電子回路論第3回

Electric Circuits for Physicists #3

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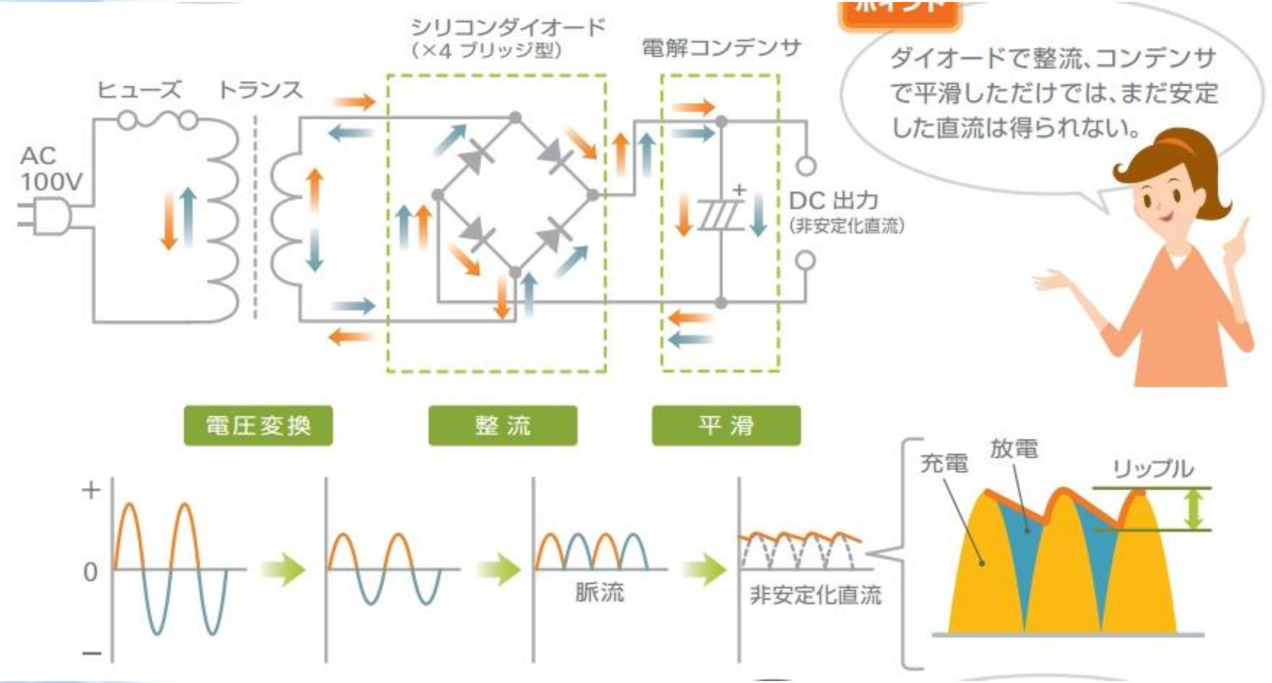
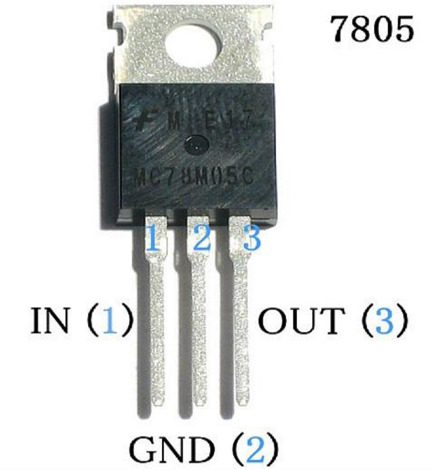
電源の雑知識 (続き）

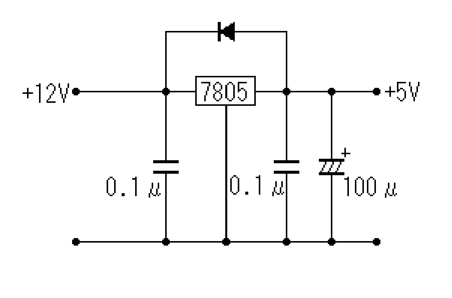
Miscellaneous knowledge on power supplies (continued)

