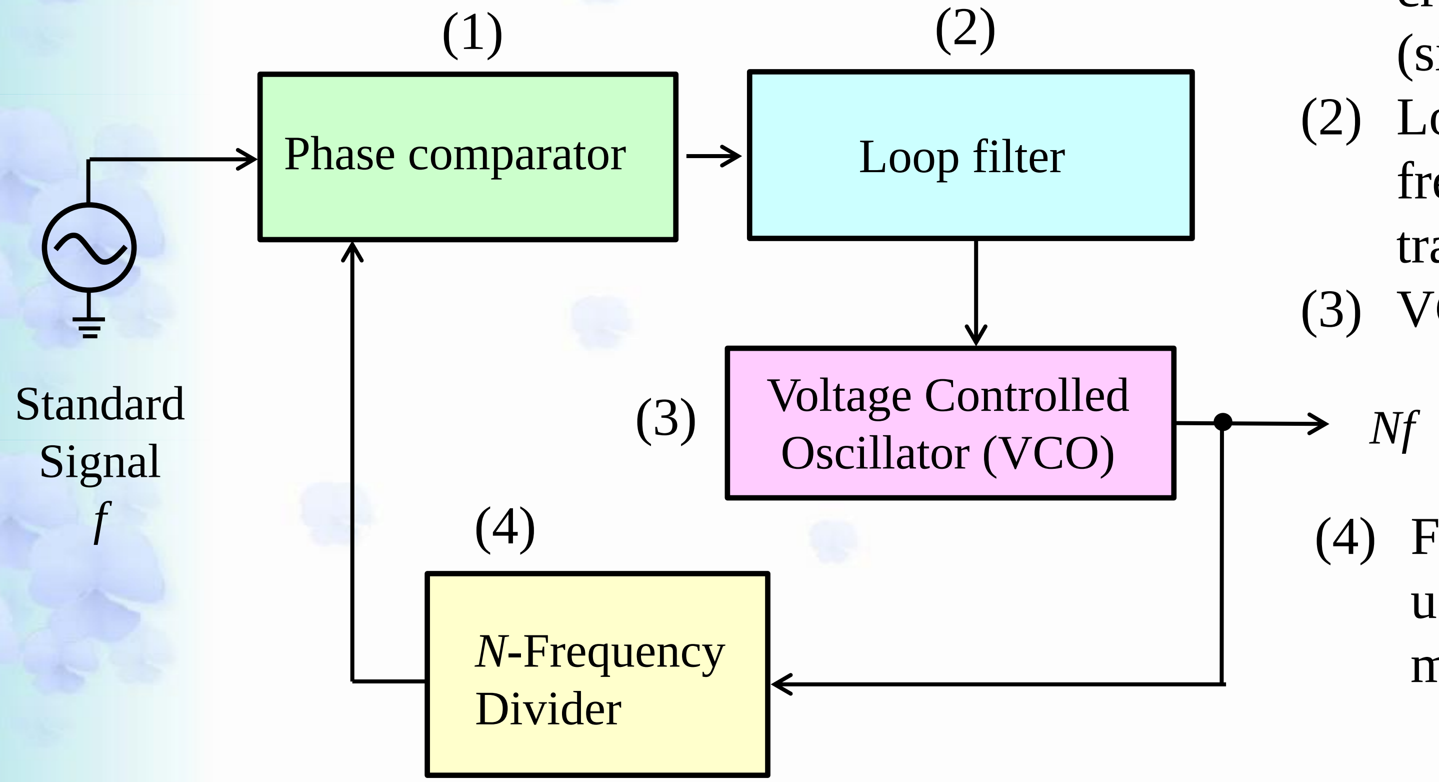
6.3.3 Angle modulation (circuit example)



Voltage Controlled Oscillator (VCO)

Phase Lock Loop (PLL)

