電子回路論第6回
Electric Circuits for Physicists #6

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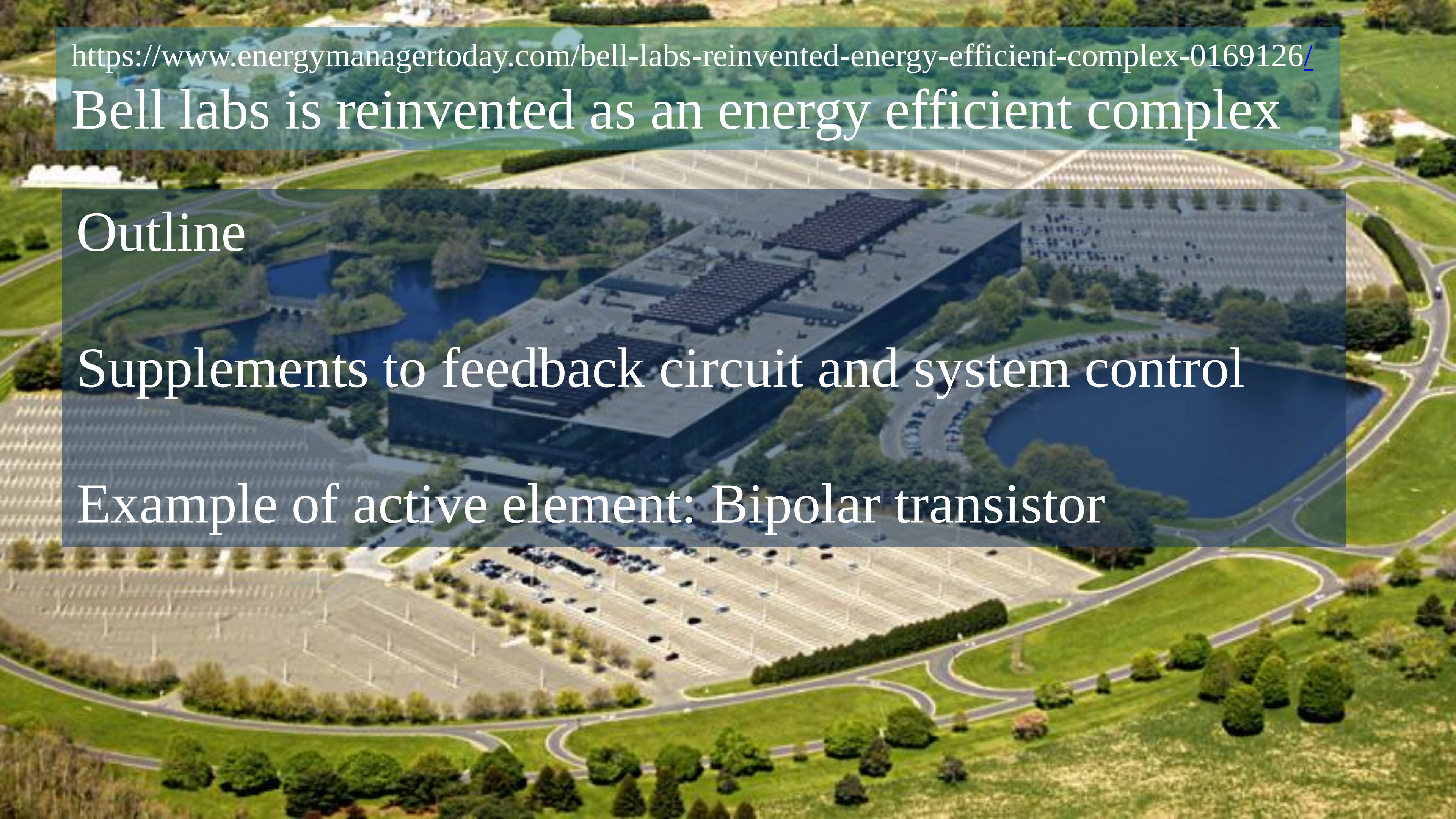


https://www.energymanagertoday.com/bell-labs-reinvented-energy-efficient-complex-0169126/

Bell labs is reinvented as an energy efficient complex

Outline

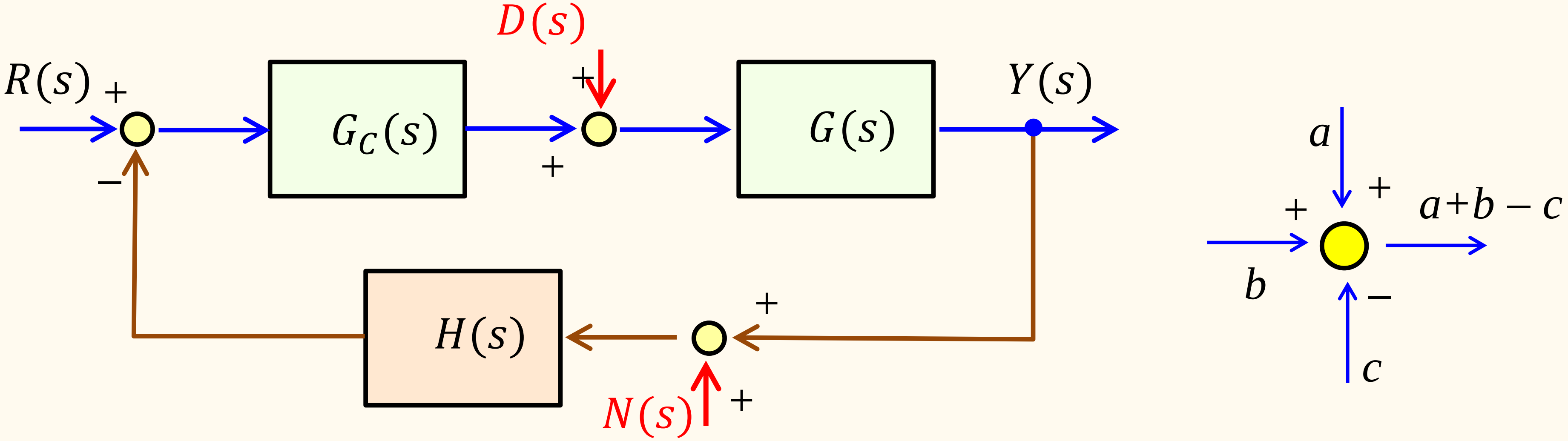
Supplements to feedback circuit and system control

Example of active element: Bipolar transistor

Disturbance and noise on feedback circuits

Circuit treatment of fluctuations:

- Prepare external power sources
- Express them as transfer functions

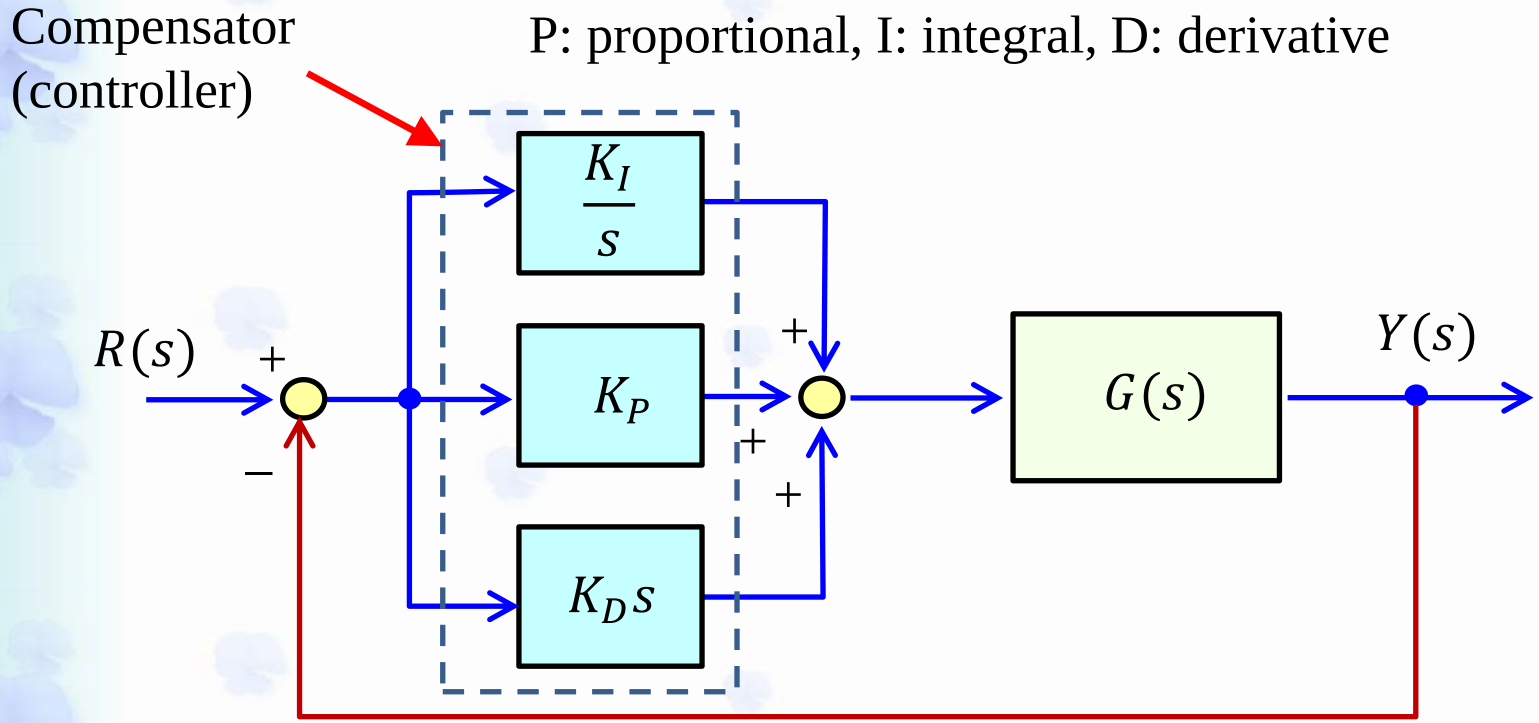


$$Y(s) = \frac{G(s)}{F(s)}\left[G_C(s)R(s) + D(s) + G_C(s)H(s)N(s)\right]$$

$$F(s) \equiv 1 + G_C(s)G(s)H(s)$$

PID control

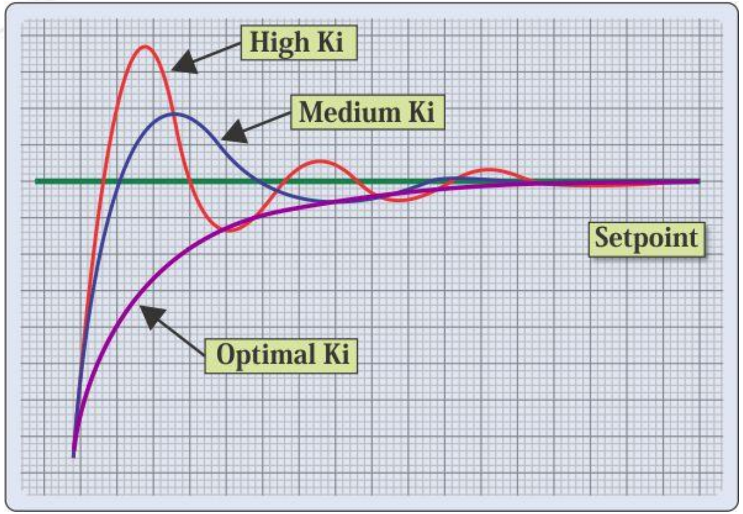
P: proportional, I: integral, D: derivative



$$G_c(s) = K_p + \frac{K_I}{s} + K_D s$$

PID controllers

invensys
EUROTHERM

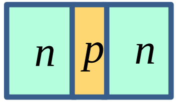


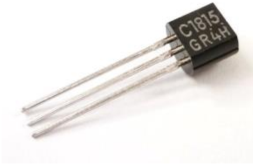
4.4 Example of active element: Transistors

John Bardeen, William Shockley,
Walter Brattain 1948 Bell Labs.