DC Stabilized Power Supply 直流安定化電源

Series (Dropper) regulation







From TDK web page

Series regulator power supply

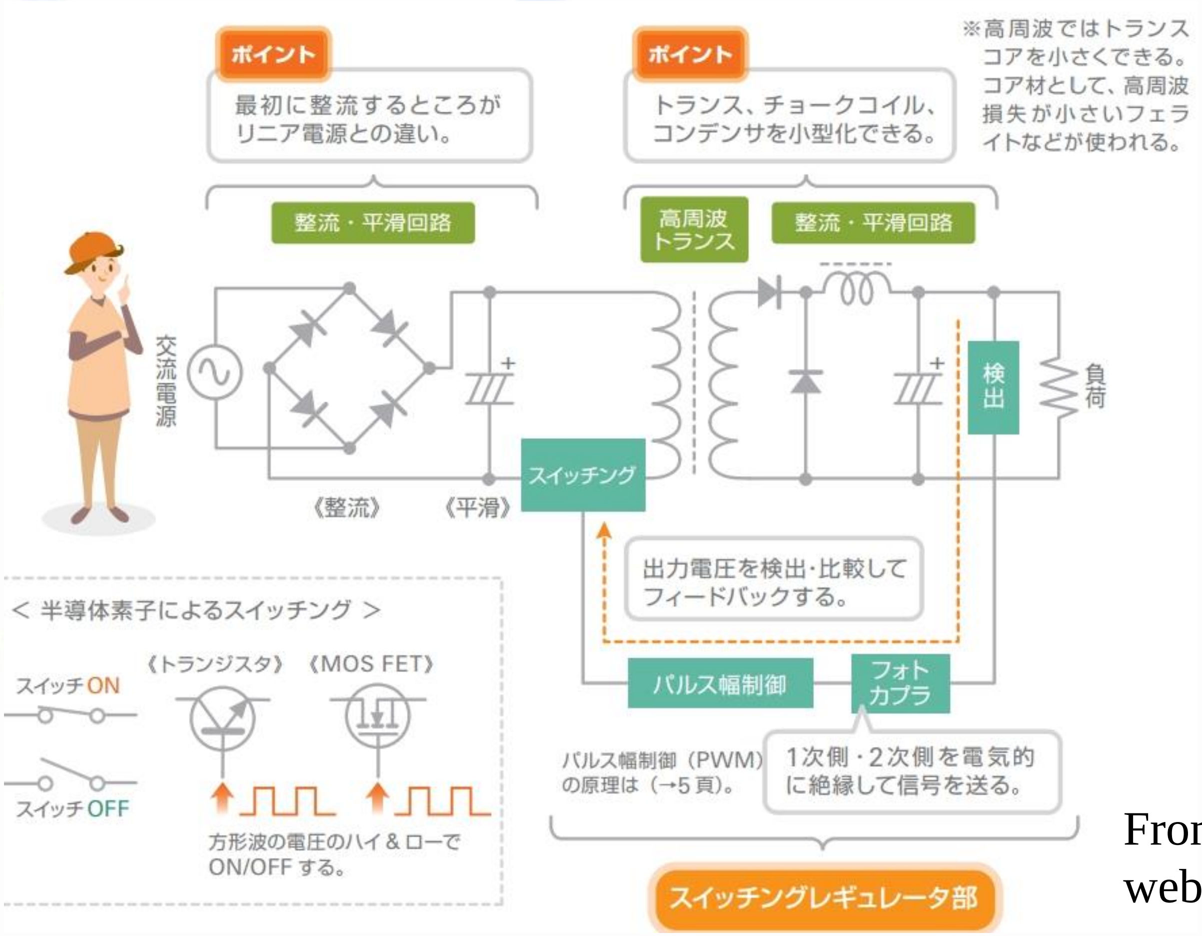
Uni-polar

Dual tracking

High precision

Bi-polar current source

Switching regulation

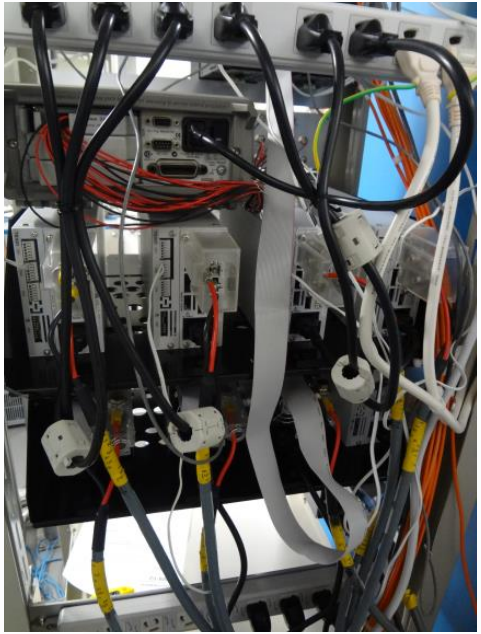


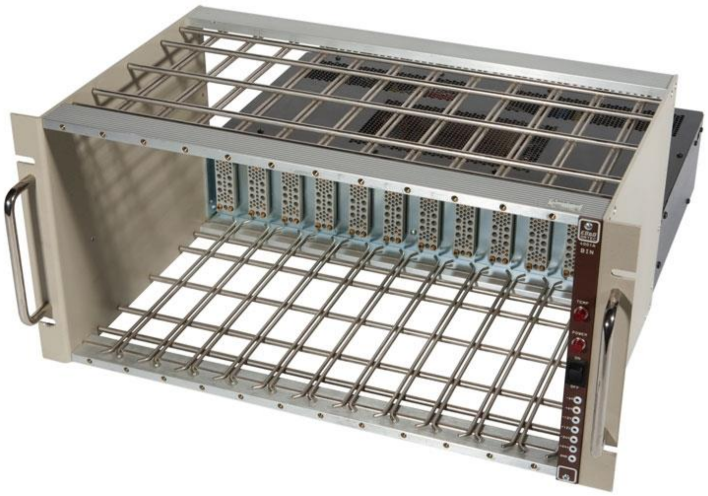
From TDK
web page

Switching regulator power supply

Molecular beam epitaxy
Control panel

Bin 電源ピン





Complicated power lines → Bin

Outline

2.5 Theorems for paired terminal circuits

Superposition, Ho-Thevenin, Reciprocity

2.6 Duality

2.7 Passive devices (elements) and active devices

Ch.3 Transfer function and transient response

3.1 Transfer function of single-pair terminal circuits

Resonance circuit

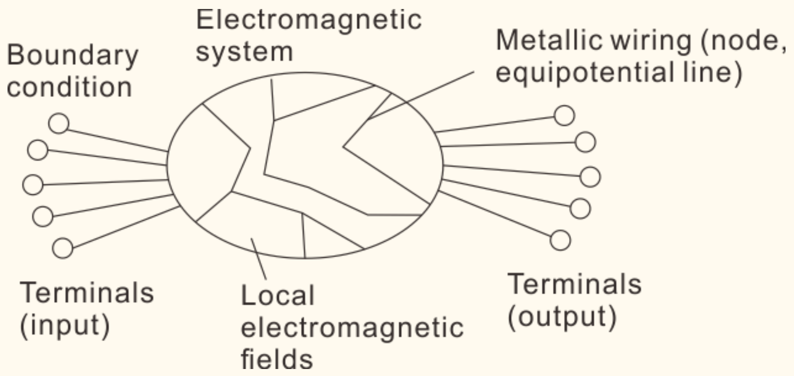
Bode plot

General properties

Appendix B   Bridges and balance circuits

Appendix C   General properties of resonance circuits

Concept of terminal pair

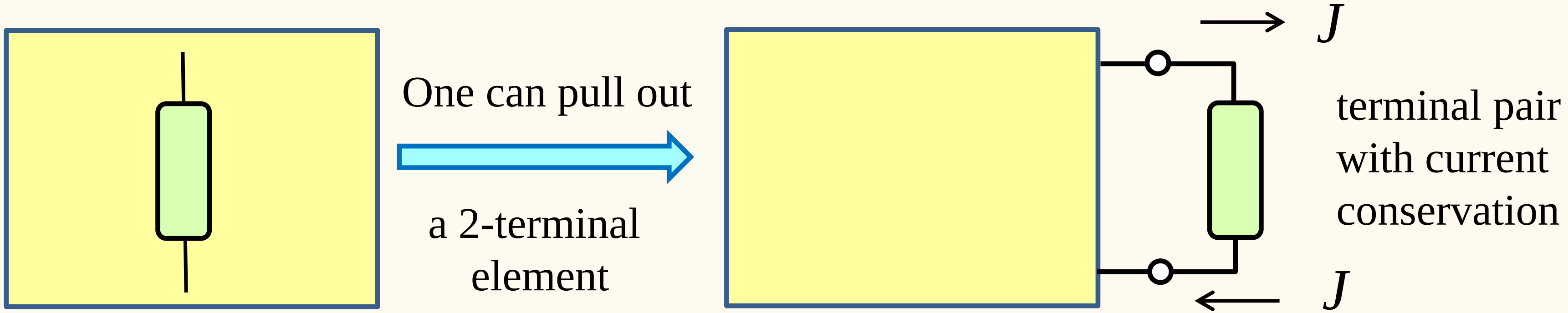


Electric circuit: a viewpoint

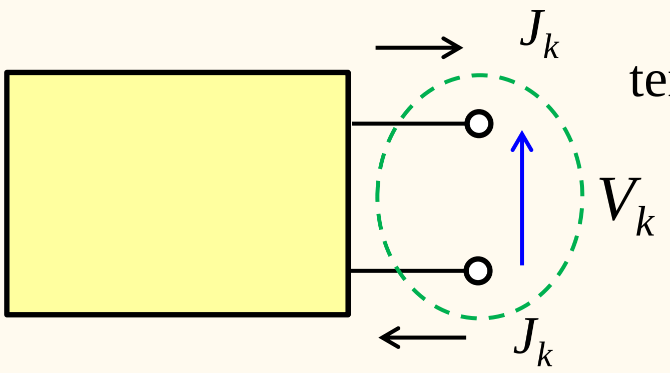
- Local electromagnetic field
- Lumped constant circuit

Two-terminal elements : linear response: Impedance

: power (energy) source (electromotive force)