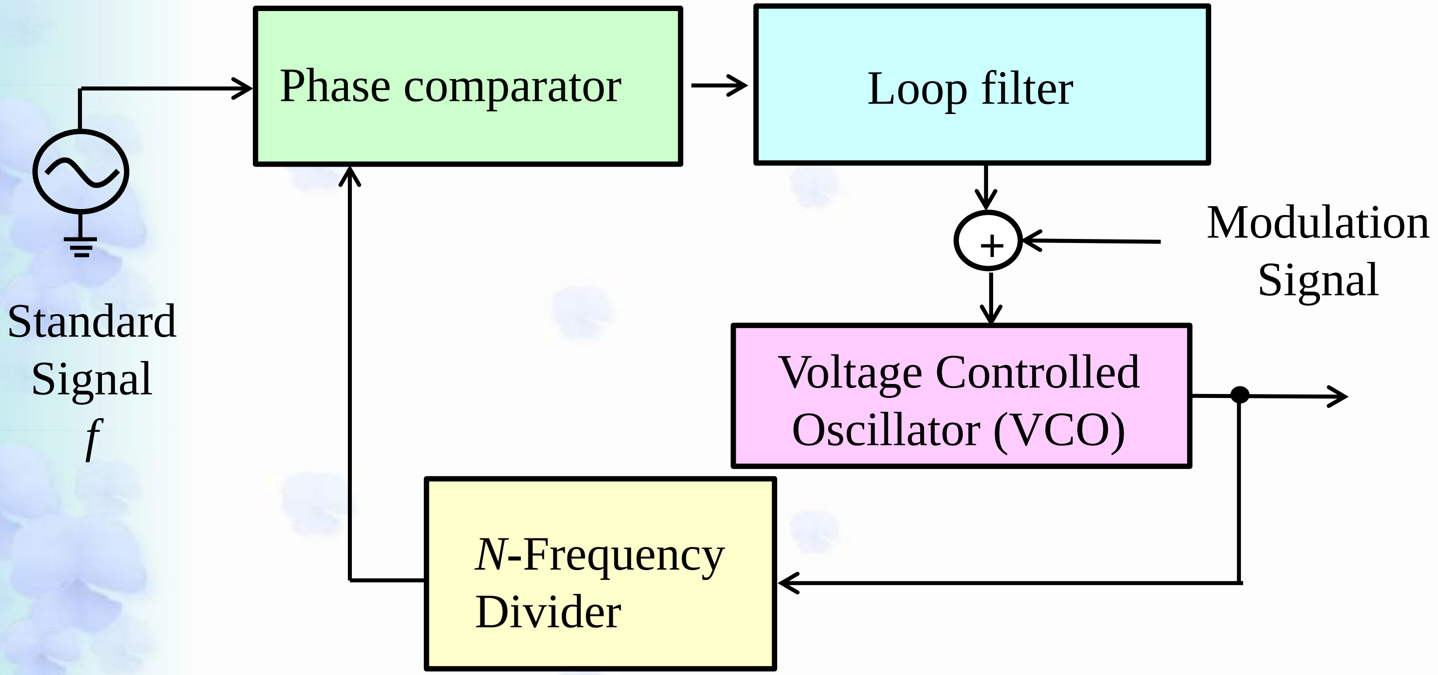
Signal flow block diagram of a phase lock loop (PLL) circuit



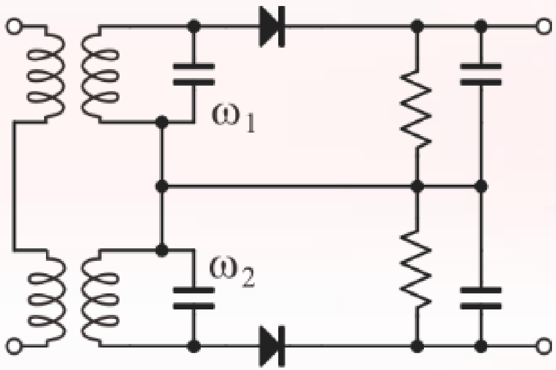
(1) Phase comparator: creates error signal (signal subtraction)
(2) Loop filter: permits frequency band to transmit.
(3) VCO is an oscillator

(4) Frequency divider is used for frequency multiplication.

