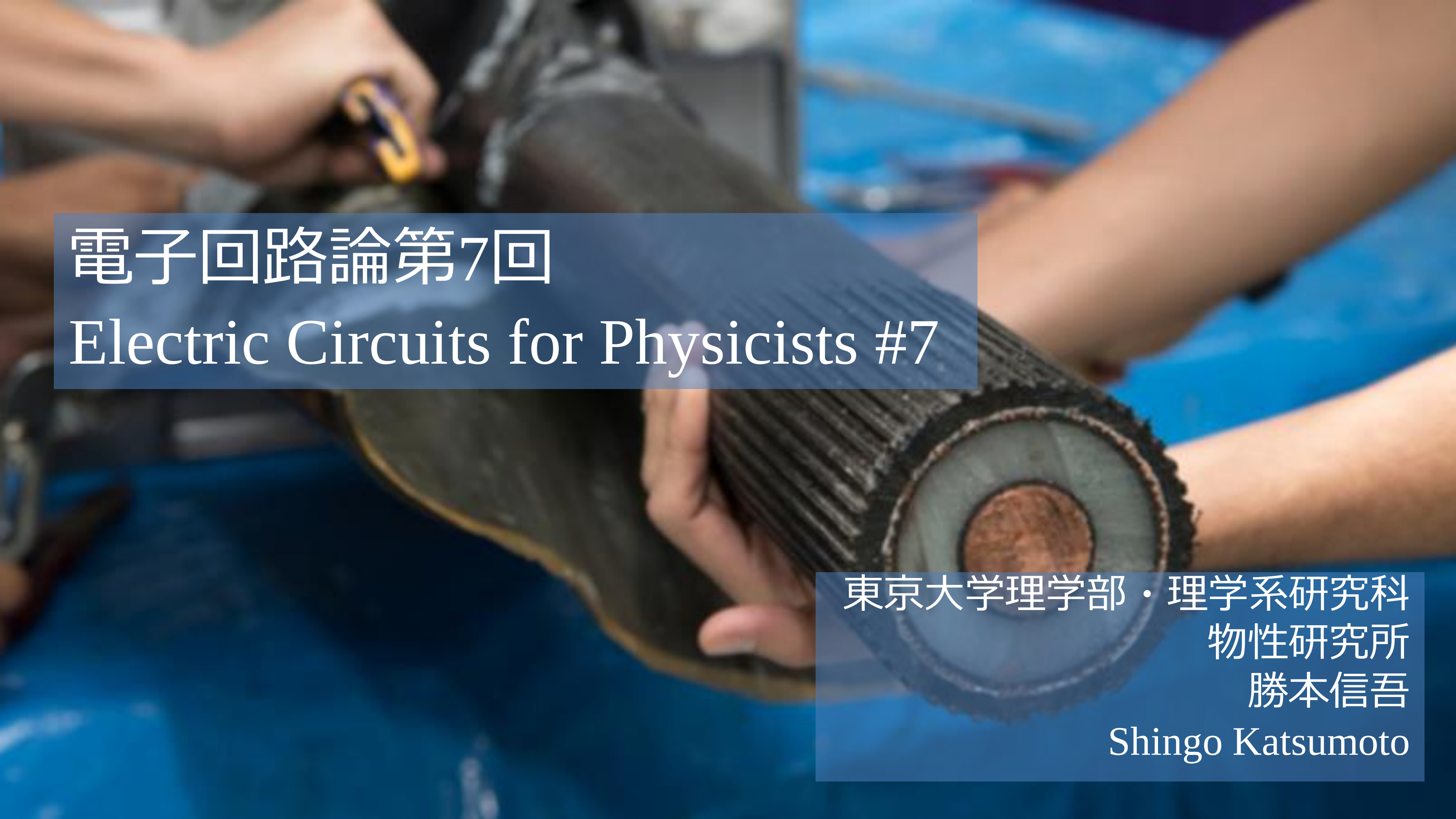

電子回路論第7回

Electric Circuits for Physicists #7

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Outline

4.5 Field Effect Transistors (FETs)

Ch.5 Distributed constant circuits

5.1 Transmission lines

5.1.1 Coaxial cables

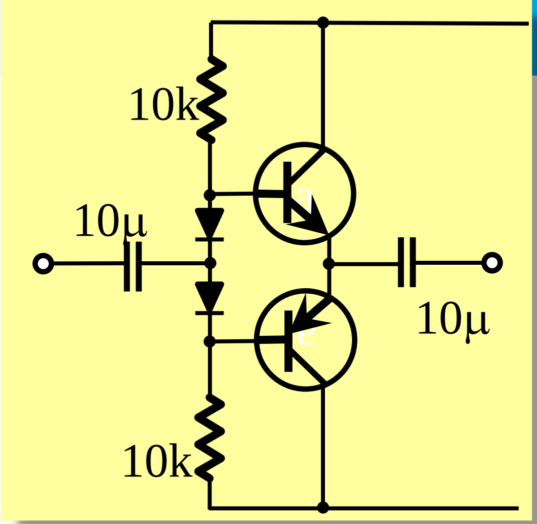
5.1.2 Lecher lines

5.1.3 Micro-strip lines

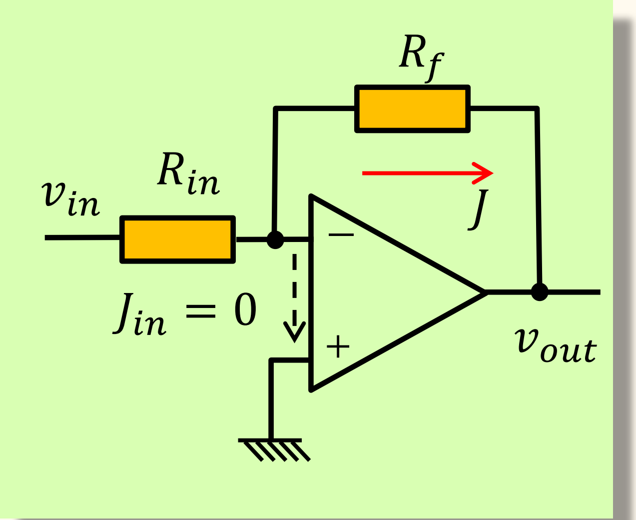
5.2 Wave propagation through transmission lines

5.2.2 Connection and termination of transmission lines

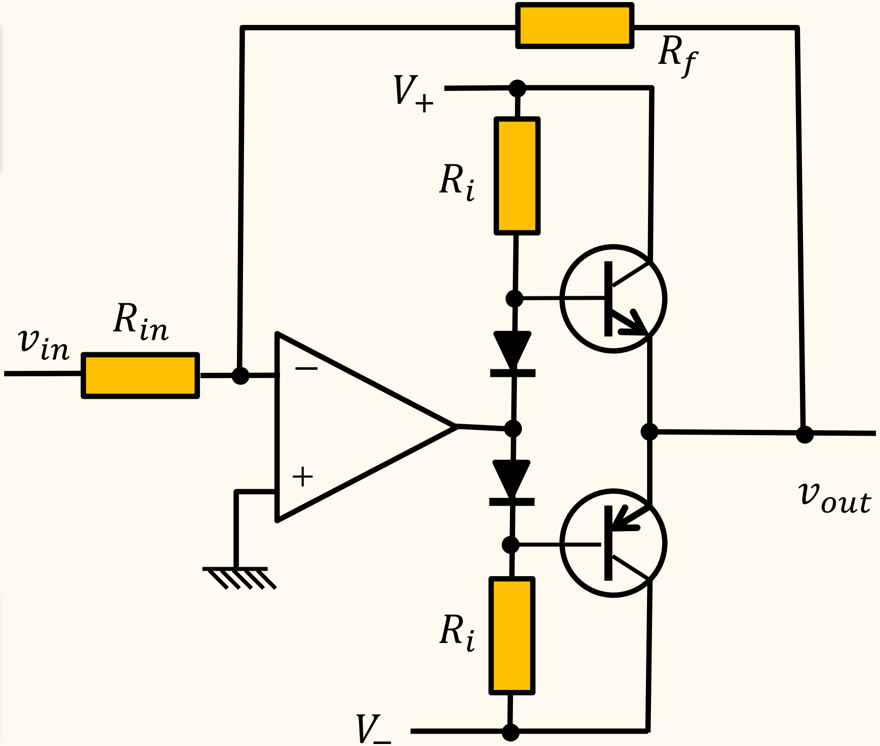
Combination of op-amp and discrete transistors



Complementary push-pull

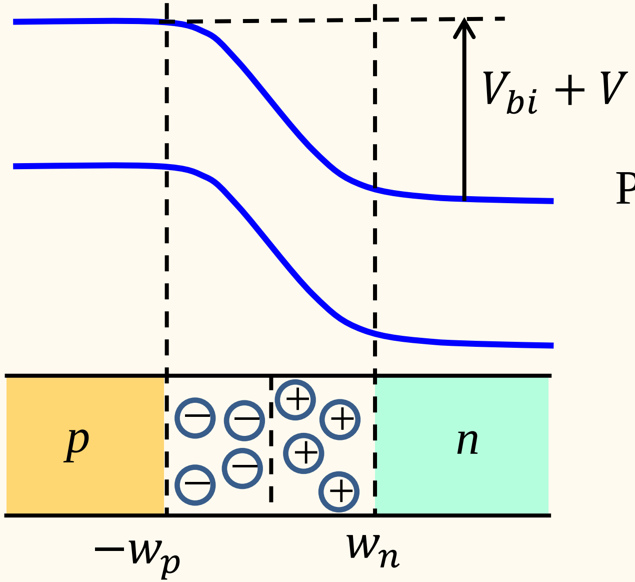


Inversion amplifier



Voltage, current booster

pn-junction in reverse bias region



Built-in potential → Depletion layer

Poisson equation $$\frac{d^2\phi}{dx^2} = -aq(x) \quad (a \equiv (\epsilon\epsilon_0)^{-1})$$

