電子回路論 第2回
Electric Circuits for Physicists #2

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Ch.1 Electromagnetic field and electric circuits

- Metals: super-screening material

  (but not superconducting. The difference is important in designing superconducting circuits.)

- Local electromagnetic field

  → Lumped constant circuits (集中定数回路)

  local magnetic fields (parts) are connected by metallic wires

  → Circuit diagrams

- Resistors, Capacitors and Inductors

Ch.2 Introduction to linear response systems

Outline

1. Transfer function (伝達関数) (continued)
2. Representative passive devices in the linear treatment
3. Impedance, admittance and other parameters in the linear treatment
4. Power sources
5. Circuit networks
6. Four terminal (two terminal-pair) circuits
7. Circuit theorems

Linear response: Transfer function

$$\underbrace{w(t)}_{\text{output}} = \int_{-\infty}^{\infty} \underbrace{u(t-t')}_{\text{input}}\xi(t')dt' \qquad \xi(t,t') \equiv \mathscr{R}\{\delta(t-t')\}$$: impulse response

$$W(\omega) = U(\omega)\underbrace{\Xi(\omega)}_{\text{Transfer function}} \qquad \left(X(\omega) \equiv \mathscr{F}\{x(t)\} \equiv \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt \quad \text{:Fourier transform} \right)$$

Transfer function: $\Xi(\omega) \equiv \mathscr{F}\{\underbrace{\mathscr{R}\{\delta(t)\}}_{\text{impulse response}}\} = \int_{-\infty}^{\infty} dte^{-i\omega t}\mathscr{R}\{\delta(t)\}$

This is a consequence of linear response: No mode-mode coupling

Extension to the complex plane

Laplace transform: $X(s) = \mathscr{L}\{x(t)\} \equiv \int_0^\infty e^{-st} x(t) dt$

We can expect this integral should converge for the responses because of the causality.

$$W(s) = U(s)\Xi(s)$$

$\Xi(s)$ is also called transfer function.

Expansion to the complex plane: $s \rightarrow \sigma + i\omega$

On the imaginary axis (the frequency space)

$$W(i\omega) = U(i\omega)\Xi(i\omega)$$

This style is often adopted in the field of electronics.

Impedance

Assignment of physical quantities to input and output for three representative passive elements.

input : current I_{12} output: voltage V_{12}

$$V_{12} = \mathscr{R}\{I_{12}\} = \hat{A} I_{12} = \begin{cases} V_{12} = R I_{12} & \text{resistor} \\ V_{12} = \dfrac{q(t)}{C} = \dfrac{1}{C} \displaystyle\int^{t} I_{12}(t')dt' & \text{capacitor} \\ V_{12} = L \dfrac{dI_{12}}{dt} & \text{inductor} \end{cases}$$

Impedance as a transfer function

Transfer function
in ω space

$$\Xi(i\omega) = \begin{cases} \int_{-\infty}^{\infty} e^{-i\omega t}\left[R\delta(t)\right] dt = R & \text{resistor} \\ \int_{-\infty}^{\infty} e^{-i\omega t}\left[\frac{1}{C}\int^{t}\delta(t')dt'\right] dt = \frac{1}{i\omega C} & \text{capacitor} \\ \int_{-\infty}^{\infty} e^{-i\omega t}\left[L\frac{d}{dt}\delta(t)\right] dt = i\omega L & \text{inductor} \end{cases}$$

$$\mathcal{V}_{12}(i\omega) = \underbrace{Z(i\omega)}_{\text{impedance}}\mathcal{I}_{12}(i\omega)$$

Impedance can be extended to the complex plane.

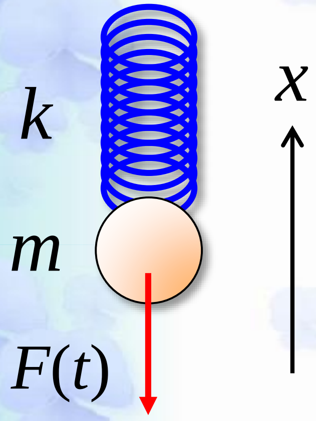
Admittance

input : voltage V_{12} output: current I_{12}

$$\mathcal{I}_{12}(i\omega) = Y(i\omega)\mathcal{V}_{12}(i\omega)$$

transfer function: Admittance $Y(i\omega) = \dfrac{1}{Z(i\omega)}$

Example of equivalent circuit

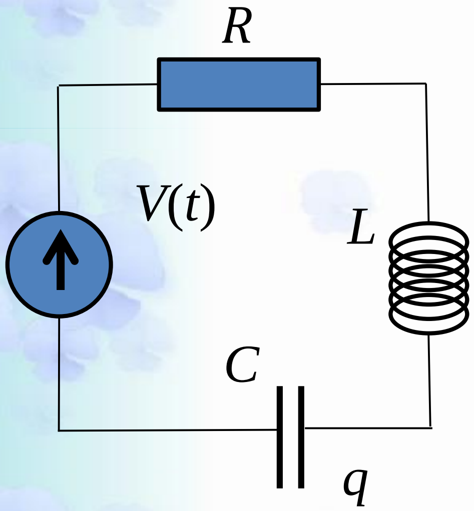


Spring pendulum

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F(t)$$

RLC circuit with electromotive force

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = V(t)$$



Electric circuits can simulate a wide range of classical dynamical systems.

Parallelism

2.2 Power sources

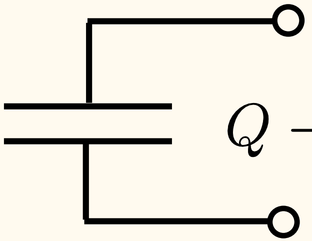
Power (energy) sources:

An active device: electric power source, electromotive force

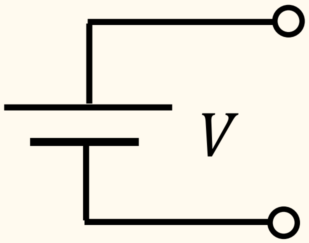
Realistic power source can be simulated with
ideal power source + non-ideal factors (simulated by circuit elements)

Ex) ideal voltage source: keeps the voltage to the specified value whatever load is connected.

Simulated as limits of passive elements:



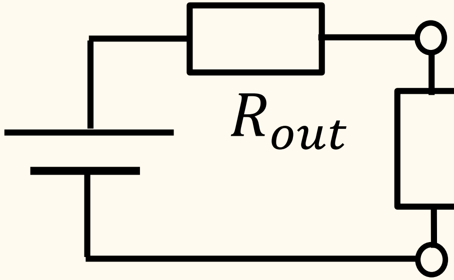
$$Q \to \infty, \quad C \to \infty, \quad V = \frac{Q}{C} = \text{const.}$$



2.2 Power sources

There are some voltage drops in real voltage sources when finite currents are extracted.