6.3.3 Angle modulation (circuit example)



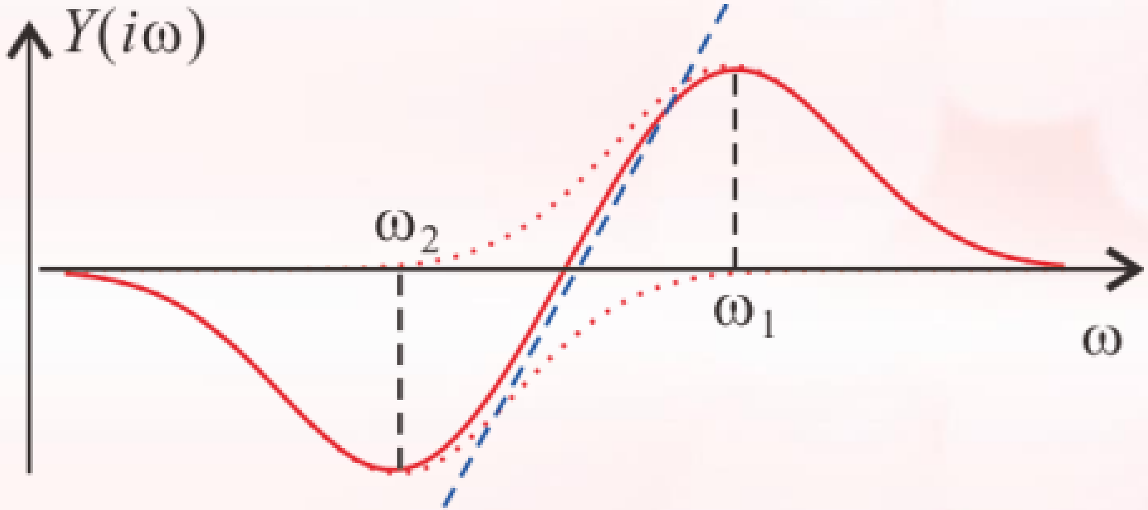
6.3.4 Angle modulation (frequency demodulation)



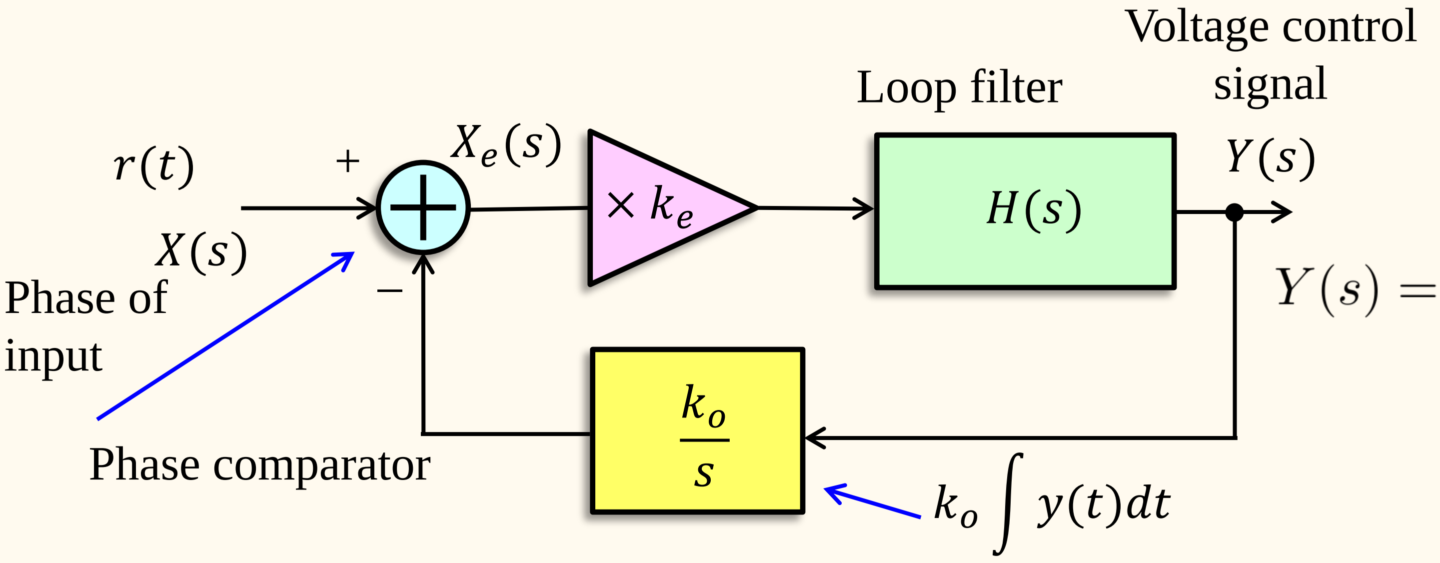
Doubly tuned circuit

Two transformers are connected in anti-phase direction.