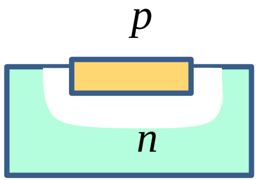
Two types of transistors

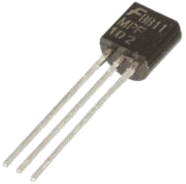
Bipolar junction transistor



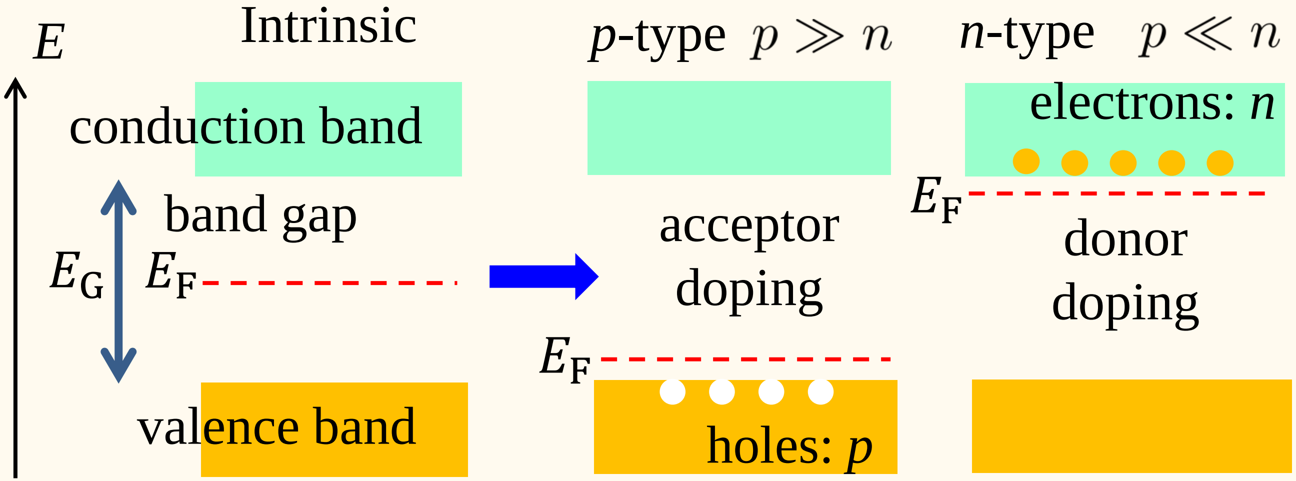


Field effect transistor



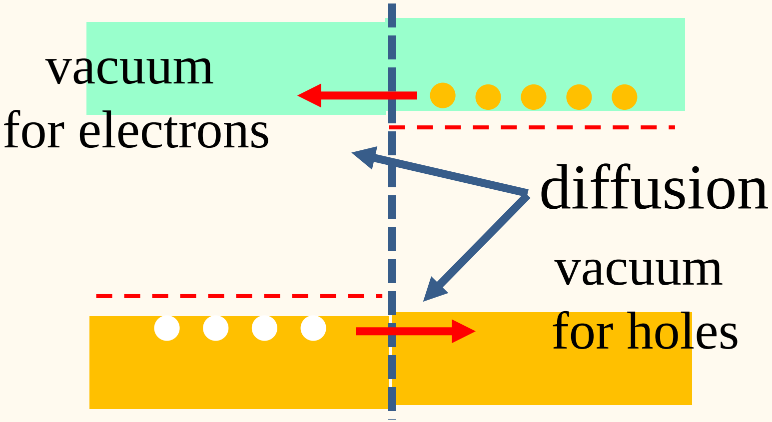


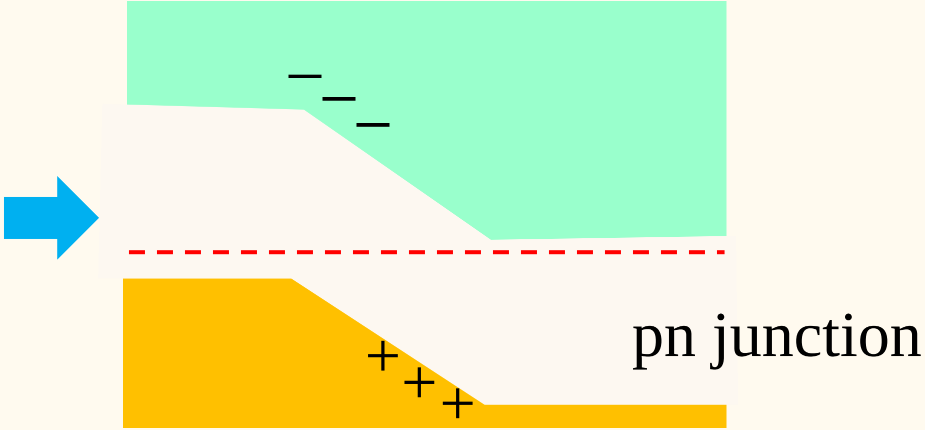
Three types of semiconductors



$$np = n_i^2$$

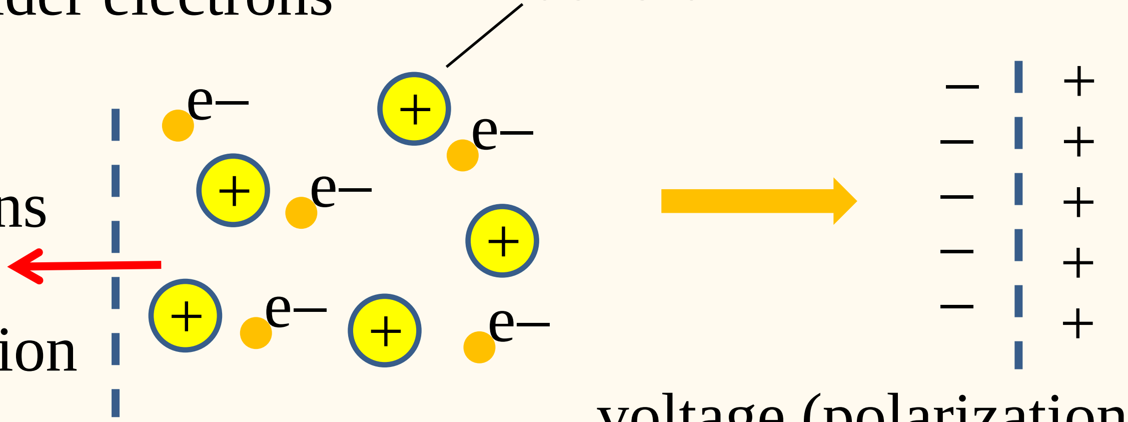
	Majority carrier	Minority carrier
p-type	hole	electron
n-type	electron	hole





pn junction thermodynamics

Consider electrons



Vacuum
for electrons

diffusion

voltage (polarization) → energy cost

Helmholtz free energy:

$$\boxed{F = \underline{U} - T\underline{S}}$$

Voltage (internal energy cost)

Diffusion (entropy)

Minimization of F → Built-in (diffusion) voltage V_{bi}

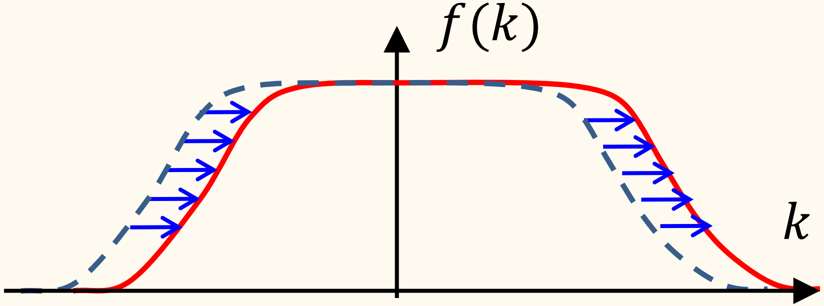
Two kinds of “current” in semiconductors

$$\boxed{F = U - TS}$$

U ➡ drift current, $-TS$ ➡ diffusion current

Drift current: Motive force (electric field) gives rise to non-uniformity in the momentum space.

$$-e\boldsymbol{E}\cdot\frac{\partial f}{\partial \boldsymbol{p}} = -\frac{f - f_0}{\tau(\boldsymbol{p})}$$



f: distribution function in phase space

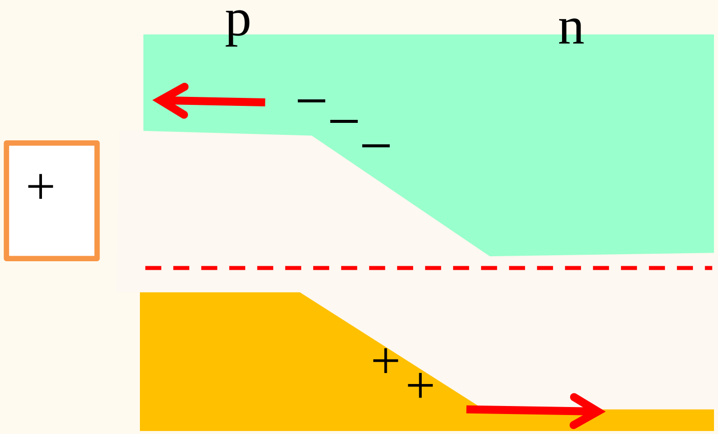
Diffusion current: There are some origins (particle sink/source) to cause non-uniformity in the real space (spatial density), which causes flow of charges.

$$\frac{\boldsymbol{p}}{m}\cdot\frac{\partial f}{\partial \boldsymbol{r}}$$

$$\boldsymbol{j} = -eD\frac{\partial \rho(\boldsymbol{r})}{\partial \boldsymbol{r}}$$

