電子回路論第4回

Electric Circuits for Physicists #4

東京大学理学部・理学系研究科
物性研究所
勝本信吾
Shingo Katsumoto

Review

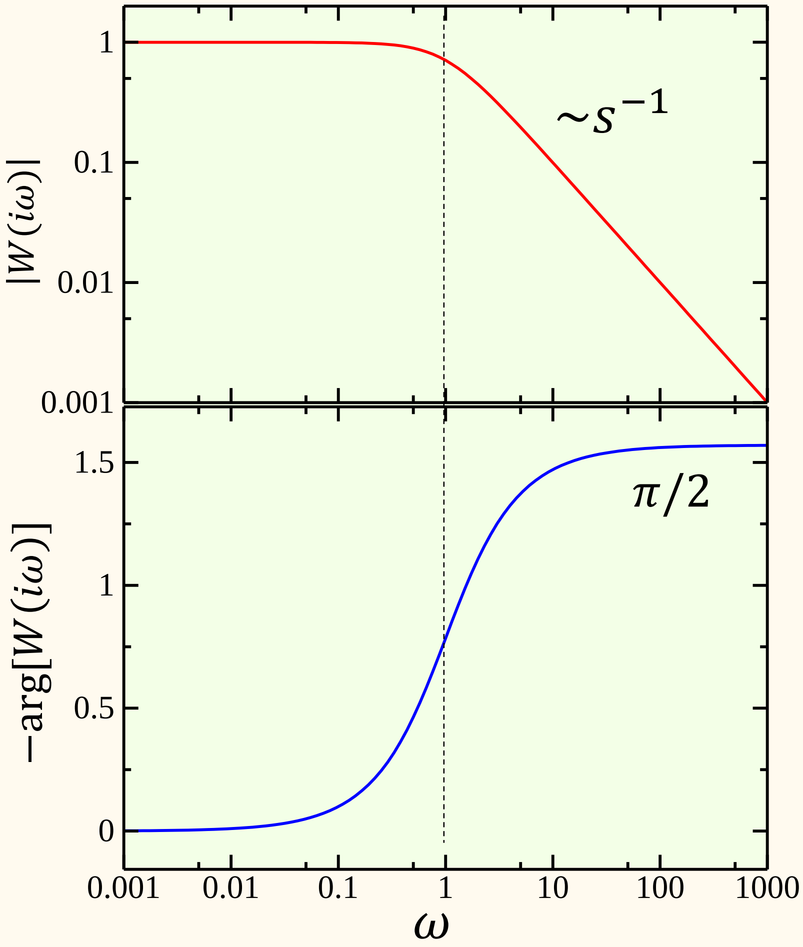
Last week we introduced

Resonance: Represented as a zero-point in impedance. A pole in admittance.

$$Z(s_{\mathrm{res}}) \sim Z_0(s - s_{\mathrm{res}}), \quad Y(s) \sim \frac{A}{s - s_{\mathrm{res}}} \quad (s \sim s_{\mathrm{res}})$$

Bode diagram: Plots of the absolute value and the argument of a transfer function as a function of frequency.

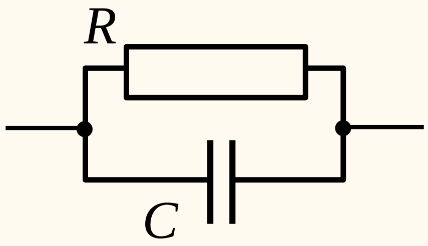
A Pole on the Real Axis



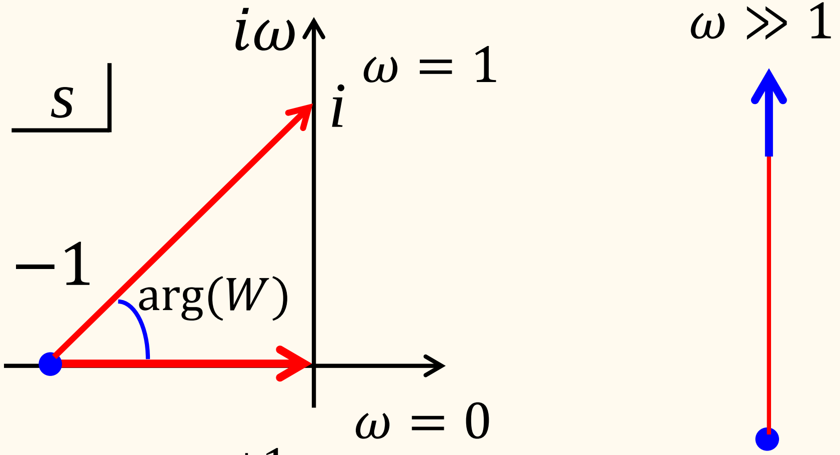
Lear system with transfer function

$$W(s) = \frac{1}{s+1}$$

Circuit example: Z of

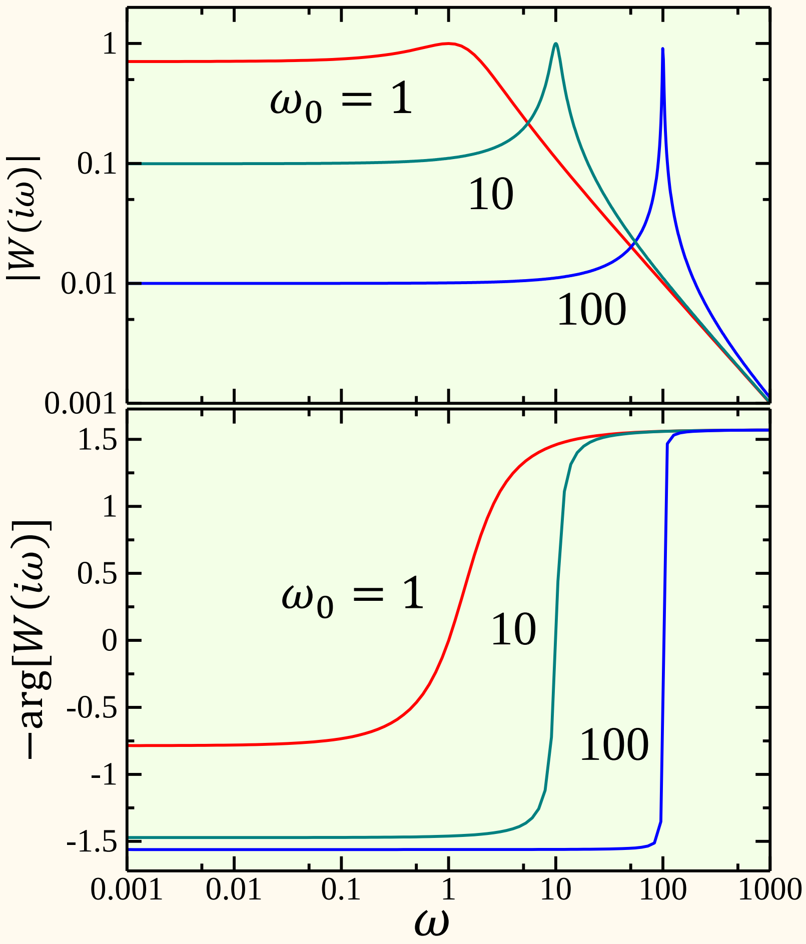


$s = \sigma + i\omega$ (dimensionless)



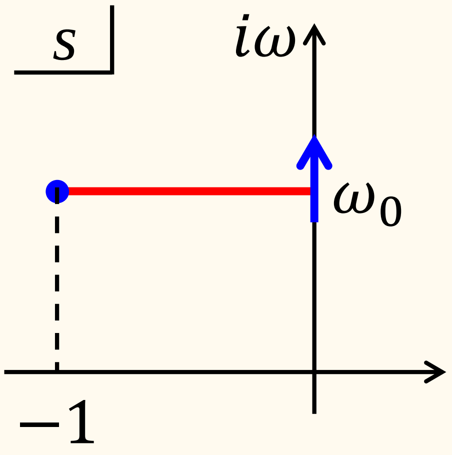
arrows: $s + 1$

A Pole with Finite Imaginary Part



$$W(s) = \frac{1}{s + 1 - i\omega_0} \qquad (*1)$$

Corresponds to resonance.
(This approximately holds for $\omega \sim \omega_0$)



(*1 Complex poles in transfer function always appear as conjugate pairs.)