FM → AM → demodulation



6.3.4 Angle modulation (frequency demodulation by PLL)



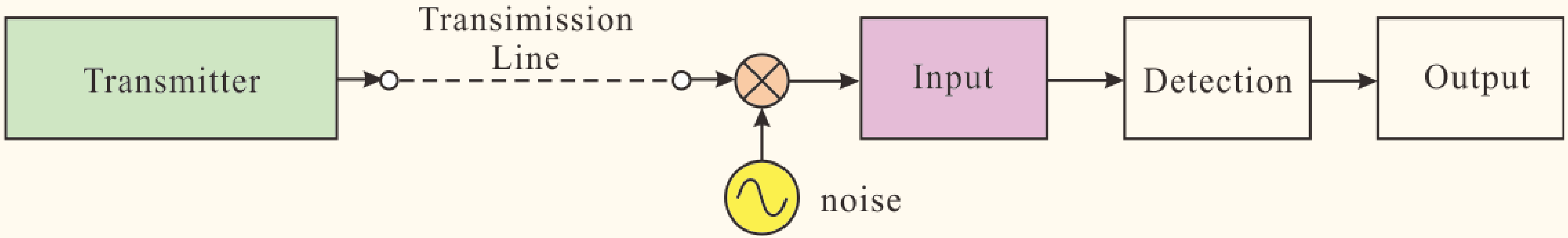
$$Y(s) = \frac{s k_e H(s)}{s + k_e k_o H(s)} X(s)$$

$g(t)$: Frequency modulation signal (original)

$$\phi(t) = k_f \int_{-\infty}^{t} g(\tau) d\tau, \quad sX(s) = k_f G(s)$$

$$\therefore \; Y(s) = \frac{k_f k_e H(s)}{s + k_e k_o H(s)} G(s) \approx \frac{k_f}{k_o} G(s)$$

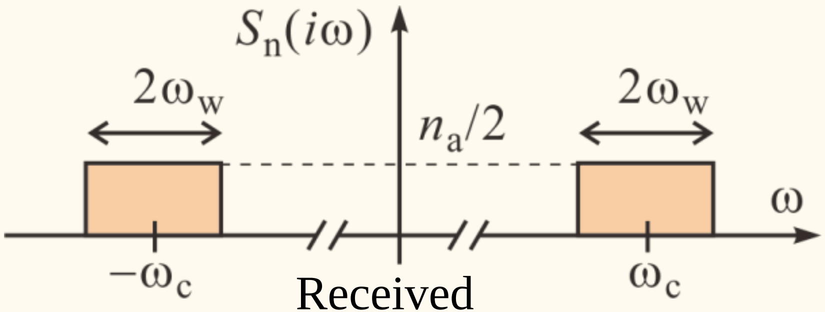
6.3.5 Modulation and noise



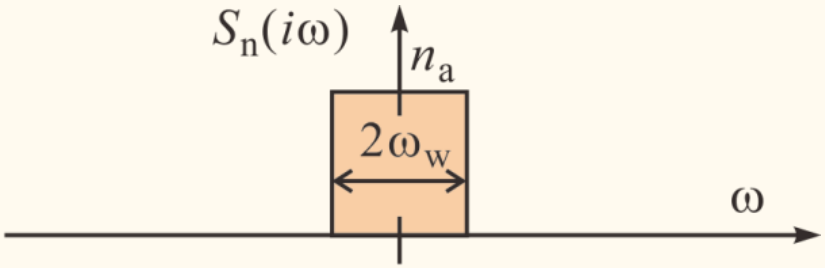
Received signal $$r(t) = A_r[1 + mf(t)]\cos\omega_c t + n_i(t)$$

Demodulated output $$g(t) = A_r m f(t) + n_o(t)$$

Averaged signal power $$\text{received: } S_{pr} = \frac{A_r^2}{2} + \frac{(A_r m)^2}{2}\langle f^2\rangle, \quad \text{output: } S_{po} = A_r^2 m^2 \langle f^2\rangle$$



Received



Demodulated

6.3.5 Modulation and noise

ω_W : Noise bandwidth (assumption: white)

Noise power: $$2 \times \frac{n_a}{2 \times 2\pi} \times 2\omega_w = \frac{n_a \omega_w}{\pi}, \qquad \frac{n_a \times 2\omega_w}{2\pi} = \frac{n_a \omega_w}{\pi}$$

Received Demodulated

$$\left.\frac{S}{N}\right|_{\text{in}} = \frac{\pi[A_r^2 + (A_r m)^2 \langle f^2 \rangle]}{2 n_a \omega_w}, \qquad \left.\frac{S}{N}\right|_{\text{out}} = \frac{\pi A_r^2 m^2 \langle f^2 \rangle}{n_a \omega_w} = 2\eta \left.\frac{S}{N}\right|_{\text{in}}$$

$$\eta = \frac{m^2 \langle f^2 \rangle}{1 + m^2 \langle f^2 \rangle}$$: is called "power transmission efficiency"

$$0 < m \leq 1 \ \rightarrow \eta < \frac{1}{2}$$

$$\text{Input sinusoidal: } \langle f^2 \rangle = \frac{1}{2} \ \rightarrow \ \eta < \frac{1}{3}$$