D: diffusion constant

4.4.1 I-V characteristics of pn junctions



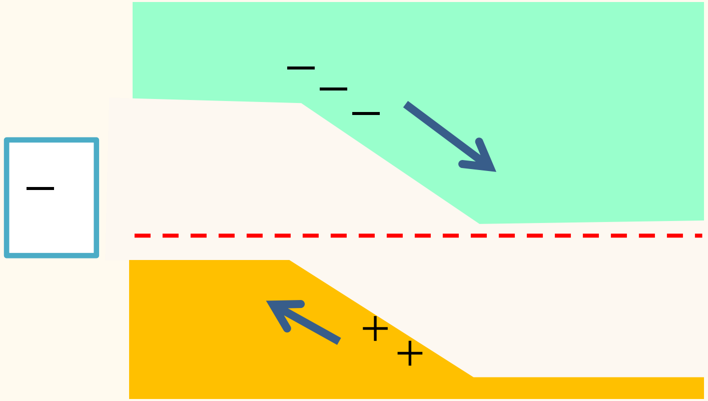
Forward bias

−

overcomes V_{bi}

: go

Minority carrier injection

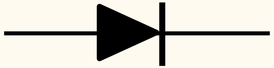


Reverse bias

+

enhances V_{bi}

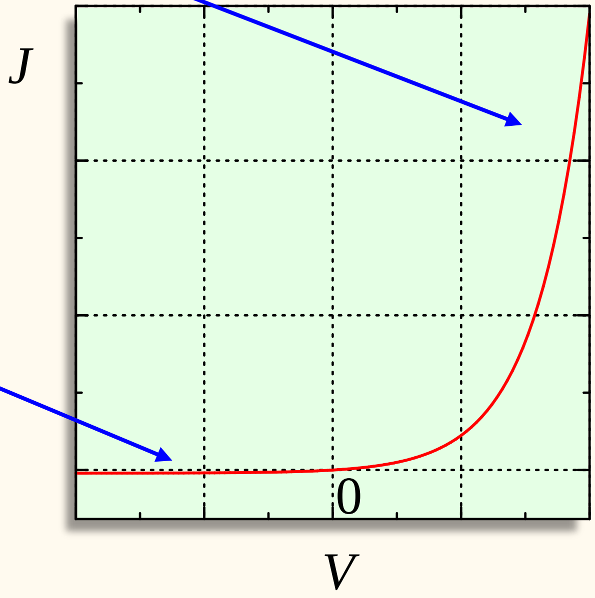
: no go



circuit symbol

Shockley theory

$$J = J_0\left[\exp\left(\frac{eV}{k_{\mathrm{B}}T}\right) - 1\right]$$

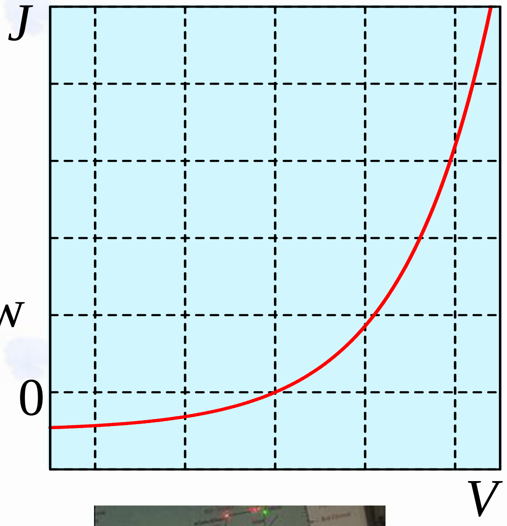


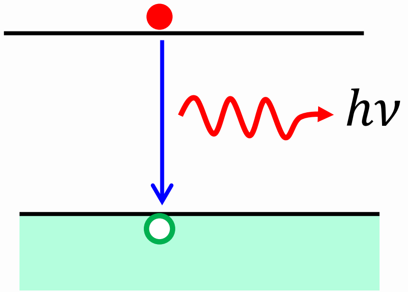
Injection of minority carriers

$$J = e\underline{(v_n n_p + v_p p_n)}\left[\underline{\exp\frac{eV}{k_{\mathrm{B}}T} - 1}\right]$$

minority carrier current

Barrier overflow



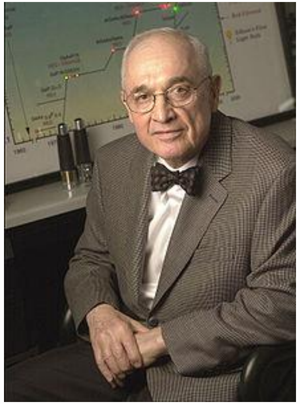


Fate of injected minority carriers:
Radiative recombination

Photo: A. Mahmoud
Isamu Akasaki

Photo: A. Mahmoud
Hiroshi Amano

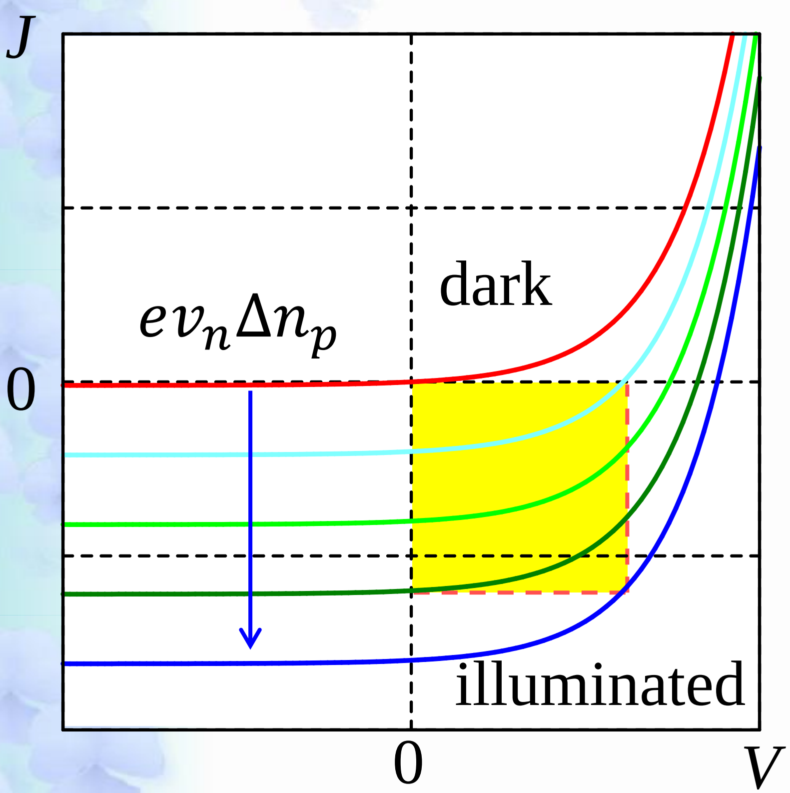
Photo: A. Mahmoud
Shuji Nakamura

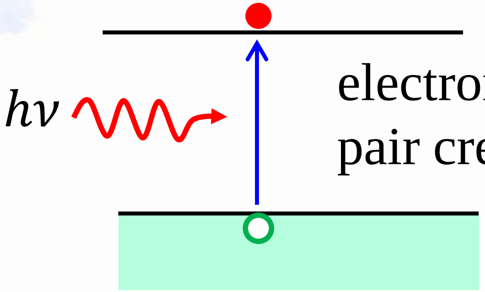


Nick Holonyak Jr.

light emitting diode

Solar cells (injection of minority carriers with light)





electron-hole
pair creation

$$J_{e0} = e v_n n_p \left[\exp \frac{eV}{k_{gB} T} - 1 \right]$$

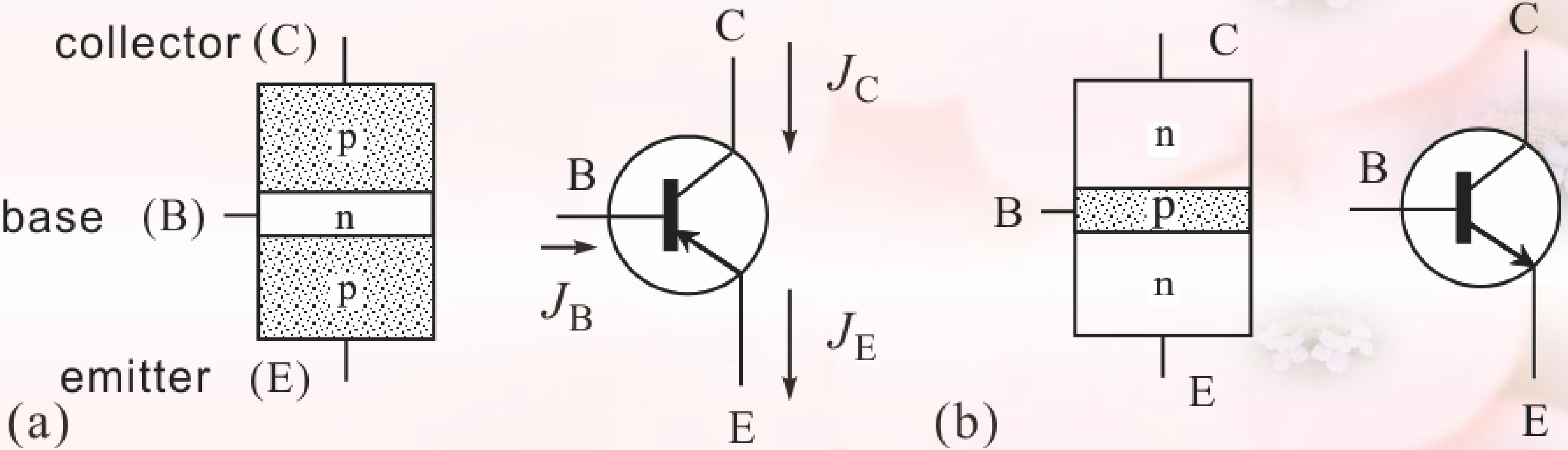
$$J_e = e v_n n_p \exp \frac{eV}{k_{\mathrm{B}} T} - e v_n (n_p + \Delta n_p)$$
$$= J_{n0} - e v_n \Delta n_p$$

external injection

Minority carriers which diffuse to the junction region are swept out to the other side.