Ideal voltage source
+resistor

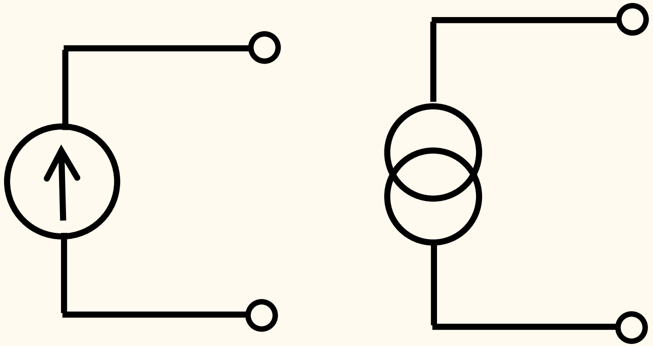


$$J = \frac{V}{R + R_L} \qquad V_{out} = \frac{R_L}{R_L + R} V$$

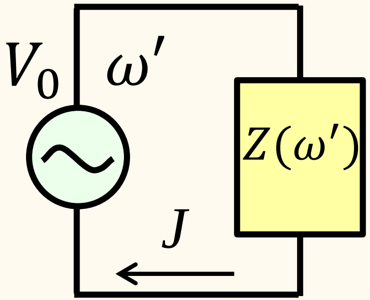
R_L: Load resistance, R_{out}: Output resistance (impedance)

Ideal current source

$$R \to \infty,\ V \to \infty,\ J = \frac{V}{R} = \text{const.}$$



Power consumption



Energy dissipation per unit time: $P = VJ$ Power consumption

Alternating current (AC) voltage source: $V_0 e^{i\omega' t}$

$$\mathcal{V}(i\omega) = \mathscr{F}\{V\} = 2\pi V_0 \delta(\omega - \omega')$$

$$J(\omega', t) = \int_{-\infty}^{\infty} \frac{2\pi V_0}{Z(i\omega)} \delta(\omega - \omega') e^{i\omega t} \frac{d\omega}{2\pi} = \frac{V_0 e^{i\omega' t}}{Z(i\omega')} = \frac{V(\omega', t)}{Z(i\omega')}$$

$$V = V_0 \cos \omega' t, \quad W(\omega', t) = V(\omega', t) J(\omega', t) = \frac{V_0^2}{Z(\omega')} \cos^2 \omega' t$$

complex instantaneous power

$$\overline{W}(\omega') = \frac{V_0^2}{2Z(\omega')}$$

time average

$$P(\omega') \equiv \mathrm{Re}[\overline{W}(\omega')]$$ Effective power (有効電力)

$$Q(\omega') \equiv \mathrm{Im}[\overline{W}(\omega')]$$ Reactive power (無効電力)

Power consumption (2)

$|\overline{W}(\omega')|$ Apparent power (皮相電力）

$$I_{\mathrm{M}} \equiv \frac{\mathrm{Re}[\overline{W}(\omega')]}{|\overline{W}(\omega')|} = \cos\left[\arg(\overline{W}(\omega'))\right] \equiv \cos\phi$$

$\boxed{\text{Moment (力率)}}$

ϕ: Phase shift between voltage and current

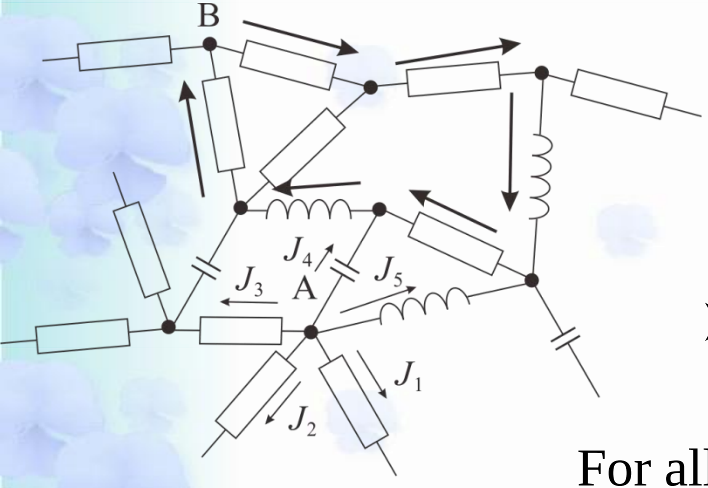
$$\overline{W}(\omega') = |\overline{W}(\omega')|e^{i\phi}$$: generally holds

$$\overline{W}(\omega') = V^*(\omega')J(\omega')$$

$$W = P = \frac{V_0^2}{2R}$$

$\frac{V_0}{\sqrt{2}}$: effective value

2.3.1 Kirchhoff's law



- Kirchhoff's first law

At all nodes $\sum_i J_i = 0$

↑ Charge conservation $\underbrace{\frac{\partial \rho}{\partial t}}_{=0} + \mathrm{div}\boldsymbol{J} = 0$

- Kirchhoff's second law

For all looping paths $\sum_j V_j = 0$

↑ Single-valuedness of electric potential

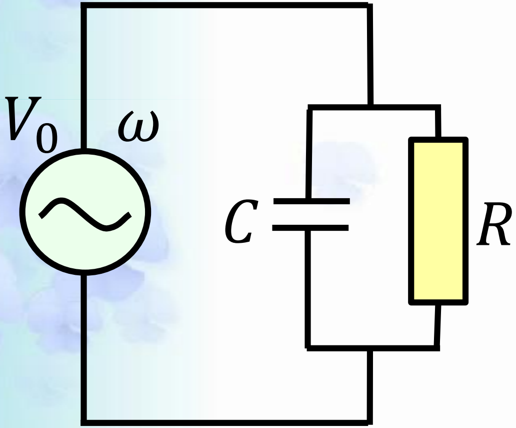
Kirchhoff's law (2)

From Kirchhoff's law, synthetic admittance and impedance are

for parallel connection: $$Y_{\mathrm{tot}} = \sum_{i=1}^{n} Y_i, \qquad Z_{\mathrm{tot}} = \left(\sum_{i=1}^{n} Z_i^{-1} \right)^{-1}$$

for series connection: $$Y_{\mathrm{tot}} = \left(\sum_{i=1}^{n} Y_i^{-1} \right)^{-1}, \qquad Z_{\mathrm{tot}} = \sum_{i=1}^{n} Z_i$$

Ex.)