6.3.5 Modulation and noise

In the case of angle modulation

In phase — Out of phase

$$r(t) = \underbrace{A_r \cos[\omega_c t + \phi(t)]}_{\text{Signal}} + \underbrace{n_l(t)\cos\omega_c t - n_r(t)\sin\omega_c t}_{\text{Noise}}$$

$$= A_r \cos[\omega_c t + \phi(t)] + A_n(t)\cos[\omega_c t + \phi_n(t)]$$

$$= V_r(t)\cos[\omega_c t + \theta(t)] \quad (\theta(t) = \phi(t) + \phi_{no}(t))$$

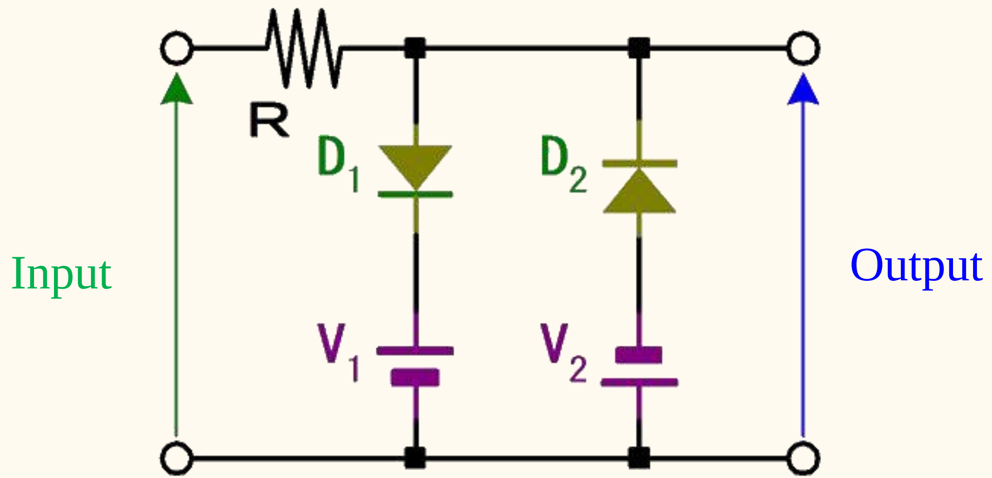
Phase noise: $\phi_{no}(t)$

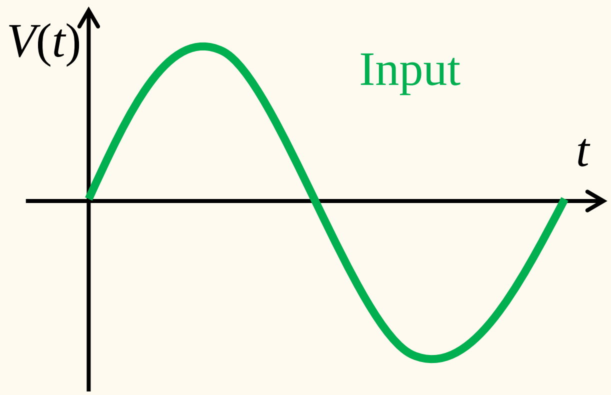
$$V_r(t) = \sqrt{A_r^2 + A_n^2(t) + 2A_r A_n(t)\cos[\phi_n(t) - \phi(t)]},$$

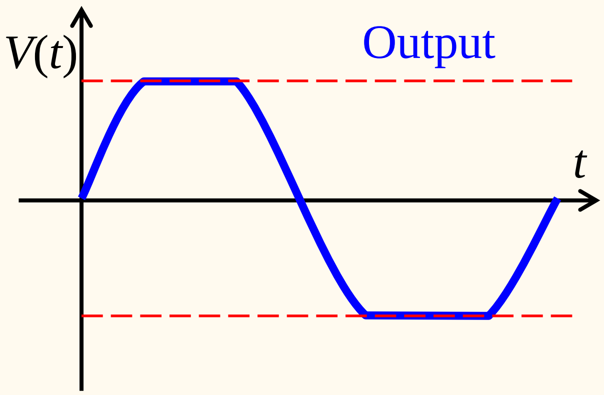
$$\phi_{no}(t) = \arctan\frac{A_n(t)\sin[\phi_n(t) - \phi(t)]}{A_r + A_n(t)\cos[\phi_n(t) - \phi(t)]}$$

Time-dependent part in $V_r(t)$ can be cut with a limiter circuit.