Gerald Pearson, Daryl Chapin →
and Calvin Fuller at Bell labs. 1954

4.3.2 Bipolar transistors



PNP type

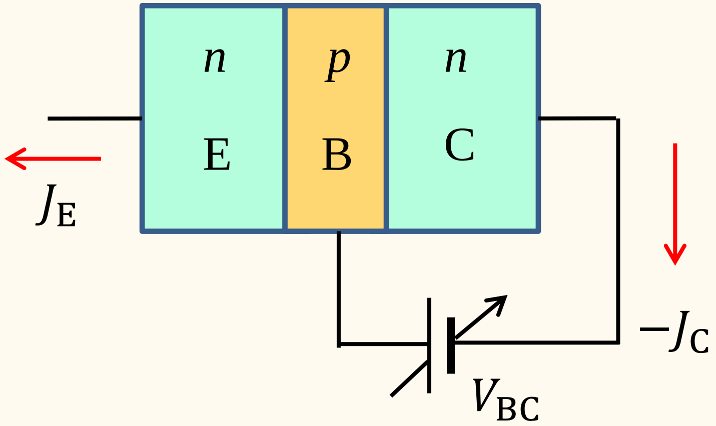
$$L_\mathrm{B} < L_h$$

NPN type

$$L_\mathrm{B} < L_e$$

Similar characteristics PNP and NPN: complementary

Base-Collector characteristics

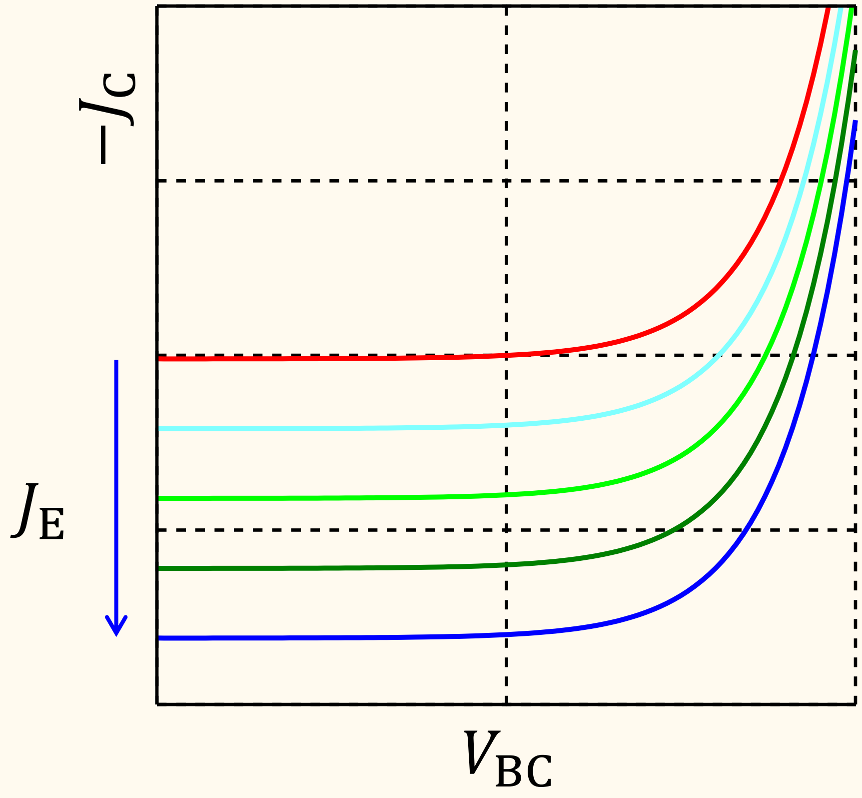


J_C increases with V_{BC}.

Light illumination in solar cells

⇕

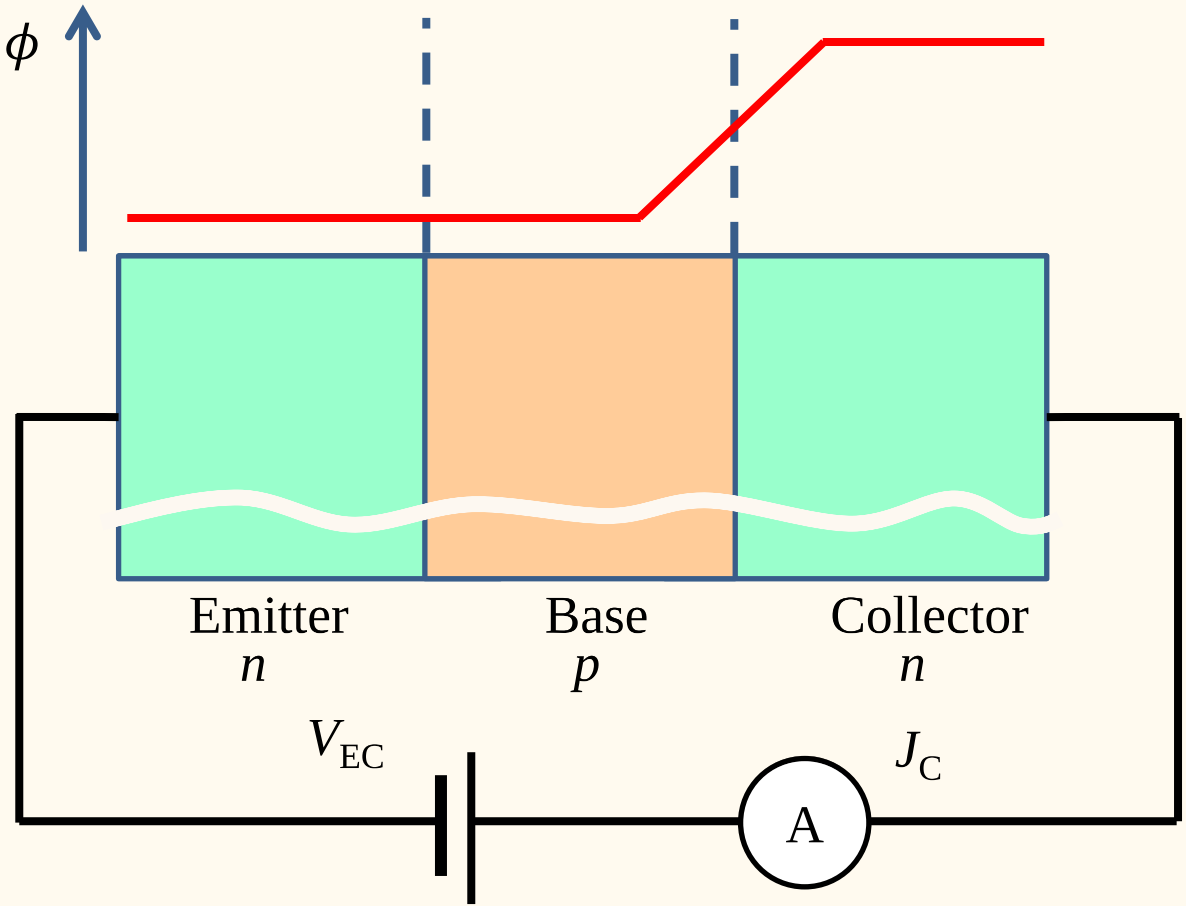
Emitter current



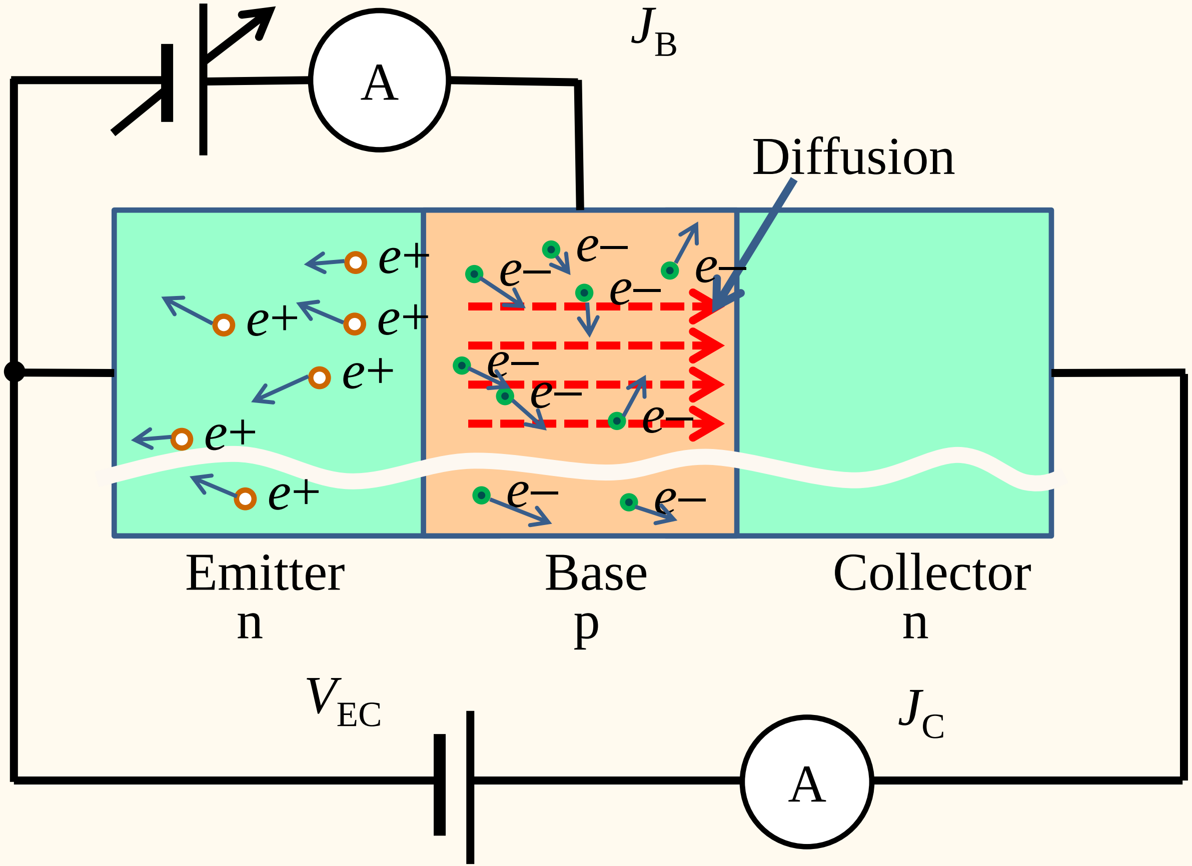
What is common?

Minority carrier injection

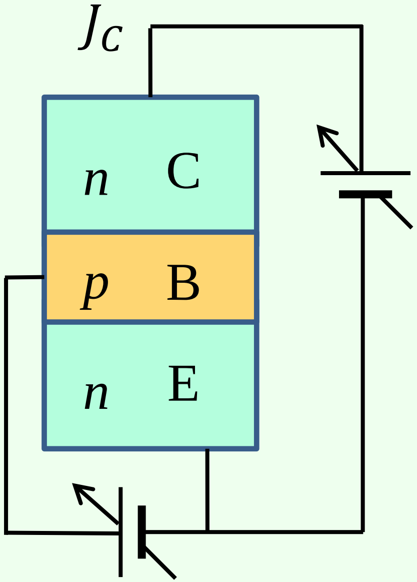
How a bipolar transistor amplifies signal?