Zeros and Poles of Transfer Function

$$W(s) = B\frac{(s-\beta_1)\cdots(s-\beta_m)}{(s-\alpha_1)\cdots(x-\alpha_n)} \qquad \{\alpha_j\}: \text{ Poles}, \quad \{\beta_j\}: \text{ Zeros}$$

(generally represented in rational formula)

$$\ln|W(i\omega)| = \ln B + \sum_{j=1}^{m} \ln|i\omega - \beta_j| - \sum_{j=1}^{n} \ln|i\omega - \alpha_j|,$$

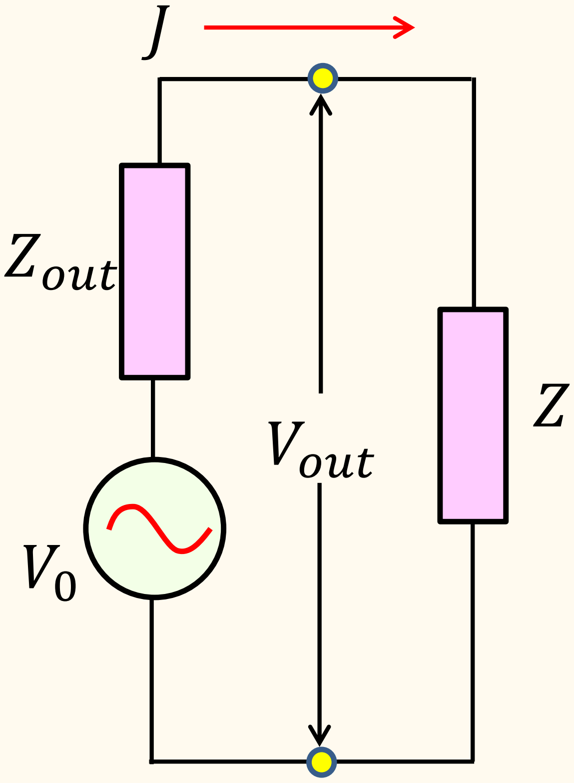
Bode plot

$$\arg[W(i\omega)] = \arg(B) + \sum_{j=1}^{m} \arg(i\omega - \beta_j) - \sum_{j=1}^{n} \arg(i\omega - \alpha_j)$$

Bode plots are sum of those for single poles and zeros.

3.2 Terminal pair circuits as signal transfer interfaces

3.2.1 Impedance matching



Efficient transfer of energy from electro-motiveforce

$$V_{\rm out}(i\omega) = V_0(i\omega) - Z_{\rm out}(i\omega)J(i\omega)$$

$$P = {\rm Re}(V_{\rm out}^* J) = {\rm Re}\left(\frac{Z^* V_0^*}{Z^* + Z_{\rm out}^*}\frac{V_0}{Z + Z_{\rm out}}\right)$$

$$= \frac{|V_0|^2}{|Z + Z_{\rm out}|^2}{\rm Re}(Z)$$

Hence the maximum power transfer $P_{\rm max} = \dfrac{|V_0|^2}{4{\rm Re}(Z_{\rm out})^2}$

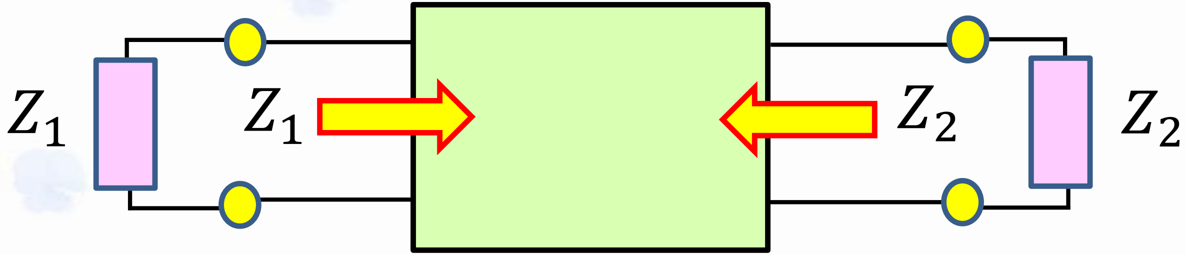
can be obtained under the impedance matching condition:

Impedance matching condition $Z = Z_{\rm out}^*$

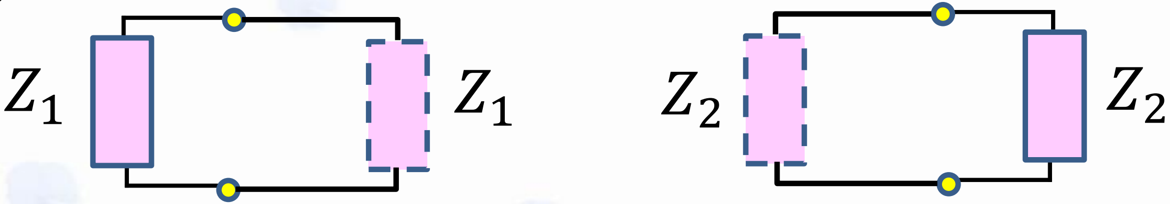
When Z is real: $Z = Z_{\rm out}$

3.2.3 Image parameters (matching with 4-terminal circuits)

If the 4-terminal parameters can be tuned to give the situation as

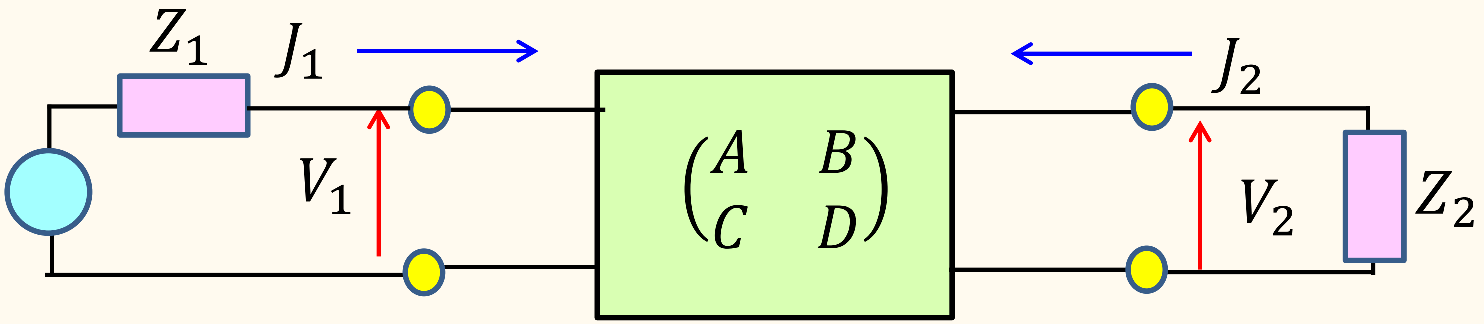


That is



Z_1 and Z_2 are called image impedances.

Conditions for image parameters



From definition:

$$\begin{cases} V_1 = AV_2 - BJ_2, \\ J_1 = CV_2 - DJ_2 \end{cases}$$

$$V_2 = -J_2 Z_2$$

Image (mirror) condition

$$\begin{cases} Z_1 = \dfrac{V_1}{J_1} = \dfrac{AZ_2 + B}{CZ_2 + D} \\ Z_2 = \dfrac{DZ_1 + B}{CZ_1 + A} \end{cases}$$

When $ABCD \neq 0$,

$$Z_1 = \sqrt{\frac{AB}{CD}}, \quad Z_2 = \sqrt{\frac{DB}{CA}}$$

Image parameters

$$\frac{V_1}{V_2} = \sqrt{\frac{A}{D}}(\sqrt{AD} + \sqrt{BC}), \quad \frac{J_1}{-J_2} = \sqrt{\frac{D}{A}}(\sqrt{AD} + \sqrt{BC})$$

The 4-terminal interface costs some power loss:

$$e^\theta \equiv \sqrt{\frac{V_1 J_1}{-V_2 J_2}} = \sqrt{\frac{Z_1}{Z_2}} \frac{J_1}{-J_2} = \sqrt{\frac{Z_2}{Z_1}} \frac{V_1}{V_2} = \sqrt{AD} + \sqrt{BC}$$

$\boldsymbol{\theta}$: Image propagation constant $\quad \theta = \alpha + i\beta \ \ (\alpha, \beta \in \mathbb{R})$

$$\alpha = \frac{1}{2} \ln \left| \frac{V_1 J_1}{V_2 J_2} \right| \quad \text{(image attenuation constant),}$$

$$\beta = \frac{1}{2} \arg \left[\frac{V_1 J_1}{-V_2 J_2} \right] \quad \text{(image phase shift)}$$

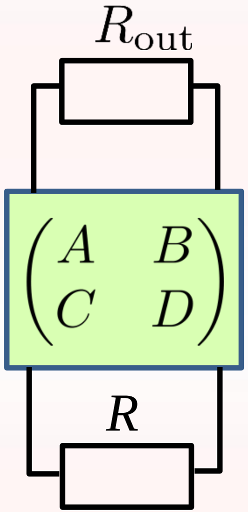
Image parameters

$$A = \sqrt{\frac{Z_1}{Z_2}} \cosh\theta, \quad B = \sqrt{Z_1 Z_2} \sinh\theta,$$

$$C = \frac{1}{\sqrt{Z_1 Z_2}} \sinh\theta, \quad D = \sqrt{\frac{Z_2}{Z_1}} \cosh\theta$$

Z_1, Z_2, θ: Image parameters

3.2.4 Impedance matching with two terminal-pair circuits



$$ABCD \neq 0 \qquad R_{\text{out}} = \sqrt{\frac{AB}{CD}}, \quad R = \sqrt{\frac{BD}{AC}}$$