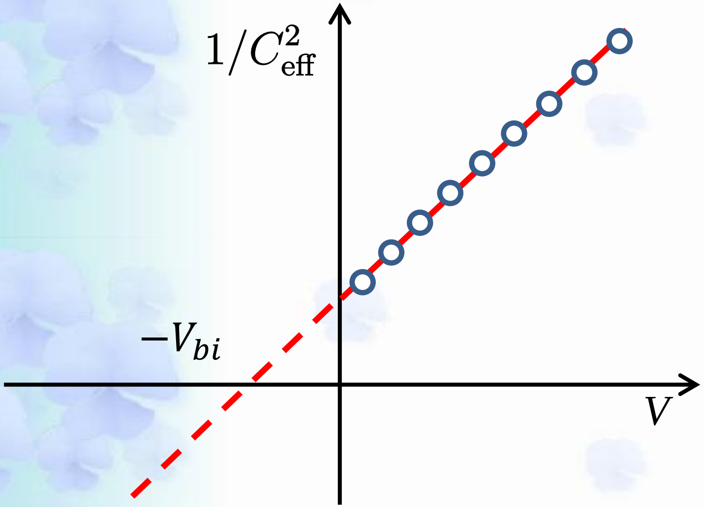
Space charge density: $$\begin{cases} q = -eN_A & (-w_p \le x \le 0), \\ q = eN_D & (0 \le x \le w_n) \end{cases}$$

Boundary condition: $$\phi(-w_p) = 0, \quad \phi(-\infty) = 0$$

$$\left.\frac{d\phi}{dx}\right|_{-w_p} = 0,\ \phi(w_n) = V + V_{\rm bi}, \quad \left.\frac{d\phi}{dx}\right|_{w_n} = 0$$

Solution: $$\phi(x) = \begin{cases} (aeN_A/2)(x + w_p)^2 & (-w_p \le x \le 0), \\ V + V_{\rm bi} - (aeN_D/2)(x - w_n)^2 & (0 \le x \le w_n) \end{cases}$$

Varicap diode

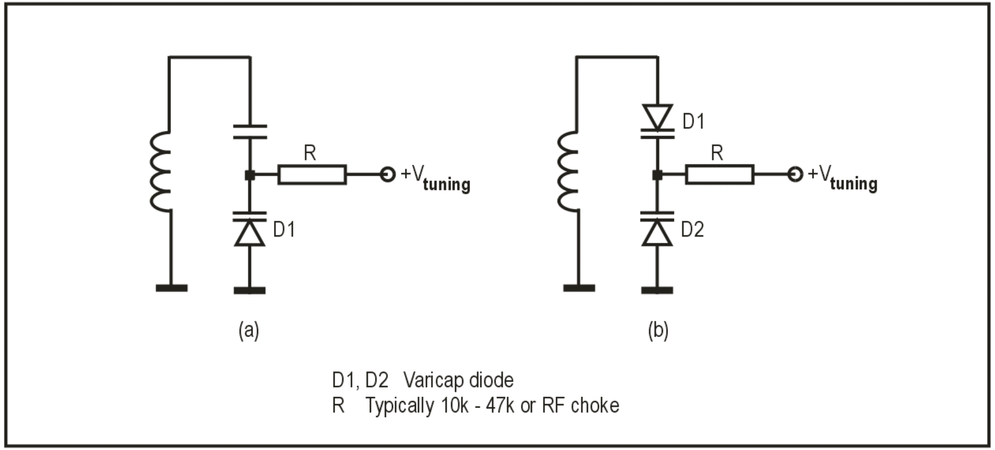


$$w_d = w_p + w_n = A\sqrt{V + V_{\mathrm{bi}}}$$

Reverse bias voltage widens depletion layer.

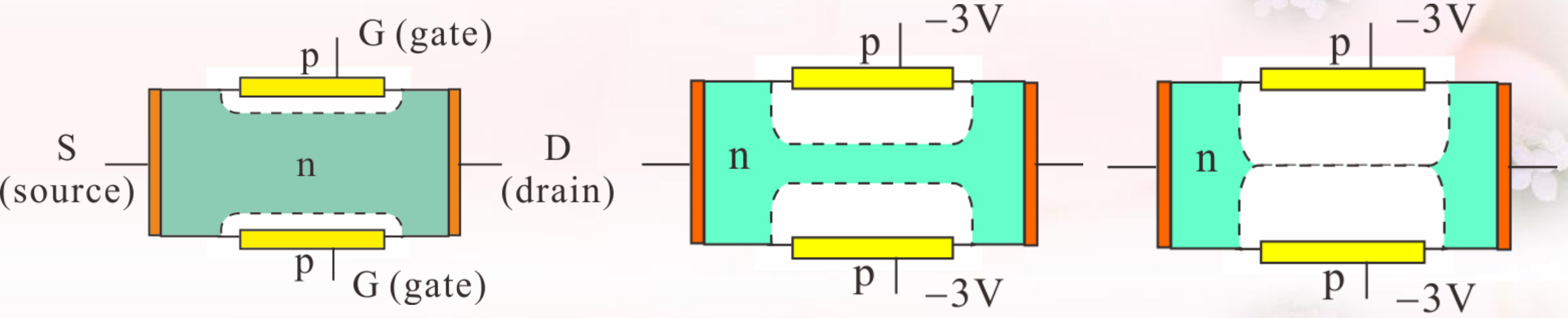
$$\frac{1}{C_{\mathrm{eff}}^2} = \frac{2}{\epsilon\epsilon_0 e N_{\mathrm{D}}}(V + V_{\mathrm{bi}})$$

Varicap diode

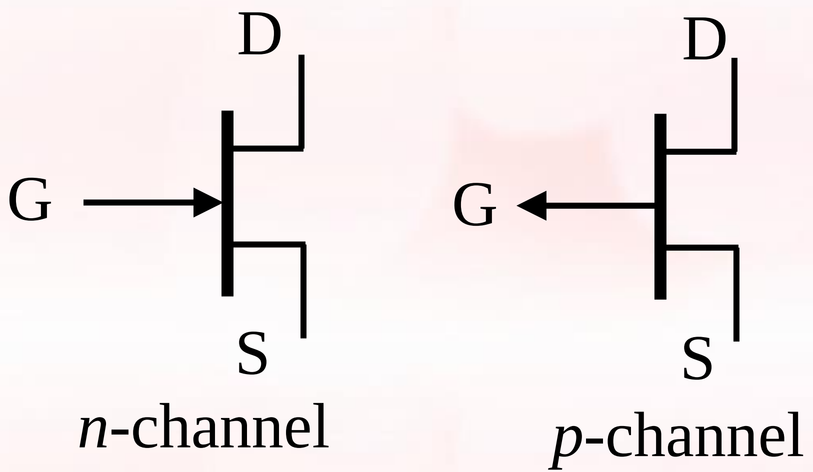


Frequency modulation (FM), Phase lock loop (PLL)

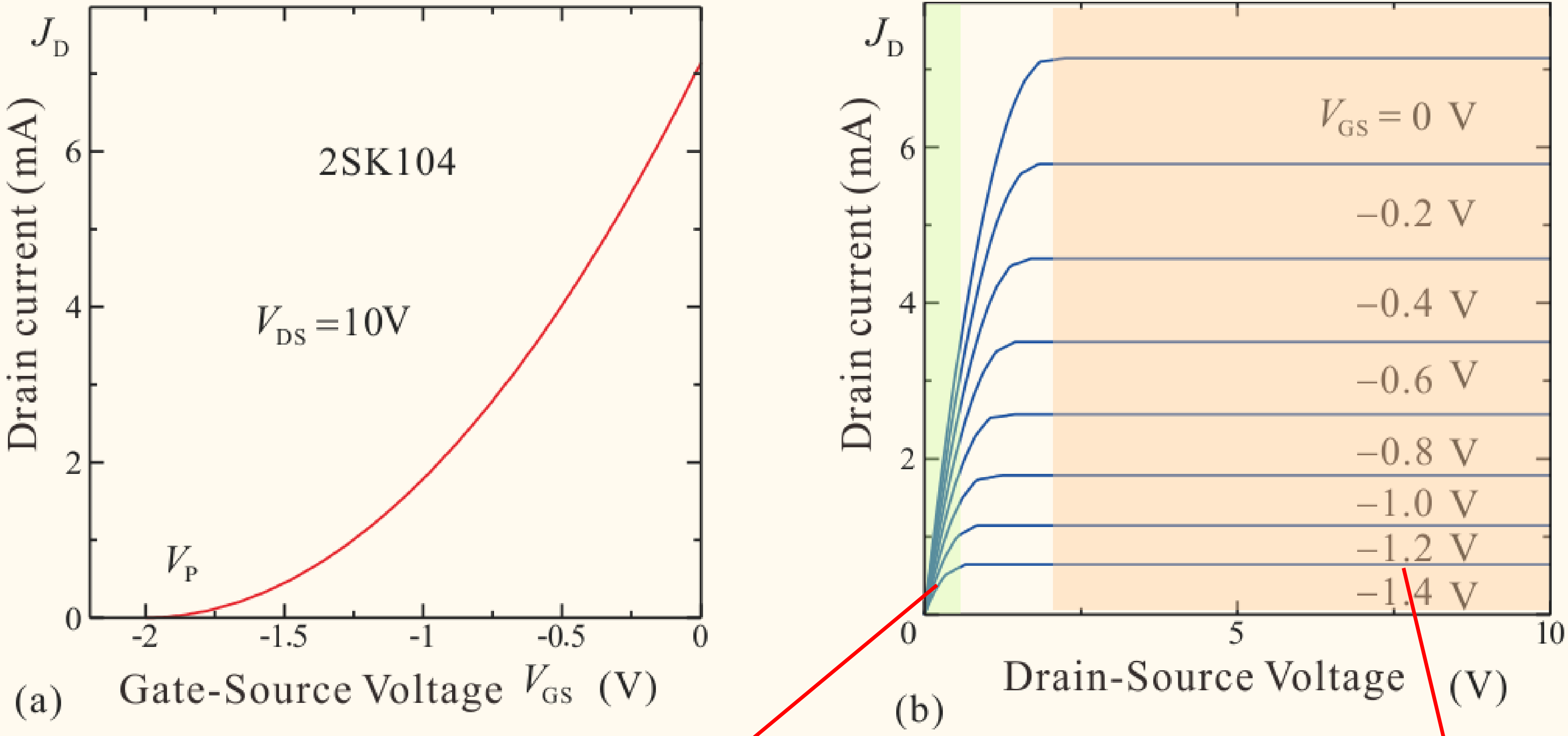
4.4 Field effect transistor (FET)