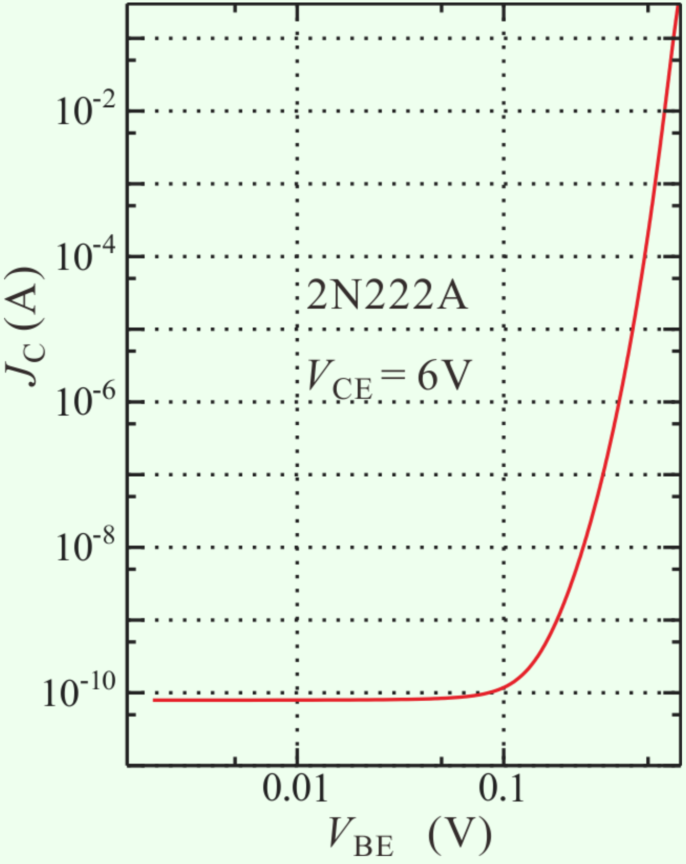


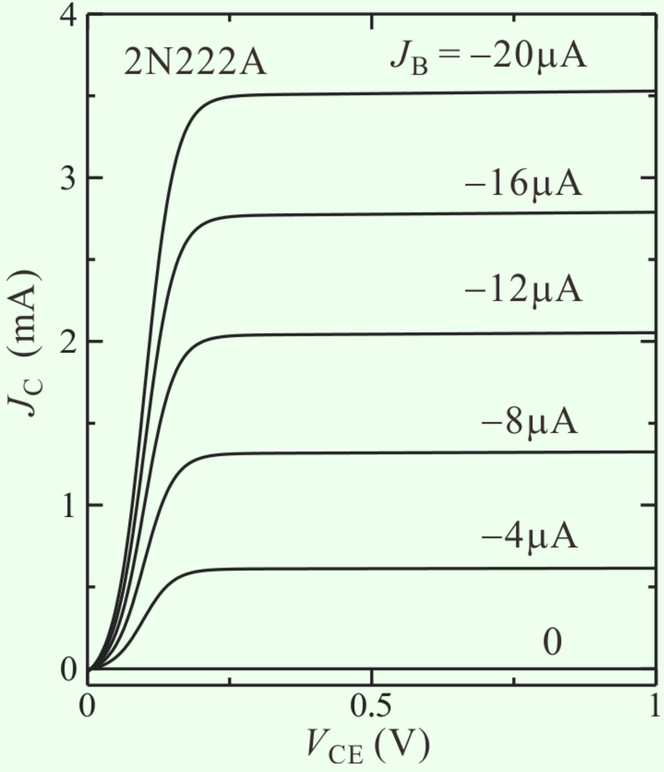
How a bipolar transistor amplifies signal?



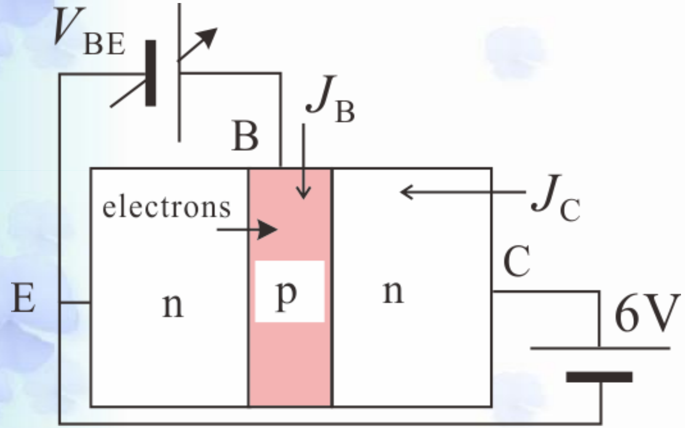
Collector-Emitter characteristics





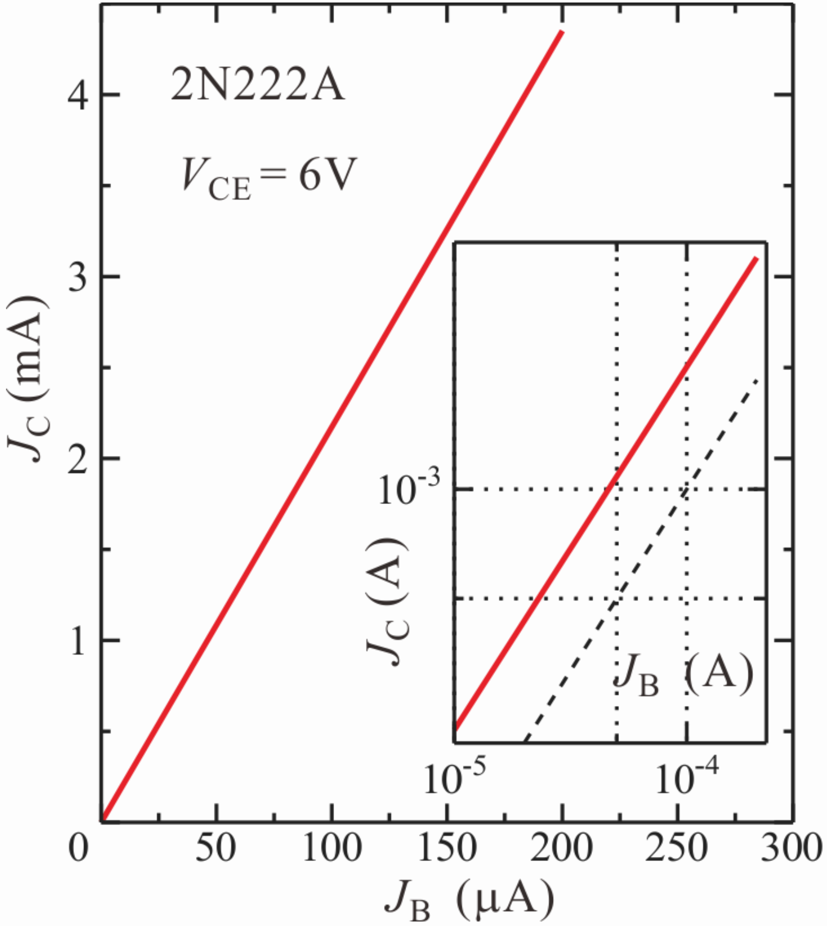


Current amplification: Linearize with quantity selection



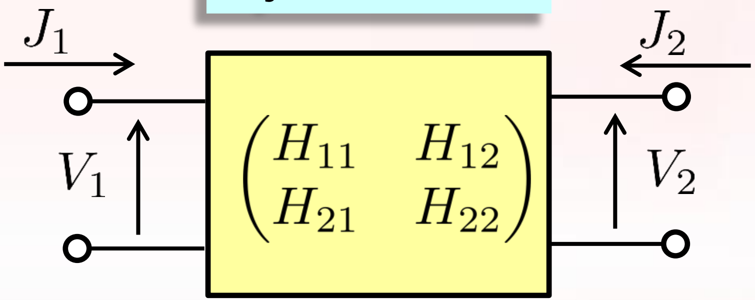
$$J_C = \underline{h_{FE}}\, J_B$$

Emitter-common current gain



Linear approximation of bipolar transistor

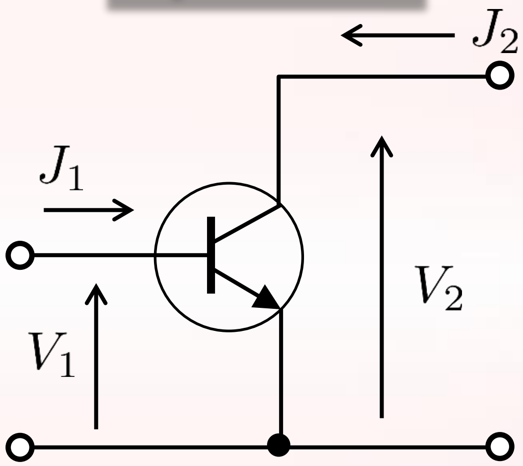
Hybrid matrix



$$\begin{pmatrix} V_1 \\ J_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} J_1 \\ V_2 \end{pmatrix}$$

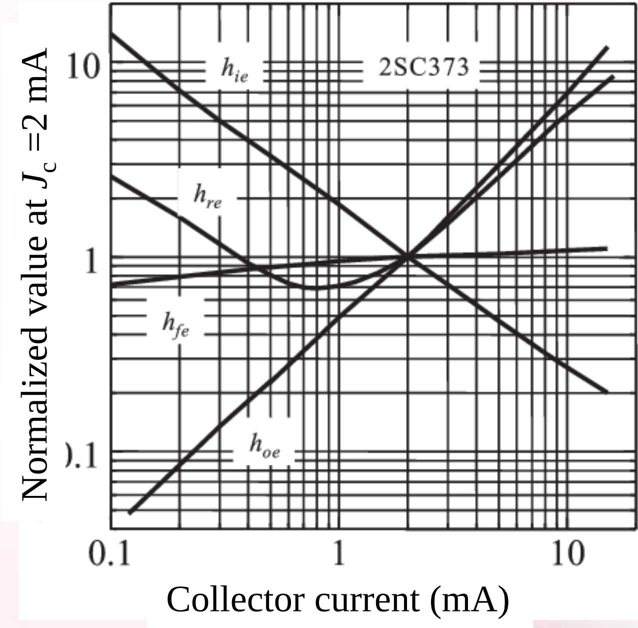
Linear approximation for non-linear devices:
Take derivative to have local linear matrices.

h-parameters



$$\begin{pmatrix} v_1 \\ j_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} j_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} h_i & h_r \\ h_f & h_o \end{pmatrix} \begin{pmatrix} j_1 \\ v_2 \end{pmatrix}$$

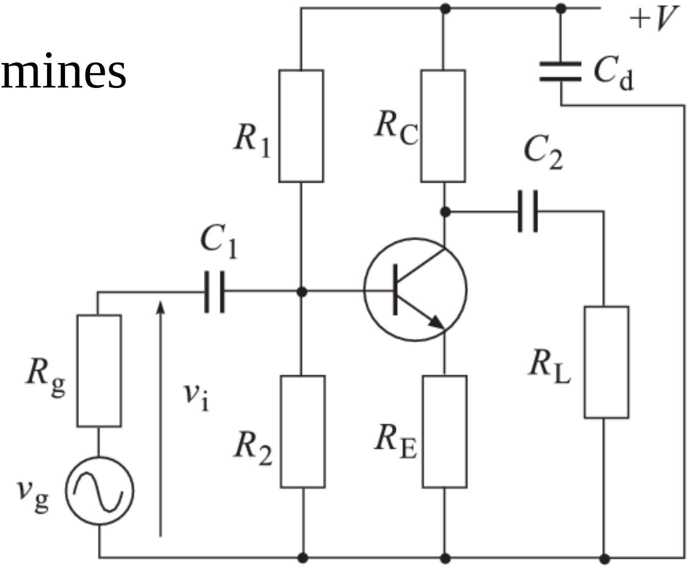
(lower case:
local linear approximation)



Concept of bias circuits for non-linear devices

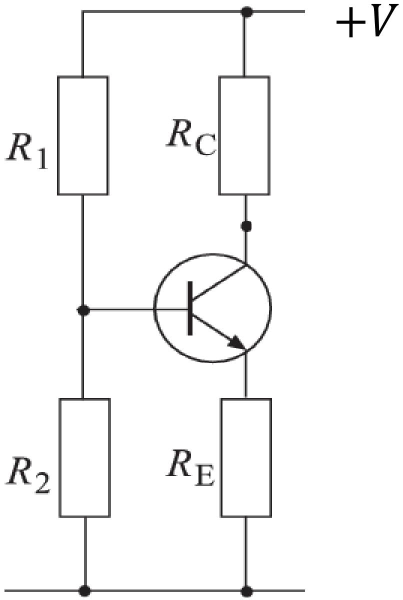
Bias circuit determines the action point.