$$Z(i\omega) = \left(\frac{1}{R} + i\omega C \right)^{-1}, \quad Y(i\omega) = \frac{1}{R} + i\omega C$$

$$P(\omega) = \frac{V_0^2}{R} \cos^2 \omega t, \quad Q(\omega) = \omega C V_0^2 \cos^2 \omega t$$

$$\frac{P(\omega)}{Q(\omega} = \frac{1}{\omega C R} = \tan \delta$$: Dissipation factor

2.3.3 Superposition theorem

Network: node, (directional) branch : directional graph (digraph)

All the branches: electromotive force E_i, resistance R_i

$$A\{(R)\}\begin{pmatrix} J_1 \\ \vdots \\ J_m \end{pmatrix} = \begin{pmatrix} E_1 \\ \vdots \\ E_m \end{pmatrix} \qquad \boldsymbol{R} = \begin{pmatrix} R_1 \\ \vdots \\ R_m \end{pmatrix}$$

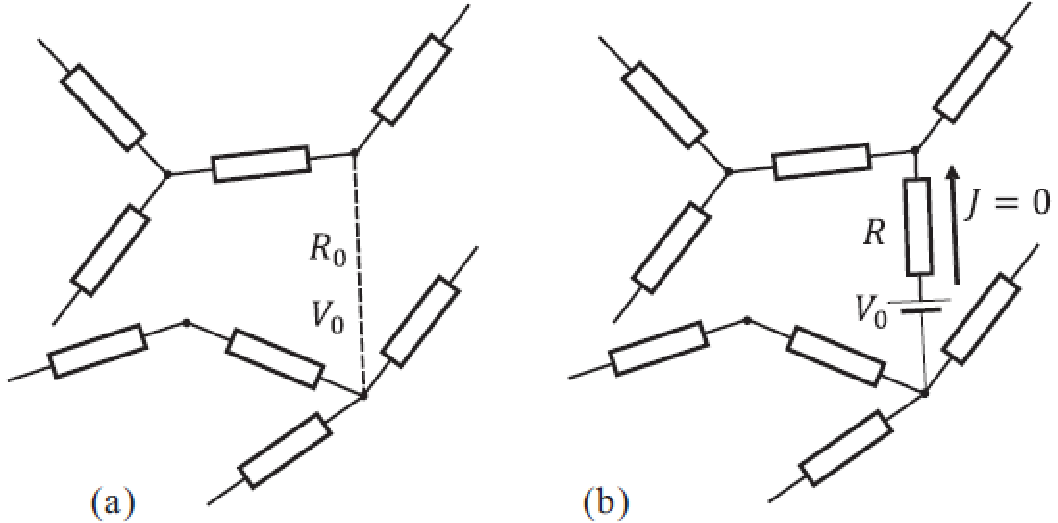
Superposition theorem:

The total current distribution is the superposition of those for single electromotive forces.

2.3.4 Ho (鳳) – Tevenin’s theorem

- Pick up two nodes in the network under consideration.
- The voltage between these two nodes is V_0.
- Set all the electromotive forces to zero and measure the resistance between the two nodes. Assume that the result is R_0.

Now connect the two nodes with resistance R and reset the electromotive forces to the original values. Then the current through resistance R is



$$J = \frac{V_0}{R + R_0}$$

2.3.5 Tellegen's theorem

Consider network with n nodes, m branches.

$i = 1, \cdots, n$: index of nodes, $j = 1, \cdots, m$: index of branches

incidence matrix $$a_{ij} = \begin{cases} 1: & i \text{ is the start of } j, \\ -1: & i \text{ is the end of } j, \\ 0: & \text{others} \end{cases}$$

$\forall j: \quad \sum_{i=1}^{n} a_{ij} = 0$: redundancy in $\{a_{ij}\}$ $\rightarrow (n-1) \times m$ matrix D : irreducible incidence matrix

- Kirchhoff's first law: $D\boldsymbol{J} = 0$ $\quad J_j$: current along branch j

vector $\boldsymbol{W}$: W_i electrostatic potential of node i,
vector $\boldsymbol{V}$: V_j voltage across branch j

2.3.5 Tellegen's theorem (2)

$i \xrightarrow{j} k$ $\qquad V_j = W_i - W_k = a_{ij}W_i - a_{kj}W_k$

$\boldsymbol{V} = {}^t\mathcal{D}\boldsymbol{W}$ $\quad ({}^t\mathcal{D}\text{: transpose})$ $\quad$ (Kirchhoff's second law)

$$\sum_{i=1}^{m} V_i J_i = ({}^t\mathcal{D}\boldsymbol{W}) \cdot \boldsymbol{J} = {}^t\boldsymbol{W}\mathcal{D}\boldsymbol{J} = 0 \qquad \therefore \boldsymbol{V} \perp \boldsymbol{J}$$

Tellegen's theorem

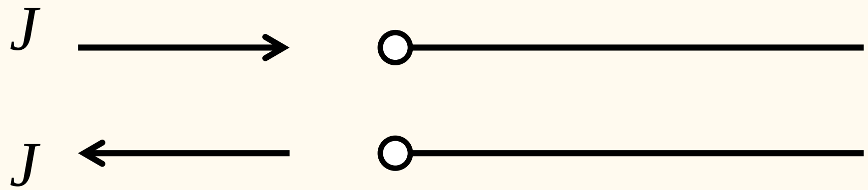
No assumption on

- Species of elements
- Linearity of the circuits

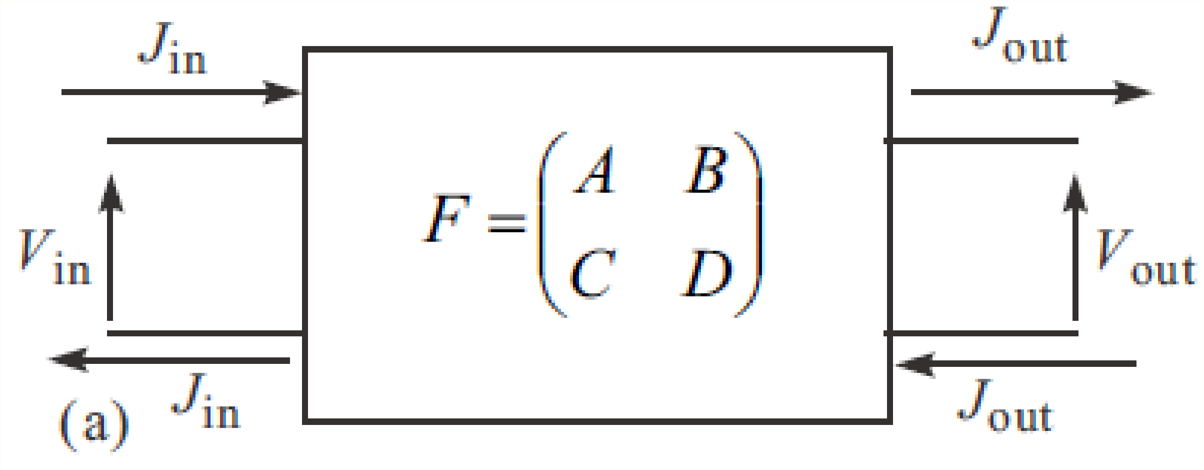
4-terminal (2-terminal pair) circuits

Terminal pair (端子対)