Circuit symbols



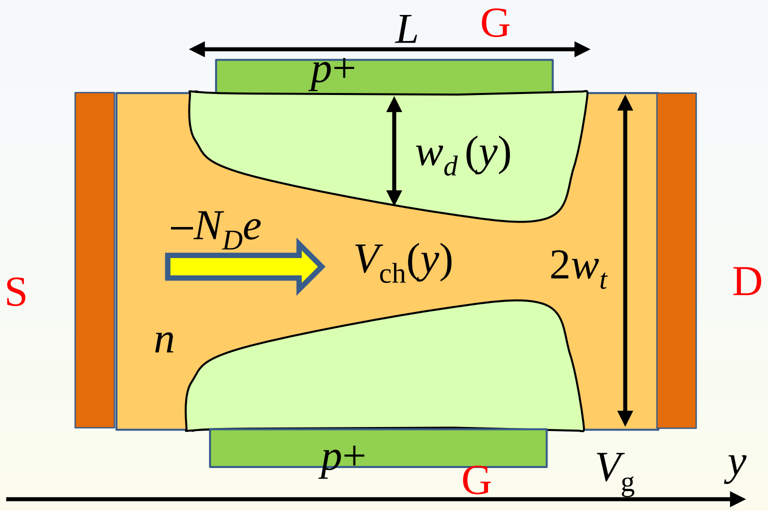
Static characteristics of FET



Ohmic area

Space charge limited area

Space-charge limitation of source-drain current



Travel distance (y) dependent potential

$$V(y) = V_g + V_{vi} - V_{ch}(y)$$

Depletion layer width

$$w_d(y) = \sqrt{\frac{2\epsilon\epsilon_0 V(y)}{eN_D}}$$

$$J_{ch} = eN_D\mu_n \frac{dV_{ch}}{dy} \cdot 2[w_t - w_d(y)]W$$

conductivity electric field channel width

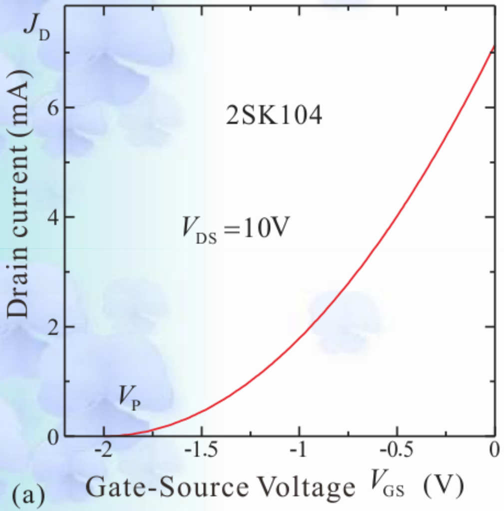
$$J_{ch}L = \int_0^L J_{ch}dy = 2eN_D\mu_n W \int_0^L (w_t - w_d)\frac{dV}{dy}dy = 2w_t eN_D\mu_n W \int_{V_0}^{V_L} \left(1 - \frac{w_d}{w_t}\right) dV$$

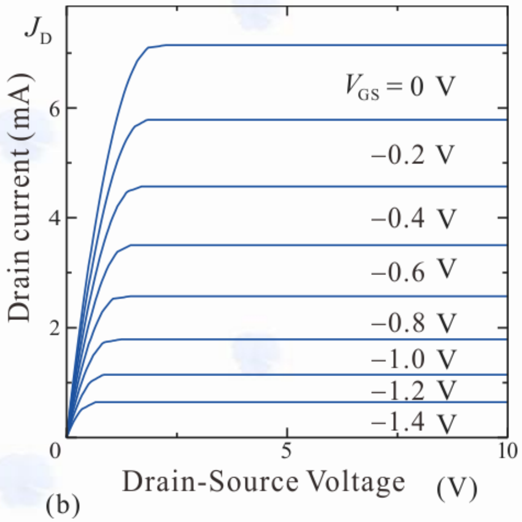
$$w_d(V_c) = w_t \qquad \therefore V_c = \frac{eN_D w_t^2}{2\epsilon\epsilon_0}$$

Approx. for $w_d < w_t/2$

$$J_{ch} = \frac{2N_D e\mu_n W w_t}{L}\left[V_L - V_0 + \frac{2}{3\sqrt{V_c}}\left(V(V_0)^{3/2} - V(V_L)^{3/2}\right)\right]$$

Static characteristics of FET





$$J_{\mathrm{G}} \simeq 0, \quad J_{\mathrm{D}} = f(V_{\mathrm{G}}, V_{\mathrm{DS}})$$

$$g_{\mathrm{m}} \equiv \left(\frac{\partial J_{\mathrm{D}}}{\partial V_{\mathrm{GS}}}\right)_{V_{\mathrm{DS}}=\mathrm{const.}}$$

transconductance

$$r_{\mathrm{d}} \equiv \left(\frac{\partial V_{\mathrm{DS}}}{\partial J_{\mathrm{D}}}\right)_{V_{\mathrm{GS}}=\mathrm{const.}}$$

drain resistance

Local linear approximation: $$j_{\mathrm{d}} = g_m v_{\mathrm{gs}} + \frac{v_{\mathrm{ds}}}{r_{\mathrm{d}}}$$

$$v_{\mathrm{ds}} = -r_{\mathrm{d}} g_{\mathrm{m}} v_{\mathrm{gs}} + r_{\mathrm{d}} j_{\mathrm{d}}$$

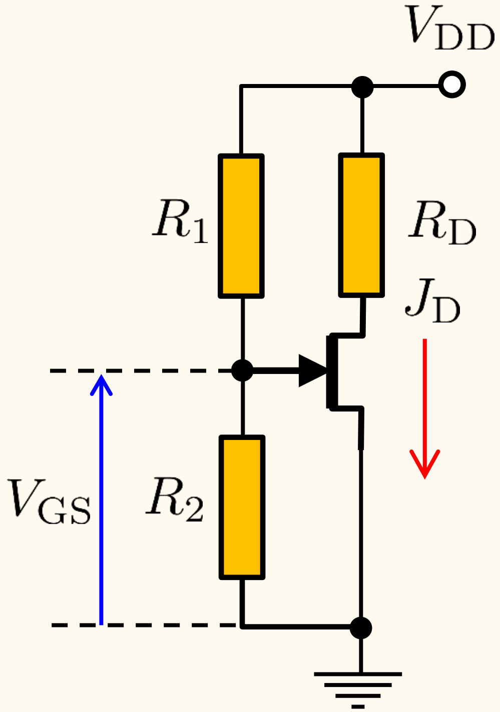
Amplification factor (voltage gain)

$$\boxed{\mu \equiv r_{\mathrm{d}} g_{\mathrm{m}}}$$

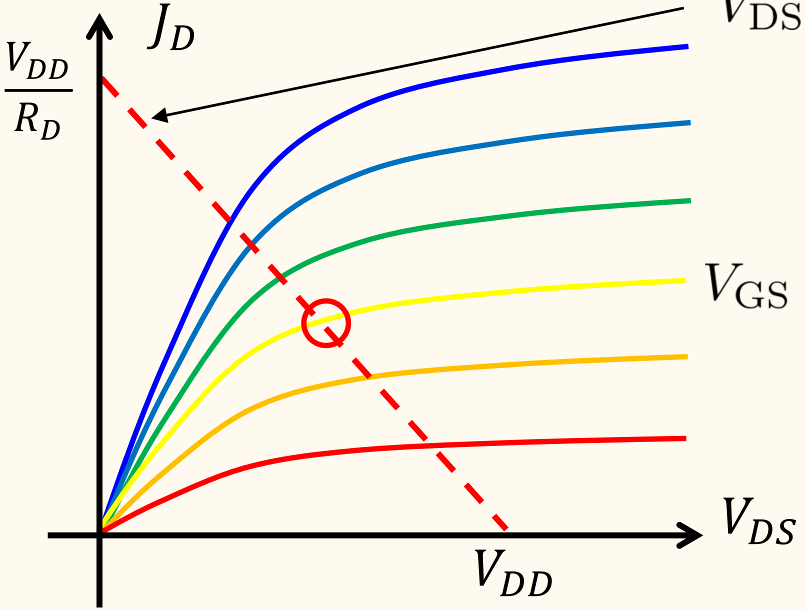
Biasing circuits for FETs: Fixed bias circuit

Local linear approximation → action point (the center of parameters)

The action point is determined by resistors and dc power sources.