Two terminal-pair circuit



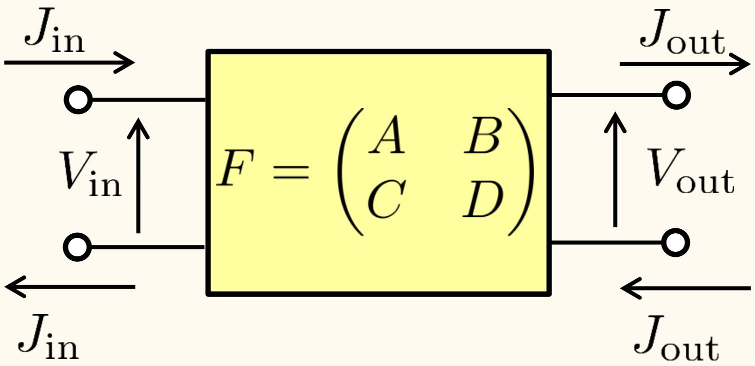
terminal pair index: k

current : J_k

voltage : V_k

} Terminal pair parameters

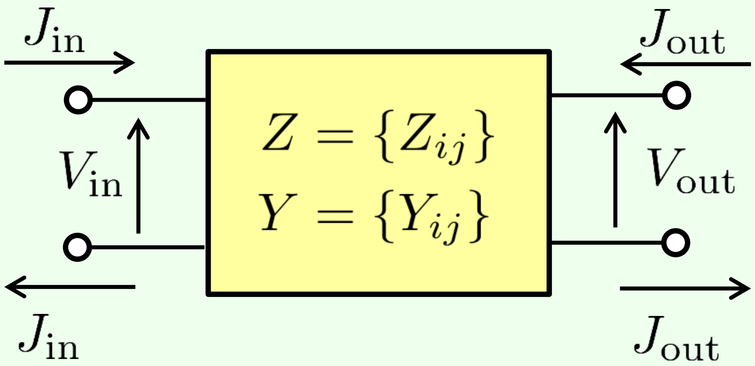
Linear relations between terminal-pair parameters: **terminal-pair matrices**



F-matrix (cascade matrix)

$$\begin{pmatrix} V_{\mathrm{in}} \\ J_{\mathrm{in}} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_{\mathrm{out}} \\ J_{\mathrm{out}} \end{pmatrix} \equiv F \begin{pmatrix} V_{\mathrm{out}} \\ J_{\mathrm{out}} \end{pmatrix}$$

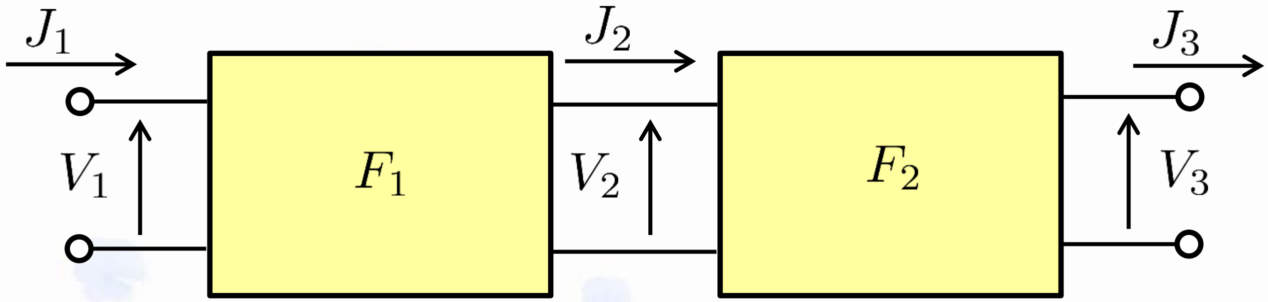
Impedance matrix, Admittance matrix



Impedance matrix
$$\begin{pmatrix} V_{\text{in}} \\ V_{\text{out}} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} J_{\text{in}} \\ J_{\text{out}} \end{pmatrix} \equiv Z \begin{pmatrix} J_{\text{in}} \\ J_{\text{out}} \end{pmatrix}$$

Admittance matrix
$$\begin{pmatrix} J_{\text{in}} \\ J_{\text{out}} \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_{\text{in}} \\ V_{\text{out}} \end{pmatrix} \equiv Y \begin{pmatrix} V_{\text{in}} \\ V_{\text{out}} \end{pmatrix}$$

Cascade connection of 4-terminal circuits



$$\begin{pmatrix} V_1 \\ J_1 \end{pmatrix} = F_1 \begin{pmatrix} V_2 \\ J_2 \end{pmatrix} = F_1 F_2 \begin{pmatrix} V_3 \\ J_3 \end{pmatrix}$$

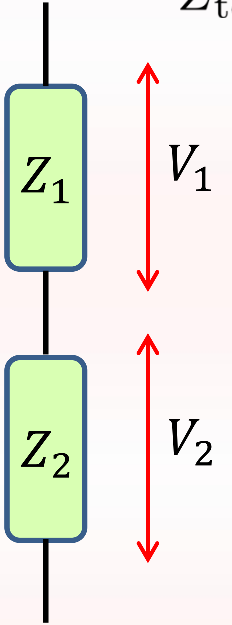
$$F_{\text{cascade}} = \prod_{i=1}^{N} F_i$$

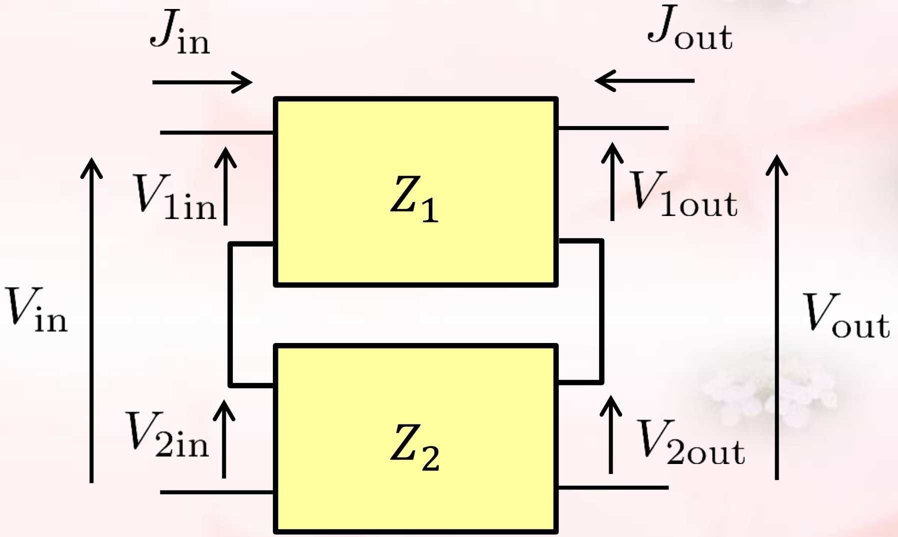
Series connections of 4-terminal circuits

Series connection of 2-terminal elements

Stack along voltage direction

$$Z_{\mathrm{tot}} = Z_1 + Z_2$$





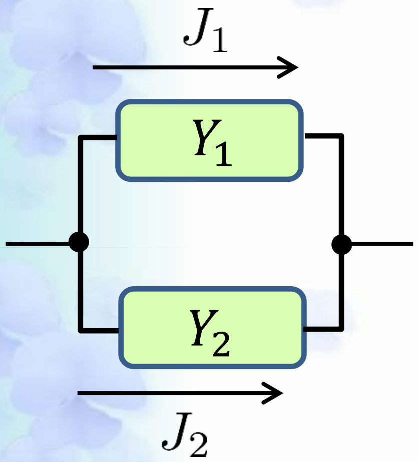
$$\begin{pmatrix} V_{\mathrm{in}} \\ V_{\mathrm{out}} \end{pmatrix} = \begin{pmatrix} V_{1\mathrm{in}} \\ V_{1\mathrm{out}} \end{pmatrix} + \begin{pmatrix} V_{2\mathrm{in}} \\ V_{2\mathrm{out}} \end{pmatrix} = (Z_1 + Z_2) \begin{pmatrix} J_{\mathrm{in}} \\ J_{\mathrm{out}} \end{pmatrix}$$

