電子回路論第5回

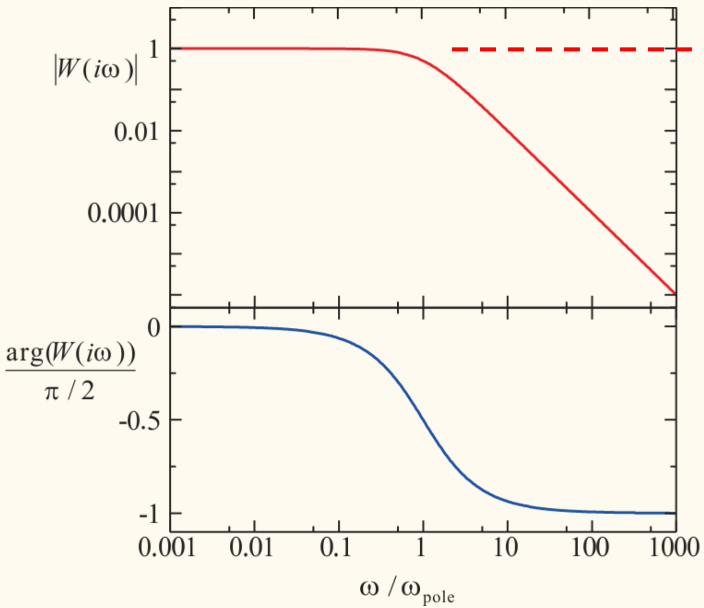
Electric Circuits for Physicists #5

東京大学理学部・理学系研究科
物性研究所
勝本信吾
Shingo Katsumoto

Chapter 4 Amplification circuits

Linear amplifier

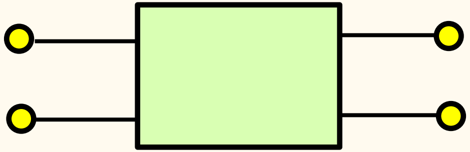
passive
filter



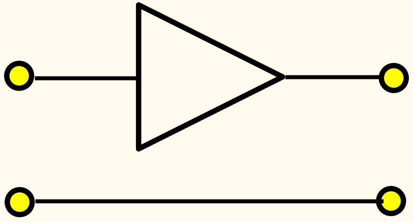
maximum gain = 1

gain >1 ⟶ amplifier

four terminal circuit model

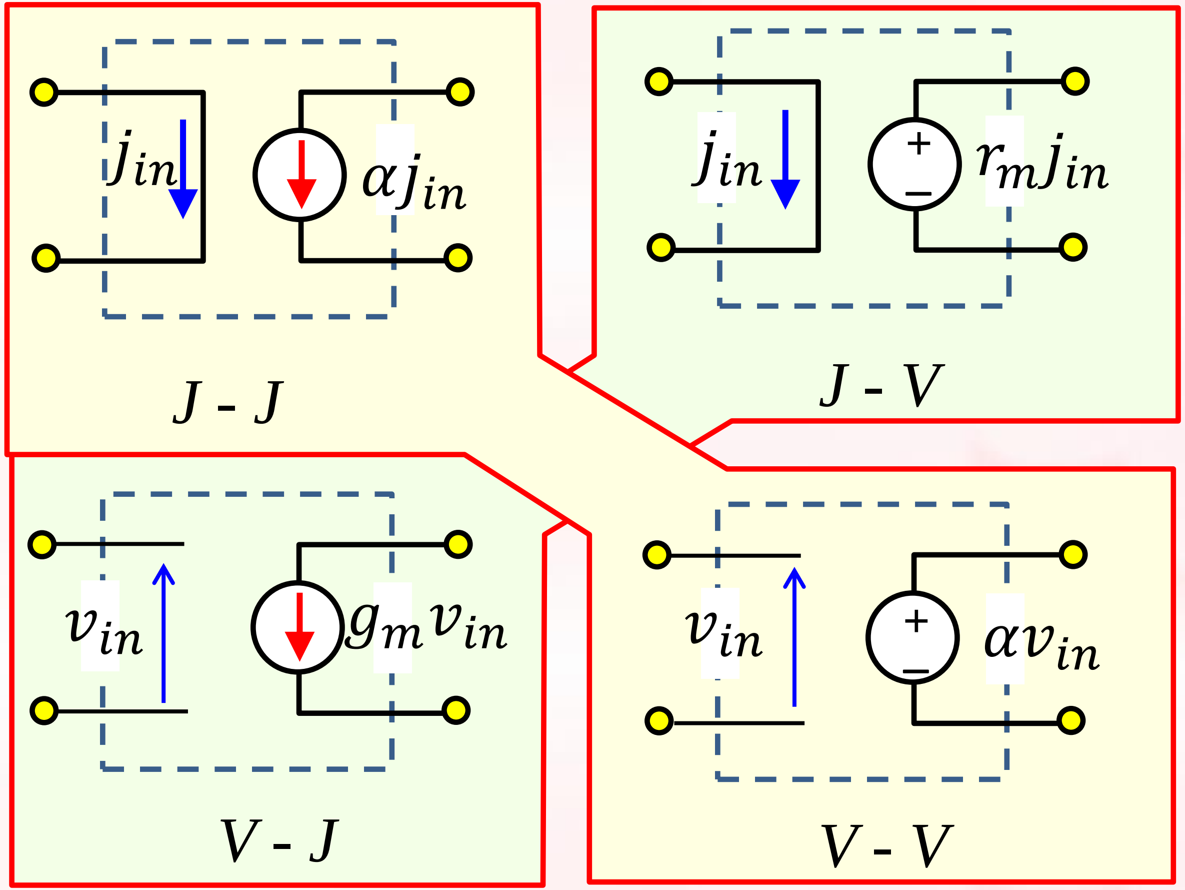


Circuit symbol



Which quantity is "amplified"?

Controlled power source models



Off-diagonal: *J-V*, *V-J*

$$v_{\rm out} = r_{\rm m} j_{\rm in}$$

$$j_{\rm out} = g_{\rm m} v_{\rm in}$$

Transducers

$r_{\rm m}$: trans (mutual) resistance

$g_{\rm m}$: trans (mutual) conductance

Diagonal: *J-J*, *V-V*

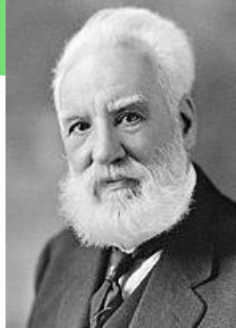
$$j_{\rm out} = \alpha j_{\rm in}$$

$$v_{\rm out} = \alpha v_{\rm in}$$

Amplifiers with gain = $|\alpha|$

Gain, and "Unit" for gain

Voltage gain: $\left|\frac{v_{out}}{v_{in}}\right|$ Current gain: $\left|\frac{j_{out}}{j_{in}}\right|$ Power gain: $\left|\frac{v_{out}j_{out}}{v_{in}j_{in}}\right|$



Alexander Graham Bell
1847 - 1922

When we say "the gain of the amplifier …", the gain usually means power gain.

quantity Q, unit Q_0 : Q in log scale: $L = \log_{10} \frac{Q}{Q_0}$ (B, bel)

cf. deca- 10

dB : (decibel)

1/10

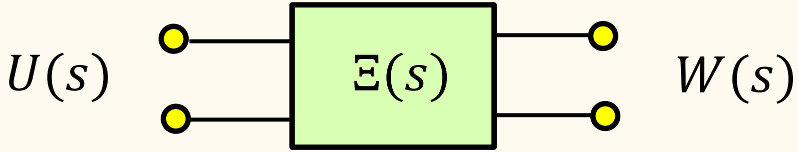
From: G. Bell

$$G = 10 \times \log_{10} \left(\frac{v_{\rm out}}{v_{\rm in}}\right)^2 = 20 \log_{10} \frac{v_{\rm out}}{v_{\rm in}}$$

dB units: dBm (1mW: 0dBm), dBv (1V: 0dBv), etc.

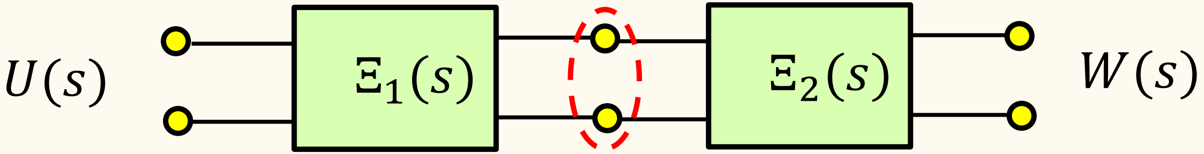
Linear response and transfer function diagram

Linear response: $W(s) = \Xi(s)U(s)$

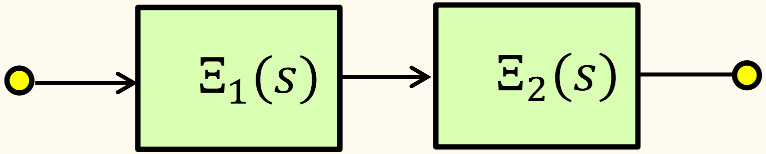


Cascade connection of linear response systems:

$$W(s) = \Xi_{\mathrm{tot}}(s)U(s) = \Xi_2(s)[\Xi_1(s)U(s)] = [\Xi_2(s)\Xi_1(s)]U(s)$$