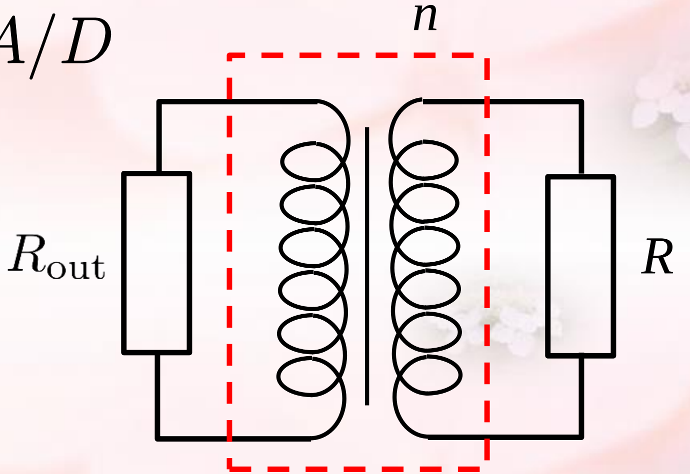
$$A = D = 0 \qquad R_{\text{out}} = \frac{AR + B}{CR + D}, \quad R = \frac{DR_{\text{out}} + B}{CR_{\text{out}} + A}$$

$$\rightarrow RR_{\text{out}} = B/C$$

$$B = C = 0 \qquad R_{\text{out}}/R = A/D$$

Matching transformer

$$n = \sqrt{\frac{R}{R_{\text{out}}}}$$



3.2.5 Fidelity and distortion in wave transformation

When a linear response $w(t) = \mathscr{L}\{u(t)\}$ is just a time delay with τ_0, that is

$$w(t) = A_0 u(t - \tau_0) \qquad \therefore W(i\omega) = A_0 e^{-i\omega\tau_0} U(i\omega),$$

$$\Xi(i\omega) = A_0 e^{-i\omega\tau_0}$$

No distortion conditions are: $(1)\ |\Xi(i\omega)| = A_0,\ (2)\ \arg[\Xi(i\omega)] = -\omega\tau_0$

In other words the conditions can be described as

(1) A_0 does not depend on frequency: No filter effect

(2) $\phi(\omega) \equiv \arg[\Xi(i\omega)], \quad \tau(\omega) \equiv -\dfrac{d\phi(\omega)}{d\omega}$:group delay $\tau(\omega) = \tau_0$

No dispersion in group delay

Effect of distortion in amplitude

(1) Sinusoidal amplitude distortion (amplitude modulation)

$$A(\omega) = a_1 \cos(\tau_1 \omega) + a_0, \ \phi(\omega) = -\tau_0 \omega$$

$$\begin{aligned} w(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\omega) U(i\omega) e^{i(\omega t + \phi(\omega))} d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega U(i\omega) \{a_1 \cos(\tau_1 \omega) + a_0\} e^{i\omega(t-\tau_0)} \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega U(i\omega) \left[a_0 + \frac{a_1}{2}(e^{i\tau_1\omega} + e^{-i\tau_1\omega})\right] e^{i\omega(t-\tau_0)} \\ &= a_0 u(t-\tau_0) + \frac{a_1}{2}[u(t-\tau_0 \underline{+\tau_1}) + u(t-\tau_0 \underline{-\tau_1})] \end{aligned}$$

Paired echo

Effect of distortion in group delay

Sinusoidal group delay distortion

$$A(\omega) = A_0, \; \phi(\omega) = -\tau_0\omega + b_1 \sin(\tau_1\omega)$$

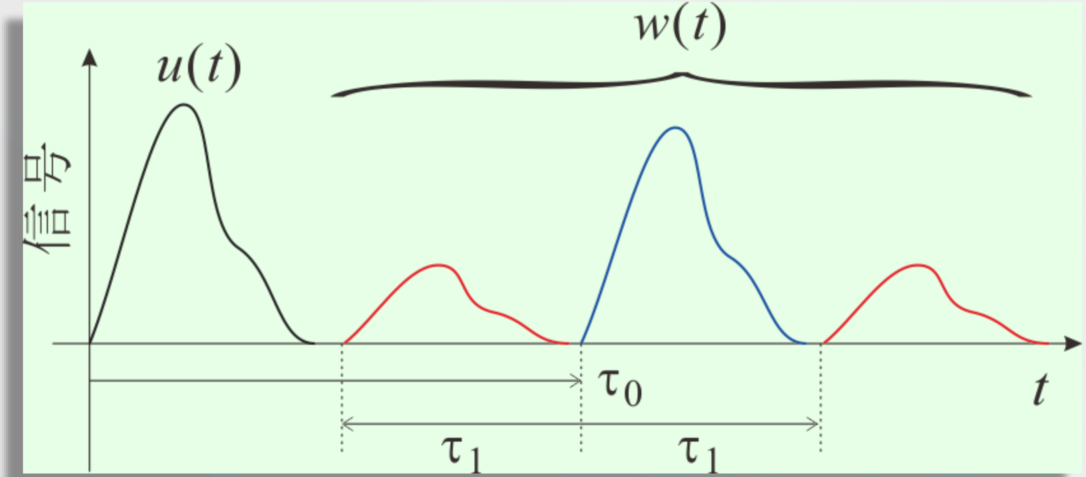
$$\exp[ib_1 \sin(\tau_1\omega)] \approx 1 + \frac{ib_1}{2i}(e^{i\tau_1\omega} - e^{-i\tau_1\omega})$$

$$w(t) = A_0[u(t - \tau_0) + \frac{b_1}{2}\{u(t - \tau_0 \underline{+ \tau_1}) \underline{-} u(t - \tau_0 \underline{- \tau_1})\}]$$

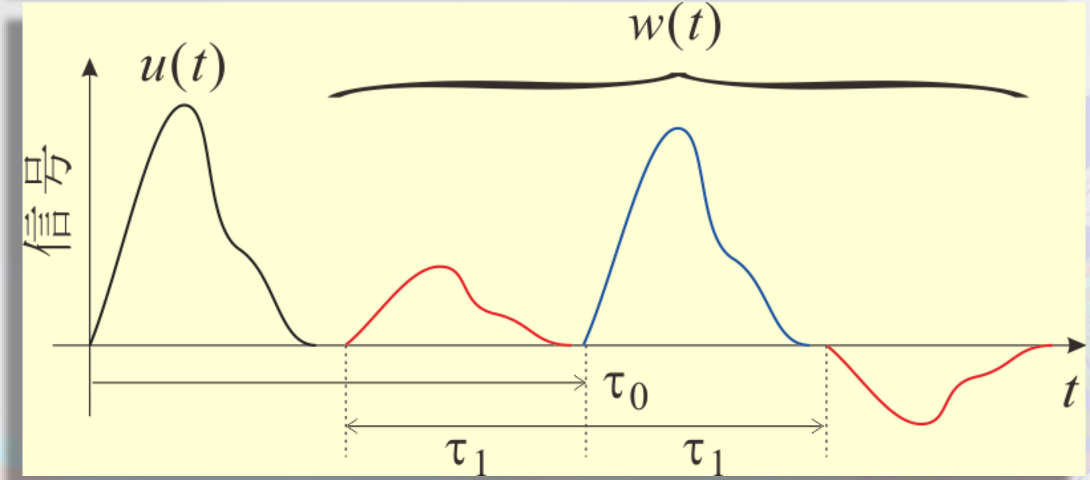
Paired echo with inverted sign

Distortion (paired echo)