Signal lines can be separated with reactive elements.



For bias (dc) circuits

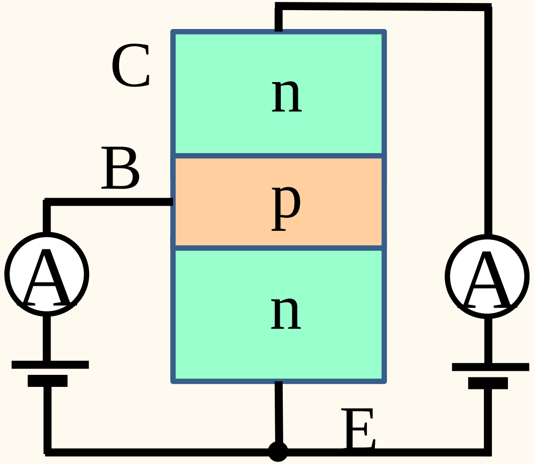
All the capacitors can be viewed as break line.

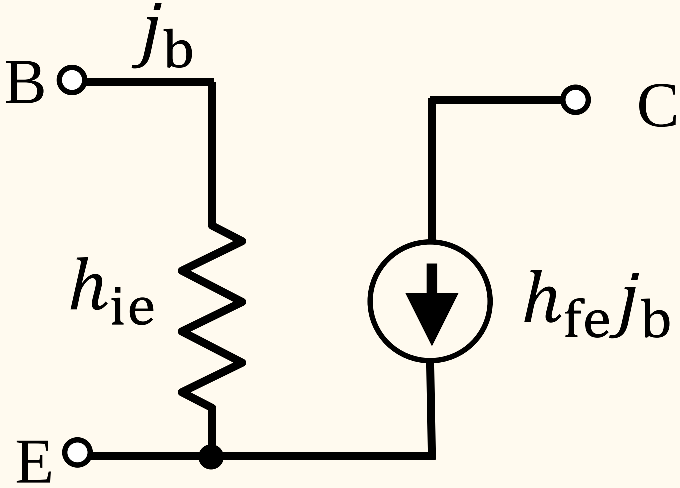


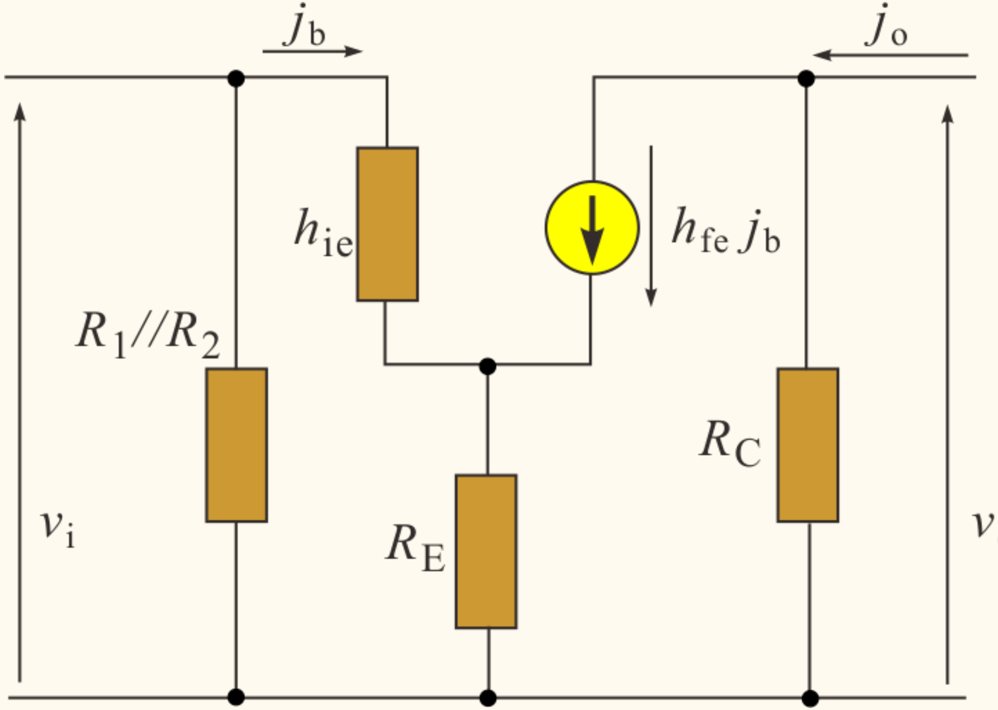
For small amplitude (high-frequency) circuits

All the capacitors can be viewed as short circuits.

Concept of equivalent circuit

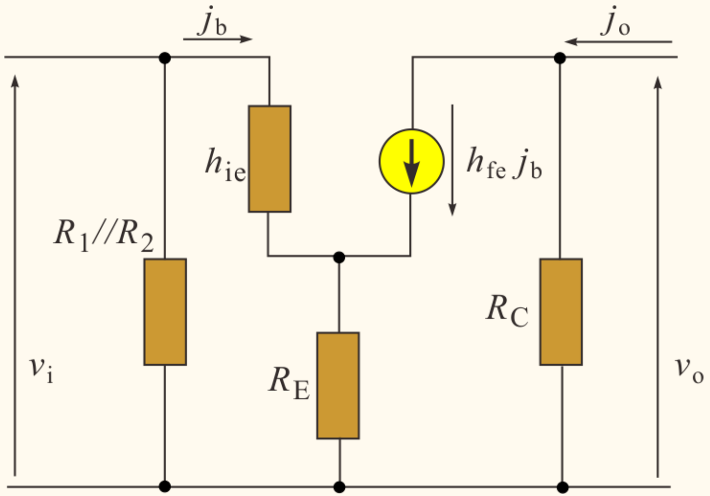




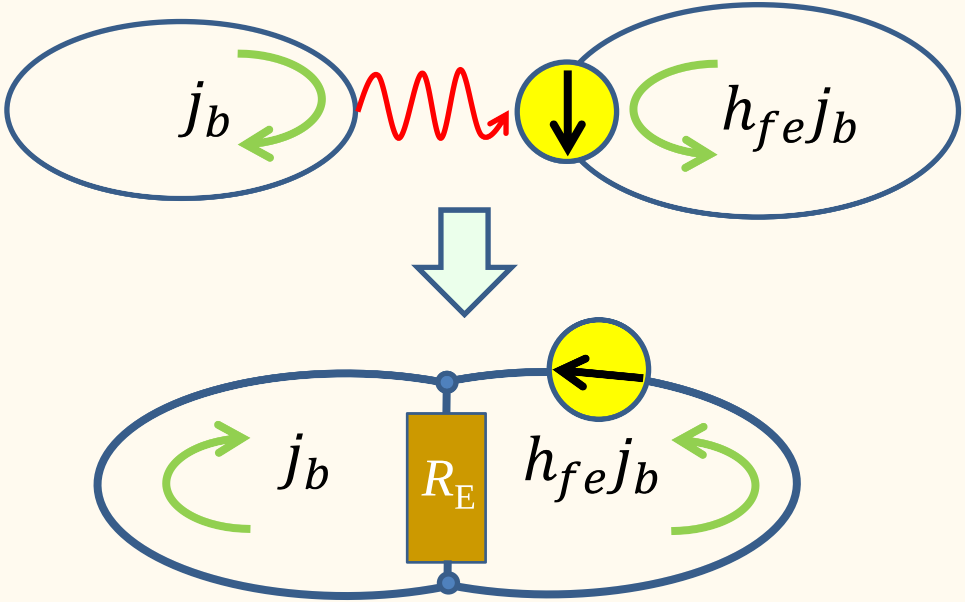


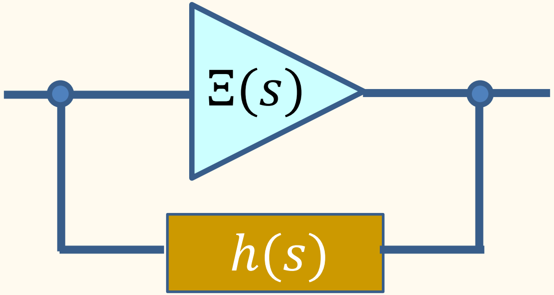
But resistors for bias circuit still work in signal circuit as long as they are not short circuited by capacitors.

Concept of equivalent circuit: Feedback?

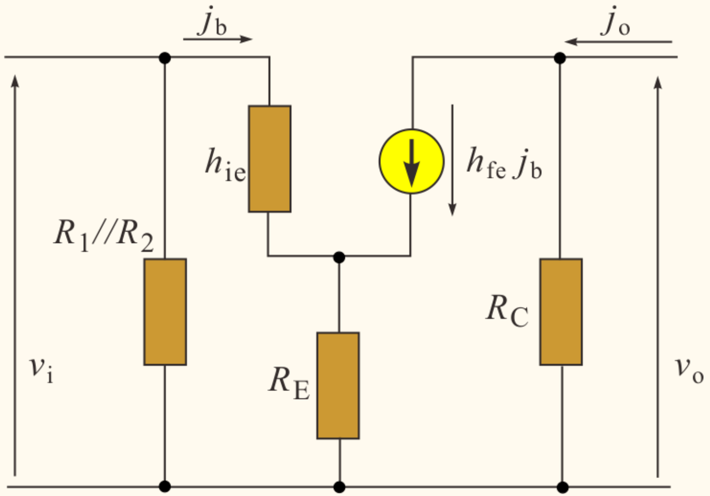


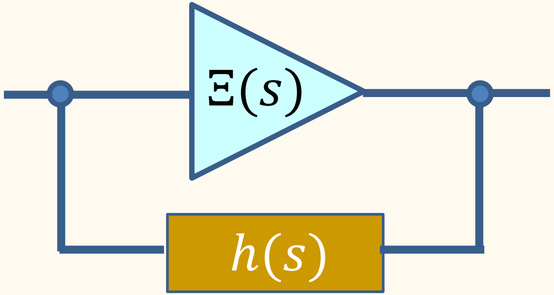
In the case of current-current linear circuit, a way for feedback is to have a common part the current loops.





Concept of equivalent circuit: Feedback?