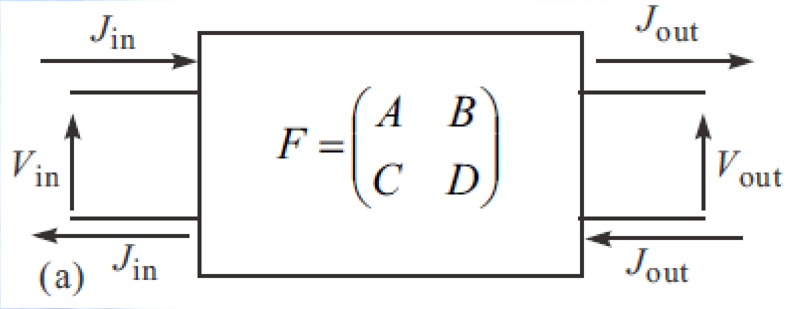
Current: circulation, no net current



2-terminal pair (4-terminal) circuit



F-matrix of 4-terminal circuit

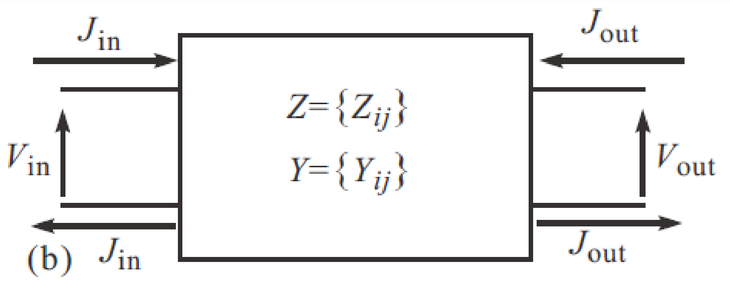


$$\begin{pmatrix} V_{\text{in}} \\ J_{\text{in}} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_{\text{out}} \\ J_{\text{out}} \end{pmatrix} \equiv F \begin{pmatrix} V_{\text{out}} \\ J_{\text{out}} \end{pmatrix}$$

$$A = \left[\frac{V_{\text{in}}}{V_{\text{out}}}\right]_{J_{\text{out}}=0}, \quad B = \left[\frac{V_{\text{in}}}{J_{\text{out}}}\right]_{V_{\text{out}}=0},$$

$$C = \left[\frac{J_{\text{in}}}{V_{\text{out}}}\right]_{J_{\text{out}}=0}, \quad D = \left[\frac{J_{\text{in}}}{J_{\text{out}}}\right]_{V_{\text{out}}=0}$$

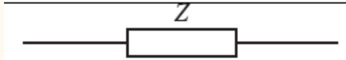
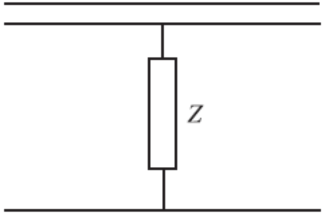
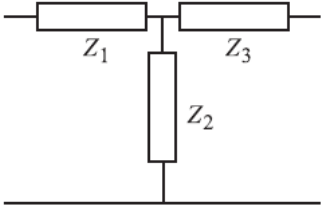
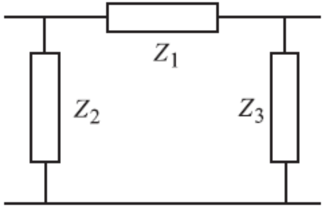
Impedance matrix, Admittance matrix



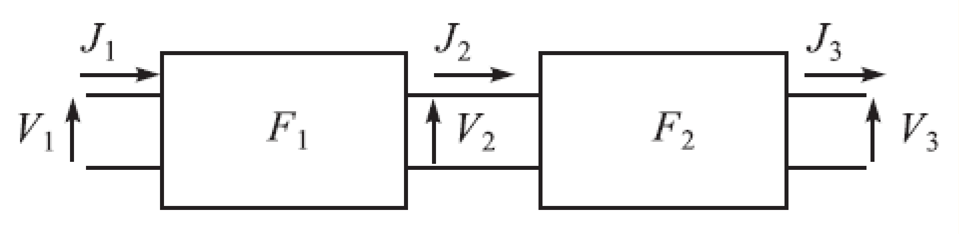
Impedance matrix $$\begin{pmatrix} V_{\text{in}} \\ V_{\text{out}} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} J_{\text{in}} \\ J_{\text{out}} \end{pmatrix} \equiv Z \begin{pmatrix} J_{\text{in}} \\ J_{\text{out}} \end{pmatrix}$$

Admittance matrix $$\begin{pmatrix} J_{\text{in}} \\ J_{\text{out}} \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_{\text{in}} \\ V_{\text{out}} \end{pmatrix} \equiv Y \begin{pmatrix} V_{\text{in}} \\ V_{\text{out}} \end{pmatrix}$$

Examples with impedances

	A	B	C	D
	1	Z	0	1
	1	0	$\frac{1}{Z}$	1
	$1+\frac{Z_1}{Z_2}$	$\frac{Z_1Z_2+Z_2Z_3+Z_3Z_1}{Z_2}$	$\frac{1}{Z_2}$	$1+\frac{Z_3}{Z_2}$
	$1+\frac{Z_1}{Z_3}$	Z_1	$\frac{Z_1+Z_2+Z_3}{Z_2Z_3}$	$1+\frac{Z_1}{Z_2}$