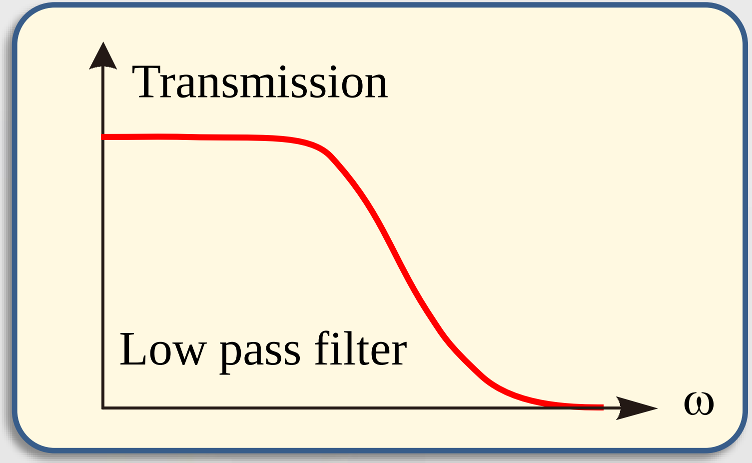
Cosine Amplitude Distortion

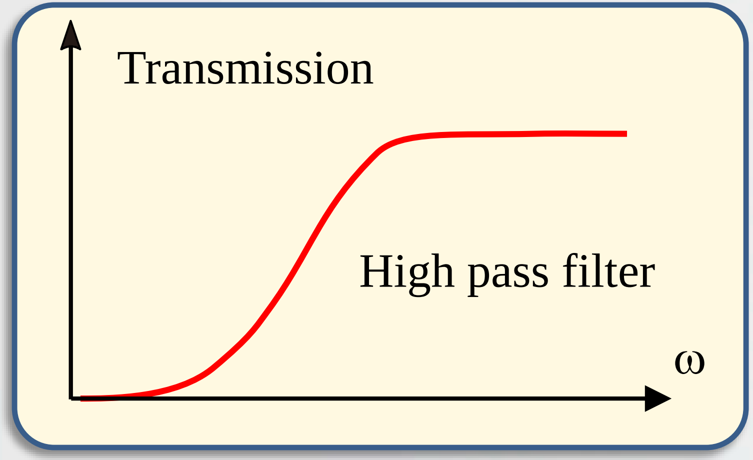


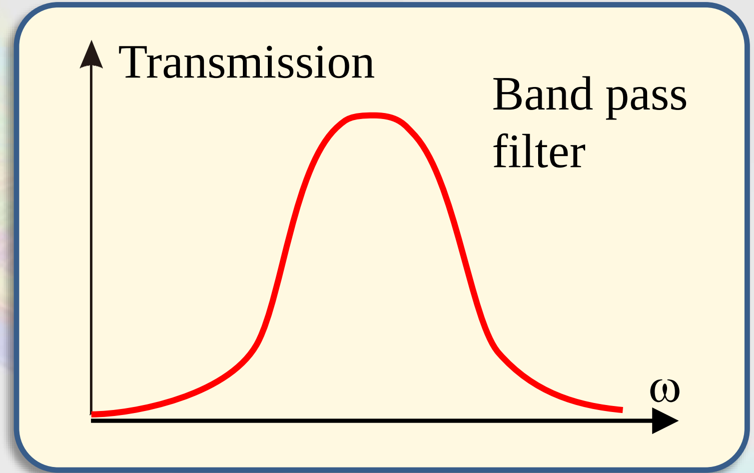
Sine Delay Distortion

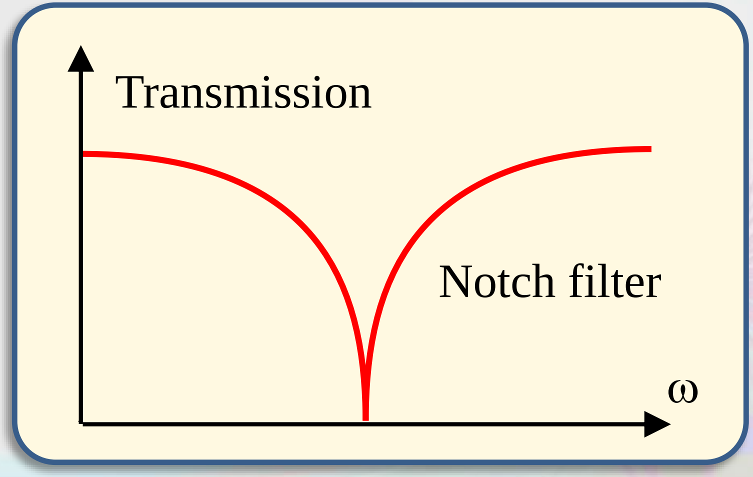


3.2.6 Filter Circuit







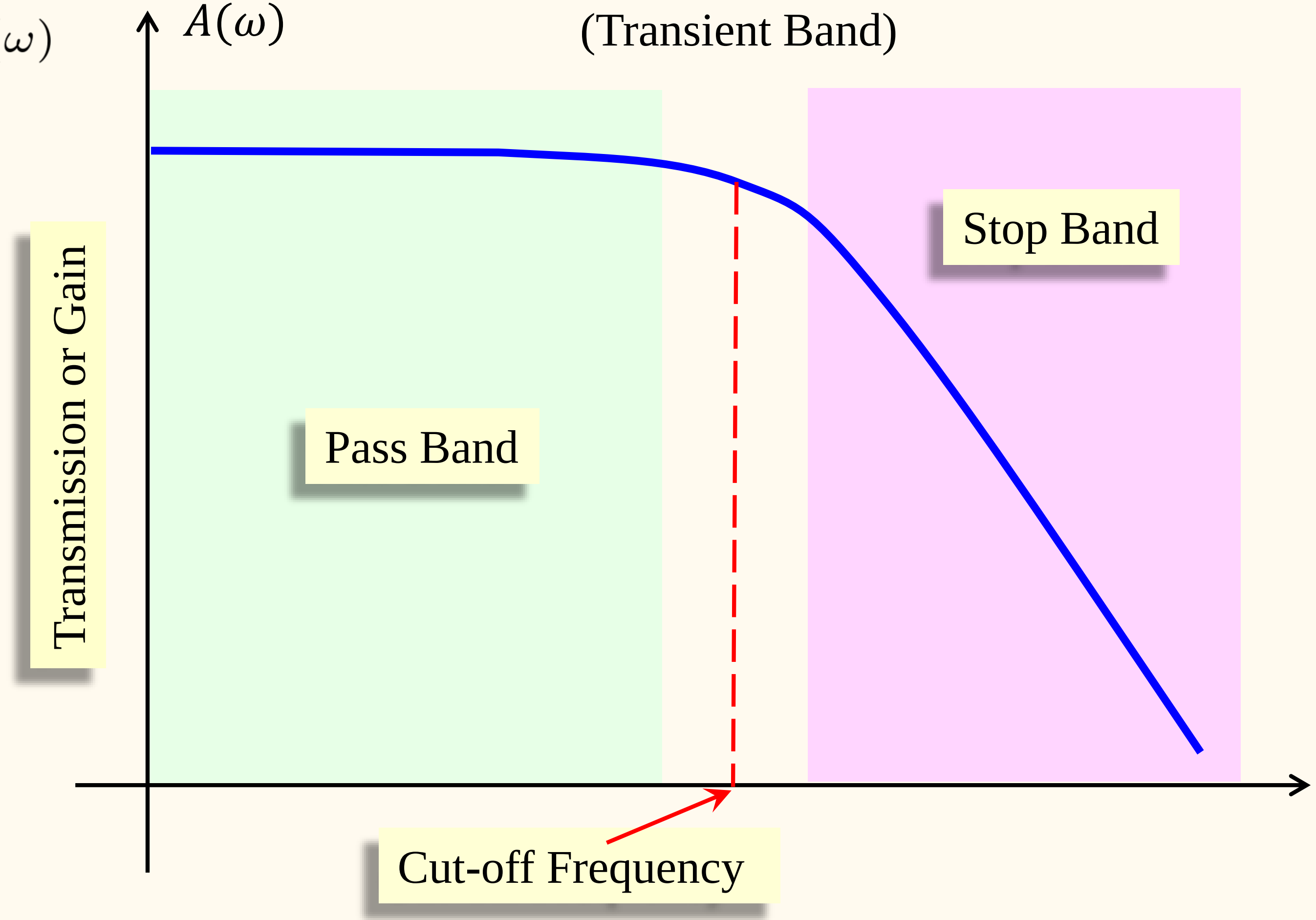


Terms for Filters

$$\Xi(i\omega) = \underline{A(\omega)}e^{i\phi(\omega)}$$

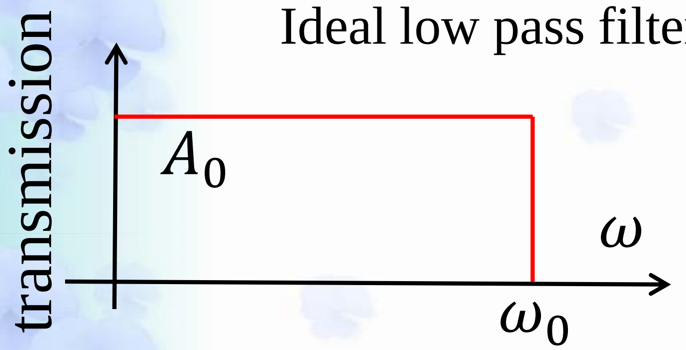
Transmission

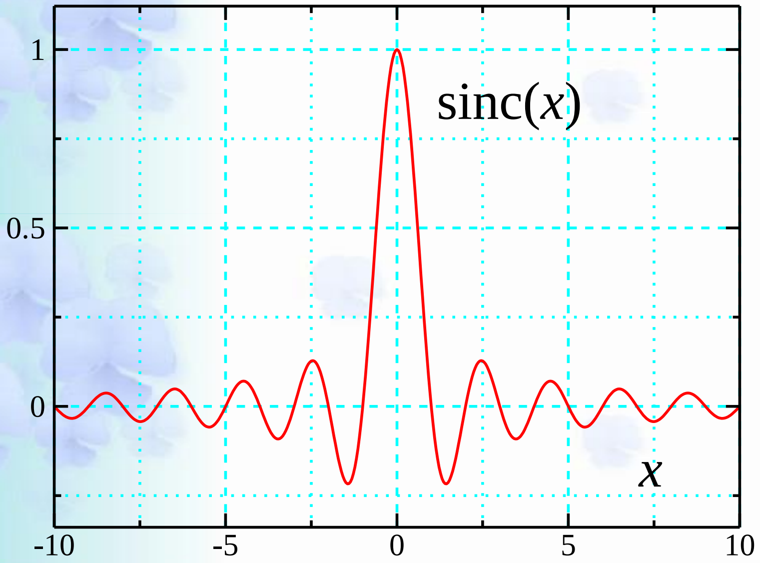
$A(\omega)$ is sometimes called as “gain”



Can an ideal filter exist?