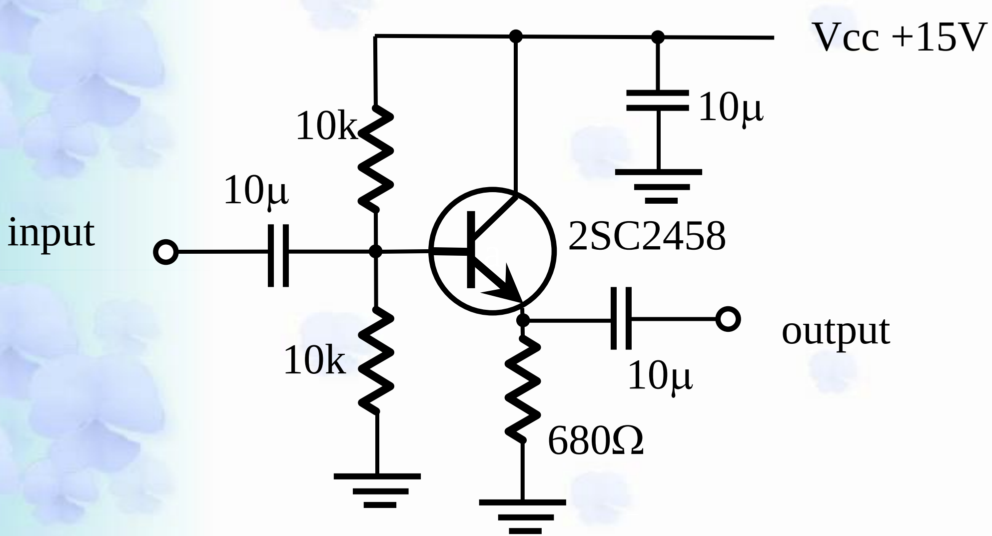


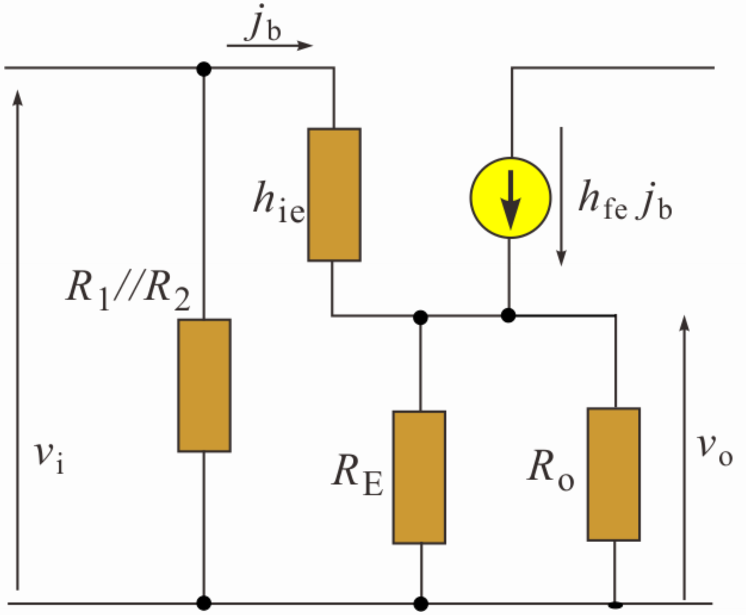


In the case of current-current linear circuit, a way for feedback is to have a common part the current loops.

$$\begin{aligned} A &= \frac{v_0}{v_i} \\ &= \frac{h_{fe} R_C}{h_{ie} + R_E(1 + h_{fe})} \\ &\approx \frac{R_C}{R_E} \qquad h_{fe} \gg 1 \end{aligned}$$

Current amplification: Emitter follower



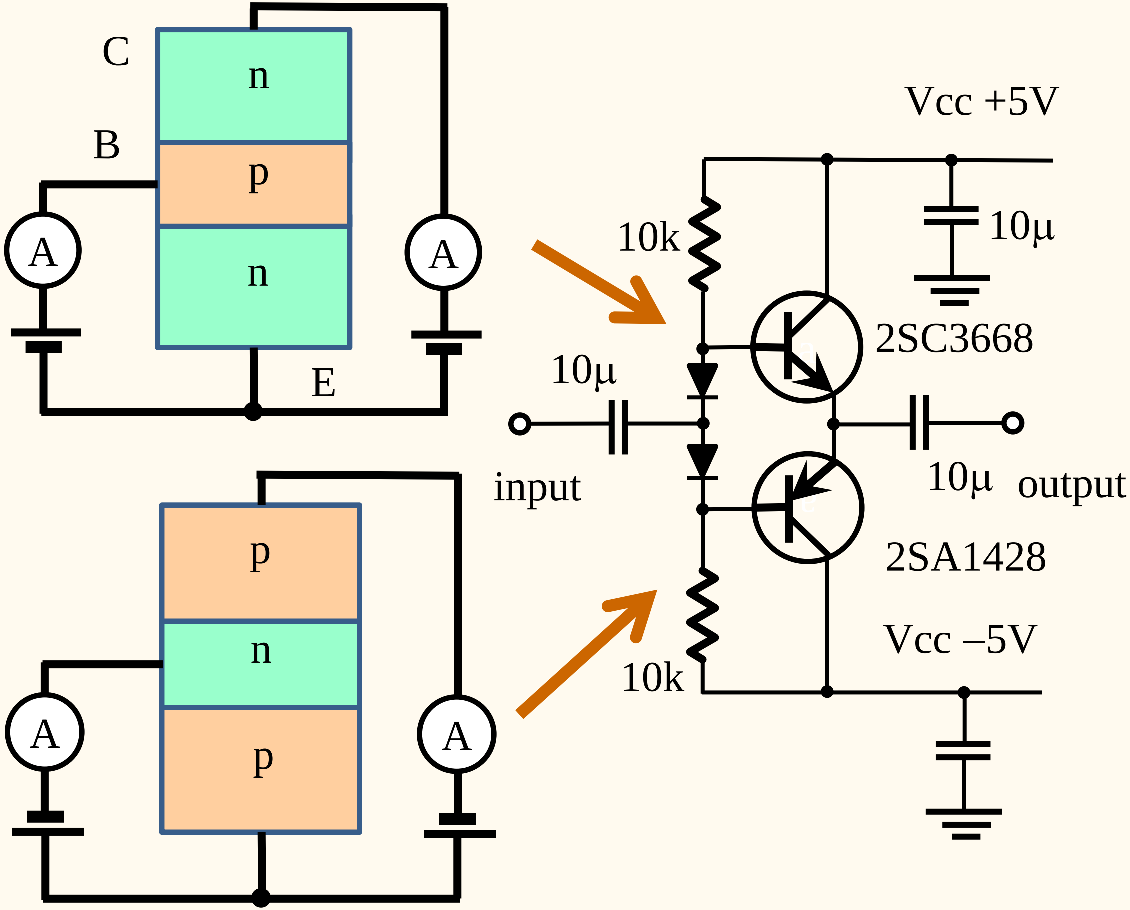


$$\frac{v_o}{v_i} = \frac{j_b(1+h_{fe})(R_E \parallel R_o)}{j_b[h_{ie} + (1+h_{fe})(R_E \parallel R_o)]}$$
$$\approx 1 \qquad (h_{fe} \gg 1)$$

v_o does not depend on load resistance

⇒ Very low output resistance

Complementary transistors

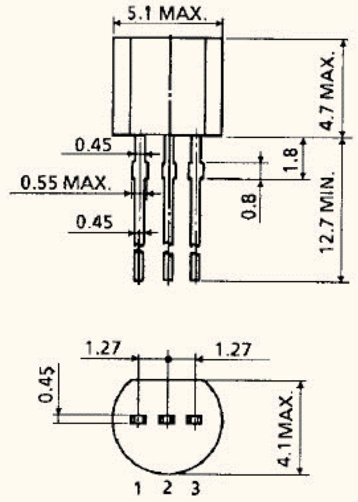


Symmetric characteristics:
Complementary

Symmetric: Small collector current (idling current) for zero input.

Example of transistor datasheet

Unit: mm



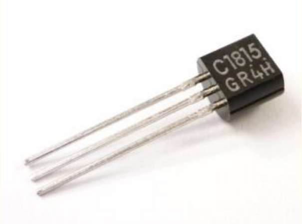
1. EMITTER
2. COLLECTOR
3. BASE

JEDEC	TO-92
JEITA	SC-43
TOSHIBA	2-5F1B

TOSHIBA 2SC1815(L)

TOSHIBA Transistor Silicon NPN Epitaxial Type (PCT process)

2SC1815(L)

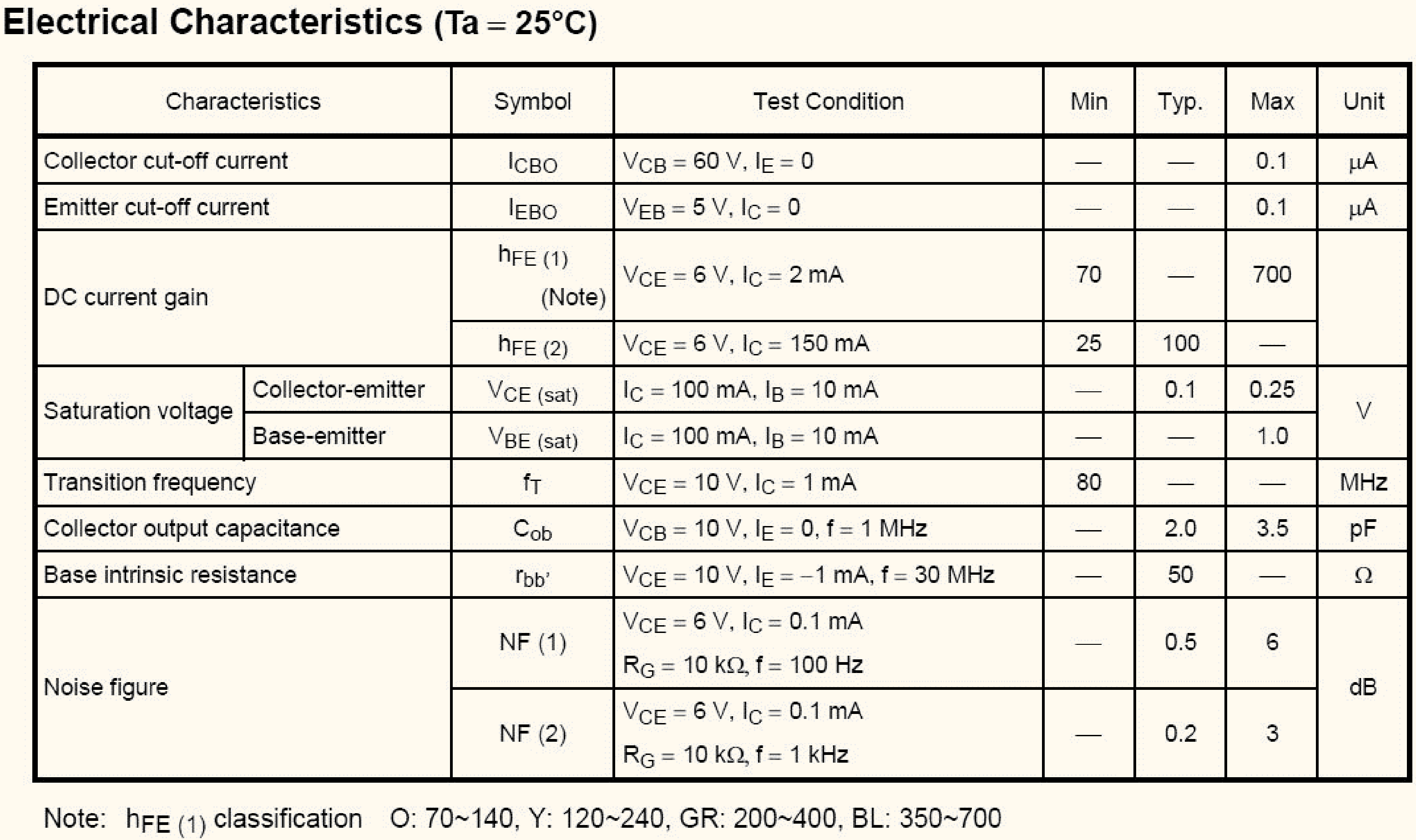


Audio Frequency Voltage Amplifier Applications

Low Noise Amplifier Applications

- High breakdown voltage, high current capability
  : $V_{CEO} = 50$ V (min), $I_C = 150$ mA (max)
- Excellent linearity of h_{FE}
  : $h_{FE\ (2)} = 100$ (typ.) at $V_{CE} = 6$ V, $I_C = 150$ mA
  : h_{FE} ($I_C = 0.1$ mA)/h_{FE} ($I_C = 2$ mA) = 0.95 (typ.)
- Low noise: NF = 0.2dB (typ.) (f = 1 kHz).
- Complementary to 2SA1015 (L). (O, Y, GR class).