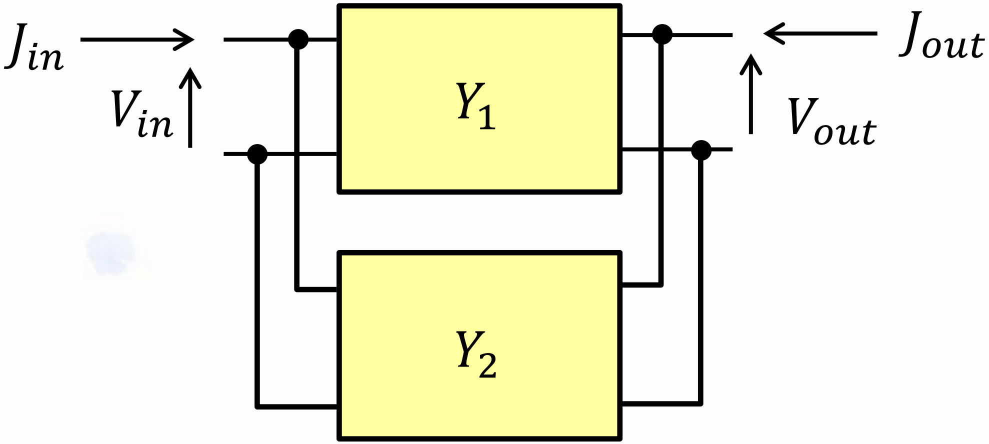
$$Z_{\mathrm{tot}} = \sum_{i=1}^{N} Z_i$$

Parallel connections of 4-terminal circuits

Parallel connection of 2-terminal elements

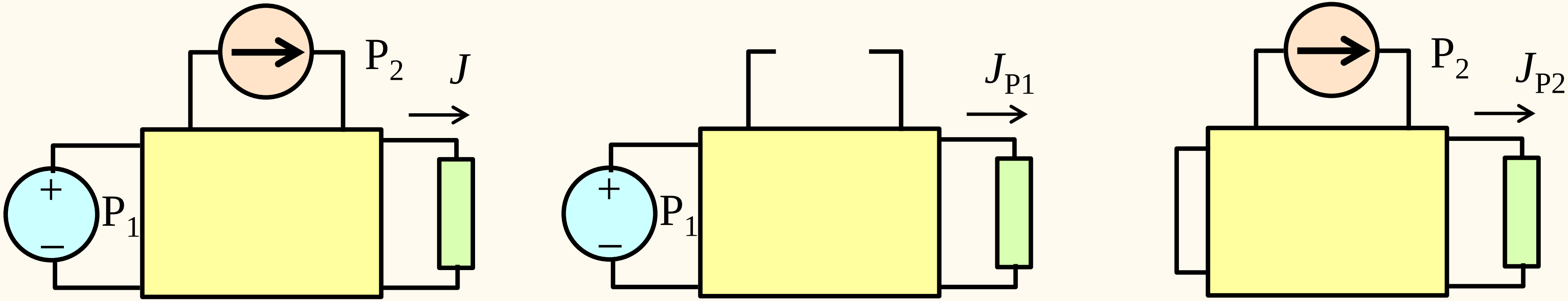
Current participation with a common voltage





$$Y_{\text{tot}} = \sum_{i=1}^{N} Y_i$$

Superposition theorem:

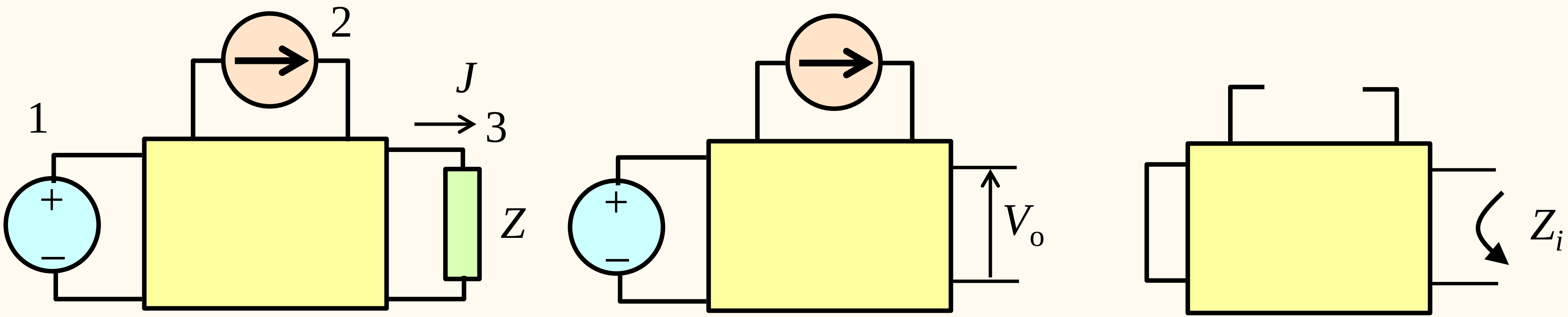


J = J_{P1} + J_{P2}

$$J = \sum_i J_{\mathrm{P}i}$$

$J_{\mathrm{p}i}$: The current caused on the output by i-th power source.

Ho-Thevenin's theorem



Consider a circuit with an open terminal pair (No.3). Obtain current *J* when the open pair is connected with impedance *Z*.

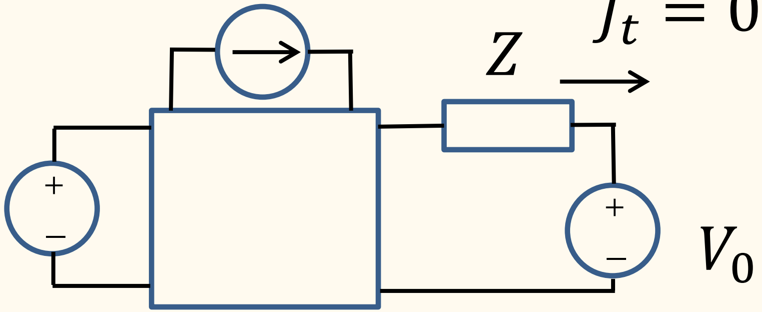
1. Measure the open terminal voltage V_0.
2. Turn off all the power sources (voltage sources: short, current sources: open). Measure the open circuit impedance Z_i.

Then

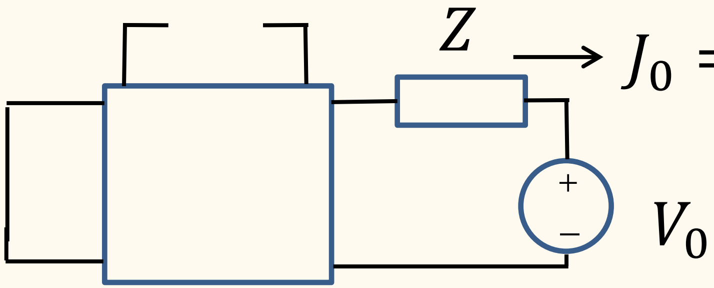
$$J = \frac{V_0}{Z + Z_i}$$

Ho-Thevenin's theorem

Because:



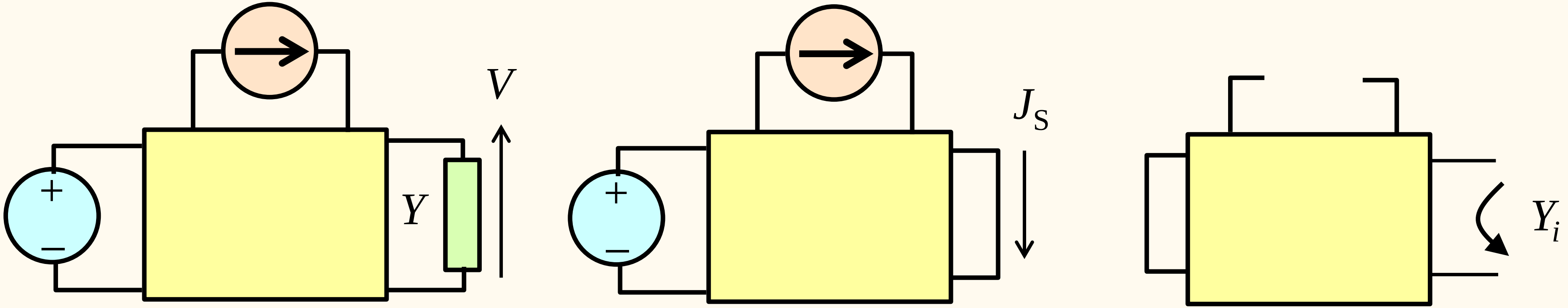
$$J_t = 0$$



$$J_0 = -\frac{V_0}{Z + Z_i}$$

$$J = J_t - J_0 = \frac{V_0}{Z + Z_i}$$

Norton's theorem

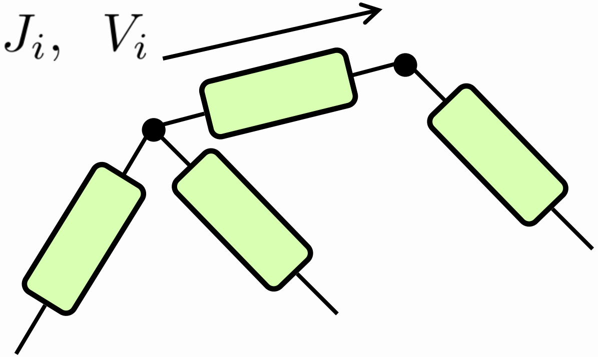


$$V = \frac{J_S}{Y + Y_i}$$

Dual theorem to Ho-Thevenin

Review: Tellegen's theorem

$i = 1, \cdots, m$: index of branches



$$\sum_{i=1}^{m} V_i J_i = 0 \quad \boldsymbol{V} \perp \boldsymbol{J}$$

Power of i-th branch

Comments

1. Power conservation law
2. Holds for any kind of circuit (irrespective of linear, or non-linear)
3. Holds for two independent circuit conditions (as long as D is the same)

D: incidence matrix

Reciprocity theorem

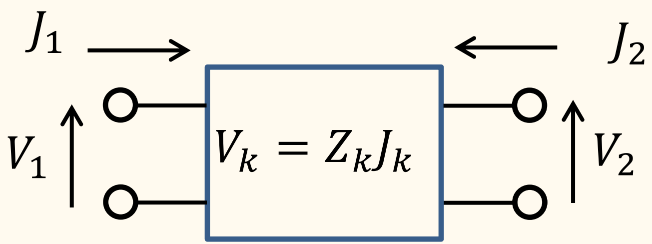
An n-terminal pair linear circuit

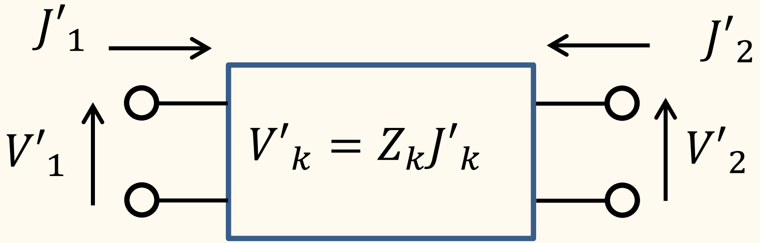
At one state $(V_1, J_1), (V_2, J_2), \cdots, (V_n, J_n)$,

at another state $(V'_1, J'_1), (V'_2, J'_2), \cdots, (V'_n, J'_n)$

$$\sum_{i=1}^{n} V_i J'_i = \sum_{i=1}^{n} V'_i J_i$$

Proof: Consider a two terminal-pair circuit with m branches.





Tellegen's theorem

$$\begin{cases} -V_1 J'_1 - V_2 J'_2 + \sum_k V_k J'_k = 0 \\ -V'_1 J_1 - V'_2 J_2 + \sum_k V'_k J_k = 0 \end{cases}$$

But $V_k J'_k = V'_k J_k = Z_k J_k J'_k$

$$\therefore V_1 J'_1 + V_2 J'_2 = V'_1 J_1 + V'_2 J_2$$

Duality 双対性

直列接続	並列接続
開放	短絡
電場	磁場
キルヒホッフの第 2 法則	キルヒホッフの第 1 法則
電圧	電流
インピーダンス	アドミッタンス
抵抗	コンダクタンス
静電容量	インダクタンス
鳳-テブナンの定理	ノートンの定理

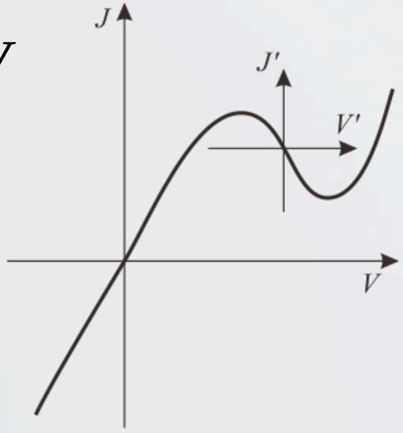
Duality

Series	Parallel
Open	Short
Voltage	Current
Impedance	Admittance
Capacitance	Inductance
Electric field	Magnetic field
Resistance	Conductance
Ho-Thevenin	Norton
Kirchhoff's 2nd law	Kirchhoff's 1st law

2.7 Definition: Passive elements and active elements

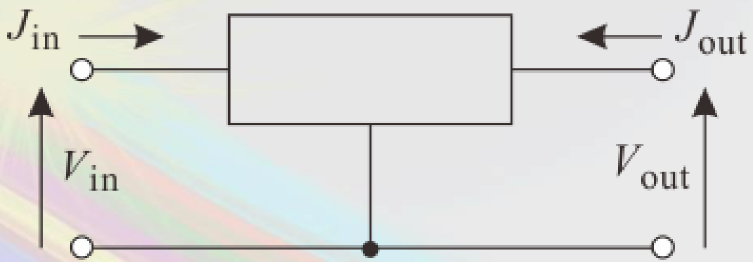
Two terminal: current J, voltage V

$JV \geq 0$: passive element
$JV < 0$: active element



Locally active two-terminal element

More than three-terminal: treat as a terminal pair circuit



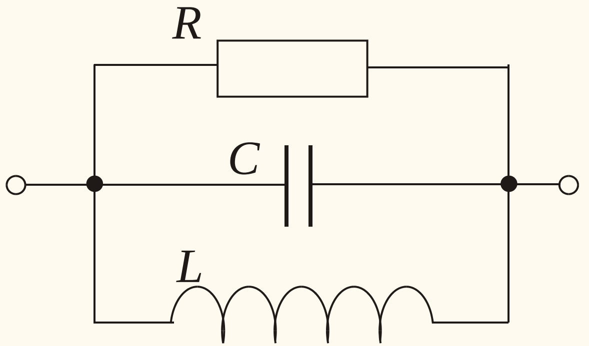
$$P = J_{\text{in}} V_{\text{in}} + J_{\text{out}} V_{\text{out}}$$

$P \geq 0$: passive element
$P < 0$: active element

Ch.3 Transfer function and transient response

3.1 General Properties of Resonance and Resonance Circuits

3.1.1 Resonance Phenomena



Harmonic oscillator: $$\frac{d^2q}{dt^2} = -\omega_0^2 q$$

Kirchhoff's law $$L\frac{dJ_{\mathrm{L}}}{dt} = -L\frac{d^2q_{\mathrm{L}}}{dt^2} = \frac{q}{C} = RJ_{\mathrm{R}} = R\frac{dq_{\mathrm{R}}}{dt}$$
$$dq_{\mathrm{L}} + dq_{\mathrm{R}} + dq = 0$$

$$\frac{d^2q}{dt^2} + \frac{1}{CR}\frac{dq}{dt} + \frac{1}{LC}q = \frac{d^2q}{dt^2} + \frac{1}{\tau}\frac{dq}{dt} + \omega_0^2 q = 0$$

$$q = \exp(\lambda t) \quad \lambda = \frac{1}{2\tau}\left[-1 \pm \sqrt{1 - 4(\omega_0\tau)^2}\right] \approx -\frac{1}{2\tau} \pm i\omega_0 \quad (\omega_0\tau \gg 1)$$