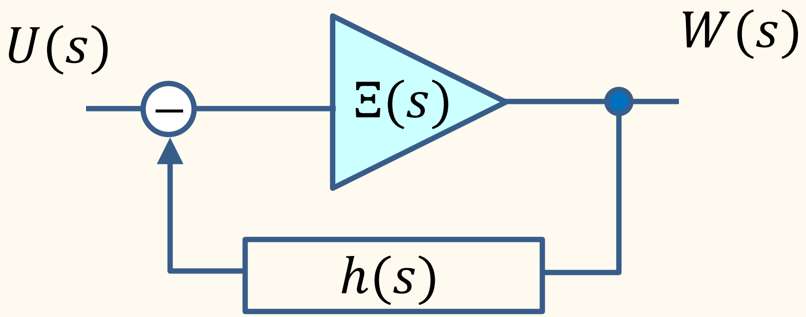


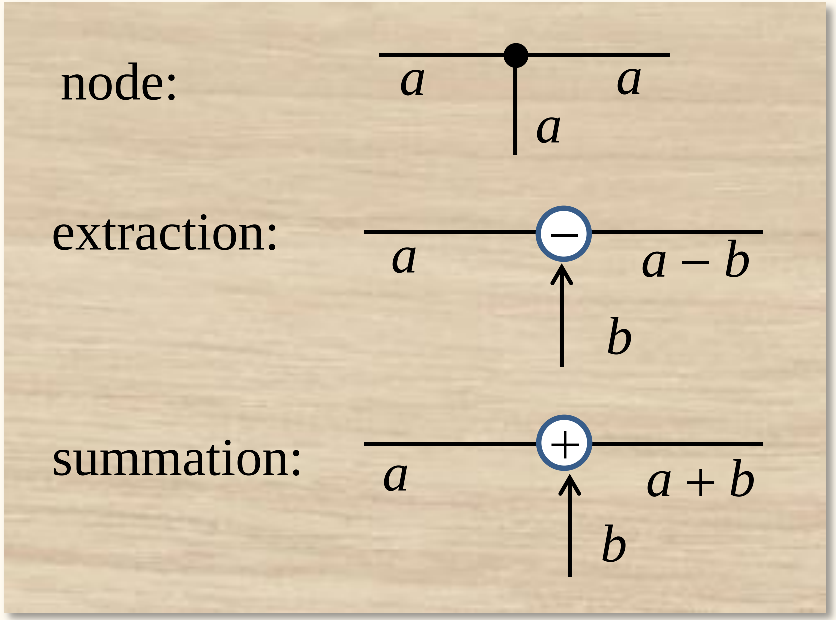
Parallel lines can be expressed as a single line. Such diagram representation can be applied to any linear response system.



(arrow is used for specifying signal flow)

Feedback





Feedforward ⟺ Feedback

$$W(s) = \Xi(s)U(s)$$

$$W(s) = \Xi(s)[U(s) - h(s)W(s)]$$

$$W(s) = \frac{\Xi(s)}{1 + \Xi(s)h(s)}U(s) \stackrel{\text{def}}{=} G(s)U(s)$$

$|1 + \Xi(s)h(s)| > 1$: Negative feedback,

< 1: Positive feedback

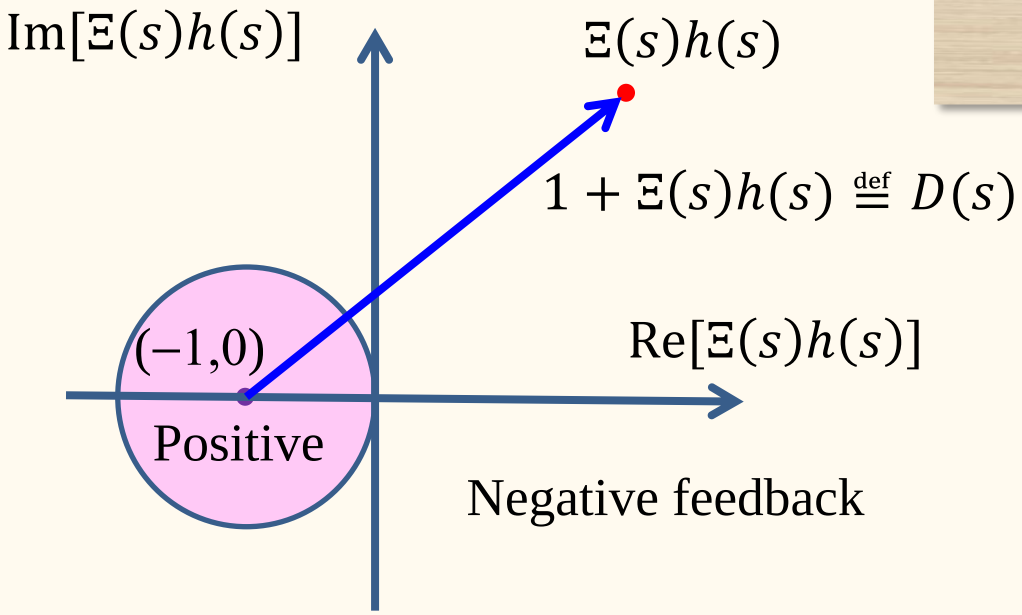
Why negative feedback?

$$|\Xi(s)| \gg 1 \rightarrow G(s) \approx \frac{1}{h(s)}$$

Can be very stable, linear. Easy to calculate.

Condition for negative feedback

$|1 + \Xi(s)h(s)| > 1$: Negative feedback
< 1: Positive feedback



$$\begin{cases} D(s) = 0 \quad (\Xi(s)h(s) = -1) \\ G(s) = \infty \end{cases}$$

Output without input: oscillation point

If $\Xi(s)h(s) = -1$ has solutions, the circuit may be unstable.

How can we judge? → Criteria (Routh-Hurwitz, Nyqust, Liapunov, …)

Stability of linear response systems with poles

Rational representation of a transfer function

$$\Xi(s) = B\frac{(s-\beta_1)\cdots(s-\beta_m)}{(s-\alpha_1)\cdots(s-\alpha_n)} \qquad \{\alpha_j\}: \text{ Poles}$$

Partial fraction expansion (ignore zeros)

$$\sim \frac{B_1}{s-\alpha_1} + \frac{B_2}{s-\alpha_2} + \cdots + \frac{B_n}{s-\alpha_n}$$

$$= \sum_{j=1}^{n} \frac{B_j}{s-s_j} \qquad \{B_j\}: \text{ Residues}$$

With inverse Laplace transform

$$\xi(t) = \sum_{j=1}^{n} B_j \exp(\alpha_j t)$$

For $\xi(t)$ to be finite with $t \to +\infty$ all the real parts of α_j should be negative.

For a linear system to be stable, all the poles of the transfer function should be in the left half of the complex plane.

Zeros and poles of $D(s)$

Assumption 1: $\Xi(s), \Xi(s)h(s)$ are stable → Poles are on the left half plane of s.

Assumption 2: $\Xi(i\omega), \Xi(i\omega)h(i\omega) \to 0$ for $|\omega| \to \infty$ (a cut of frequency should exist)

$$\Xi(s) = \frac{Q(s)}{P(s)},\ h(s) = \frac{q(s)}{p(s)}$$: $P(s), Q(s), p(s), q(s)$ polynomials

$$\deg(P) > \deg(Q), \deg(p) \geq \deg(q)$$

$$D(s) = 1 + \Xi(s)h(s) = \frac{P(s)p(s) + Q(s)q(s)}{P(s)p(s)}$$

$P(s)p(s)$ should be dominant in determining the order

$$D(s) = D_0 \frac{(s-\beta_1)\cdots(s-\beta_n)}{(s-\alpha_1)\cdots(s-\alpha_n)}$$

The same order

The numerator and the denominator are in the same order in s.

Zeros and poles of $D(s)$

$$D(s) = D_0 \frac{(s-\beta_1)\cdots(s-\beta_n)}{(s-\alpha_1)\cdots(s-\alpha_n)}$$

$\{\beta_i\}$: Zeros of $D(s)$ → Poles of $G(s)$

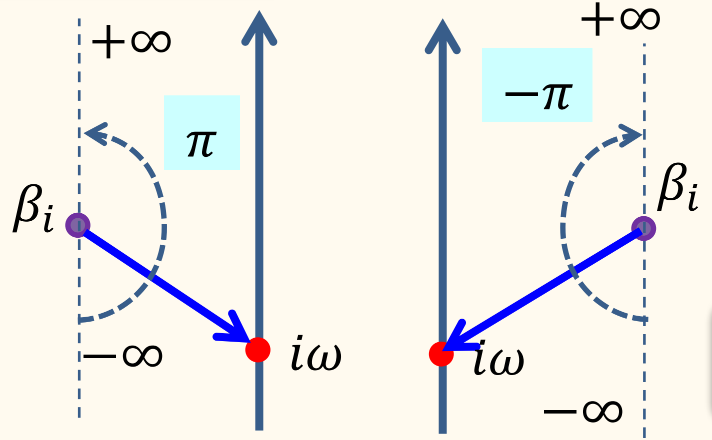
Then we say that $\exists \beta_i \in$ right half plane of s → The circuit is unstable.

Taking the argument we write:

$$\arg(D) = \sum_{i=1}^{n} \arg(s-\beta_i) - \sum_{i=1}^{n} \arg(s-\alpha_i)$$

Left half plane

Right half plane



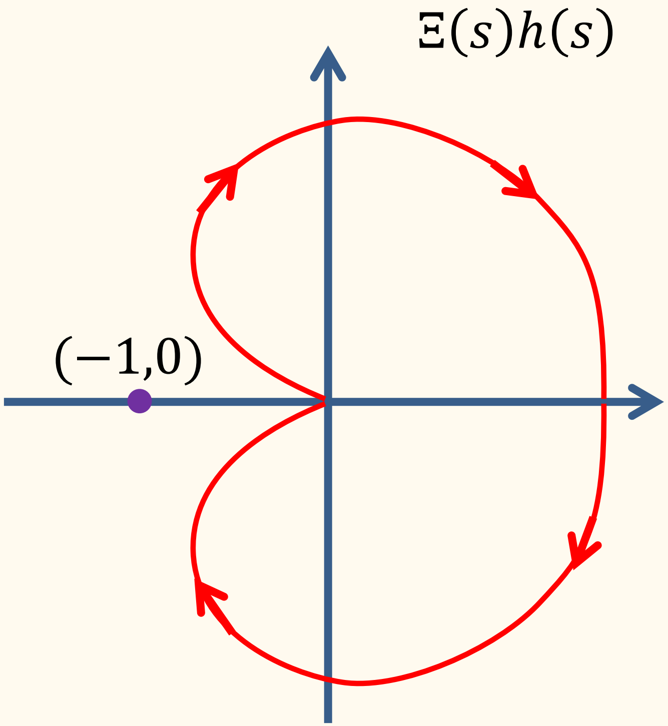
Imagine you are on the imaginary axis $s = i\omega$.
And $\omega: -\infty \to +\infty$.

Number of zeros on the right half plane: m

All the poles should be on the left plane from the assumption 1.