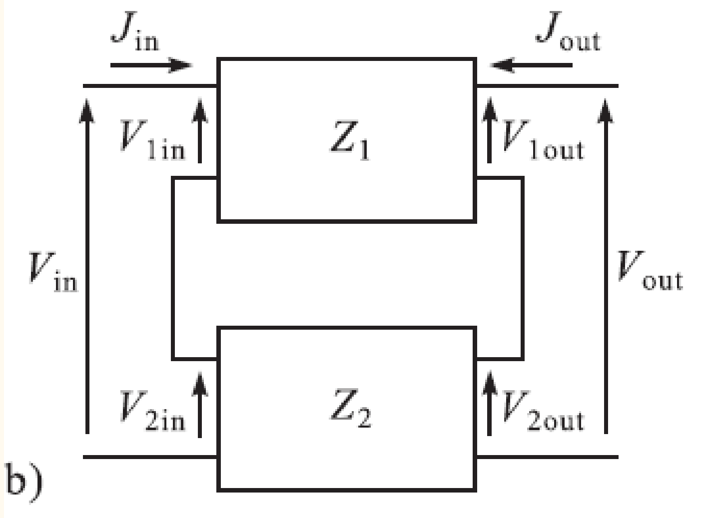
Connections of 4-terminal circuits



Cascade

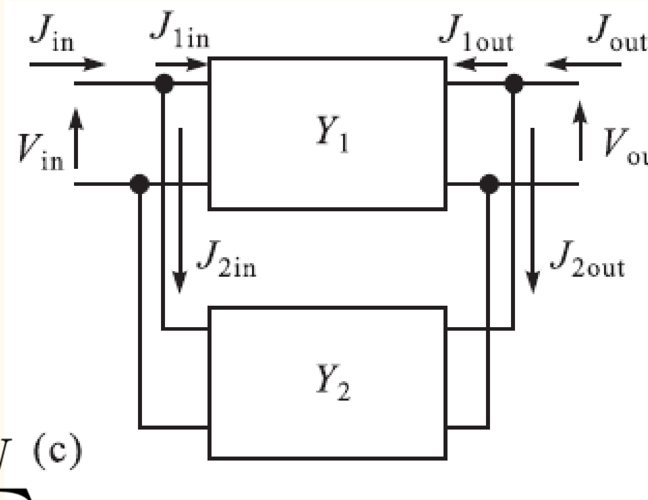
$$F_{\mathrm{tot}} = \prod_{i=1}^{N} F_i$$

Series



$$Z_{\mathrm{tot}} = \sum_{i=1}^{N} Z_i$$

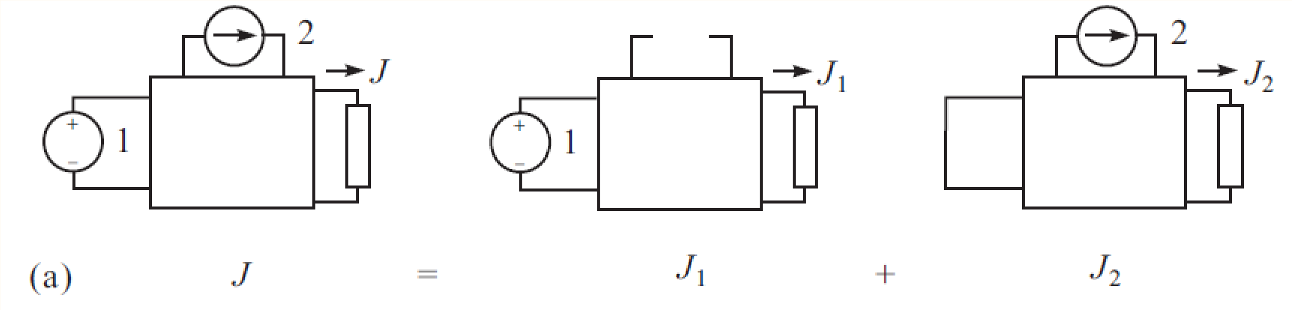
Parallel



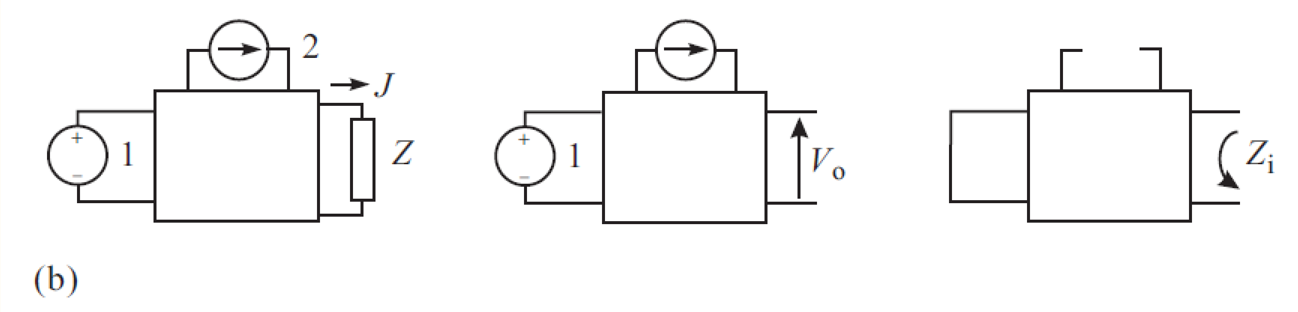
$$Y_{\mathrm{tot}} = \sum_{i=1}^{N} Y_i$$

Theorems for terminal-pair circuits

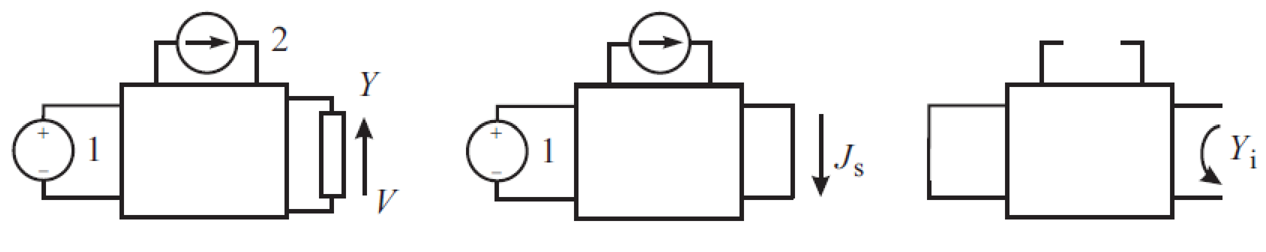
Superposition theorem



Ho-Thevenin's theorem



Norton's theorem



$$V = \frac{J_s}{Y + Y_i}$$

Duality 双対性

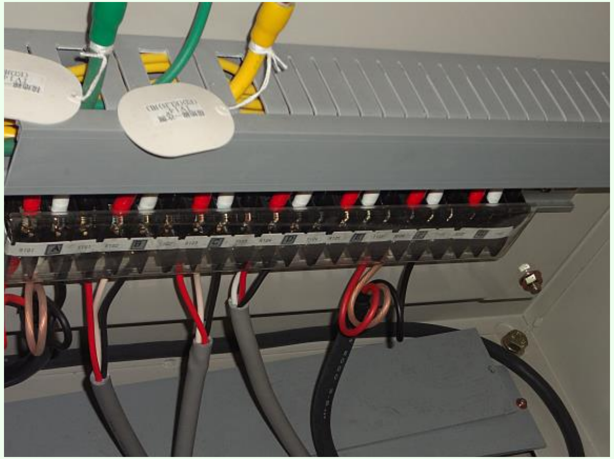
直列接続	並列接続
開放	短絡
電場	磁場
キルヒホッフの第 2 法則	キルヒホッフの第 1 法則
電圧	電流
インピーダンス	アドミッタンス
抵抗	コンダクタンス
静電容量	インダクタンス
鳳-テブナンの定理	ノートンの定理