Resonant (angular) frequency $$\omega_0 \equiv \frac{1}{\sqrt{LC}}$$

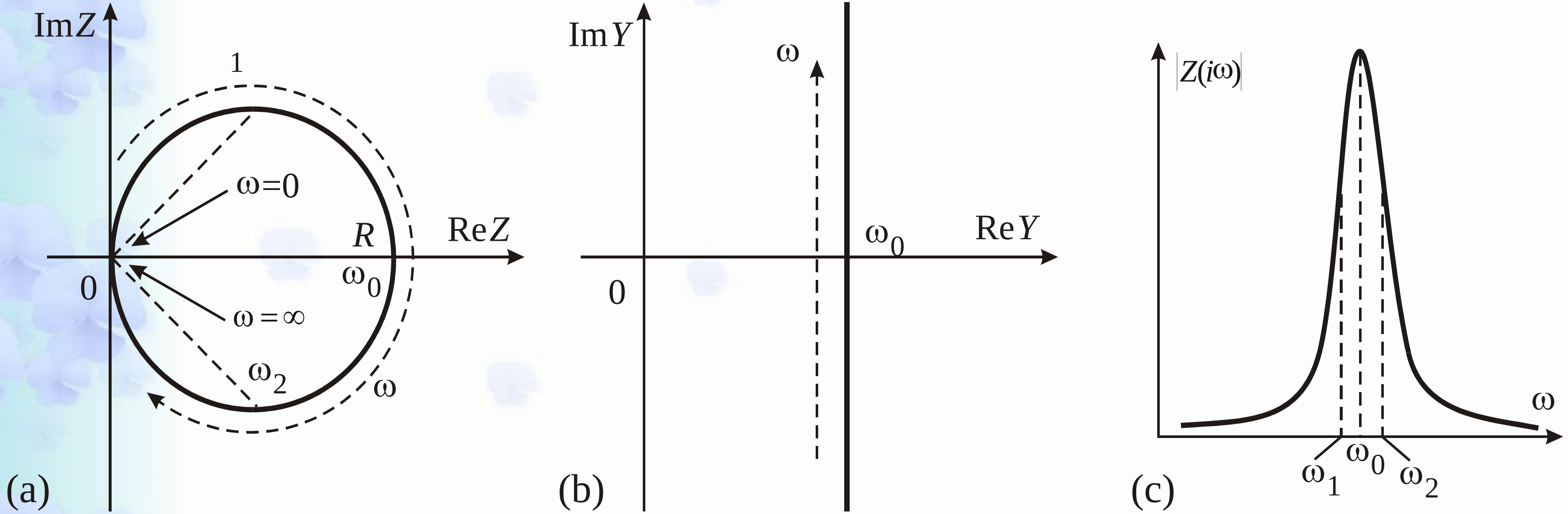
Transfer function, resonance and phase shift

$$Z_{\text{tot}}(i\omega) = \left[\frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right)\right]^{-1}$$

$$\omega_0 \equiv \frac{1}{\sqrt{LC}}$$

Resonance: Reactance =0

Total Phase Shift Change: π

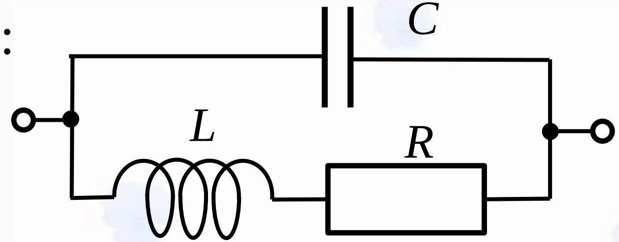


The Bode diagram (plot)

Plot of absolute value and argument of an impedance as a function of frequency.

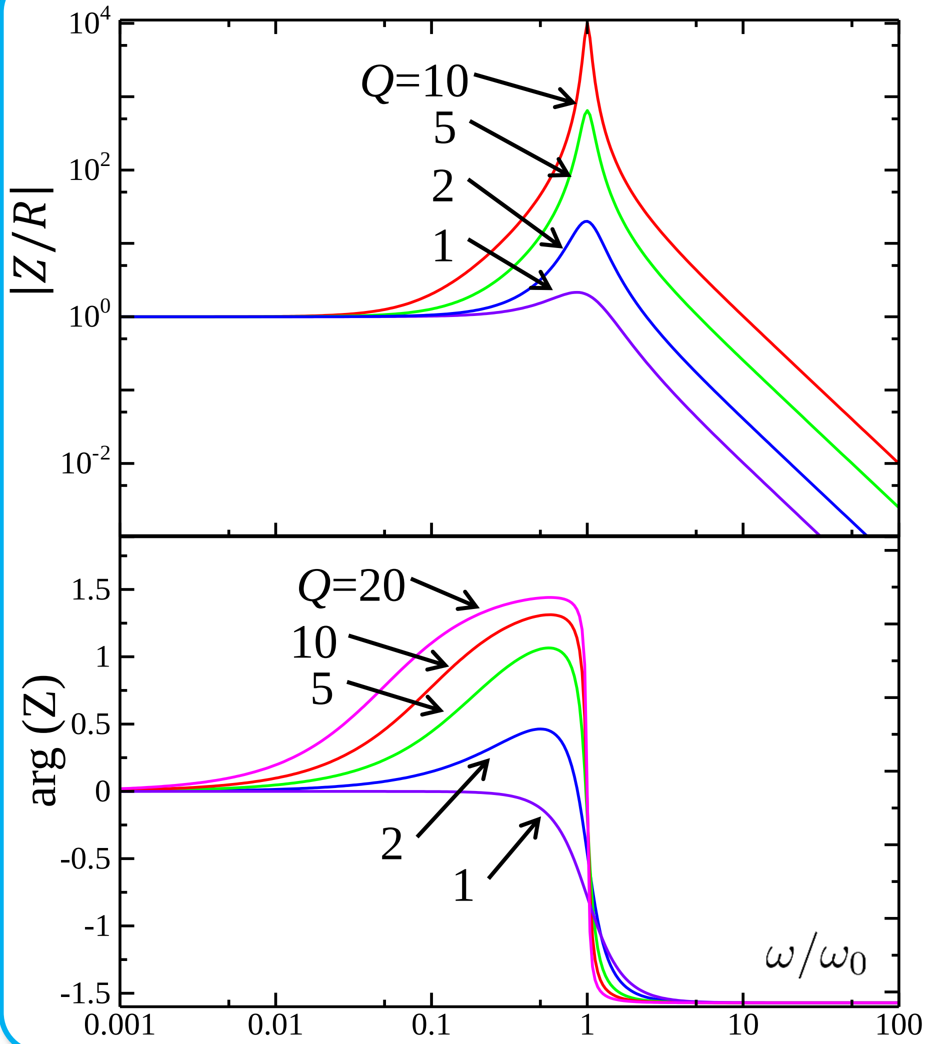
Absolute value: log-log mode
Argument: semi-log mode

Example:

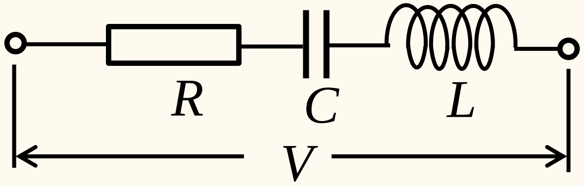


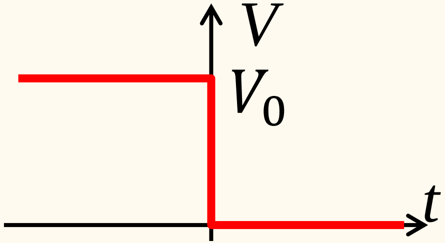
$$Q \approx \omega_0 \frac{L}{R}$$

$$Z(i\omega) = \frac{R + i\omega L}{1 - \omega^2 + i\omega CR}$$
$$= \frac{R + i\omega L}{1 - (\omega/\omega_0)^2 + i\omega CR}$$