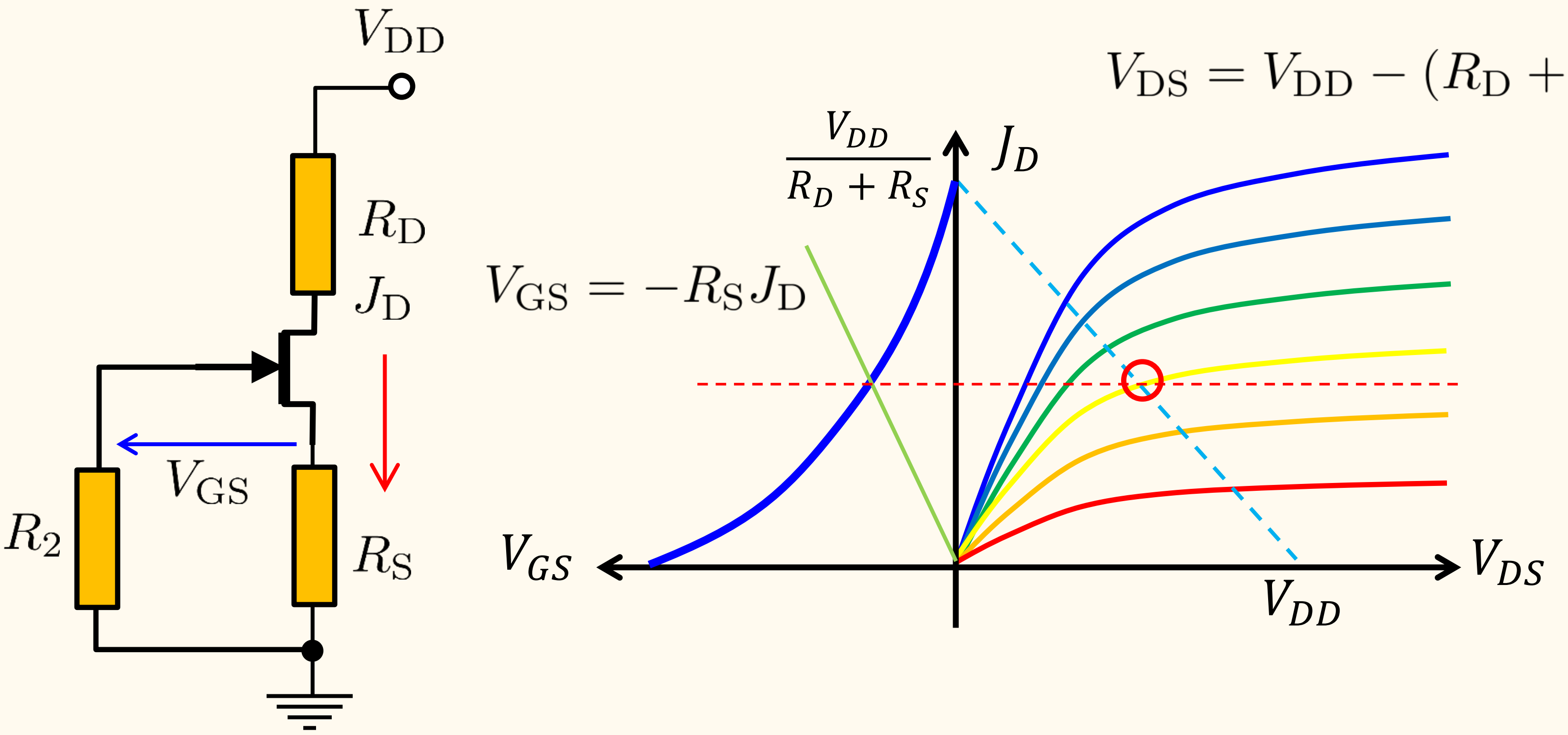
Biasing circuit



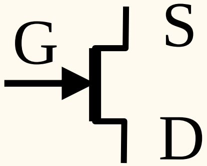
$$V_{\mathrm{DS}} = V_{\mathrm{DD}} - R_{\mathrm{D}} J_{\mathrm{D}}$$

$$V_{\mathrm{GS}} = \frac{R_2}{R_1 + R_2} V_{\mathrm{DD}}$$

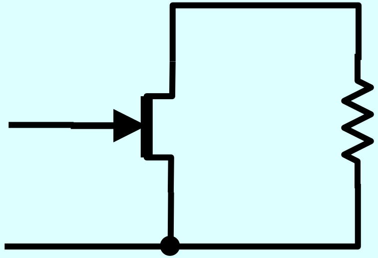
Biasing circuits for FETs: Self-biasing circuit



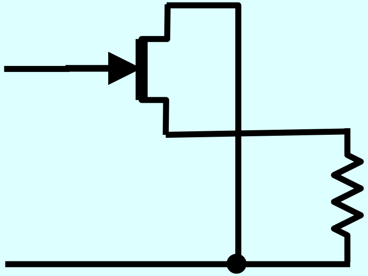
Equivalent signal circuits for FET



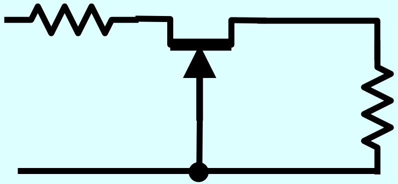
$$\mu = r_{\mathrm{d}} g_{\mathrm{m}}$$



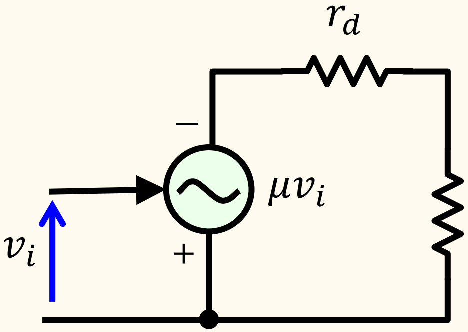
Source grounded

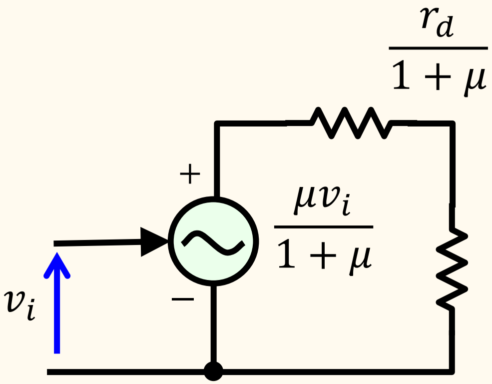


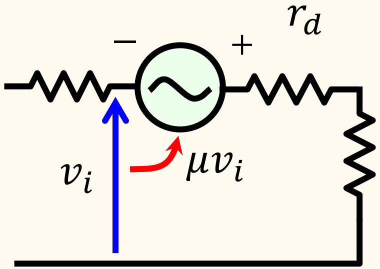
Drain grounded



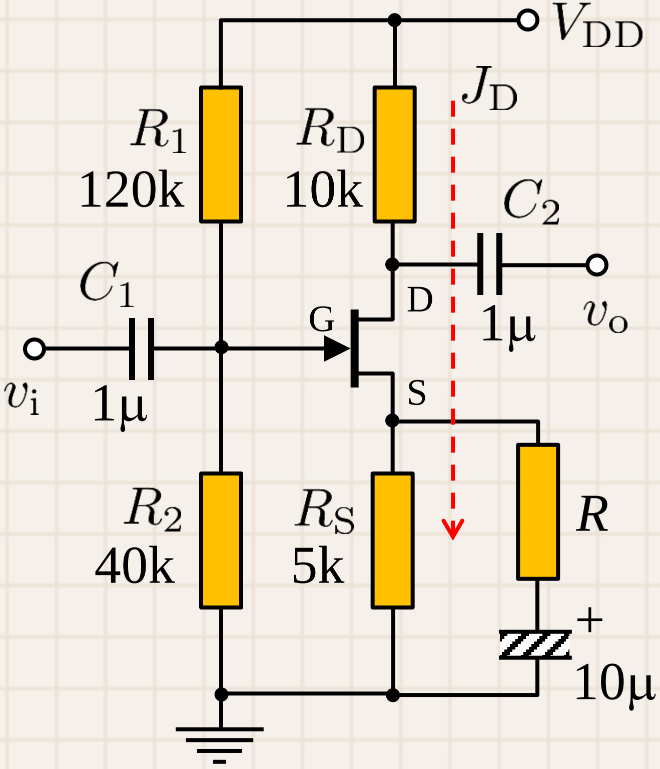
Gate grounded







Example of source-grounded FET amplifier

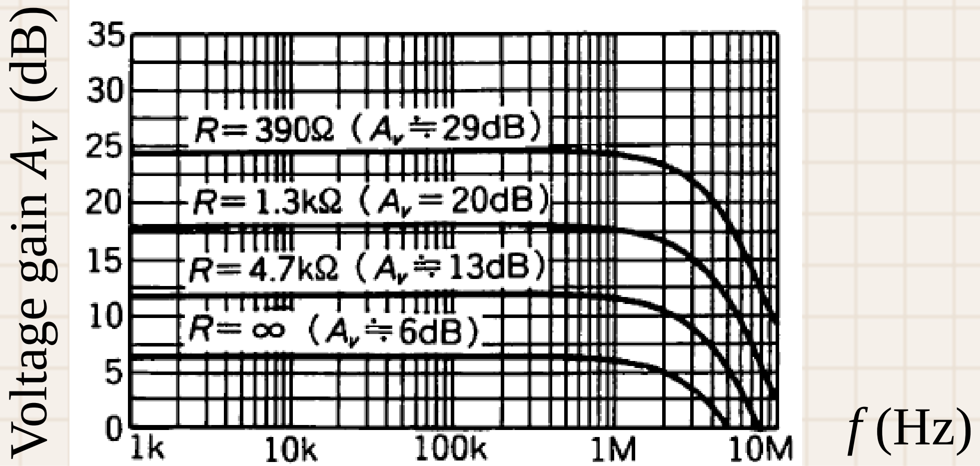


R_S is working as a negative feedback circuit.

$$\begin{cases} \delta V_{\rm GS} = \delta V_{\rm G} - R_{\rm S}\delta J_{\rm D} \\ \delta V_{\rm DS} = -\mu(\delta V_{\rm G} - R_{\rm S}\delta J_{\rm D}) \\ \delta V_{\rm D} = \delta V_{\rm DS} + R_{\rm S}\delta J_{\rm D} \end{cases}$$