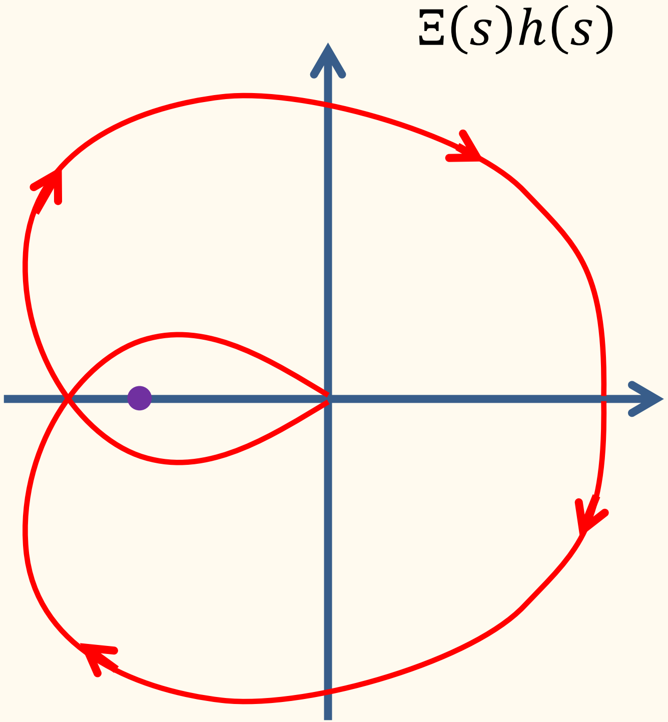
$$\Delta \arg(D) = (n-m)\pi - m\pi - n\pi = -2m\pi$$

Nyquist plot and criterion



$\Delta \arg(D) = 0$

Stable



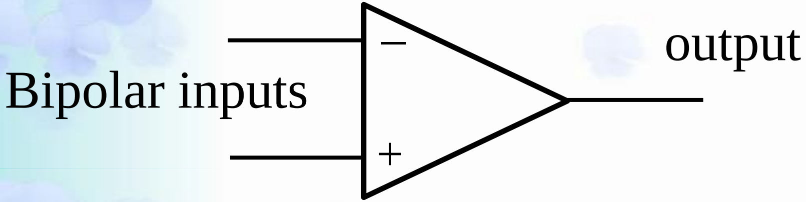
$\Delta \arg(D) = -4\pi$

Unstable

Harry Nyquist
(1889–1976)

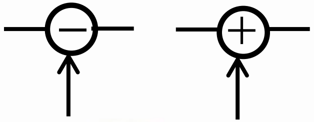
Operational amplifier (OP amp.)

circuit symbol



- Differential amplifier
- Input impedance ~∞
- Open loop gain $A_o \gg 1$
- Output resistance ≈ 0

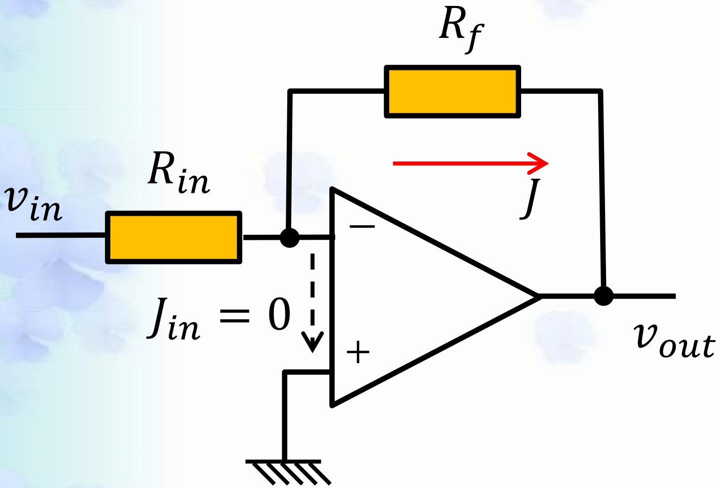
can be used for both



$$V_{\text{out}} = A_o(V_+ - V_-)$$

$$A_\text{o} \gg 1 \quad \therefore V_- \approx V_+ = 0$$

Virtual short circuit

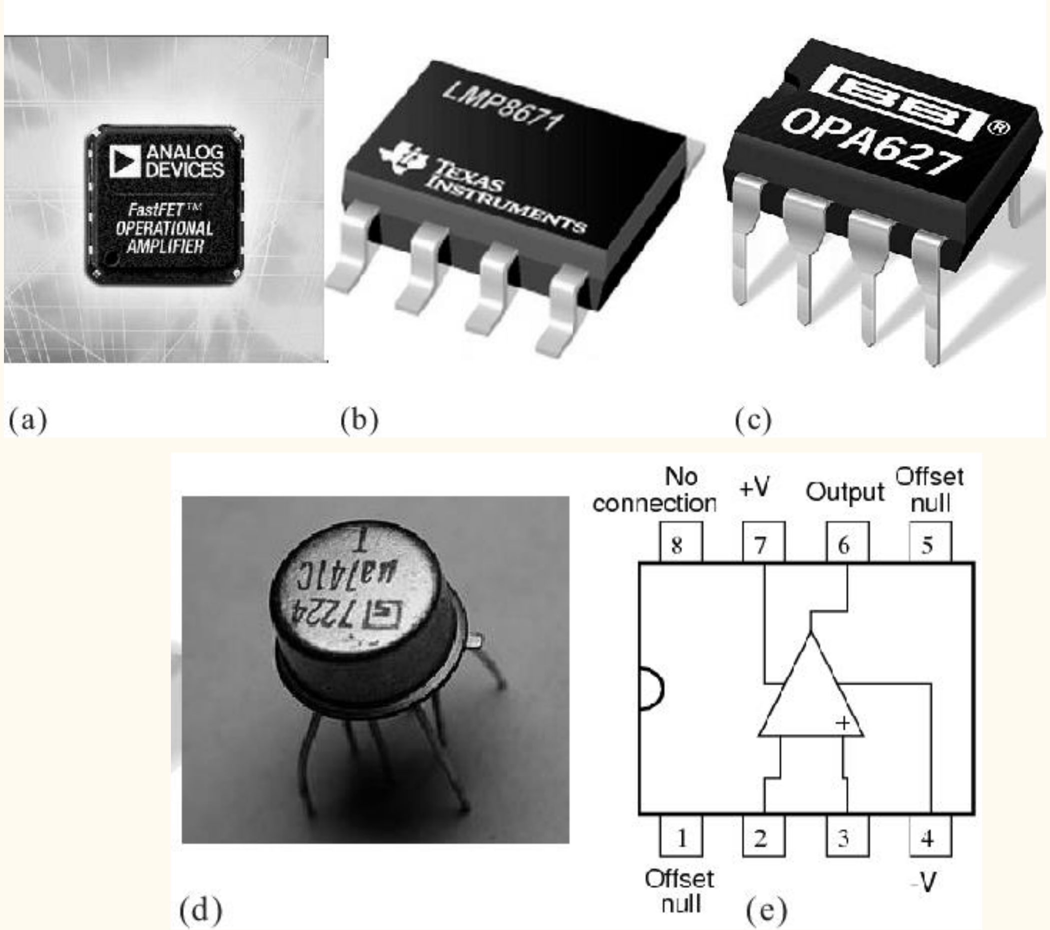


$$J = -\frac{v_{\text{out}}}{R_f} = \frac{v_{\text{in}}}{R_{\text{in}}}$$

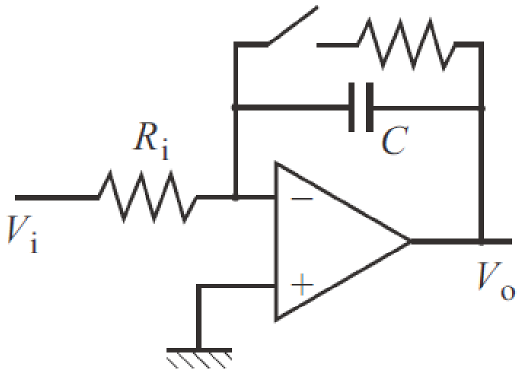
$$\therefore v_{\text{out}} = -\frac{R_f}{R_{\text{in}}} v_{\text{in}}$$