Duality

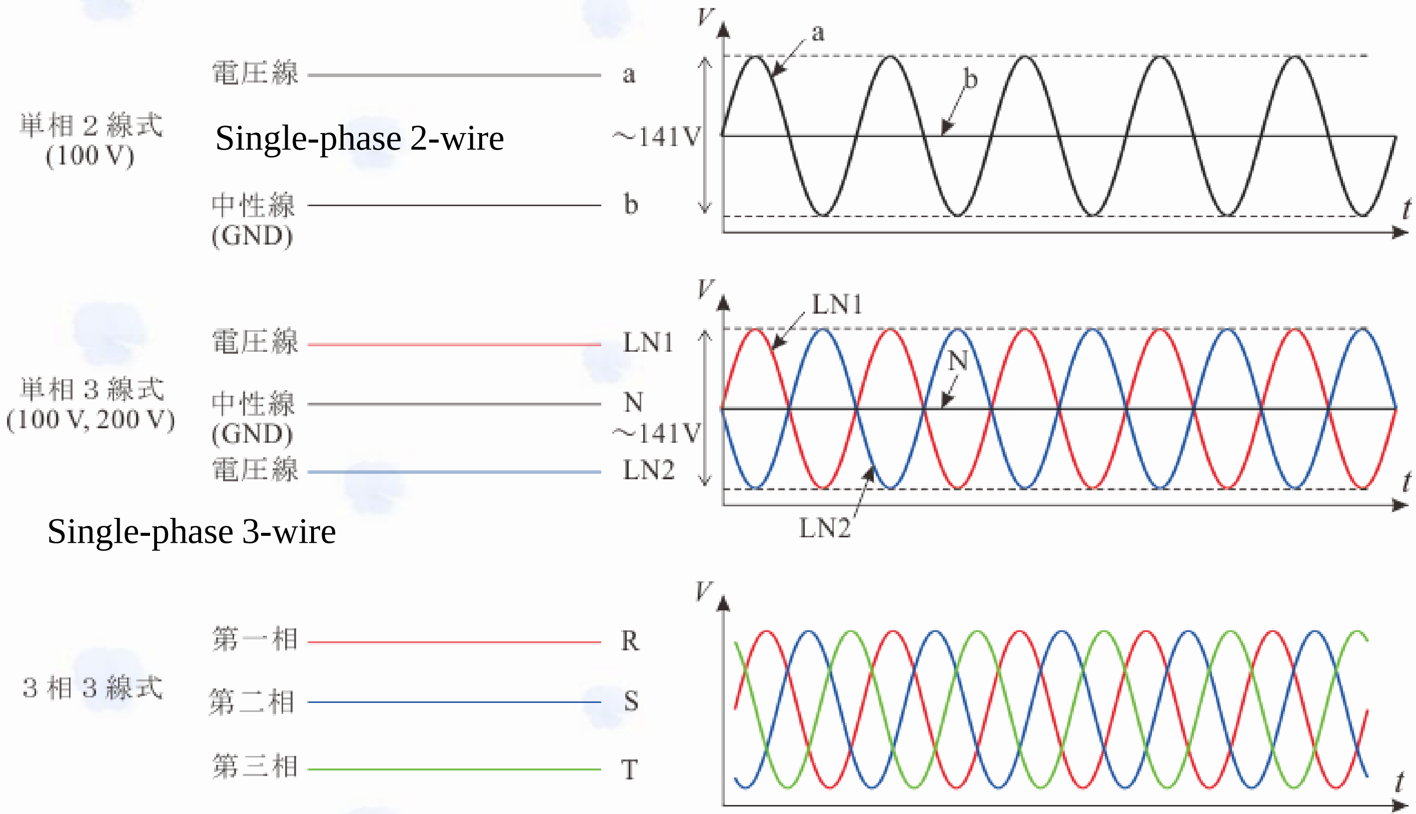
Series	Parallel
Open	Short
Voltage	Current
Impedance	Admittance
Capacitance	Inductance
Electric field	Magnetic field
Resistance	Conductance
Ho-Thevenin	Norton
Kirchhoff’s 2nd law	Kirchhoff’s 1st law