Ideal low pass filter





$$\Xi(i\omega) = H(\omega_0 - |\omega|)$$

Transfer function is written as Heaviside function

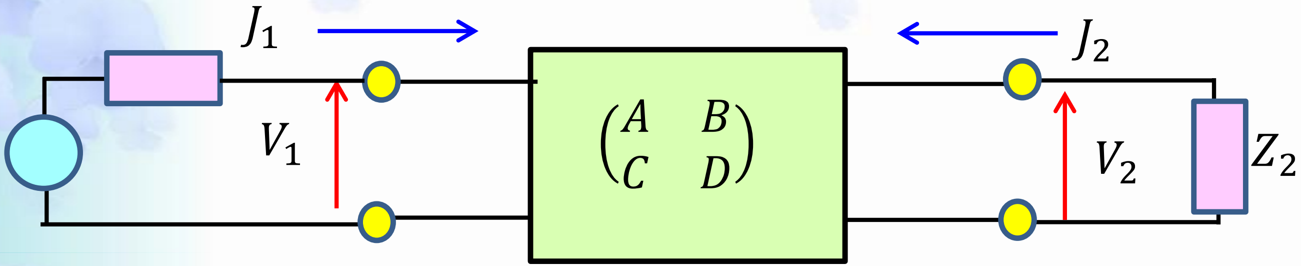
$w(t)$: output to δ-function input at $t = 0$

$$\frac{w(t)}{A_0} = \int_{-\omega_0}^{\omega_0} e^{i\omega t} \frac{d\omega}{2\pi}$$
$$= \int_{-\omega_0}^{\omega_0} \frac{d\omega}{2\pi} \cos \omega t = 2 f_0 \frac{\sin \omega_0 t}{\omega_0 t}$$
$$= 2 f_0 \mathrm{sinc}(2 f_0 t)$$

The output begins $t < 0$ hence breaks the causality.

No ideal low pass filter can exist.

Transmission



Voltage transmission coefficient:

$$T(i\omega) \equiv \frac{V_2(i\omega)}{V_1(i\omega)}$$

$$\log T = \log|T| + i\arg T = -\alpha - i\beta$$

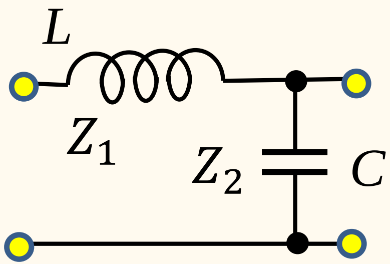
attenuation phase shift

Square root power transmission coefficient

$$S_{\mathrm{B}} \equiv \sqrt{\frac{P_0}{P_2}} = \frac{R_2 A + B + C R_1 R_2 + D R_1}{2\sqrt{R_1 R_2}}$$

Sometimes called as "gain"

An example: Constant K type filter



definition

$$Z_1 Z_2 = R^2 = K$$

:constant

$$Z_1 Z_2 = \frac{L}{C} = R^2 \quad \therefore \sqrt{\frac{L}{C}} = R$$

Image parameters are

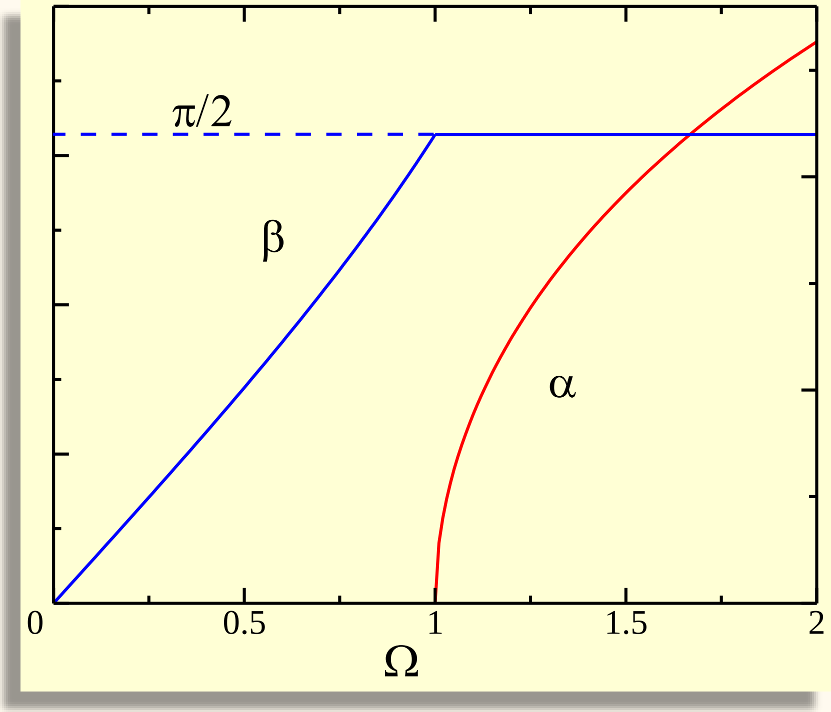
$$Z_{01} = R\sqrt{1-\omega^2 LC}, \quad Z_{02} = \frac{R}{\sqrt{1-\omega^2 LC}},$$

$$\coth\theta = \sqrt{1-\frac{1}{\omega^2 LC}} \equiv \sqrt{1-\frac{1}{\Omega^2}}$$

$$\theta = \alpha + i\beta$$

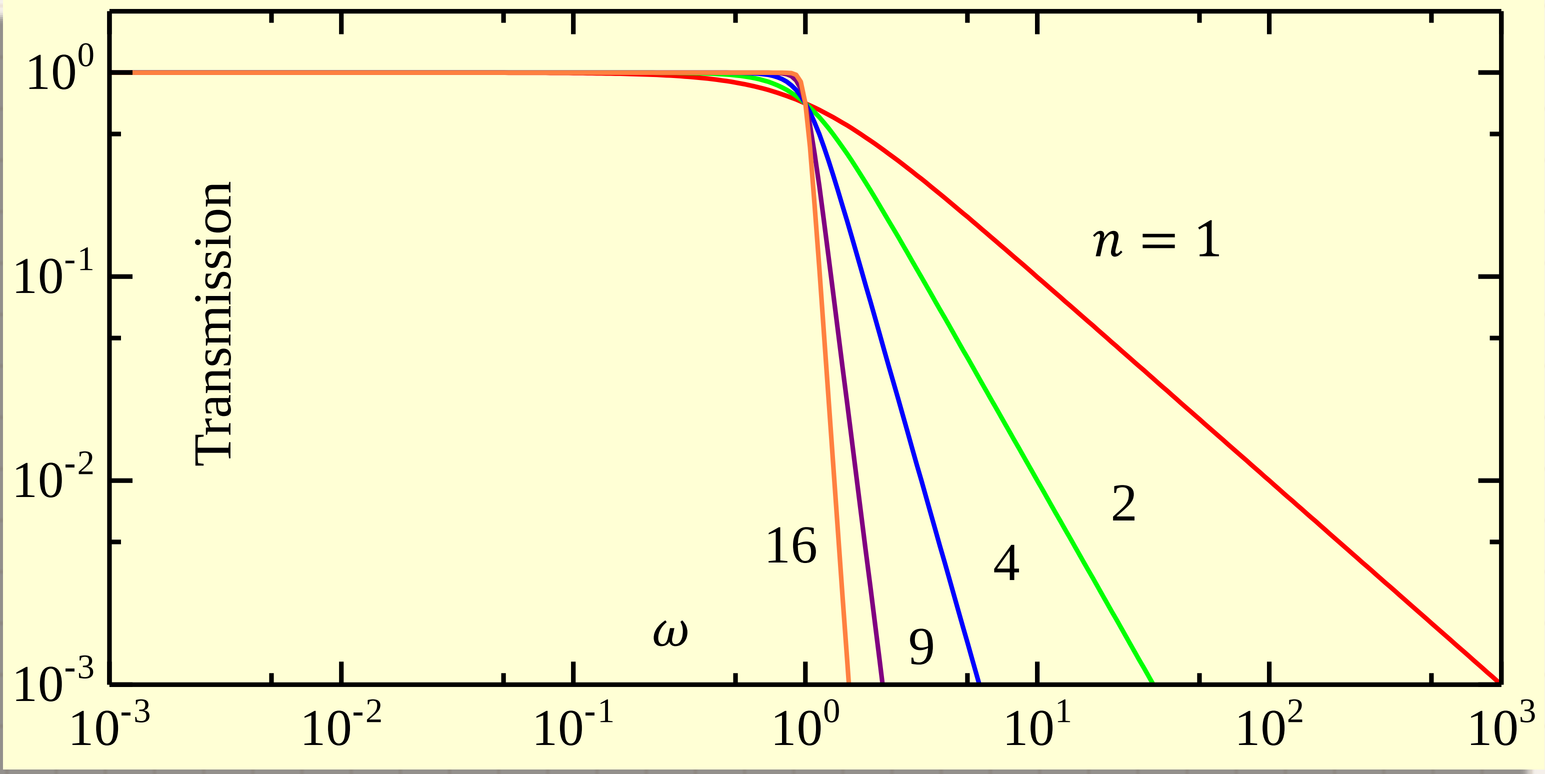
$$\cosh(\alpha+i\beta) = \cosh\alpha\cos\beta + i\sinh\alpha\sin\beta$$

$$= \sqrt{1-\Omega^2}$$



Butterworth Filter

$$G^2(i\omega/\omega_0) = |H(i\omega)|^2 = \frac{1}{1 + (\omega/\omega_0)^{2n}}$$