Inverting amplifier

OP amp. packages



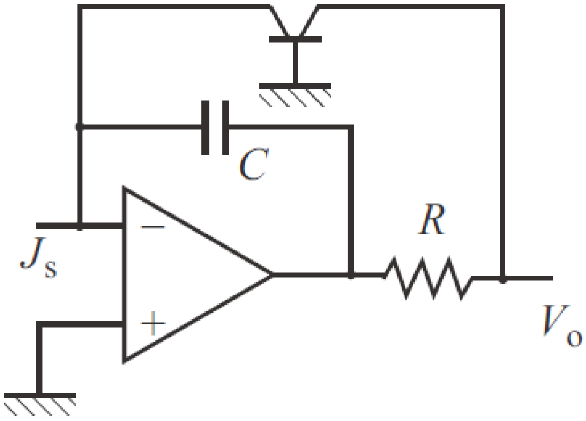
Various applications of OP amps



$$V_{out}(t) = -\frac{Q}{C} = -\frac{1}{C}\int_0^t \frac{V_i(\tau)}{R_i}d\tau$$

$$= -\frac{1}{CR_i}\int_0^t V_i(\tau)d\tau$$

Integration circuit

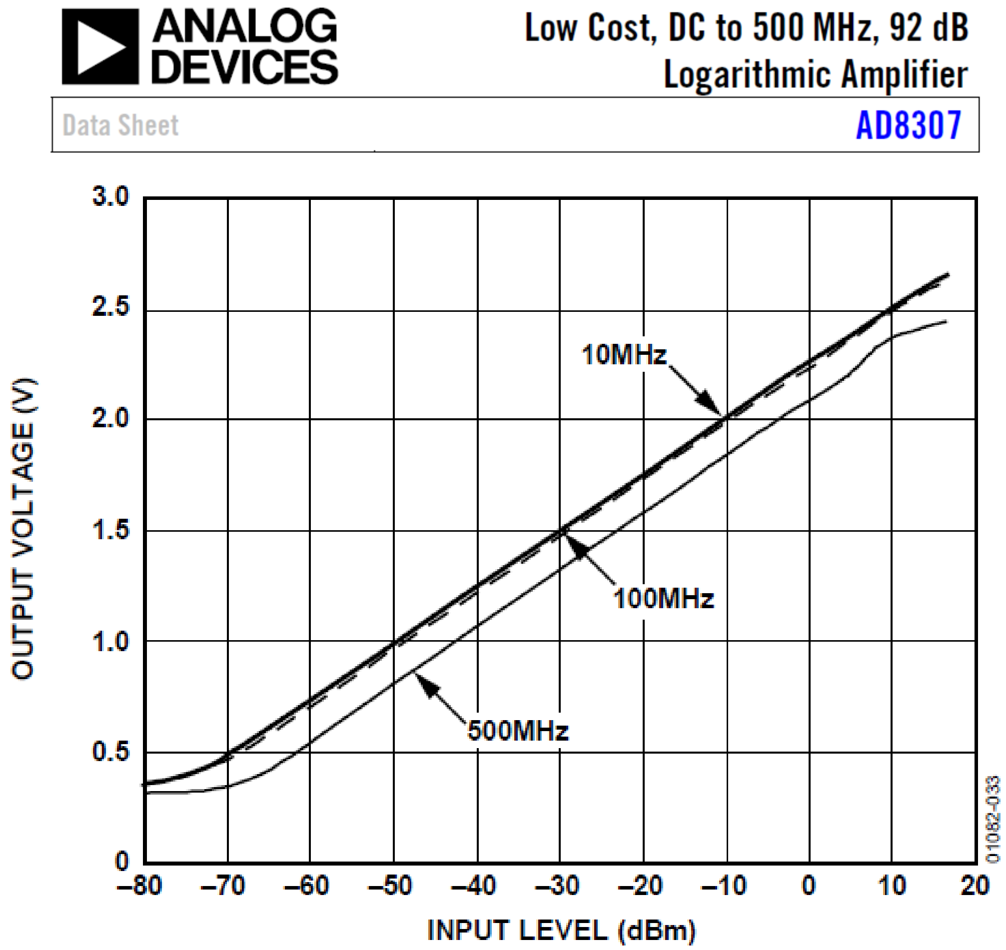


$$V_{out} = -V_{BE} = -\frac{k_B T}{e}\ln\left(\frac{J_s}{J_0}+1\right)$$

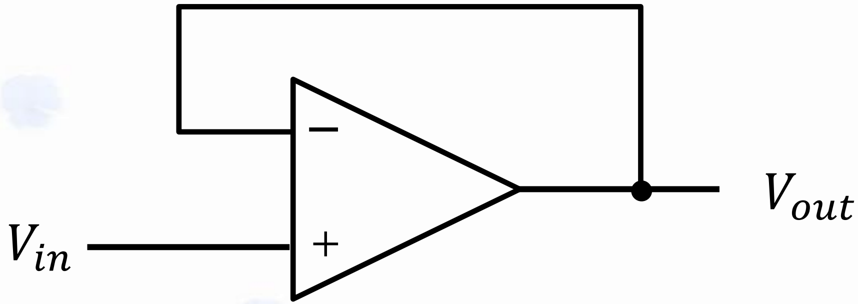
Logarithmic amplifier

Amplifiers with specialized function

Logarithmic amplifier



Voltage follower



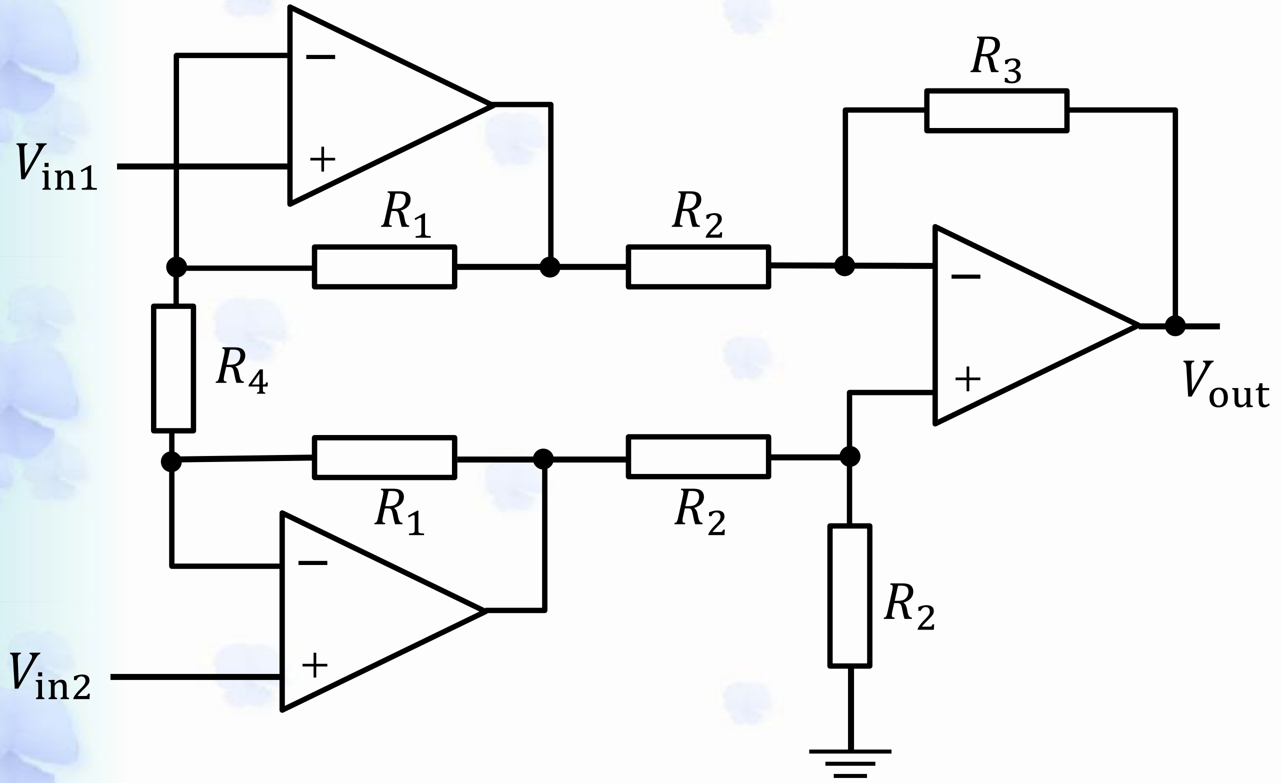
From virtual shortage, simply $V_{out} = V_{in}$

Very high input impedance, very low output impedance.

⇩

Impedance transformer

Instrumentation amplifier



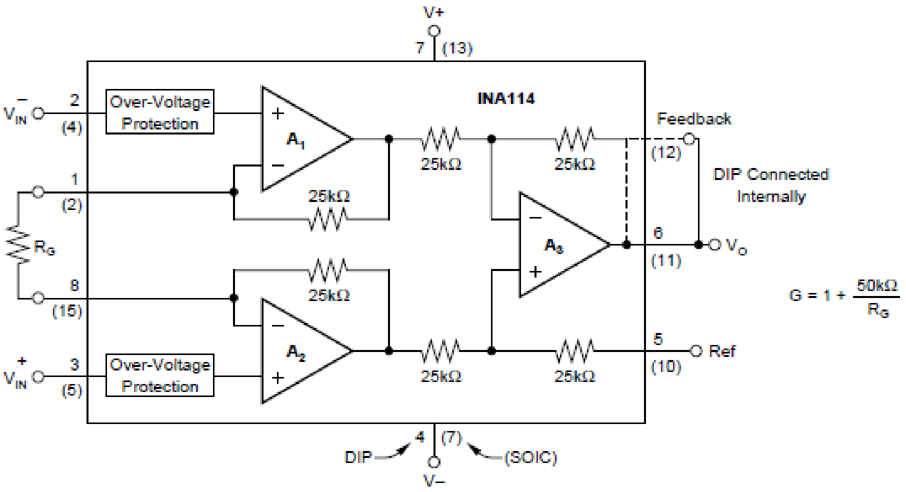
$$V_{out} = -\frac{R_3}{R_2}\frac{2R_1 + R_4}{R_4}(V_{in1} - V_{in2})$$

Instrumentation amplifier

BURR - BROWN®

INA114

Precision
INSTRUMENTATION AMPLIFIER



OP amp data sheet

ANALOG DEVICES

Ultralow Offset Voltage Operational Amplifier

Data Sheet **OP07**

FEATURES

- Low V_{OS}: 75 μV maximum
- Low V_{OS} drift: 1.3 μV/°C maximum
- Ultrastable vs. time: 1.5 μV per month maximum
- Low noise: 0.6 μV p-p maximum
- Wide input voltage range: ±14 V typical
- Wide supply voltage range: ±3 V to ±18 V
- 125°C temperature-tested dice

PIN CONFIGURATION

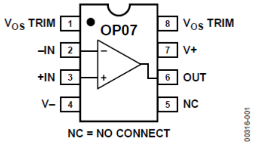
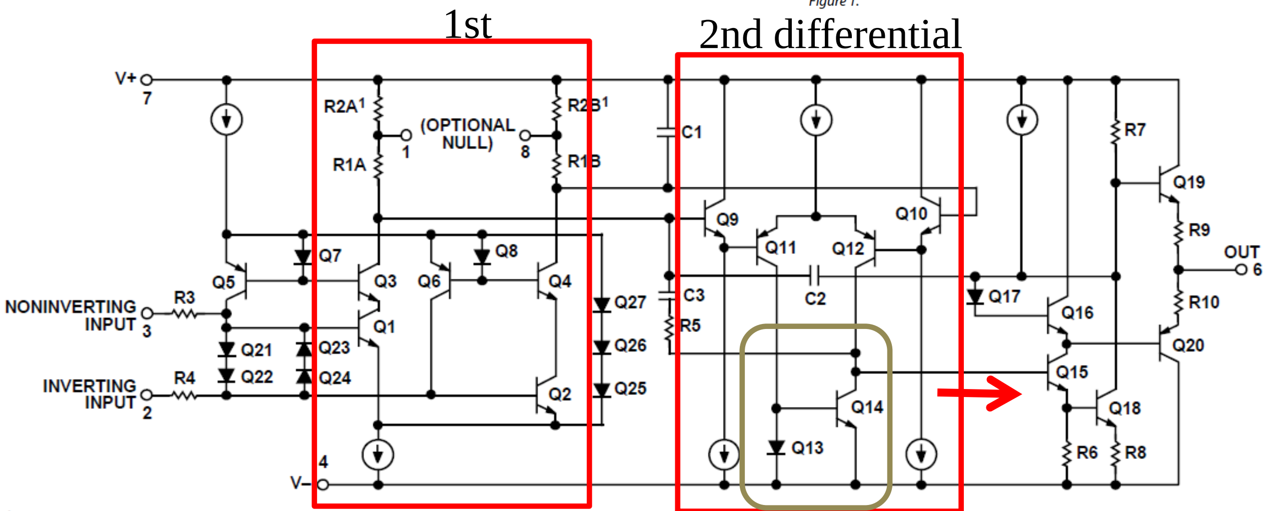


Figure 1.