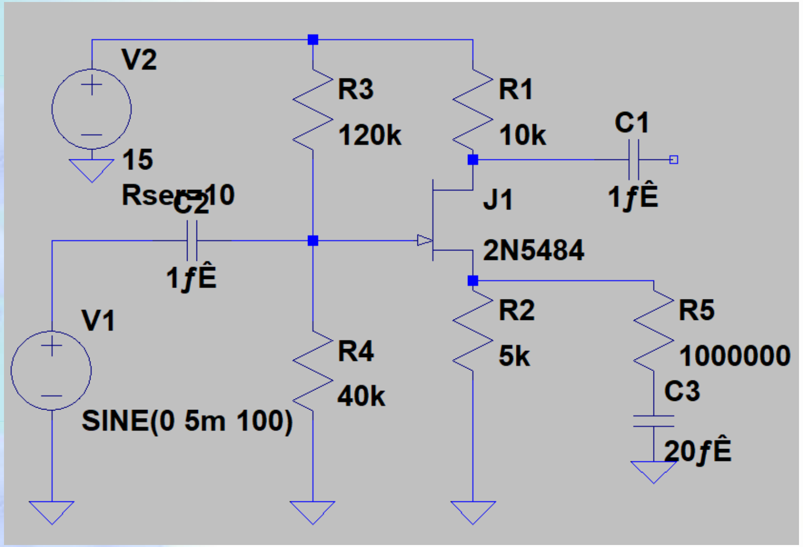
ignore $r_{\rm d}$

$$\delta V_{\rm D} = \frac{-\mu R_{\rm D}}{R_{\rm D} + R_{\rm S}(1-\mu)}\delta V_{\rm G} \approx \overset{A_V}{\boxed{\frac{R_{\rm D}}{R_{\rm S}}}}\delta V_{\rm G}$$