Transient response of resonant circuit





$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0 \quad (t > 0),$$

$$q(0) = CV_0$$

$$q(t) = CV_0 e^{st} \;\rightarrow\; Ls^2 + Rs + C^{-1} = 0$$

$$s = \left(-\omega_0 \pm \sqrt{\omega_0^2 - 4\alpha^2}\right)\frac{\omega_0}{2\alpha} \quad \alpha \equiv (CR)^{-1}$$

$$(\omega_0/2) < \alpha \;\rightarrow\; \text{imaginary part}$$

$$q(t) = CV_0 \exp[(-\gamma \pm i\omega_s)t], \quad \gamma \equiv \frac{\omega_0^2}{2\alpha}, \quad \omega_s \equiv \omega_0\left(1 - \frac{\omega_0^2}{4\alpha^2}\right)^{1/2}$$

Damped oscillation with time constant γ^{-1}, frequency ω_s

Transient response of resonance circuit (transfer function)

Synthesized impedance, admittance

$$Z_{\rm tot}(s) = sL + R + \frac{1}{sC}, \quad Y_{\rm tot}(s) = Z_{\rm tot}(s)^{-1}$$

Zero (pole) of $Z_{\rm tot}(s)$ ($Y_{\rm tot}(s)$) $\quad s = (-\omega_0 \pm \sqrt{\omega_0^2 - 4\alpha^2}\,)(\omega_0/2\alpha)$

$Z_{\rm tot}(s_0) = 0 \qquad$ Time constant: $\mathrm{Re}(s_0)$ Frequency: $\mathrm{Im}(s_0)$

Laplace transformation of voltage: $V(s)$

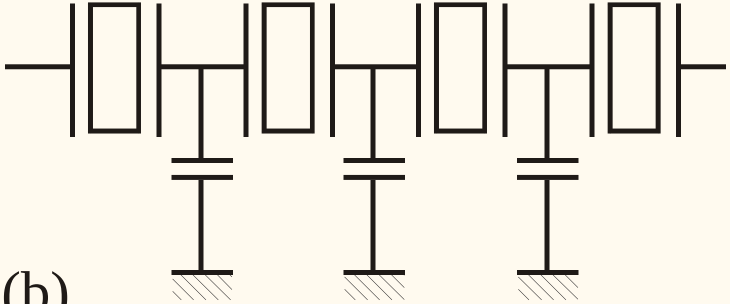
$$J(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} Y(s)V(s)e^{st}ds = \sum_i R(s_i)V(s_i)e^{s_i t} \quad (c > 0)$$

Natural current

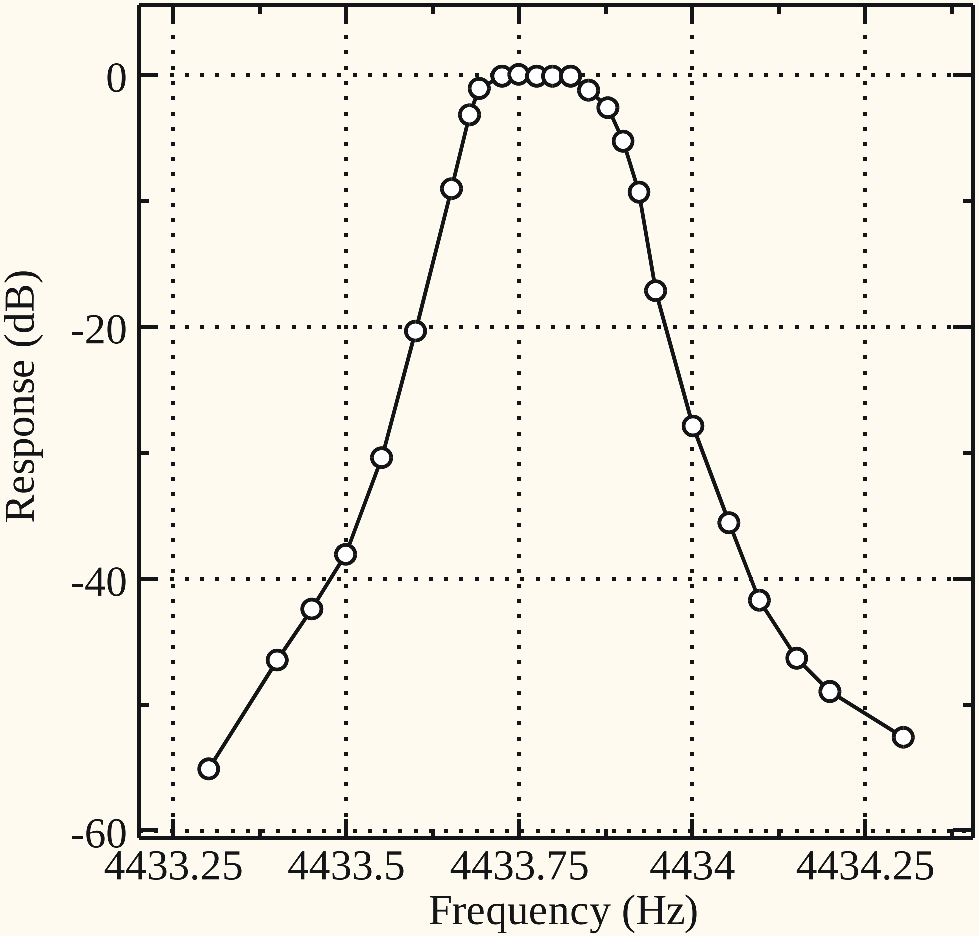
s_i: poles of $Y(s) \quad R(s_i) = Y(s)(s - s_i)|_{s=s_i}$

Quartz crystal filter

(a)



(b)

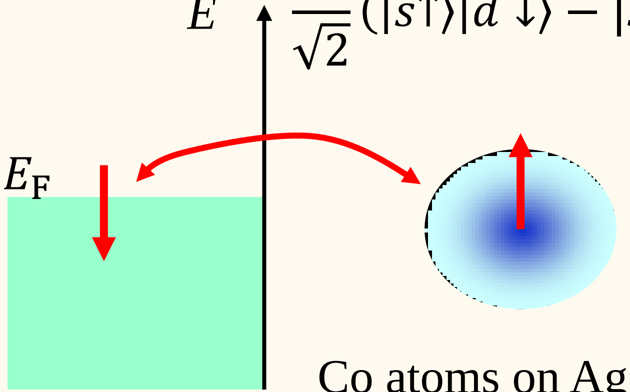


(c)

Kondo Resonance and Phase shift

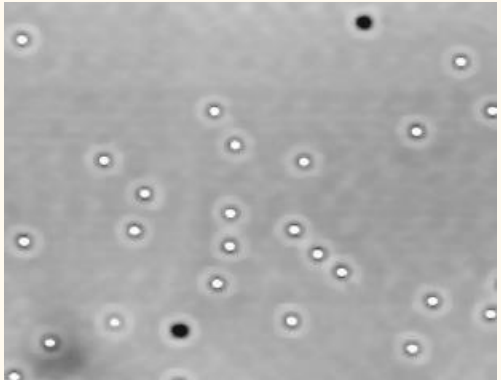

Jun Kondo

$$\frac{1}{\sqrt{2}}(|s\uparrow\rangle|d\downarrow\rangle - |s\downarrow\rangle|d\uparrow\rangle)$$