[1] R2A AND R2B ARE ELECTRONICALLY ADJUSTED ON CHIP AT FACTORY FOR MINIMUM INPUT OFFSET VOLTAGE.

Figure 2. Simplified Schematic

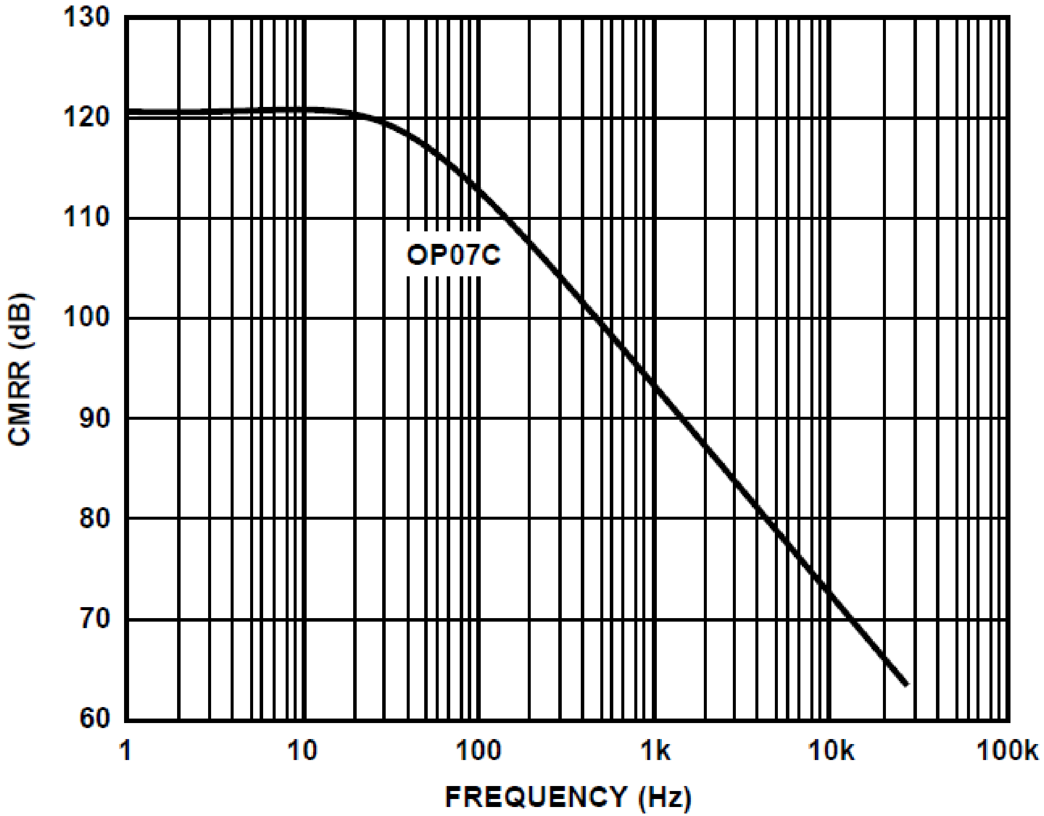
OP amp. data sheet

Parameters

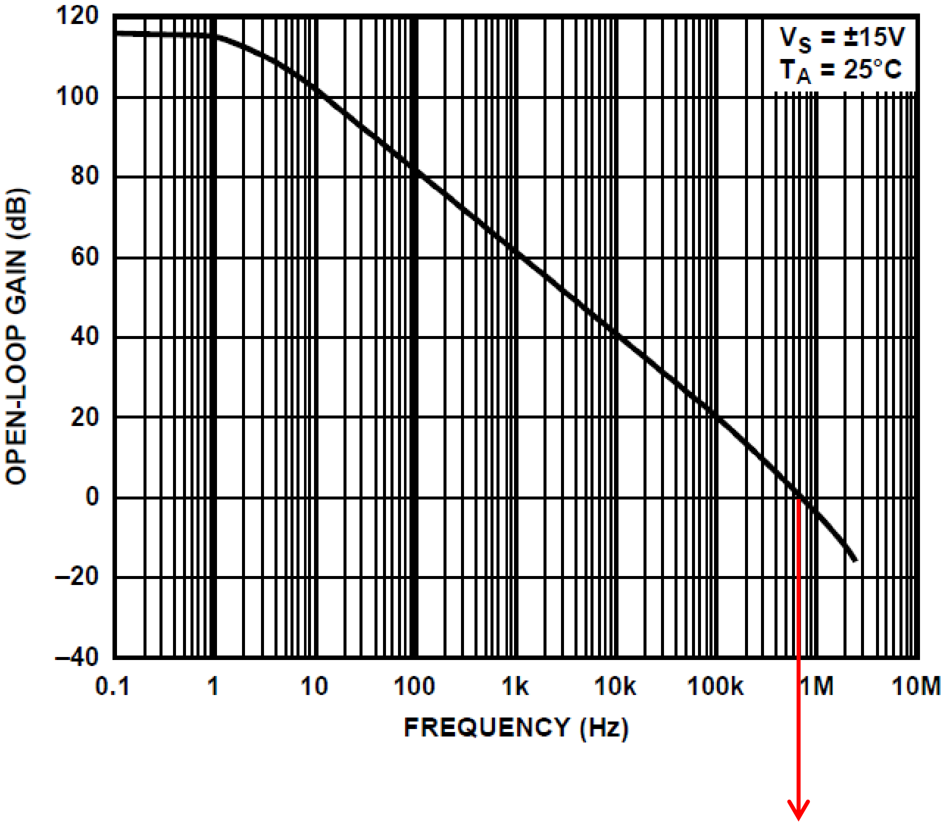
Parameter	Symbol	Conditions	Min	Typ	Max	Unit
INPUT CHARACTERISTICS						
T_A = 25°C						
Input Offset Voltage[1]	V_{OS}			60	150	μV
Long-Term V_{OS} Stability[2]	V_{OS}/Time			0.4	2.0	μV/Month
Input Offset Current	I_{OS}			0.8	6.0	nA
Input Bias Current	I_B			±1.8	±7.0	nA
Input Noise Voltage	e_n p-p	0.1 Hz to 10 Hz[3]		0.38	0.65	μV p-p
Input Noise Voltage Density	e_n	f_O = 10 Hz		10.5	20.0	nV/√Hz
		f_O = 100 Hz[3]		10.2	13.5	nV/√Hz
		f_O = 1 kHz		9.8	11.5	nV/√Hz
Input Noise Current	I_n p-p			15	35	pA p-p
Input Noise Current Density	I_n	f_O = 10 Hz		0.35	0.90	pA/√Hz
		f_O = 100 Hz[3]		0.15	0.27	pA/√Hz
		f_O = 1 kHz		0.13	0.18	pA/√Hz
Input Resistance, Differential Mode[4]	R_{IN}		8	33		MΩ
Input Resistance, Common Mode	R_{INCM}			120		GΩ

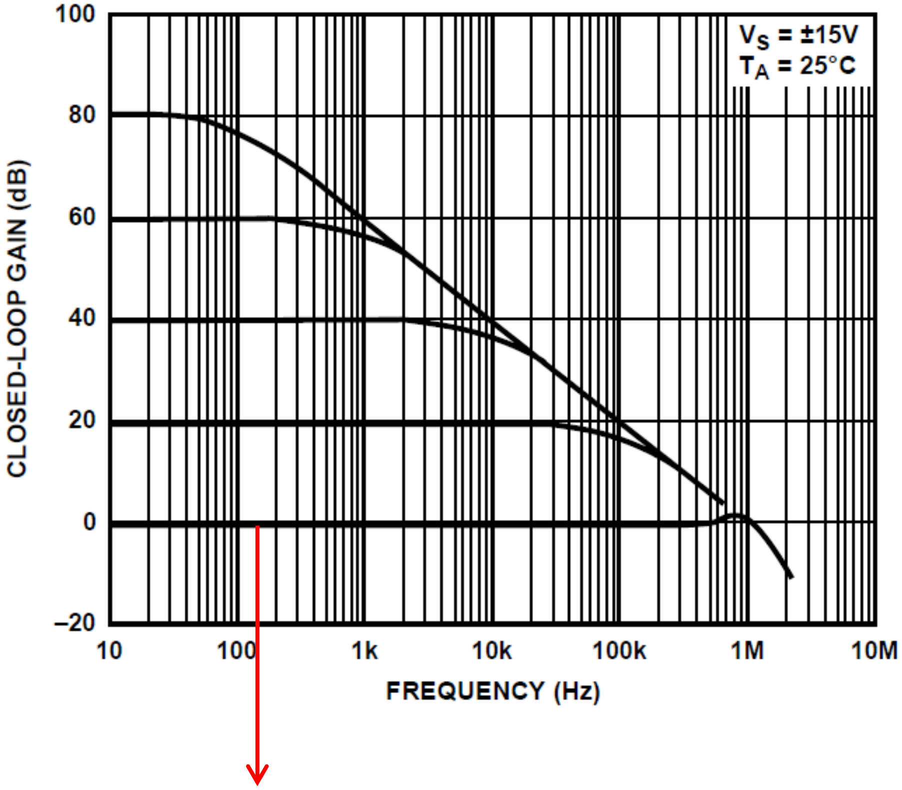
OP amp. data sheet

Common mode rejection ratio (CMRR)