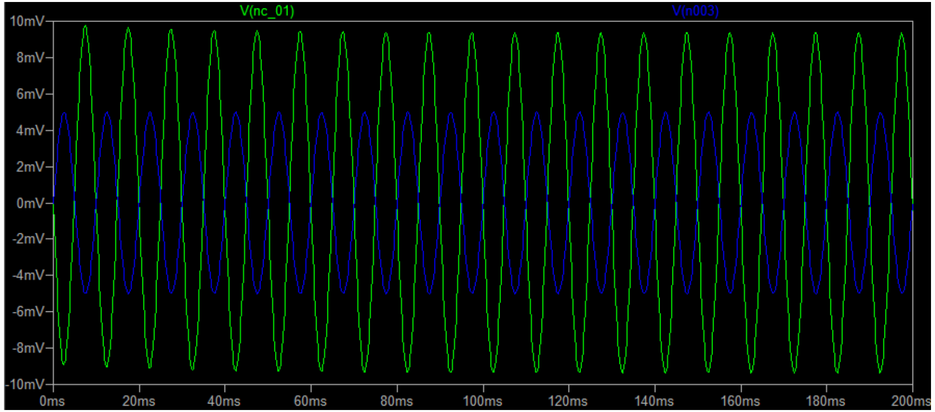


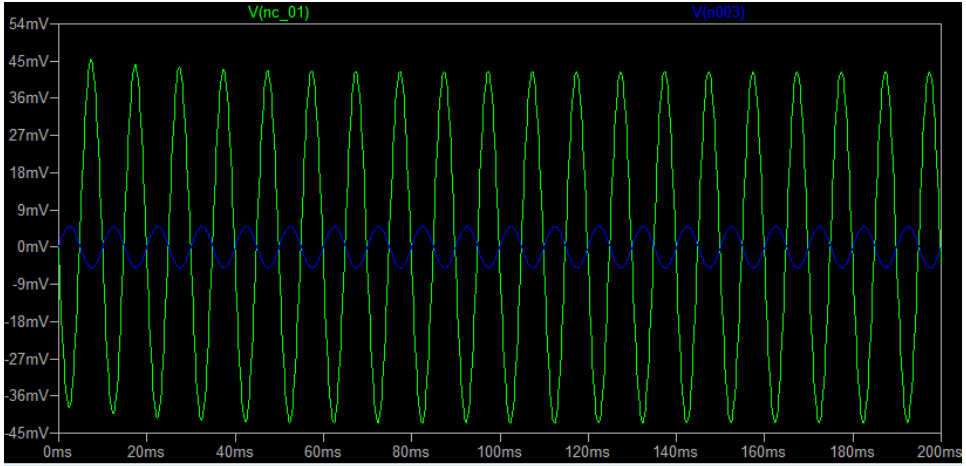
Simulation by LTSpice

$$R_5 = 1 \times 10^6 \ \ \Omega \quad A_V \approx 2$$

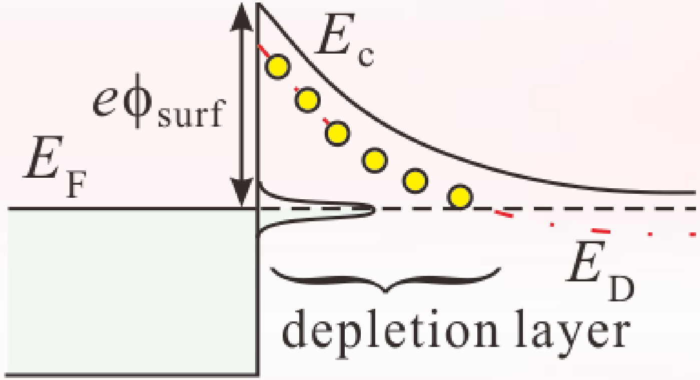


$$R_5 = 1 \times 10^3 \ \ \Omega \quad A_V \approx 9$$

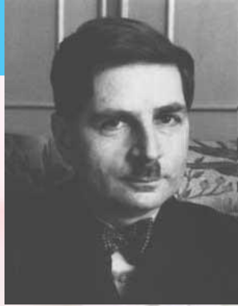


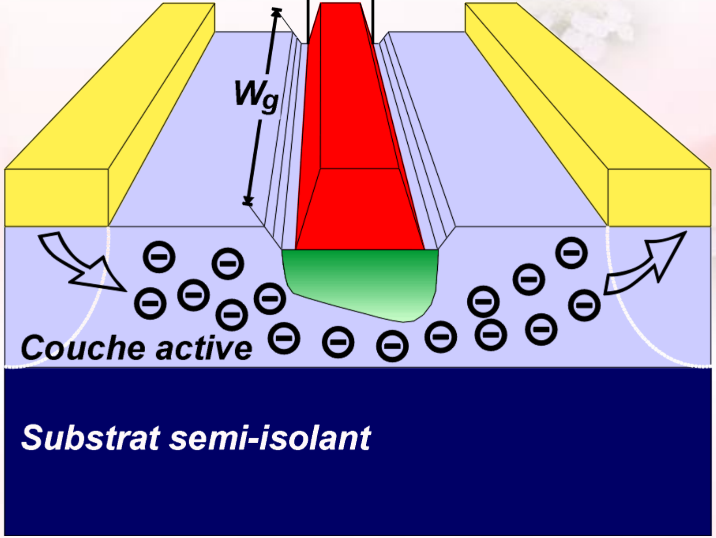


Metal-Semiconductor (MES) FET

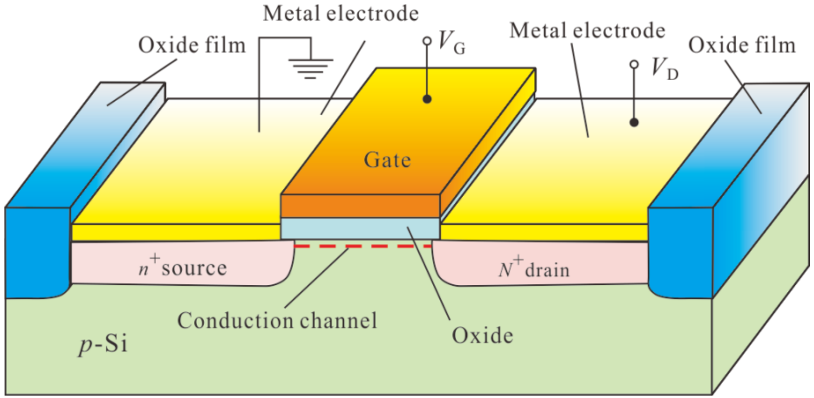


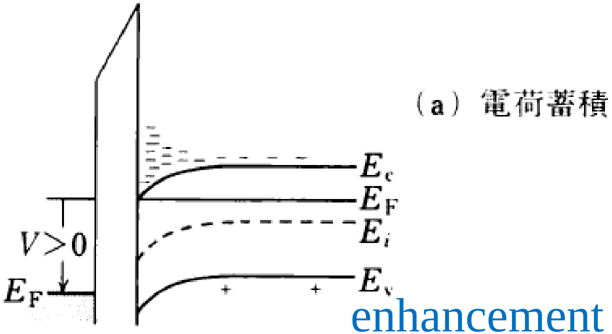
Walter Schottky
1886-1976



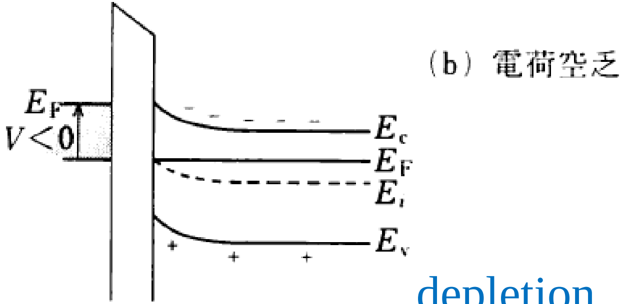


Metal-Oxide-Semiconductor (MOS) FET

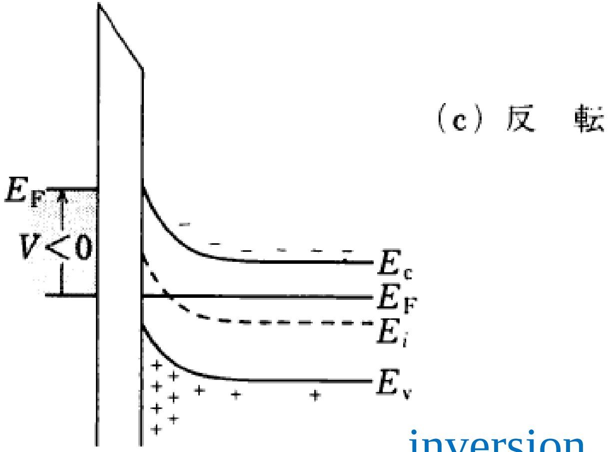




enhancement

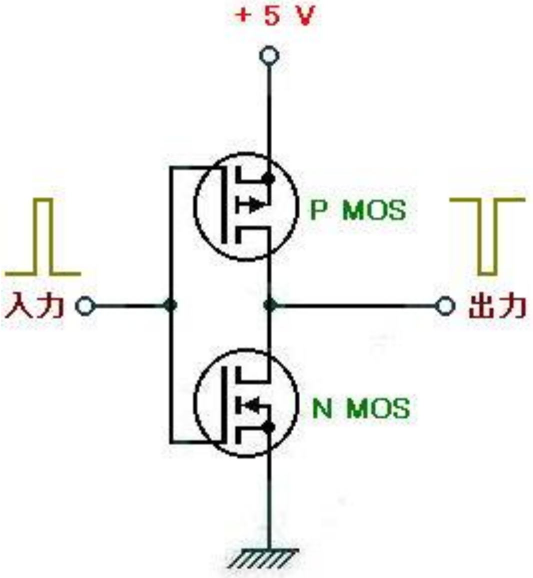


depletion



inversion

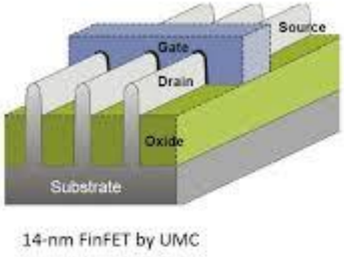
Complementary MOS logic gates



Simplified
CMOS inverter
circuit

Low leakage
current

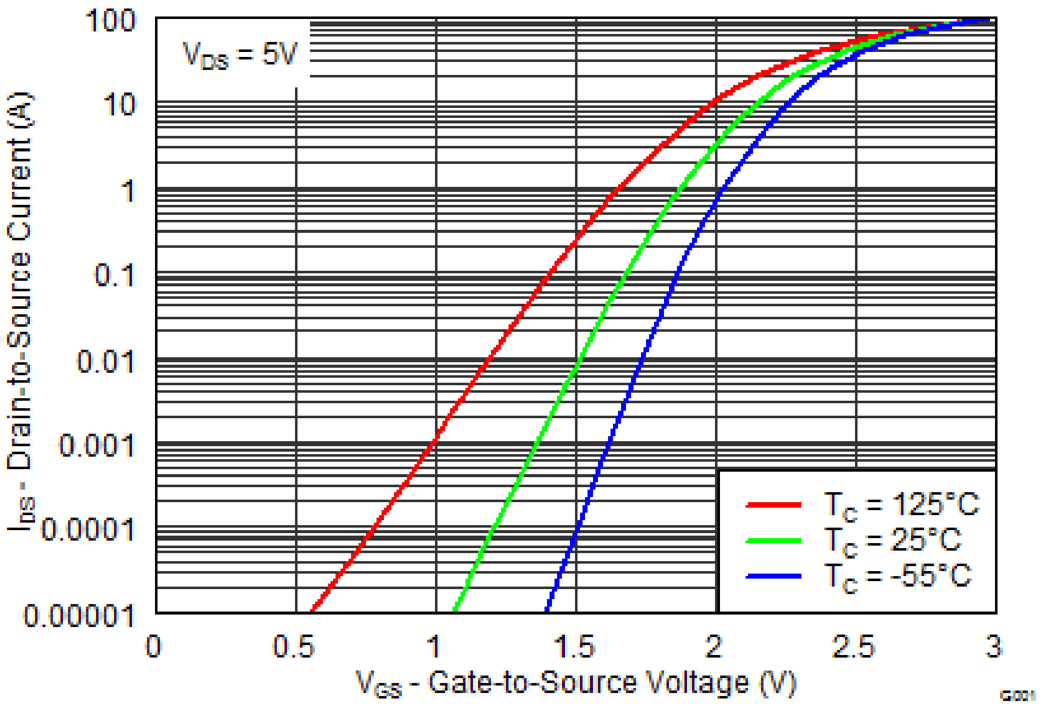
Single gate input
both on/off switch



FinFET structure

MOSFET switching characteristics

From datasheet CSD87381P power MOSFET (Texas instr.).



More than 7 orders change in J_D within 3 V change of V_{GS}.

Summary of chapter 4 Amplification circuit

Feedback (feedforward)

Transfer function diagram
Stability criteria

Operational amplifier

Elements for amplification
(non-linear treatment.
Bias circuits + signal circuits.)

Bipolar transistors (Semiconductor physics)
Field effect transistors

OP amp selection

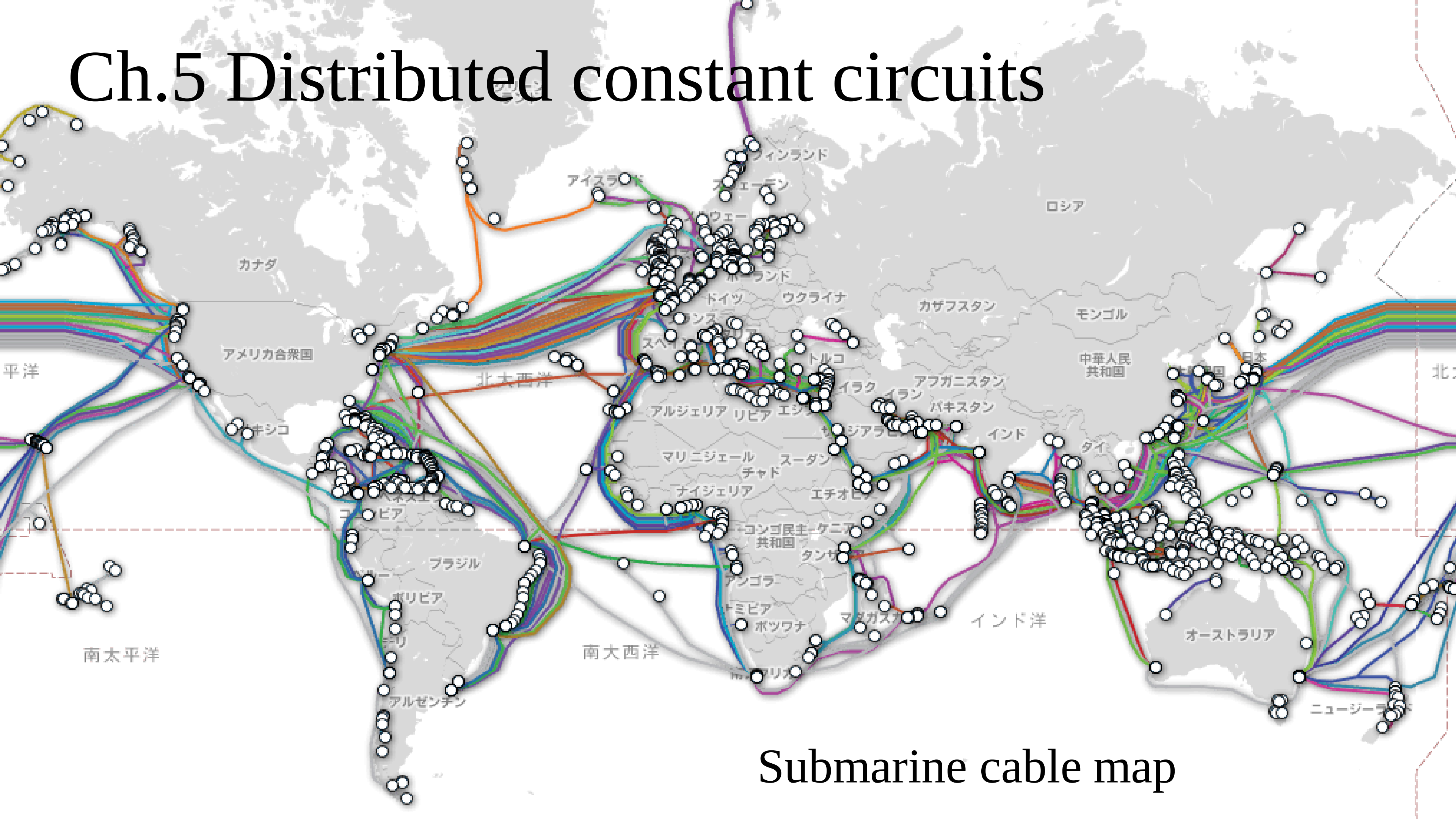
BT input

High precision
Low voltage noise
Large power output

FET input

Low bias current/ High input impedance
Low power consumption

Ch.5 Distributed constant circuits



Submarine cable map

Distributed constant circuit concept

1. In what case we need to consider distributed constant circuits?

Characteristic sizes of devices $\gtrsim$ wavelength of electromagnetic signal

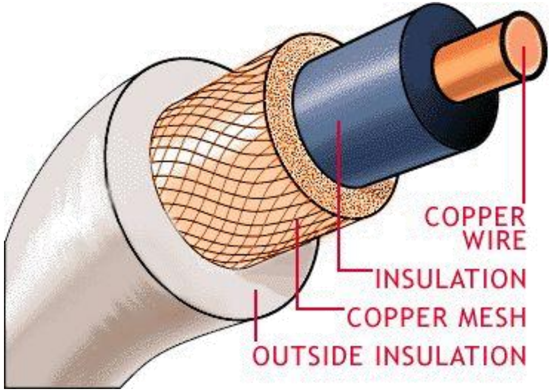
2. A typical scheme to make the shift for distributed circuit