6.3.5 Modulation and noise (amplitude limiter)







6.3.5 Modulation and noise

$$A_r \gg A_n(t) \quad \phi_{\text{no}} \cong \arctan\left[\frac{A_{\text{n}}(t)}{A_r}\sin[\phi_{\text{n}}(t)-\phi(t)]\right] \cong \frac{A_{\text{n}}(t)}{A_r}\sin[\phi_{\text{n}}(t)-\phi(t)]$$

Noise power: $N_i = \frac{n_a \omega_B}{2\pi}$ Signal power: $\frac{A_r^2}{2}$ $\frac{S_{\text{i}}}{N_{\text{i}}} = \frac{\pi A_r^2}{n_a \omega_{\text{B}}}$

Phase modulation $\phi[t, f(t)] = k_p f(t)$

Averaged signal power: $k_p^2 \langle f^2 \rangle$

Averaged noise power: $N_{\text{oPM}} \cong \frac{1}{A_r^2}\langle A_{\text{n}}(t)^2 \sin^2[\phi_{\text{n}}(t)-\phi(t)]\rangle$

$\phi_n(t)$: Uniform in [0, 2π] → ignored

$$N_{\text{oPM}} \cong \frac{1}{A_r^2}\langle A_{\text{n}}(t)^2 \sin^2 \phi(t)\rangle = \frac{n_a \omega_{\text{w}}}{\pi A_r^2}$$

6.3.5 Modulation and noise

$$f(t) = A_p \cos \omega_p t, \quad \beta \equiv k_p A_p \to S_o = \frac{\beta^2}{2}, \quad \omega_\mathrm{B} = 2(\beta + \xi)\omega_\mathrm{w} \ (1 \le \xi \le 2)$$

$$\frac{S_\mathrm{o}}{N_\mathrm{o}} = \frac{\beta^2}{2} \frac{\pi A_r^2}{n_a \omega_\mathrm{w}} = \frac{\beta^2}{2} \frac{\omega_\mathrm{B}}{\omega_\mathrm{w}} \frac{\pi A_r^2}{n_a \omega_\mathrm{B}} = \beta^2(\beta + \xi)\frac{S_i}{N_i}$$

Frequency modulation

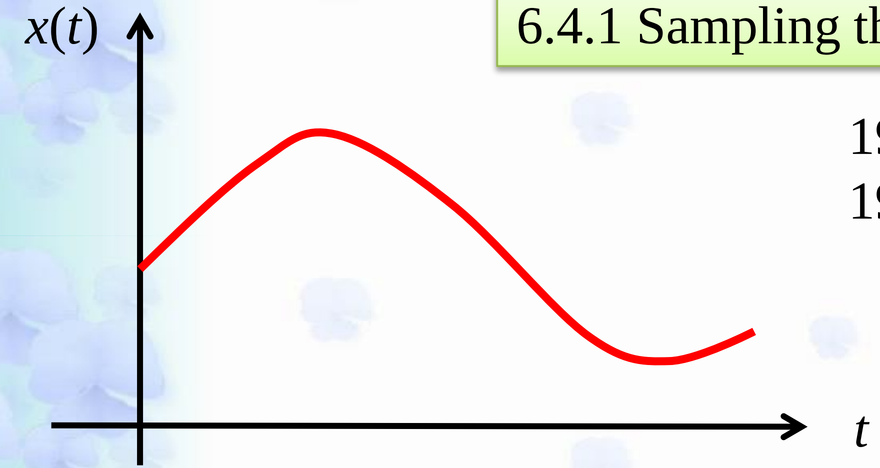
$$N_\mathrm{oFM} = \left\langle \frac{dn_\mathrm{no}}{dt} \right\rangle = \frac{1}{A_r^2} \left\langle \frac{dn_l}{dt} \right\rangle = \frac{1}{A_r^2} \int_{-\omega_\mathrm{w}}^{\omega_\mathrm{w}} n_a \omega^2 \frac{d\omega}{2\pi} = \frac{n_a \omega_\mathrm{w}^3}{3\pi A_r^2}$$