Power Sources in Lab. 電源の雑知識

AC Power from distribution board 配電盤からの電力供給



AC Power from distribution board 配電盤からの電力供給

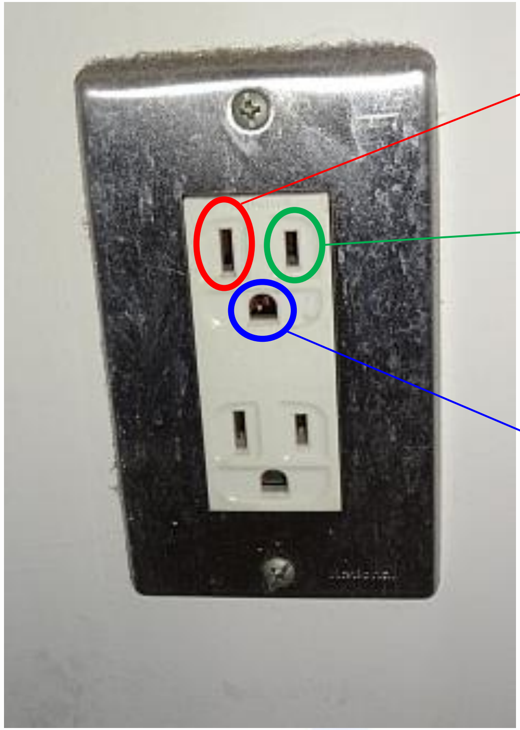


Single-phase 2-wire

Single-phase 3-wire

Three-phase 3-wire

Japanese outlet tap definition 日本式コンセント



Cold line 中性線

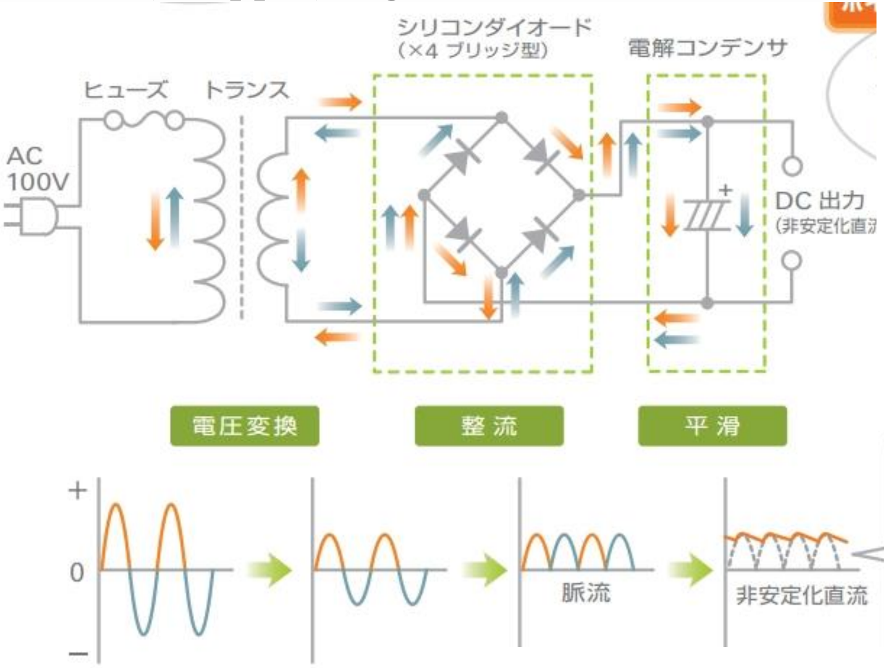
Hot line 電圧線

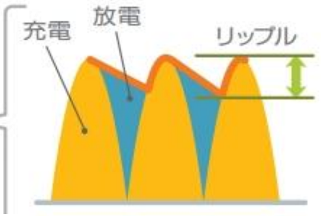
Ground 接地線

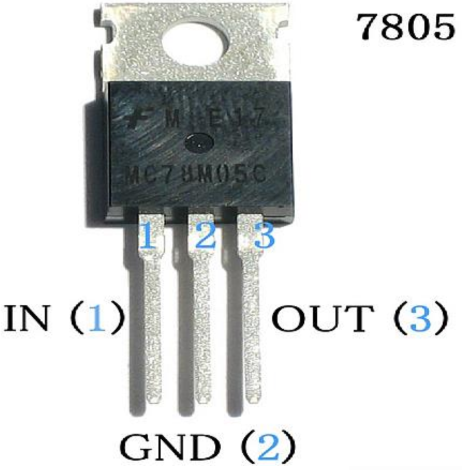
検電ドライバー
Electroscopic
Screwdriver

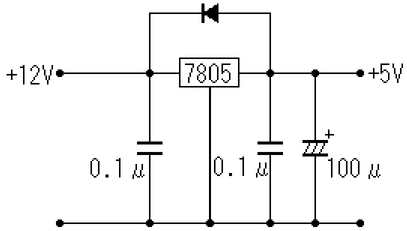
DC Stabilized Power Supply 直流安定化電源

Series (Dropper) regulation









From TDK web page

Series regulator power supply