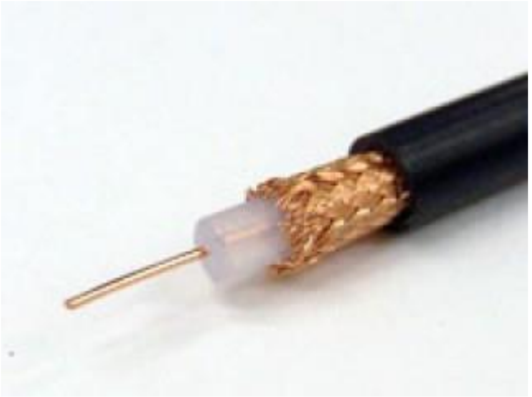
Lumped constant circuit → 1. Connection of unit circuits 2. Taking the infinitesimal limit → Distributed constant circuit

3. Distributed constant circuits : transmission lines

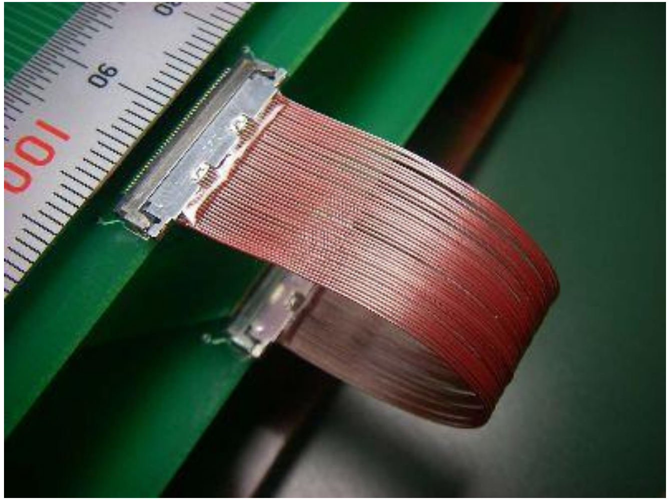
Coaxial cables, Lecher lines, micro-strip lines, waveguides, optical fibers

5.1.1 Coaxial cable

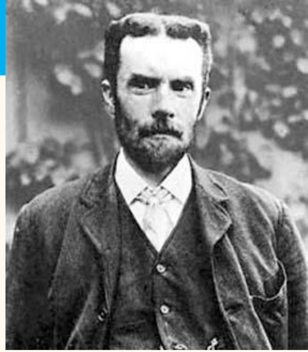




Thin coaxial cable AWG50 (ϕ25μm)

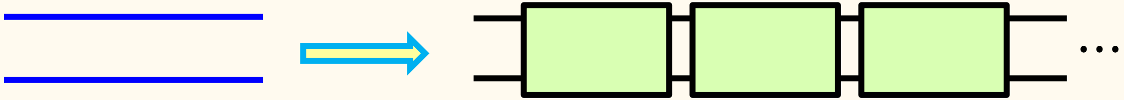


Transmission line as a series of infinitesimal terminal-pairs

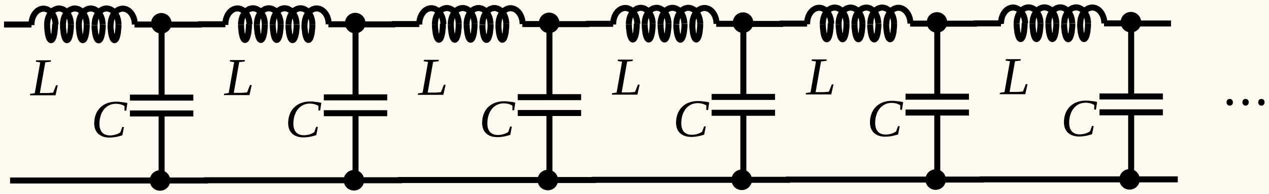


Oliver Heaviside
1850- 1925

Transmission line → divide into four terminal circuits



Each unit should have delay. Ignore energy dissipation.

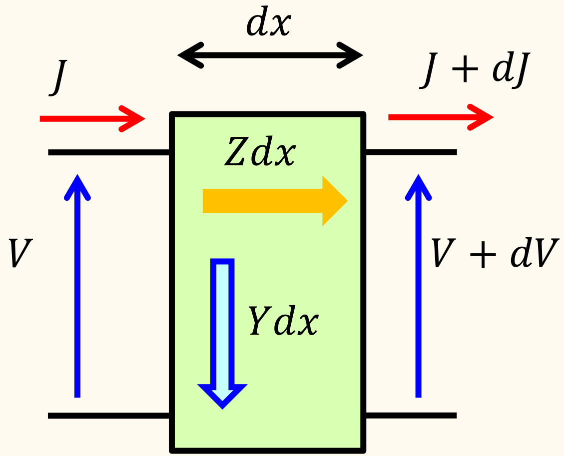


Then take the infinitesimal limit

Width → 0, Number → ∞

$$dV = -JZdx, \quad dJ = -VYdx$$

Z, Y : Impedance, Admittance per unit length



Characteristic impedance

$$\frac{d^2V}{dx^2} = YZV, \quad \frac{d^2J}{dx^2} = YZJ$$: telegraphic equation

$$J(x,t) = J(0,t)\exp(\pm\kappa x), \quad V(x,t) = V(0,t)\exp(\pm\kappa x) \quad \kappa \equiv \sqrt{YZ}$$

−: Progressive, +: Retrograde

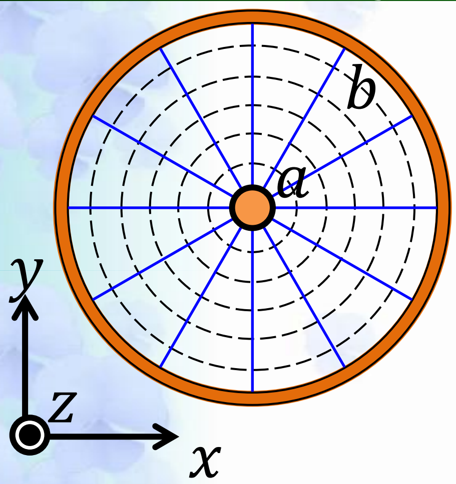
Impedance: $$\boxed{\frac{V}{J} = \mp\frac{Z}{\kappa} = \mp\sqrt{\frac{Z}{Y}}}$$

Pure reactance $Y = i\omega C$, $Z = i\omega L$ for L and C model

$$\kappa = \sqrt{-\omega^2 LC} = i\frac{\omega}{\omega_0}, \quad \omega_0 \equiv \frac{1}{\sqrt{LC}}$$ (physical dimension: velocity)

Characteristic impedance: $$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

Maxwell theory for coaxial cable



a: inner metal radius, *b*: radius of outer cylinder,
ε: insulator dielectric constant, μ: magnetic permeability

$$E = E_0(x,y)e^{i\omega t-\gamma z}, \quad H = H_0(x,y)e^{i\omega t-\gamma z}$$

From Maxwell equations

$$(\omega^2\epsilon\mu+\gamma^2)\begin{pmatrix}E_x\\E_y\end{pmatrix} = \begin{pmatrix}-\gamma\partial_x & -i\omega\mu\partial_y\\ -\gamma\partial_y & i\omega\mu\partial_x\end{pmatrix}\begin{pmatrix}E_z\\H_z\end{pmatrix},$$

$$(\omega^2\epsilon\mu+\gamma^2)\begin{pmatrix}H_x\\H_y\end{pmatrix} = \begin{pmatrix}i\omega\mu\partial_y & -\gamma\partial_x\\ -i\omega\mu\partial_x & -\gamma\partial_y\end{pmatrix}\begin{pmatrix}E_z\\H_z\end{pmatrix}.$$

In TEM (transverse electric and magnetic) mode: $E_z = H_z = 0$

For the fields along *x* and *y* to survive, $\omega^2\epsilon\mu+\gamma^2 = 0 \quad \therefore \gamma = \pm i\omega\sqrt{\epsilon\mu}$

Propagation velocity $v = \dfrac{\omega}{\omega\sqrt{\epsilon\mu}} = \dfrac{1}{\sqrt{\epsilon\mu}}$

Maxwell theory for coaxial cable

In TEM mode: $\mathrm{rot}_{xy}\boldsymbol{H} = 0, \quad \mathrm{rot}_{xy}\boldsymbol{E} = 0$ → potentials $\mathcal{U}, \ \mathcal{V}$ are available.

That is $\boldsymbol{E} = \nabla_{xy}\mathcal{U}/\sqrt{\epsilon}, \quad \boldsymbol{H} = \nabla_{xy}\mathcal{V}/\sqrt{\mu}$

$$\frac{\partial \mathcal{U}}{\partial x} = \frac{\partial \mathcal{V}}{\partial y}, \quad \frac{\partial \mathcal{U}}{\partial y} = -\frac{\partial \mathcal{V}}{\partial x}$$ Cauchy-Riemann condition

$f(w) = \mathcal{U} + i\mathcal{V}$ is an analytic function of $w = x + iy$

Characteristic impedance: $Z_0 = \dfrac{V}{J} = \dfrac{\mathcal{U}_a - \mathcal{U}_b}{J\sqrt{\epsilon}}$ $\qquad \mathcal{U}_{a,b}$: potential at *a*, *b*

This expression represents the equivalence of distributed constant circuit model and the Maxwell theory for coaxial cable.