OP amp. data sheet

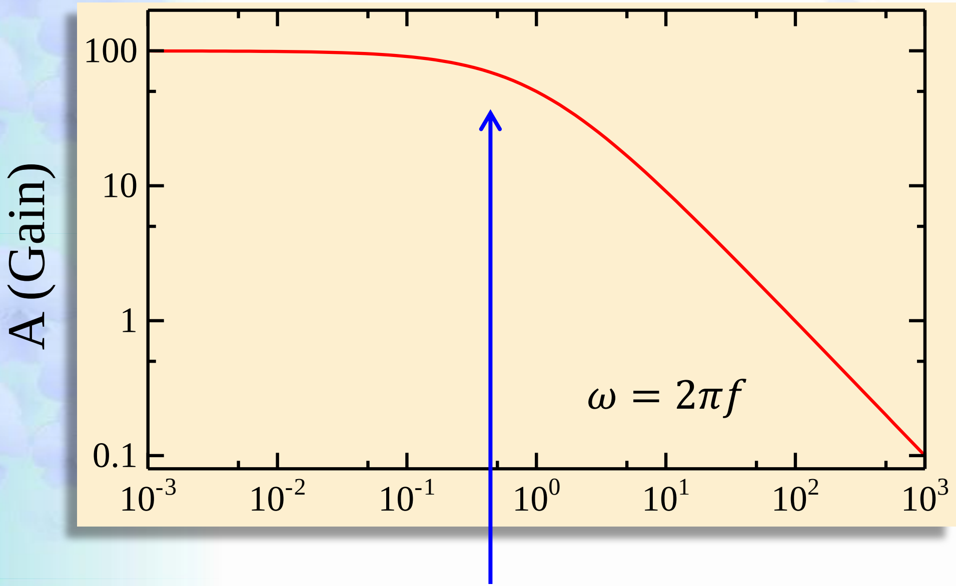


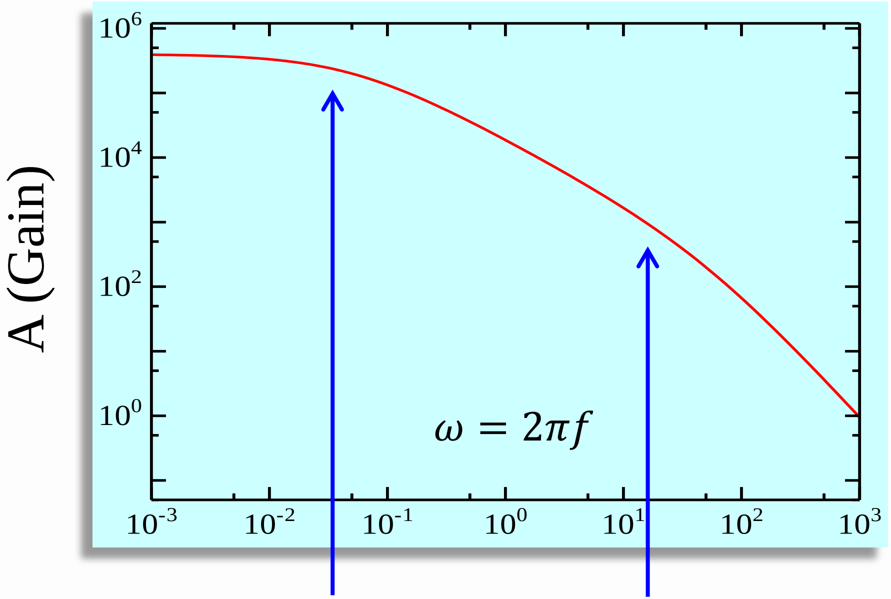


Unity gain frequency

Voltage follower

Frequency dependent characteristics of OP amps

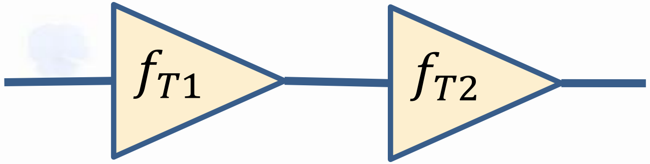




Cut-off frequency $\omega_T = 2\pi f_T$

Phase rotates by $\pi/2$

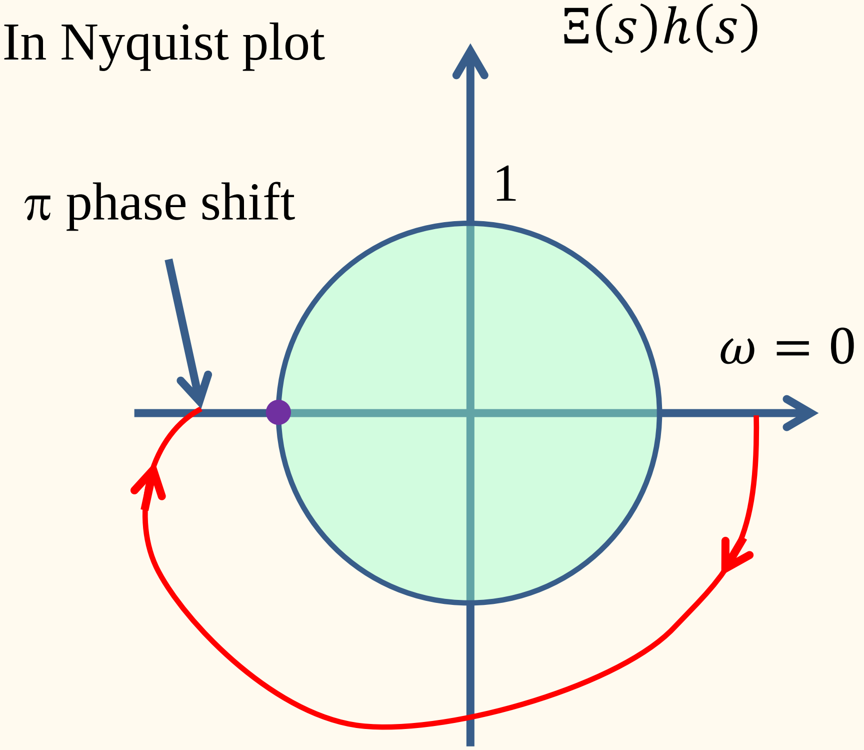
Multiple cut-off frequency:
Phase rotates more than π



If gain is larger than 1 at phase shift π :
Dangerous!

Phase compensation

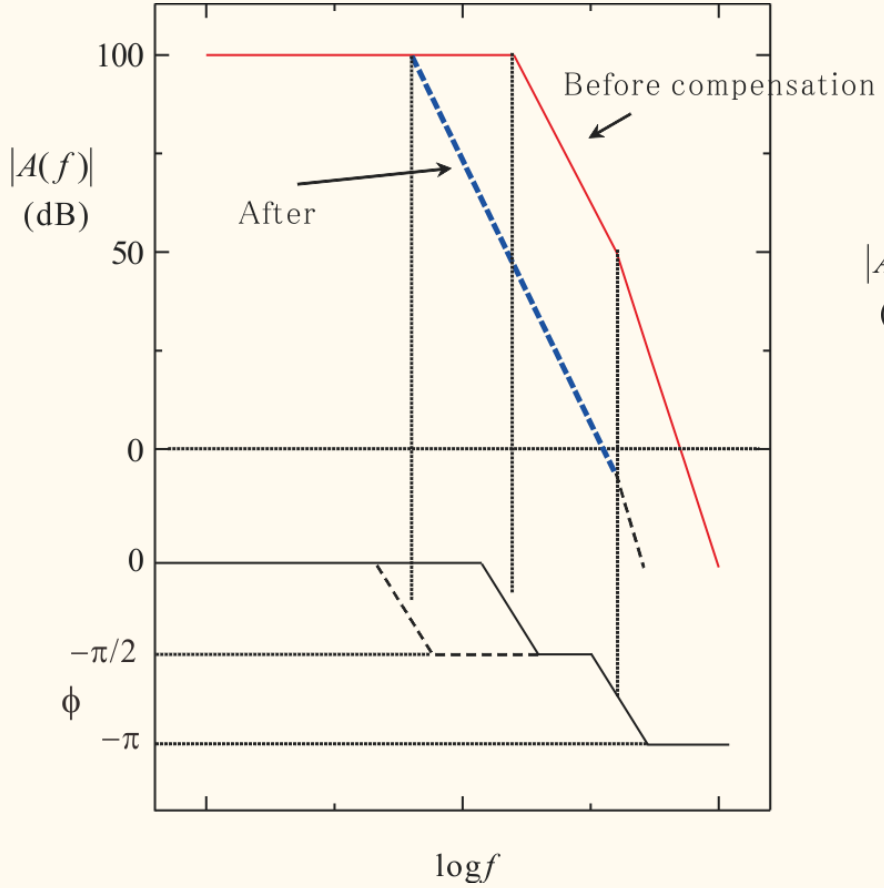
In Nyquist plot

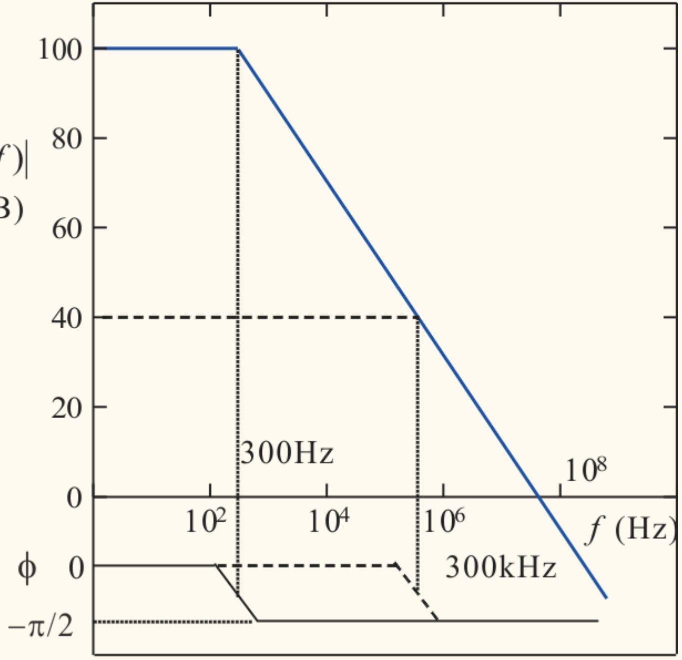


Why dangerous?