Example of transistor datasheet (2SC1815(L))

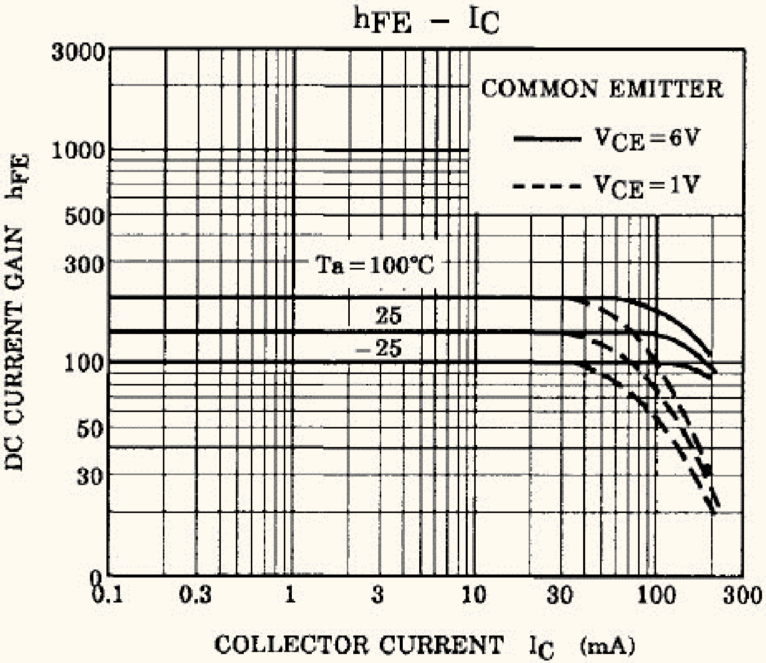


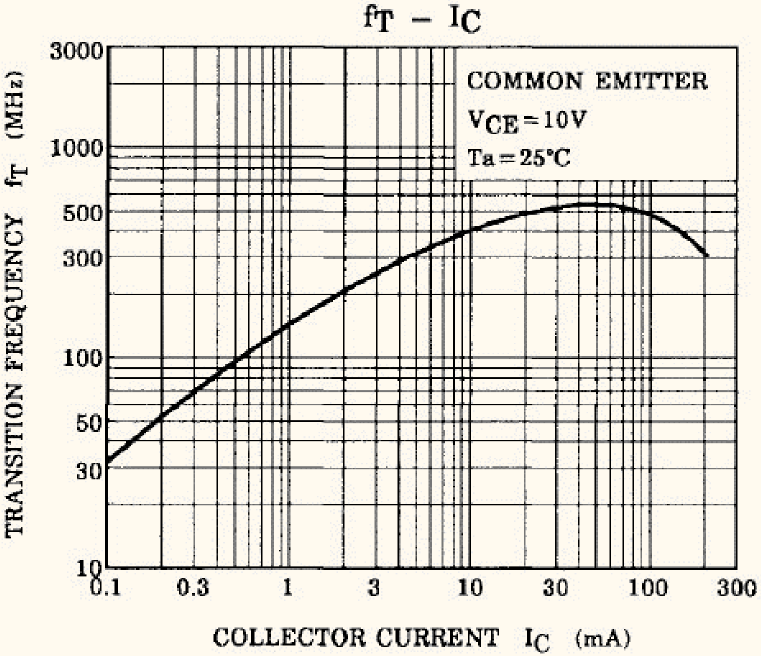
Electrical Characteristics (Ta = 25°C)

Characteristics		Symbol	Test Condition	Min	Typ.	Max	Unit
Collector cut-off current		I_{CBO}	V_{CB} = 60 V, I_E = 0	—	—	0.1	μA
Emitter cut-off current		I_{EBO}	V_{EB} = 5 V, I_C = 0	—	—	0.1	μA
DC current gain		$h_{FE\ (1)}$ (Note)	V_{CE} = 6 V, I_C = 2 mA	70	—	700	
		$h_{FE\ (2)}$	V_{CE} = 6 V, I_C = 150 mA	25	100	—	
Saturation voltage	Collector-emitter	$V_{CE\ (sat)}$	I_C = 100 mA, I_B = 10 mA	—	0.1	0.25	V
	Base-emitter	$V_{BE\ (sat)}$	I_C = 100 mA, I_B = 10 mA	—	—	1.0	
Transition frequency		f_T	V_{CE} = 10 V, I_C = 1 mA	80	—	—	MHz
Collector output capacitance		C_{ob}	V_{CB} = 10 V, I_E = 0, f = 1 MHz	—	2.0	3.5	pF
Base intrinsic resistance		$r_{bb'}$	V_{CE} = 10 V, I_E = −1 mA, f = 30 MHz	—	50	—	Ω
Noise figure		NF (1)	V_{CE} = 6 V, I_C = 0.1 mA R_G = 10 kΩ, f = 100 Hz	—	0.5	6	dB
		NF (2)	V_{CE} = 6 V, I_C = 0.1 mA R_G = 10 kΩ, f = 1 kHz	—	0.2	3	

Note: $h_{FE\ (1)}$ classification O: 70~140, Y: 120~240, GR: 200~400, BL: 350~700

Example of transistor datasheet (2SC1815(L))





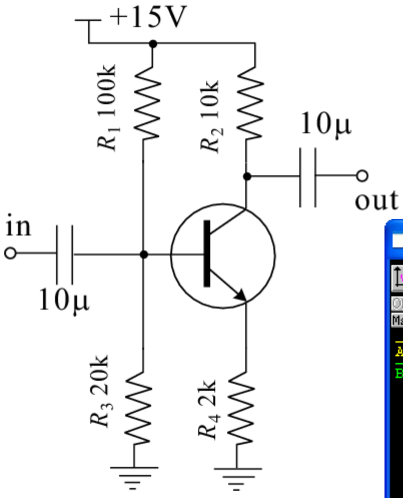
h_{fe} linear model availability in the range of J_C.

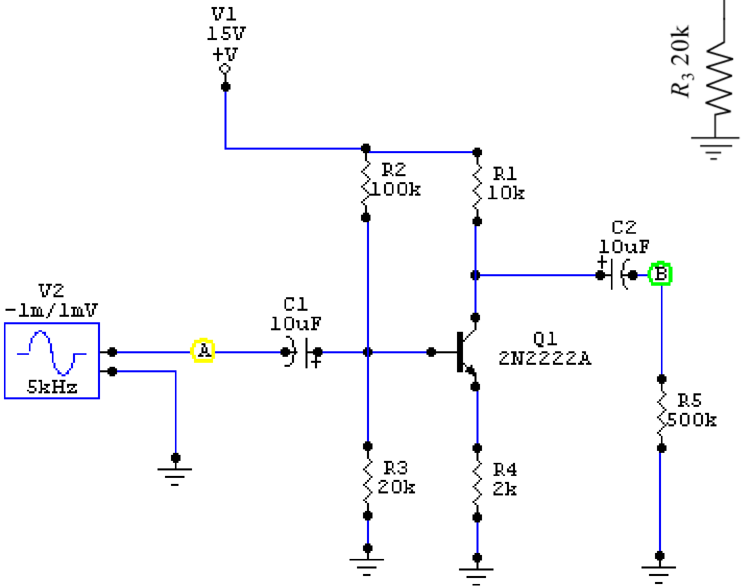
Cut-off frequency as a function of J_C

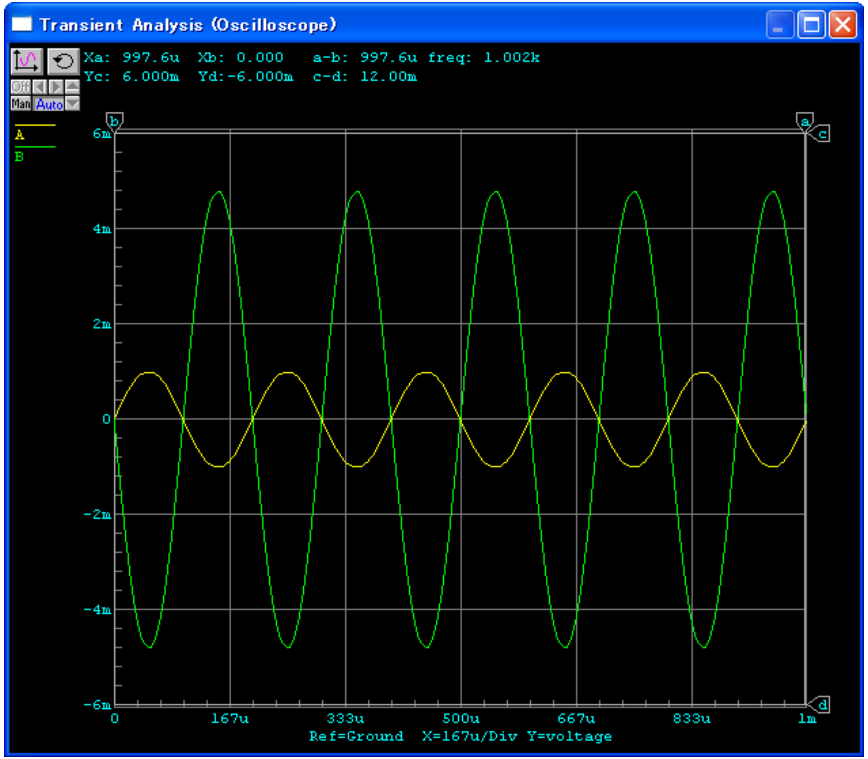
Simulation example

$$\Delta V_C = R_2 \Delta J_C \approx R_2 \Delta J_E$$

$$= R_2 \frac{\Delta V_E}{R_4} = \frac{R_2}{R_4} \Delta V$$







```
Xa: 997.6u  Xb: 0.000    a-b: 997.6u freq: 1.002k
Yc: 6.000m  Yd:-6.000m   c-d: 12.00m
```

Ref=Ground X=167u/Div Y=voltage