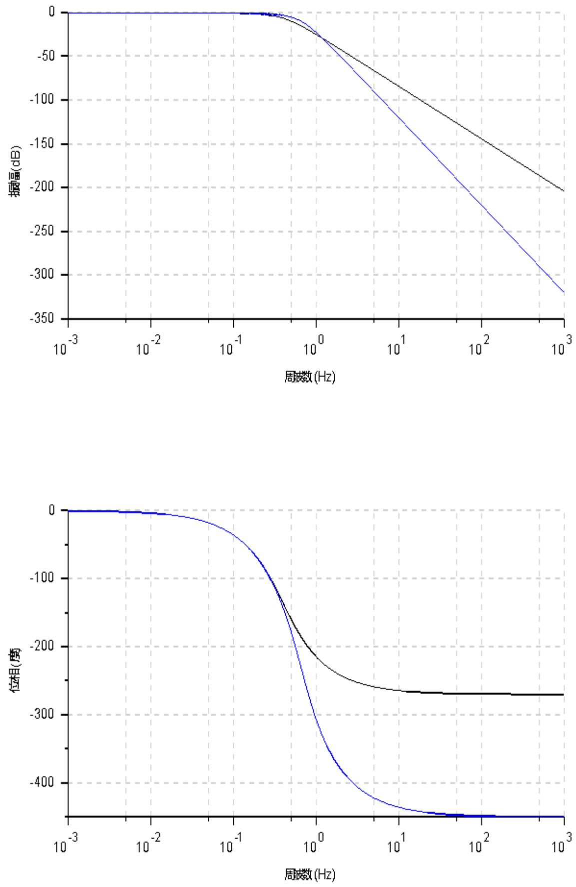
Bessel Filter

Inverse Bessel Polynomial

$$B_0 = 1, \quad B_1(s) = s + 1$$

$$B_n(s) = (2n-1)B_{n-1}(s) + B_{n-2}(s)s^2$$

$$\Xi(s) = \frac{B_n(0)}{B_n(s)}$$

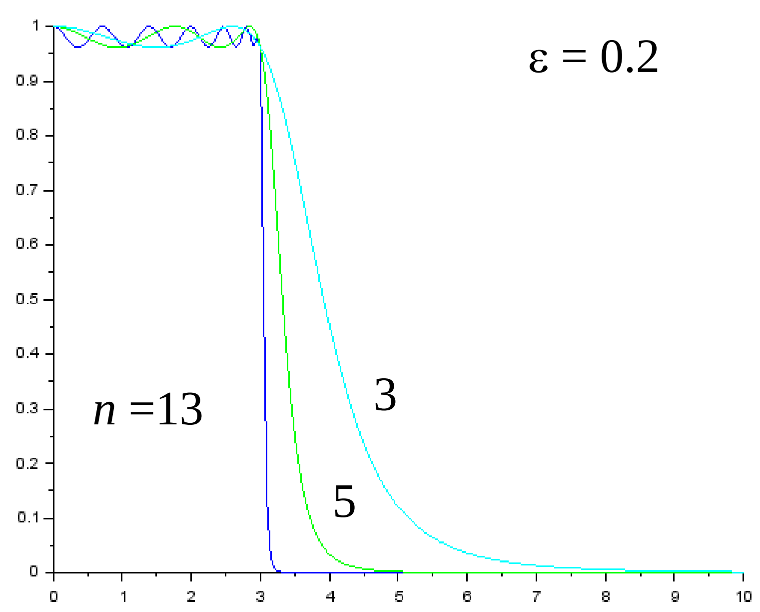


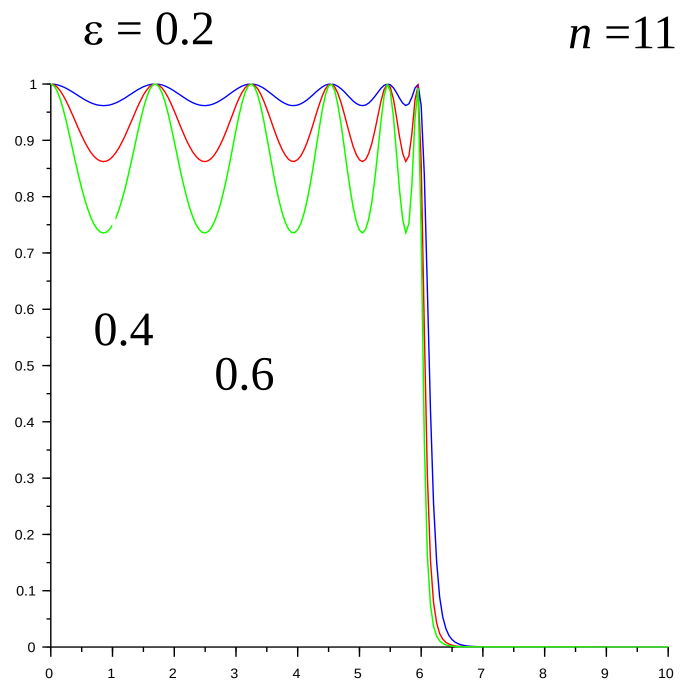
Chebyshev Filter

$$G_n(i\Omega) = |H_n(i\Omega)| = \frac{1}{\sqrt{1+\epsilon^2 T_n^2(\Omega)}}$$

ε: Ripple coefficeint

T_n: n-th order Chebyshev polynomial





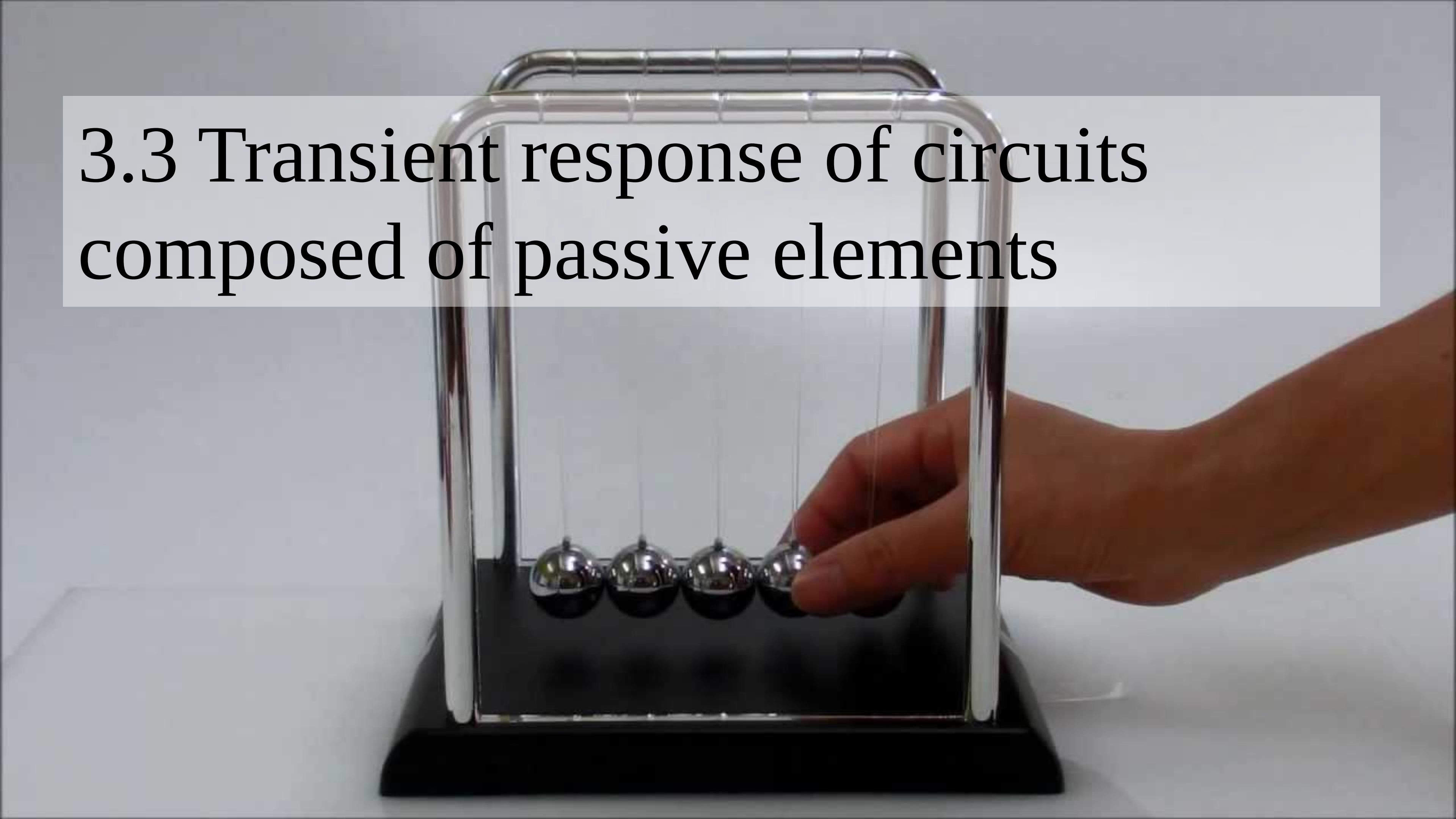
Packaged Filters

Web selection page

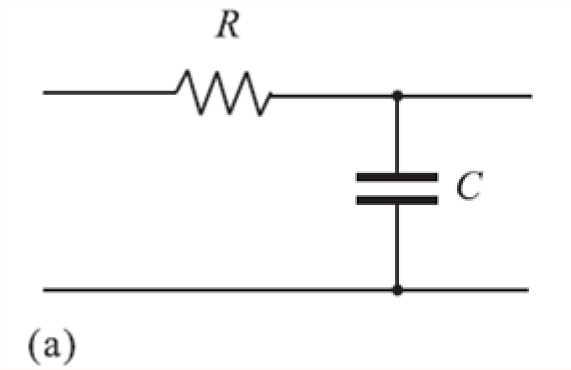
https://ww3.minicircuits.com/WebStore/RF-Filters.html

Mini-Circuits
Band Pass
19.2 – 23.6MHz 50Ohm

3.3 Transient response of circuits composed of passive elements

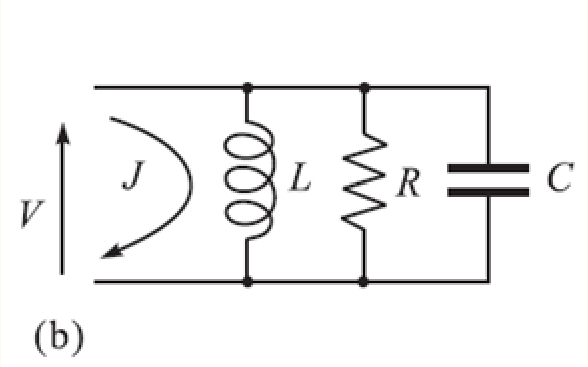


3.3.1 Classification with the number of storages



(a) Single energy storage

$$\Xi(s) = \frac{1}{1 + s/s_0}$$



(b) Double energy storage

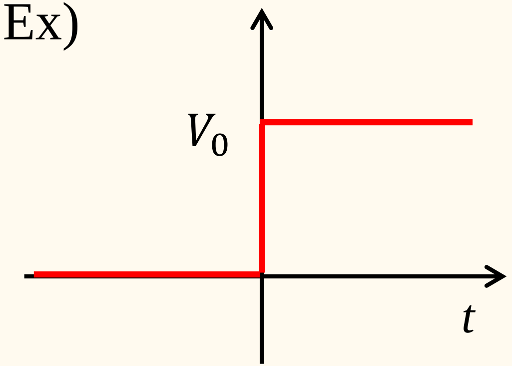
$$\Xi(s) = \frac{1}{b + s + as^{-1}}$$

Number of storages: order of polynomial in the denominator of rational expression for transfer function