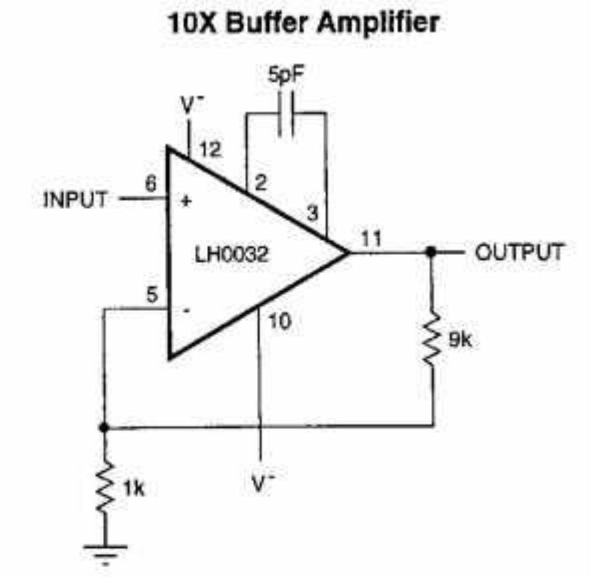
π phase shift: negative feedback → positive feedback

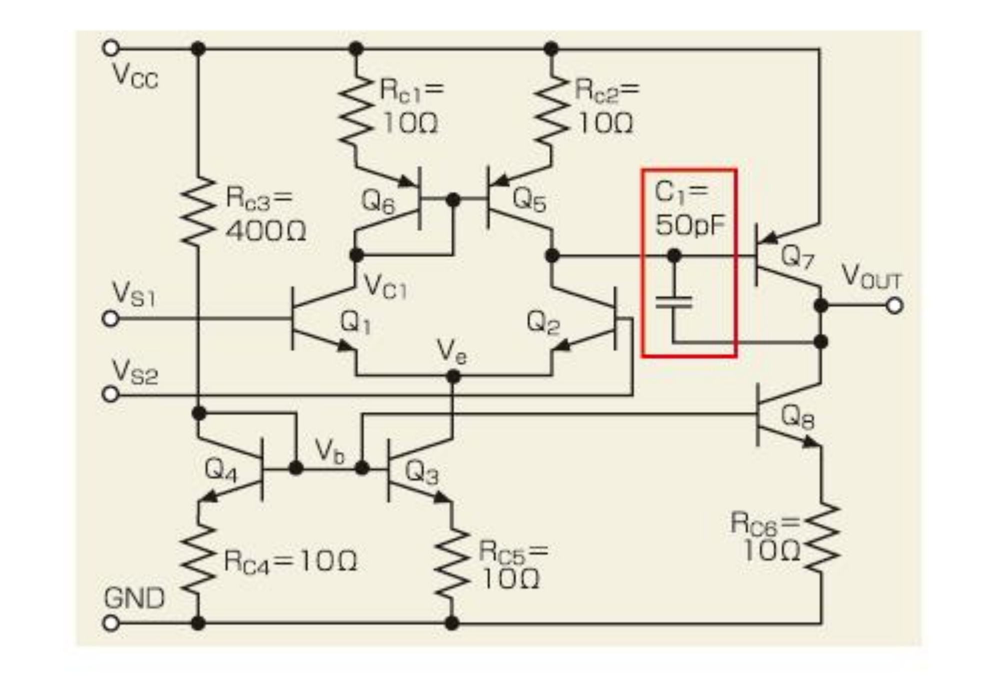
Phase compensation



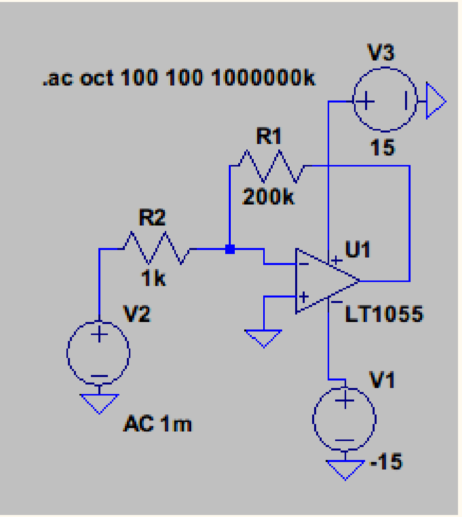


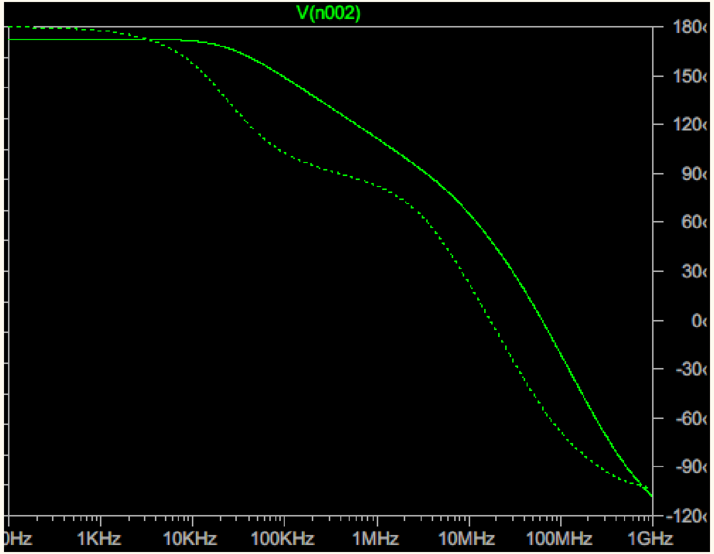
Phase compensation



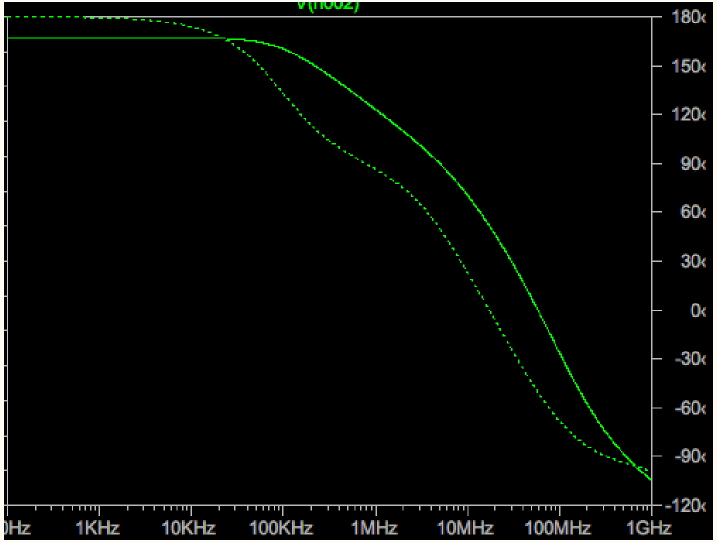


Inverting amplifier and cut-off frequency (LT Spice simulation)



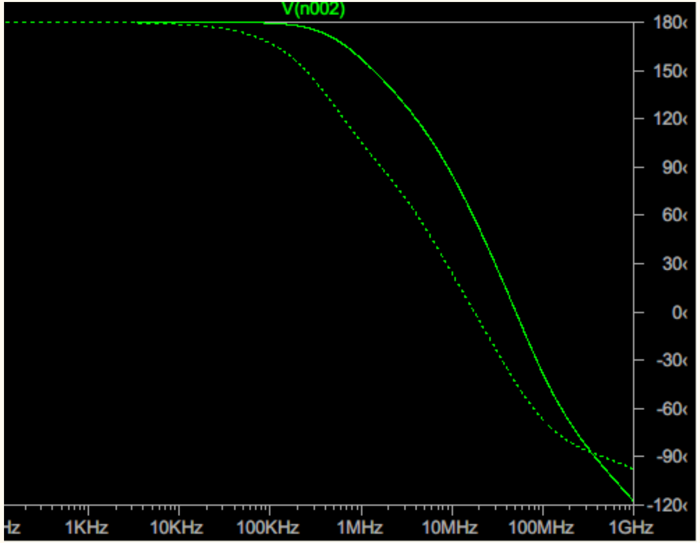


A =200 f_T =30kHz

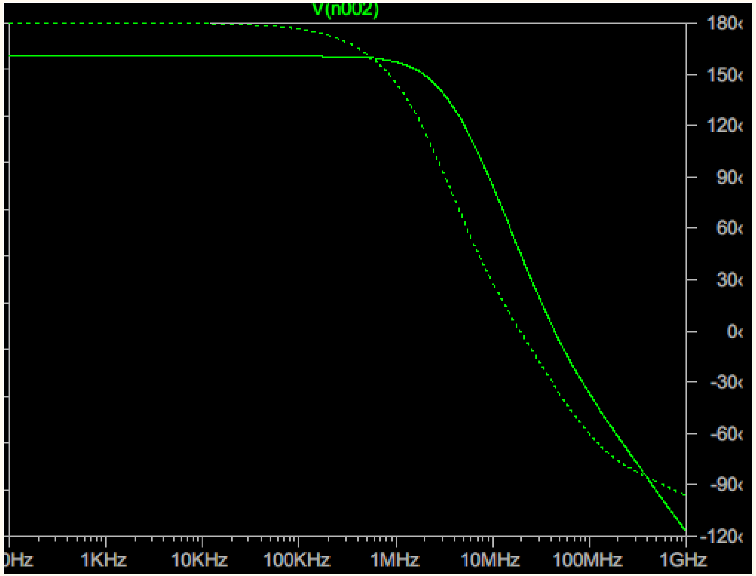


A =50 f_T =90kHz

Inverting amplifier and cut-off frequency (LT Spice simulation)