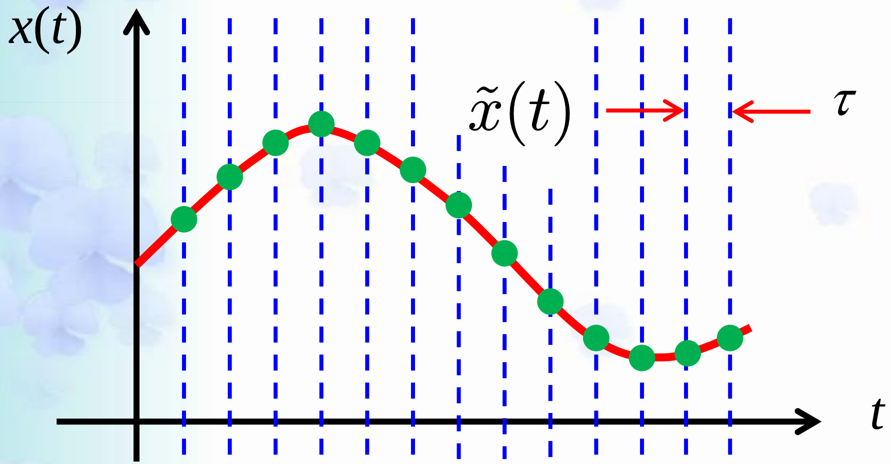
$$\beta \equiv k_f A_p / \omega_\mathrm{w} \qquad \frac{S_\mathrm{o}}{N_\mathrm{o}} = 3\beta^2(\beta + \xi)\frac{S_\mathrm{i}}{N_\mathrm{i}}$$

$$\left.\frac{S_\mathrm{o}}{N_\mathrm{o}}\right|_\mathrm{FM} = 3\beta^2 \left.\frac{S_\mathrm{o}}{N_\mathrm{o}}\right|_\mathrm{AM}, \qquad \left.\frac{S_\mathrm{o}}{N_\mathrm{o}}\right|_\mathrm{PM} = \beta^2 \left.\frac{S_\mathrm{o}}{N_\mathrm{o}}\right|_\mathrm{AM}$$

6.4 Discrete signal

6.4.1 Sampling theorem

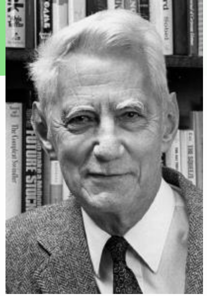




1928 H. Nyquist

1949 C. Shannon, 染谷勲

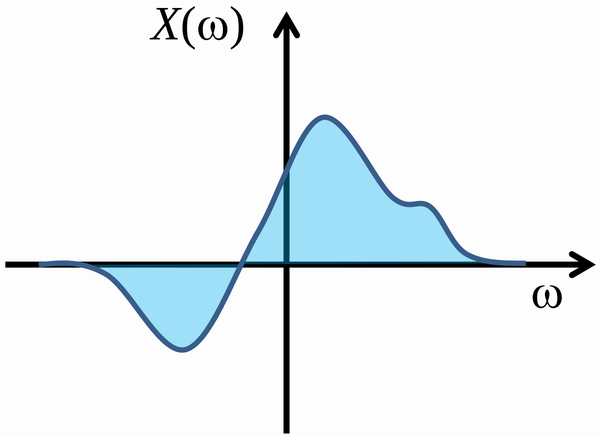
Isao Someya
1915-2007



Claude Shannon
1916-2001

$$\tilde{x}(t) = x(t)\underline{\delta_\tau(t)}$$

δ-functions with the period τ



6.4.1 Sampling theorem

$$\delta_\tau(t) = \sum_{j=-\infty}^{\infty} \delta(t - j\tau) = \sum_{n=-\infty}^{\infty} \left[\frac{1}{\tau} \int_{-\pi/\tau}^{\pi/\tau} \delta(s) ds \right] \exp\left(-in\frac{2\pi}{\tau}t\right) = \frac{1}{\tau} \sum_{n=-\infty}^{\infty} \exp\left(-in\frac{2\pi}{\tau}t\right)$$

$$\underline{\mathscr{F}\{\delta_\tau(t)\}} = \int_{-\infty}^{\infty} \left[\frac{1}{\tau} \sum_{-\infty}^{\infty} e^{-in(2\pi/\tau)t} \right] e^{i\omega t} dt = \frac{1}{\tau} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left[i \left(\omega - n\frac{2\pi}{\tau} \right) t \right] dt$$