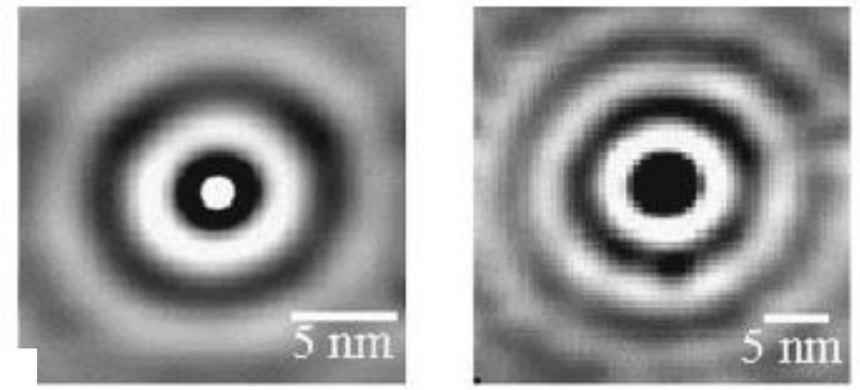


Many body resonance.
But still has the phase shift of $\pi/2$!

Co atoms on Ag (111) surface

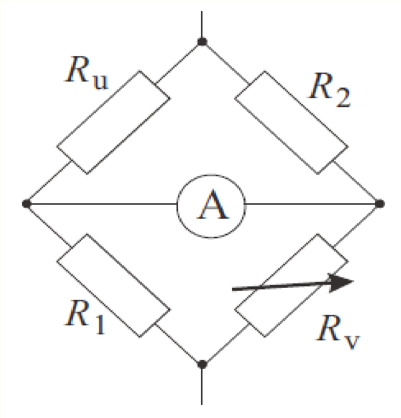


Co (magnetic)   Defect (non-magnetic)



Schneider et al., Phys. Rev. B65, 121406 (2002).

Resistance bridge 抵抗ブリッジ

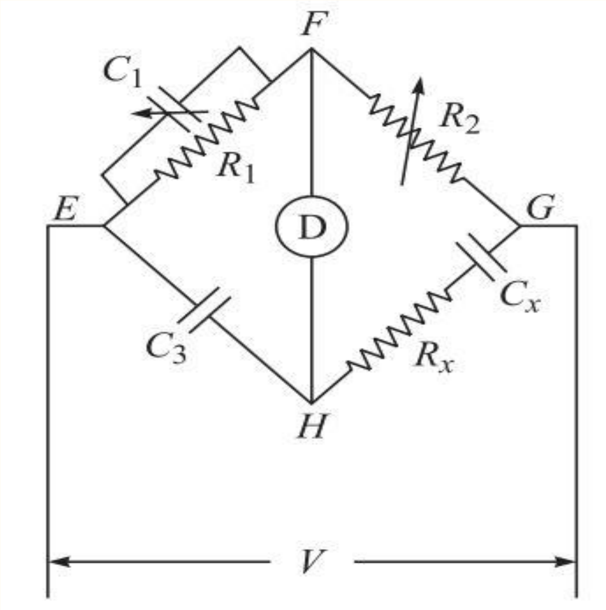


Wheatstone bridge

AVS-47 Resistance bridge

Not a “bridge” circuit!

Schering Bridge



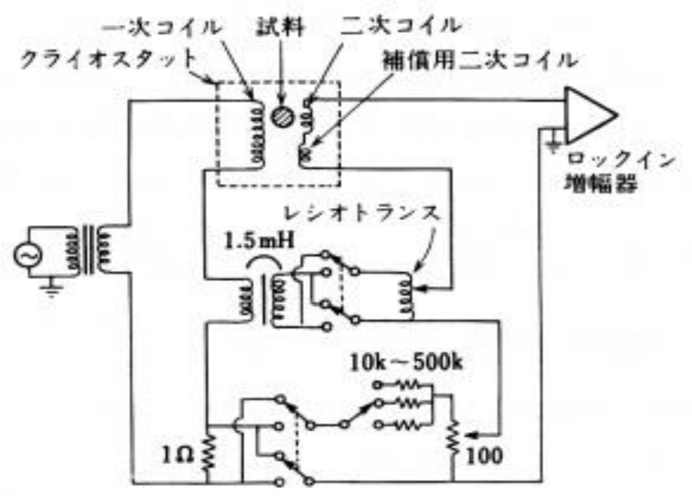
$$Z_1 Z_x = Z_2 Z_3, \quad Z_x = Z_2 Z_3 Y_1$$

$$Z_x = R_x + \frac{1}{i\omega C_x}, \quad Z_2 = R_2, \quad Z_3 = \frac{1}{i\omega C_3}, \quad Y_1 = \frac{1}{R_1} + i\omega C_1$$

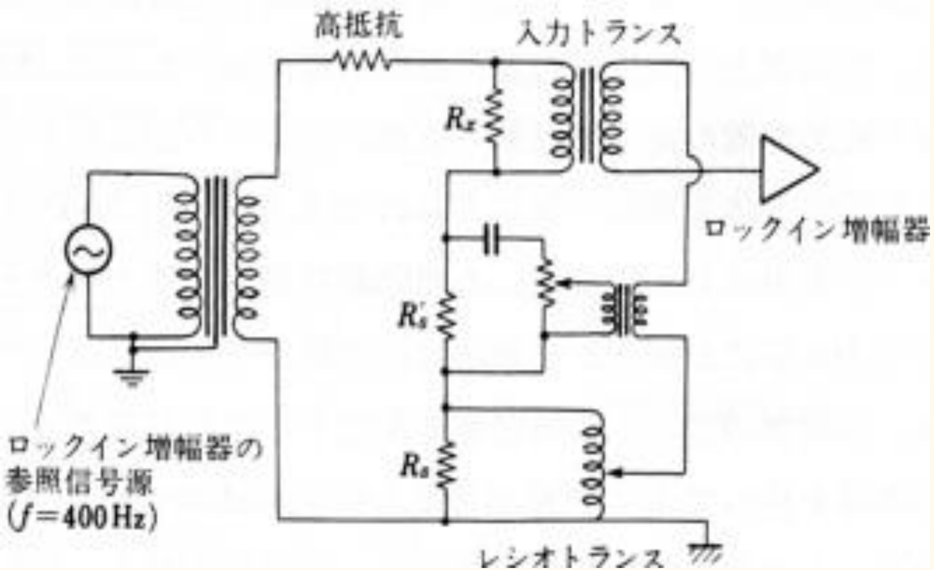
$$R_x + \frac{1}{i\omega C_x} = R_2 \frac{1}{i\omega C_3} \left(\frac{1}{R_1} + i\omega C_1 \right)$$

$$R_x = \frac{R_2 C_1}{C_3}, \quad C_x = \frac{R_1}{R_2} C_3$$

Magnetic moment measurement



Resistance measurement



Capacitance bridge キャパシタンス ブリッジ

General Radio
3-terminal
Capacitance bridge

Agilent E4981A

What is Spice?

SPICE: Simulation Program with Integrated Circuit Emphasis

A language which describes electronic circuits (corresponding to circuit diagrams).

ex) a CR circuit and a dc power source

```
* 0---R1---1---C1---2---V1---0
R1 0 1 10
C1 1 2 20
V1 2 0 5
.END
```

Graphical user interface: Circuit diagram

Linear Technology
web site

Circuit Simulator

Download LTSpice from the web site of Linear Technology

Operation example

Summary

Theorems for paired terminal circuits

Superposition, Ho-Tevenin, Reciprocity

Duality

Passive devices (elements) and active devices

Transfer function and transient response

Transfer function of single-pair terminal circuits

Resonance circuit

Bode plot

General properties