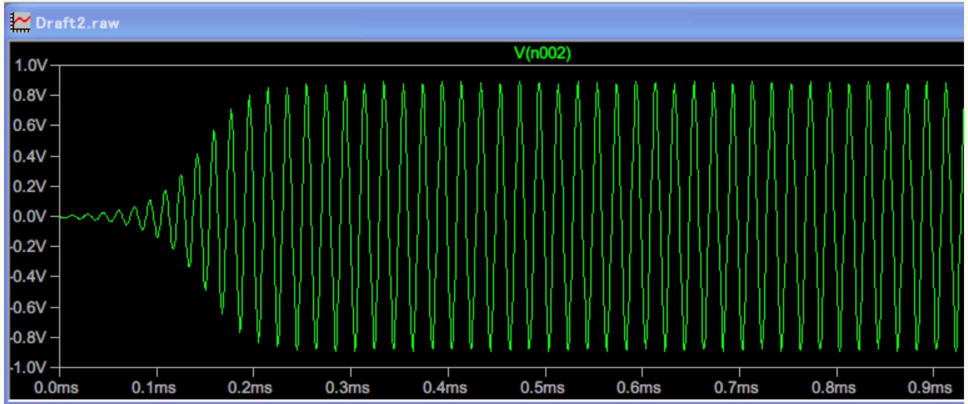


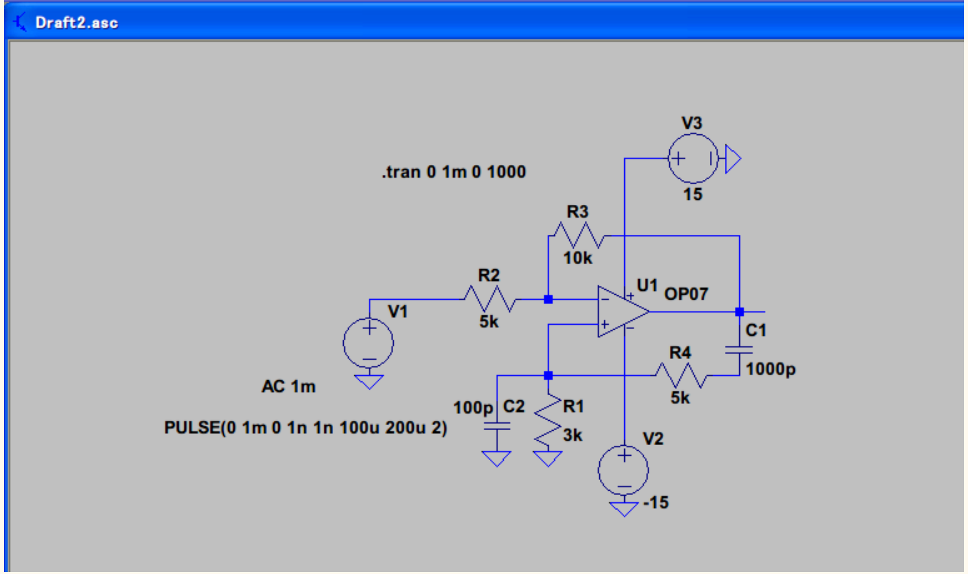
$A = 10 \; f_T = 300\text{kHz}$

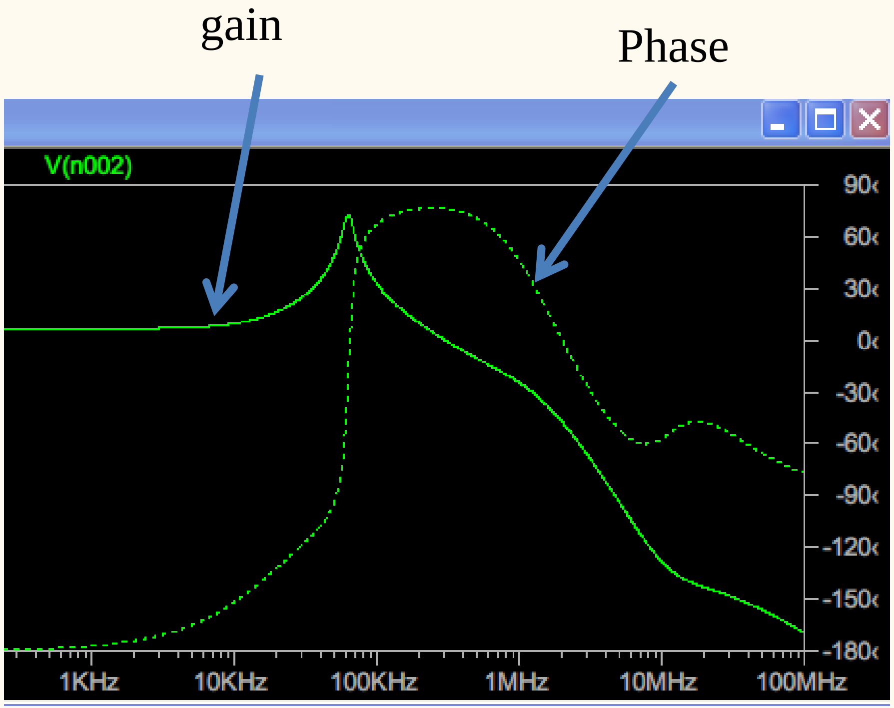


$A = 2 \; f_T = 2\text{MHz}$

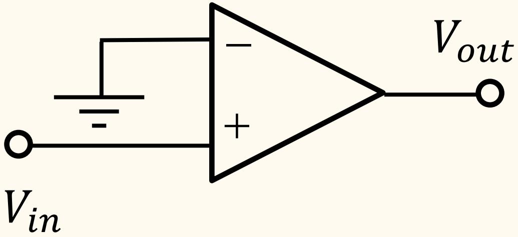
Oscillation in an OP amp. circuit



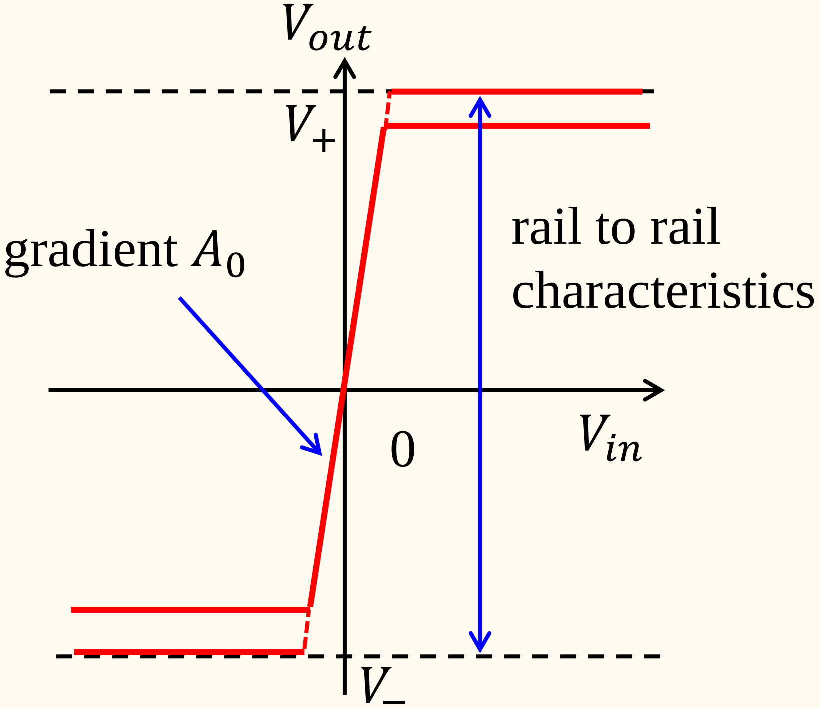




Use of OP amps at saturation voltages

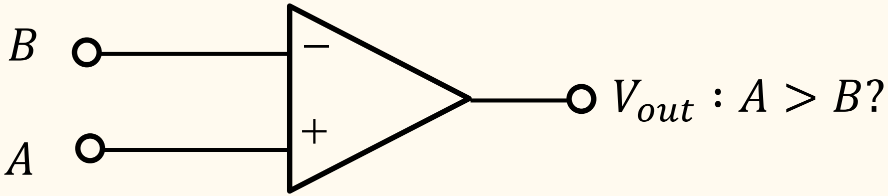


Positive feedback is applied. V_{out} goes up to the highest voltage which the OP amp can reach for positive $V_{\rm in}$.

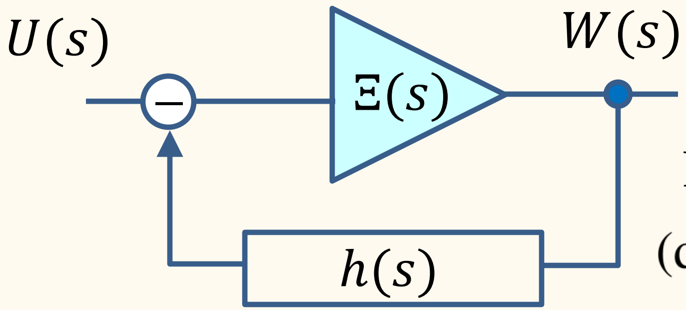


Compare V_{in} with 0

Comparator



Hurwitz criterion (no proof)



$$G(s) = \frac{\Xi(s)}{1 + h(s)\Xi(s)}$$

Adolf Hurwitz
1859 - 1919

Pole equation:

$$\text{(denominator)} = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0$$
$$= a_n (s - p_1) \cdots (s - p_n) = 0$$

$$\forall j = 0, 1, \cdots, n : \quad a_j > 0 \text{ (or } < 0)$$

Otherwise the system is unstable (no proof is given here).

Then we assume all $a_j > 0$.

Define Hurwitz matrix

$n \times n$

n is even (poles are paired).

$$H = \begin{pmatrix} a_{n-1} & a_{n-3} & a_{n-5} & \cdots & 0 \\ a_n & a_{n-2} & a_{n-4} & \cdots & 0 \\ \hline 0 & a_{n-1} & a_{n-3} & \cdots & 0 \\ 0 & a_n & a_{n-2} & \cdots & 0 \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a_0 \end{pmatrix}$$

Hurwitz criterion (no proof)

Hurwitz determinants $H_j \equiv |H[1, \cdots, j; 1, \cdots, j]|$

$$H_1 = a_{n-1},\ H_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix},\ H_3 = \begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} \\ a_n & a_{n-2} & a_{n-4} \\ 0 & a_{n-1} & a_{n-3} \end{vmatrix}, \cdots .$$

Hurwitz criterion

$$H_j > 0 \quad (j = 2, \cdots, n-1)$$

$H_1, H_n > 0$ is trivial from the assumption.

Another expression:

Divide the denominator to odd and even parts $O(s)$ and $E(s)$. If the zeros of $O(s)$ and $E(s)$ are aligned on the imaginary axis alternatively, the system is stable.

PDF password