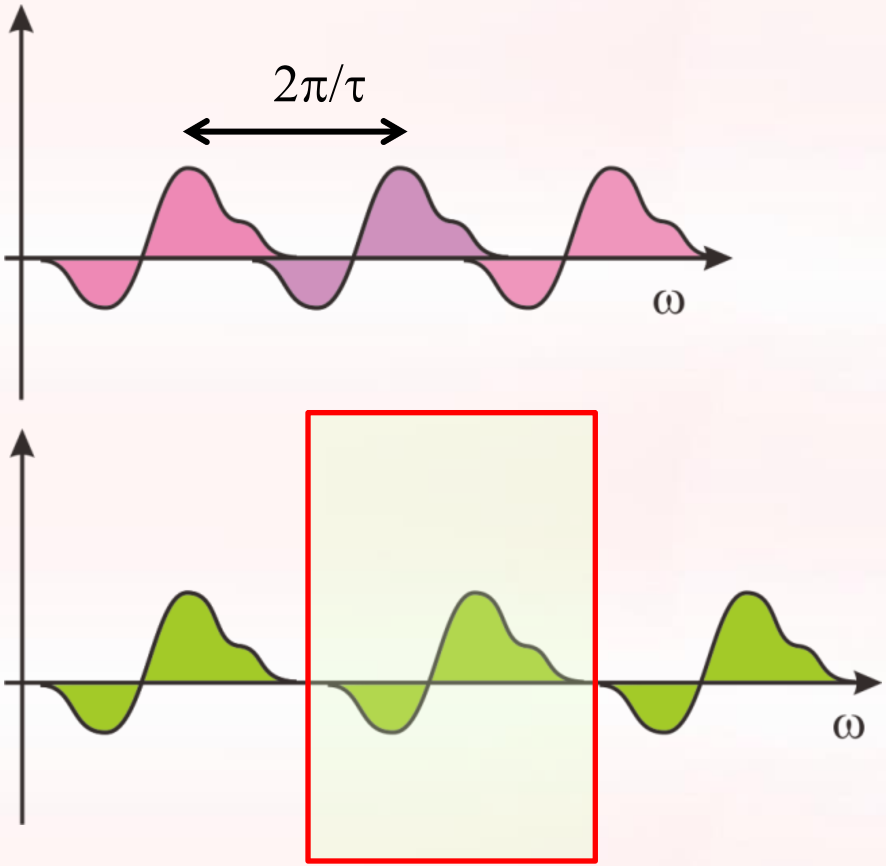
$$= \frac{2\pi}{\tau} \sum_{n=-\infty}^{\infty} \delta\left(\omega - n\frac{2\pi}{\tau}\right) = \underline{\frac{2\pi}{\tau} \delta_{2\pi/\tau}(\omega)}$$

$$\mathscr{F}\{x(t)\} = X(\omega), \; \mathscr{F}\{\tilde{x}_\tau(t)\} = \tilde{X}_\tau(\omega)$$

$$\tilde{X}_\tau(\omega) = \frac{1}{2\pi} X(\omega) * \frac{2\pi}{\tau} \delta_{2\pi/\tau}(\omega) = \frac{1}{\tau} X(\omega) * \sum_{n=-\infty}^{\infty} \delta\left(\omega - n\frac{2\pi}{\tau}\right)$$

$$= \frac{1}{\tau} \int_{-\infty}^{\infty} X(\omega') \left\{ \sum_{n=-\infty}^{\infty} \delta\left(\omega - n\frac{2\pi}{\tau} - \omega'\right) \right\} d\omega' = \frac{1}{\tau} \sum_{n=-\infty}^{\infty} X\left(\omega - n\frac{2\pi}{\tau}\right)$$

6.4.1 Sampling theorem



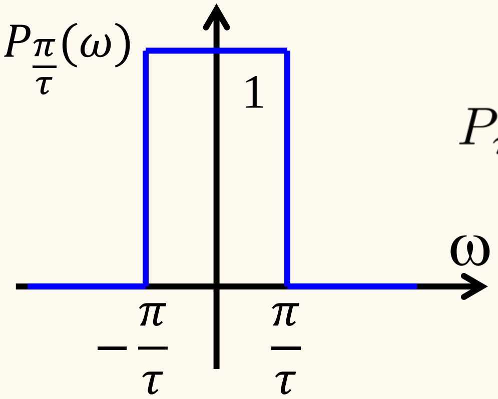
"Cutting out" the frequency spectrum

ω_h: Highest frequency in $\tilde{X}_\tau(\omega)$

$$\frac{2\pi}{\tau} > 2\omega_h, \quad \tau < \frac{\pi}{\omega_h}$$

$\frac{1}{2\tau}$: Nyquist frequency

6.4.1 Sampling theorem: reconstructing signal



$$P_{\pi/\tau}(\omega) = \begin{cases} 1 & |\omega| \leq \dfrac{\pi}{\tau}, \\ 0 & |\omega| > \dfrac{\pi}{\tau} \end{cases}$$

$$x(t) = \mathscr{F}^{-1}\{\tau P_{\pi/\tau}(\omega)\tilde{X}_\tau(\omega)\}$$