Ex) Transient response to step function input

Responses are obtained as $w(t) = \mathscr{F}^{-1}\{\Xi(i\omega)U(i\omega)\} = \int_{-\infty}^{\infty} \Xi(i\omega)U(i\omega)e^{i\omega t}\frac{d\omega}{2\pi}$

Ex)

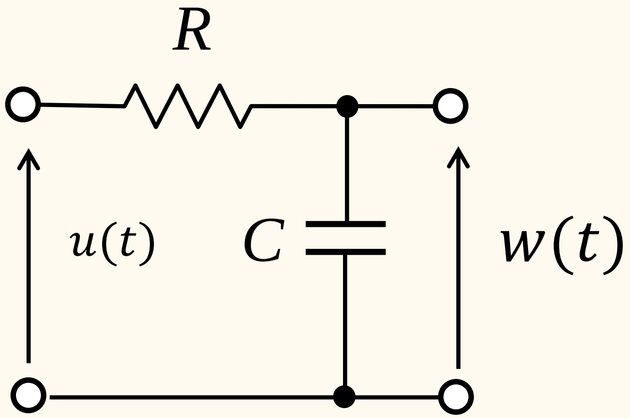


Heaviside function

$$H(x) = \begin{cases} 0 & x < 0 \\ 1/2 & x = 0 \\ 1 & x > 0 \end{cases}$$

Fourier transform

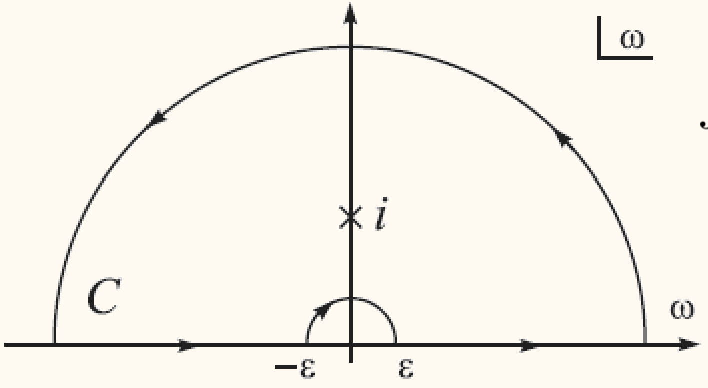
$$\mathscr{F}\{H(t)\} = \frac{1}{i\omega} + \pi\delta(\omega)$$



$$g(t) = \int_{-\infty}^{\infty} \frac{e^{i\omega t}}{1+i\omega}\left[\frac{1}{i\omega} + \pi\delta(\omega)\right]\frac{d\omega}{2\pi}$$

$$= \int_{-\infty}^{\infty} \frac{e^{i\omega t}}{(i-\omega)\omega}\frac{d\omega}{2\pi} + \frac{1}{2}$$

Ex) Transient response to step function input

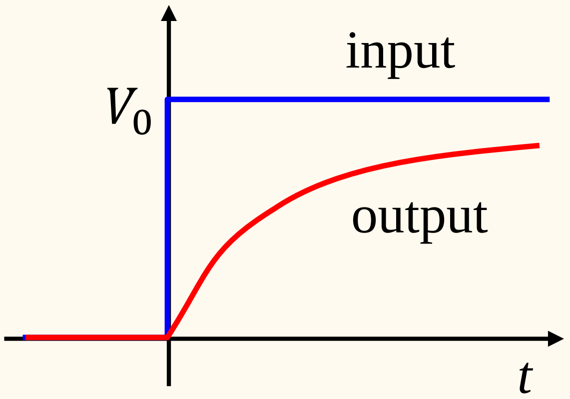


$$\int_{-\infty}^{\infty} \frac{e^{i\omega t}}{(i-\omega)\omega}\frac{d\omega}{2\pi}$$

$$= -2\pi i \frac{e^{-t}}{2\pi i} - \lim_{\epsilon\to 0}\left[\int_{\pi}^{0} \frac{e^{i\epsilon e^{i\theta} t}}{\epsilon e^{i\theta}(\epsilon e^{i\theta} - i)} \frac{i\epsilon e^{i\theta} d\theta}{2\pi}\right]$$