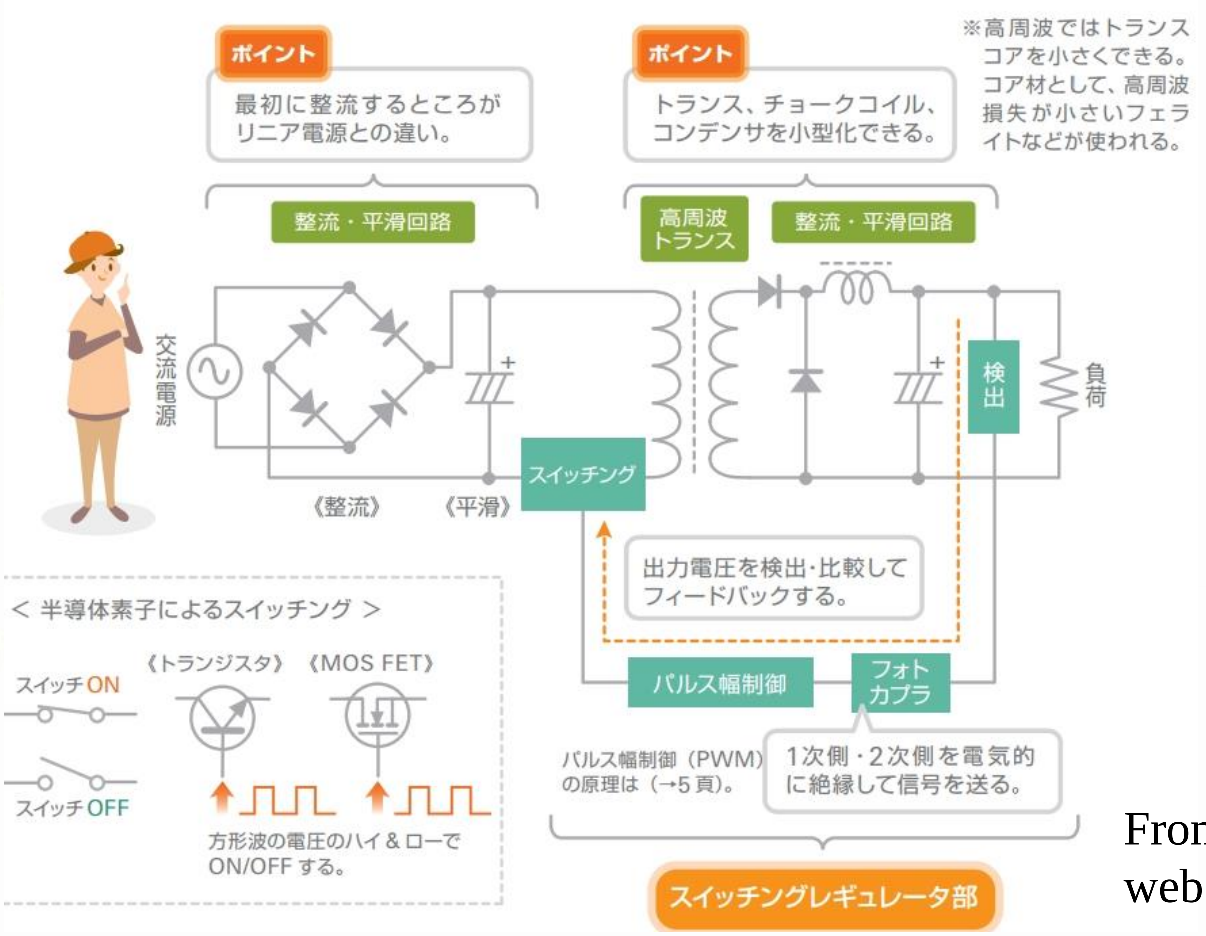
Uni-polar

Dual tracking

High precision

Bi-polar current source

Switching regulation

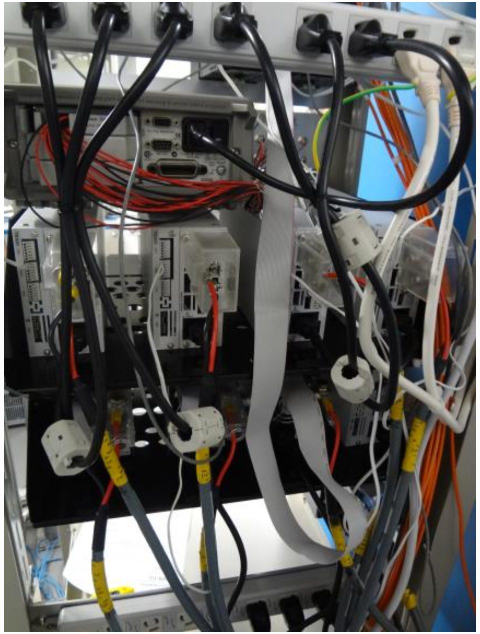


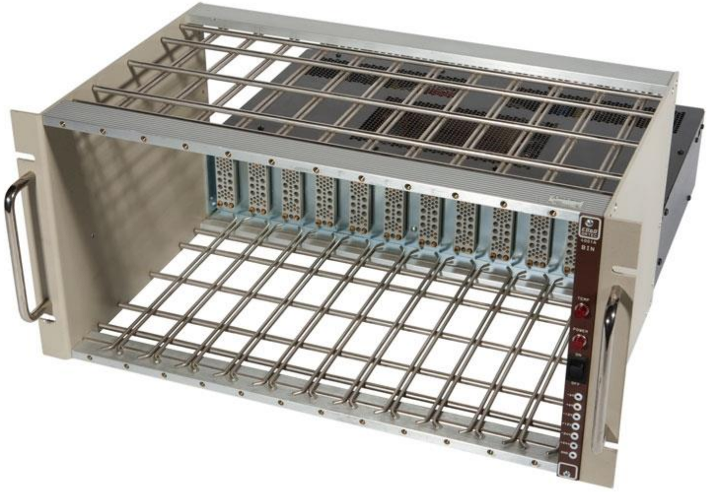
From TDK
web page

Switching regulator power supply

Molecular beam epitaxy
Control panel

Bin 電源ビン

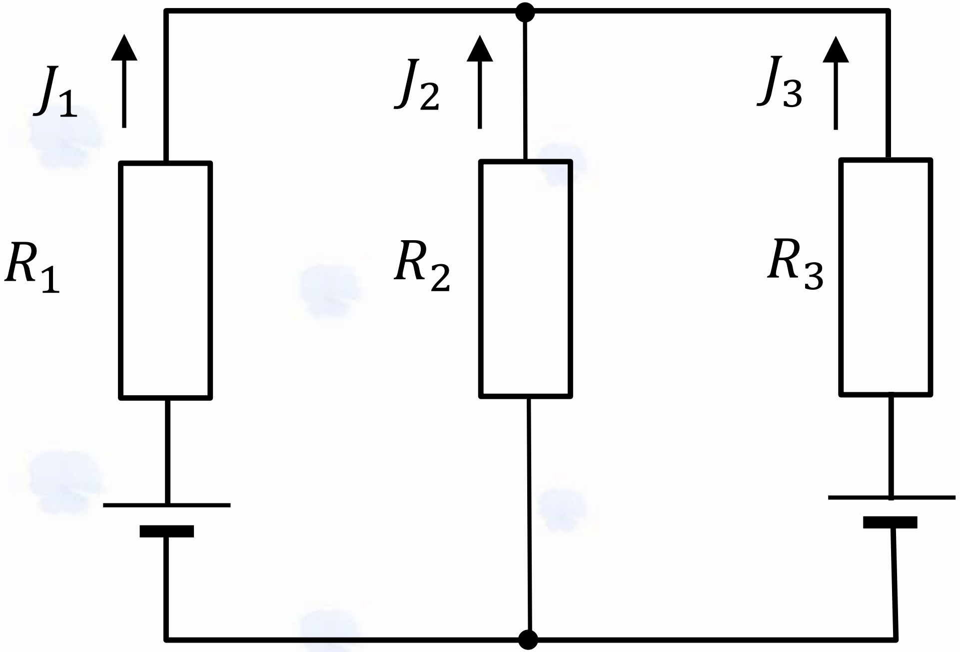




Complicated power lines → Bin

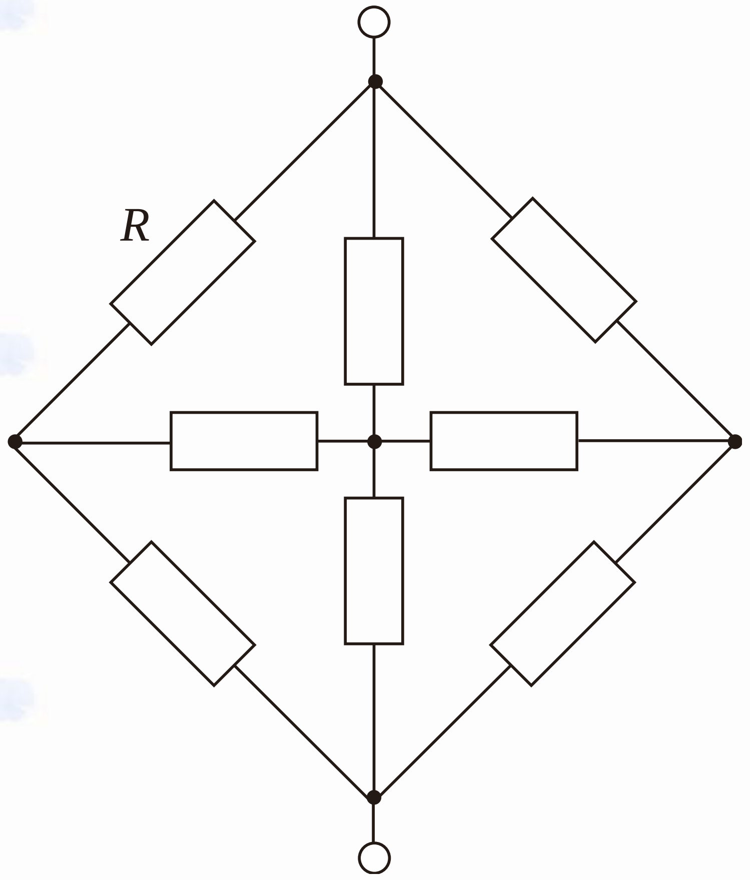
Exercise A-1

Express J_1, J_2, J_3 with other parameters.



Exercise A-2

All the resistors have the same resistance R. Obtain the combined resistance.



Exercise A-3

Obtain the effective value of voltage for the saw tooth wave.