$$= -e^{-t} - \frac{1}{2}$$

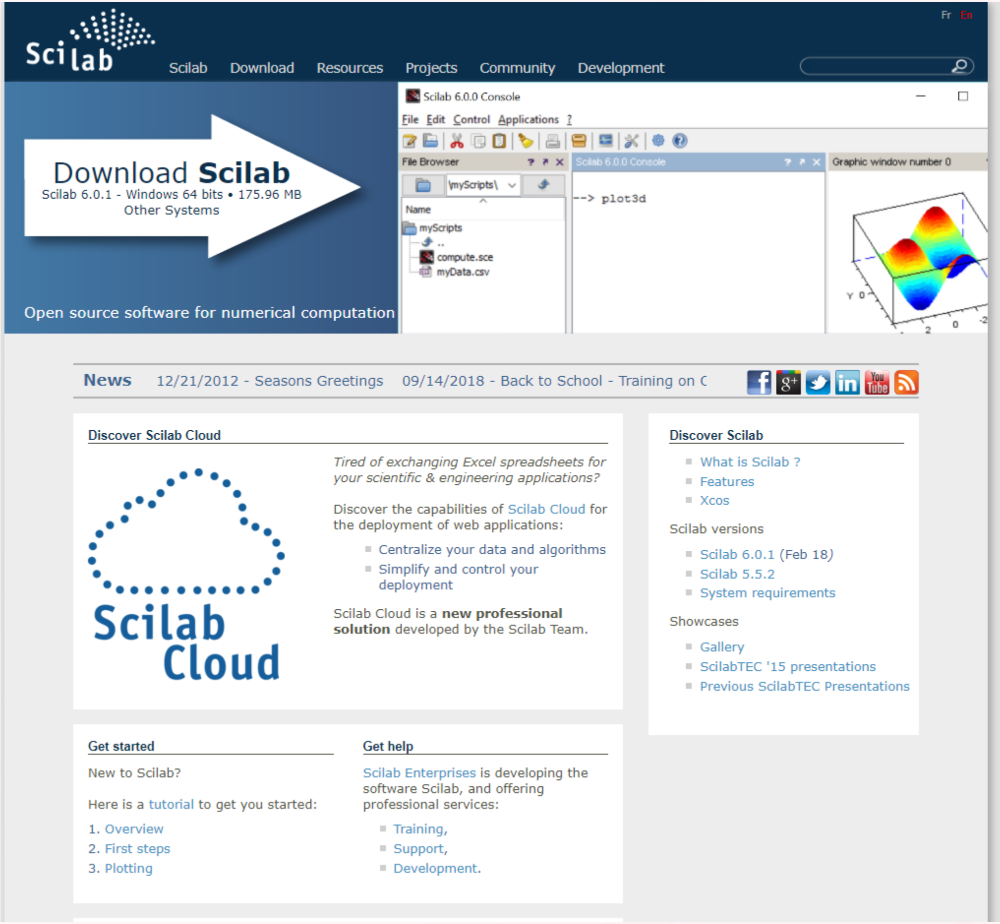
$$\therefore \quad g(t) = -e^{-t}$$



From the initial condition and renormalization of t we obtain

$$V = V_0 \left[1 - \exp\left(-\frac{t}{CR}\right)\right]$$

Introduction of freeware: Scilab



https://www.scilab.org/

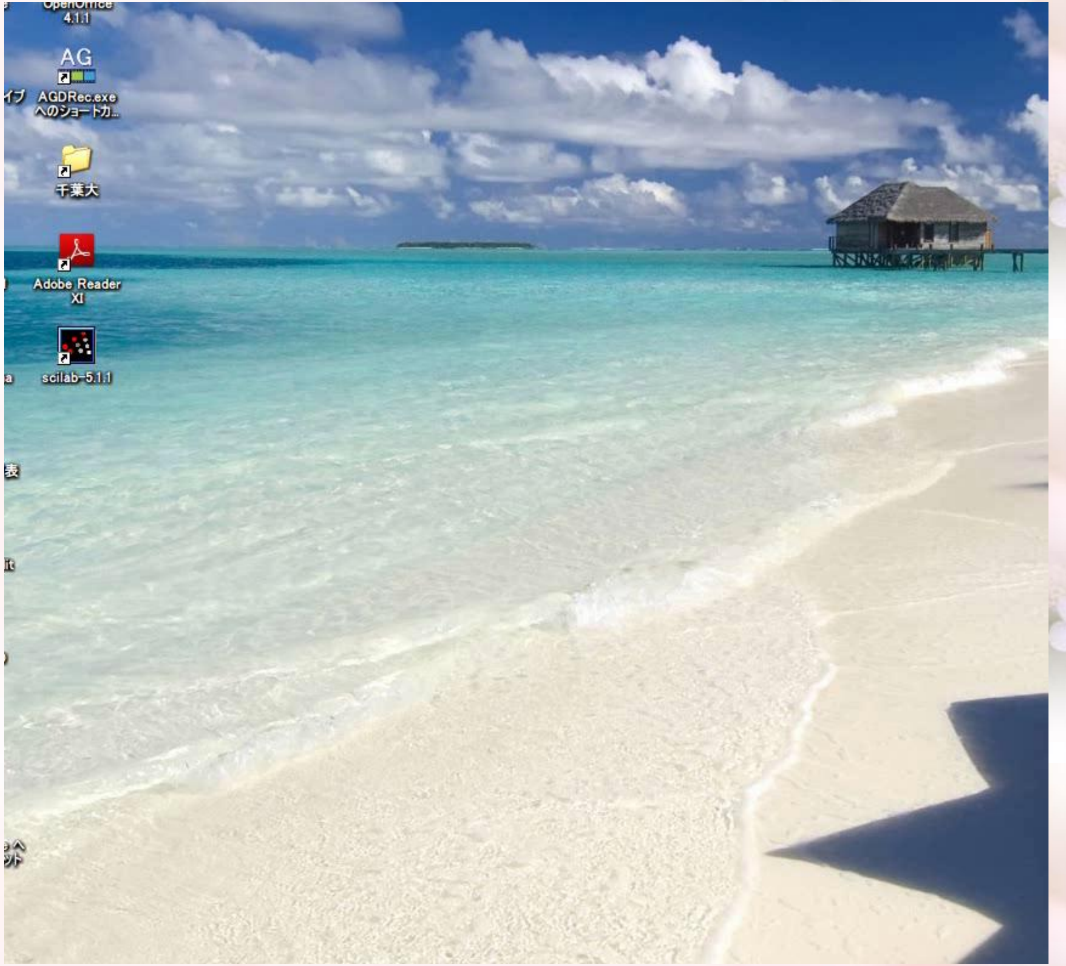
MATLAB clone but free!

Graphical linear response analysis: Scicos

Convenient set of commands for linear response analysis

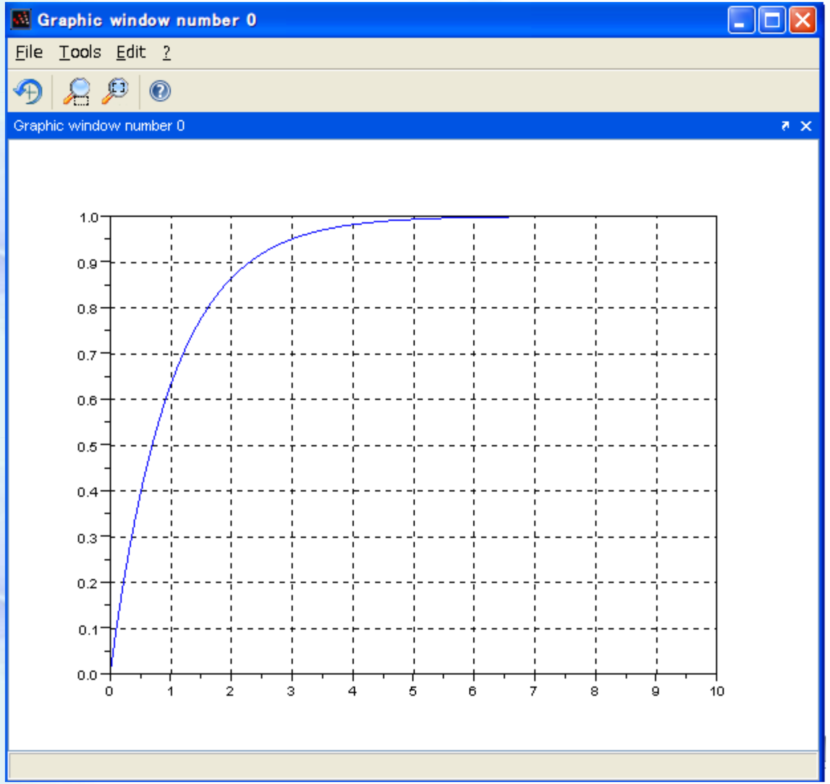
Transfer function analysis with Scilab

$$\Xi(s) = \frac{1}{1+s}$$



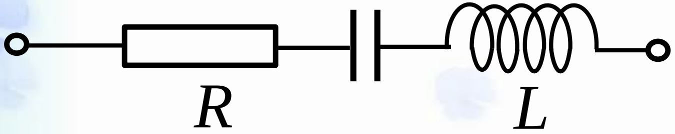
Transient response: Use of Scilab

$$\Xi(s) = \frac{1}{1+s}$$




```
-->s=poly(0,'s');

-->G=1/(1+s);

-->sys=syslin('c',G);

-->t=linspace(0,10,100);

-->y=csim('step',t,sys);

-->plot(t,y)

-->xgrid()
```

Transient response: Use of Scilab



$$Y(s) = \frac{Cs}{LCs^2 + CRs + 1}$$

```
-->G=s/(1+s+2*s*s);

-->sys=syslin('c',G);

-->y=csim('step',t,sys);

-->plot(t,y)
```