Capacitance part

$$V = \frac{q}{\epsilon}\int_a^b \frac{dr}{2\pi r} = \frac{q}{2\pi\epsilon}\log\frac{b}{a} = \frac{q}{C} \qquad \therefore C = \frac{2\pi\epsilon}{\log(b/a)}$$

Maxwell theory for coaxial cable

Inductance part

Core current J, shield current $-J$ $\quad H(r) = \frac{J}{2\pi r}, \quad B(r) = \frac{\mu J}{2\pi r}$

Flux per length: $$\Phi = \int_a^b dr B(r) = \frac{\mu J}{2\pi} \log \frac{b}{a}$$

Self inductance per length: $$L = \frac{\mu}{2\pi} \log(b/a)$$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \log\left(\frac{b}{a}\right)$$

cf. Characteristic impedance of the vacuum $\quad Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 376\Omega$

Coaxial cable 2

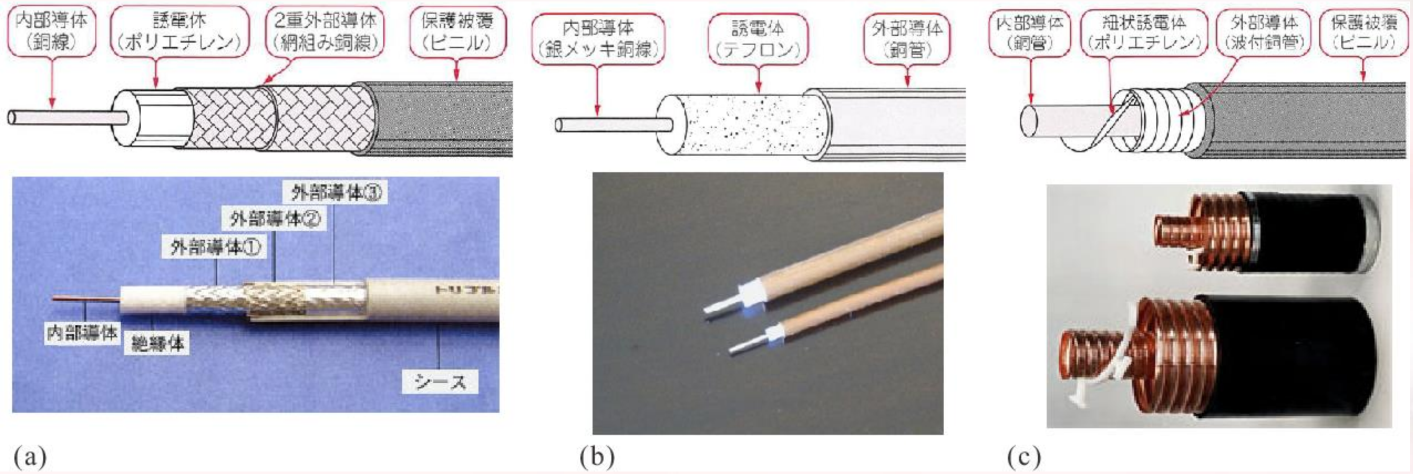


図12 同軸ケーブルの型名(JIS C3501)

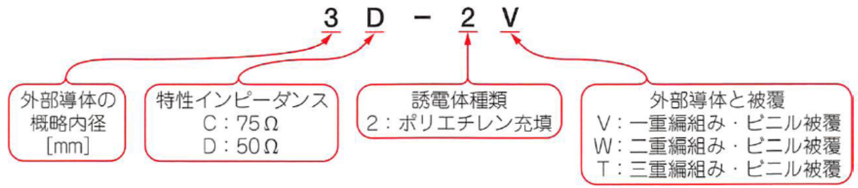
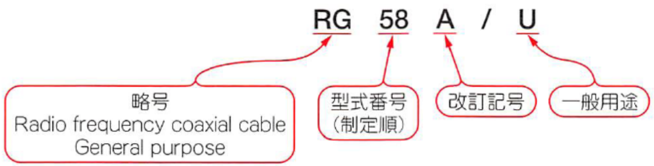
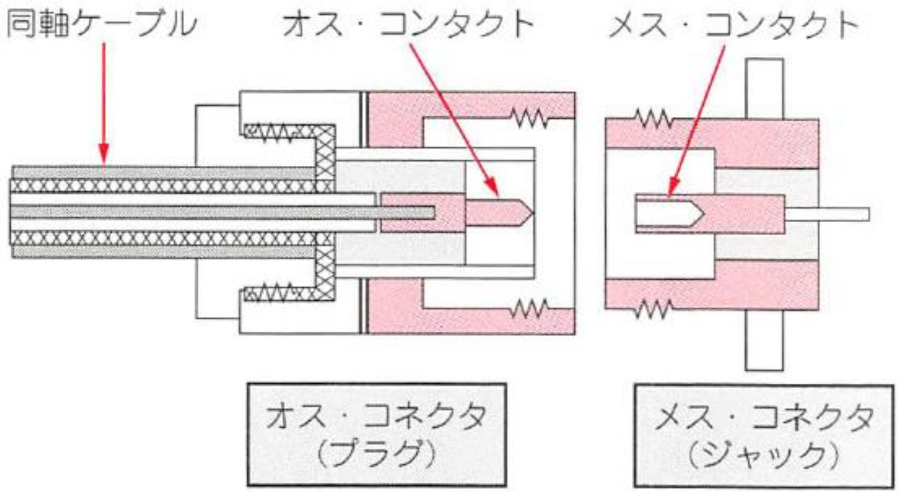


図13 MIL規格での同軸ケーブル型名の例



Coaxial connectors

図22 同軸コネクタの構造（概念図）

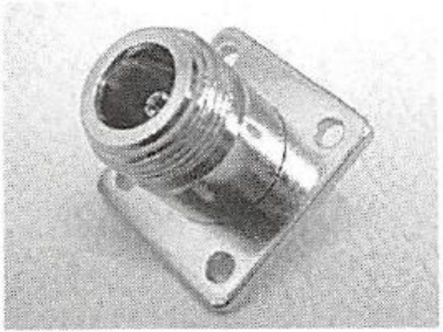


代表的な同軸コネクタの最高使用周波数例

形　式	外部導体内径	最高使用周波数
BNC	約 7 mm	2 ～ 4 GHz
N	約 7 mm	10 ～ 18 GHz
7 mm	7 mm	～ 18 GHz
SMA	4.15 mm	18 GHz
3.5 mm	3.5 mm	26.5 GHz
K	2.92 mm	40 GHz
2.4 mm	2.4 mm	50 GHz
V	1.85 mm	65 GHz
W	1.1 mm	110 GHz
1.0 mm	1.0 mm	110 GHz

Coaxial connectors

写真2 N型コネクタ

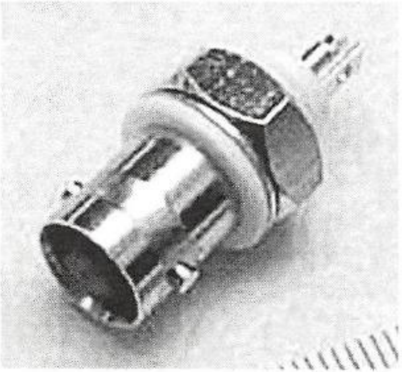


（a） フランジ付きジャック

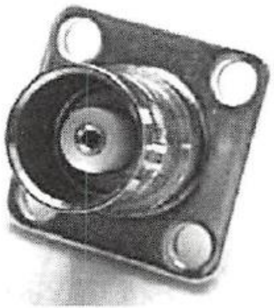
（b） プラグ

（c） プラグ［（b）を分解］

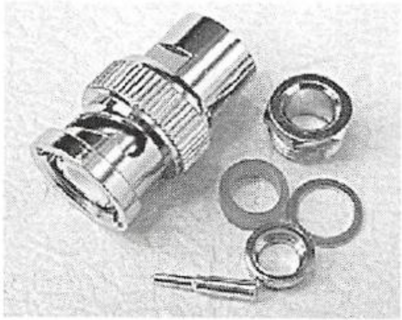
写真3 BNC型コネクタ



（a） 絶縁型ジャック
（高周波に向かない）



（b） フランジ付きジャック



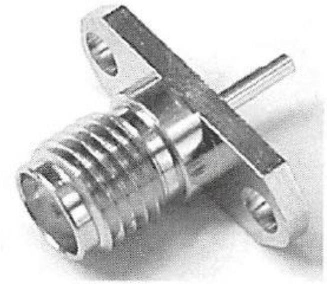
（c） プラグ

Coaxial connectors 2

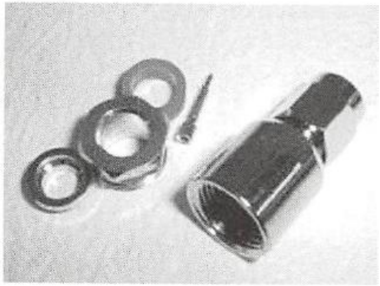
SMA-type

写真4 SMA型コネクタ

jack



(a) ジャック

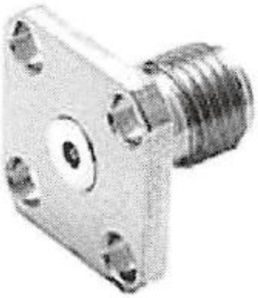


(b) プラグ

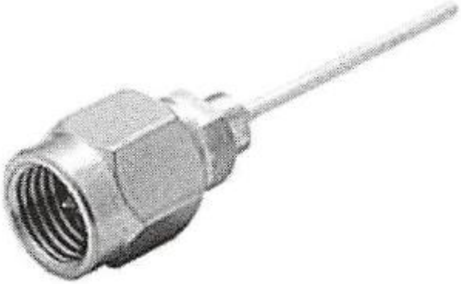
plug

K-type

写真6 K型コネクタ



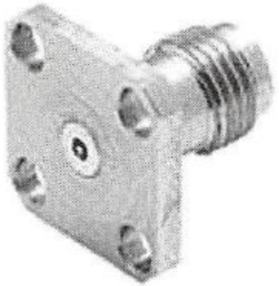
(a) ジャック



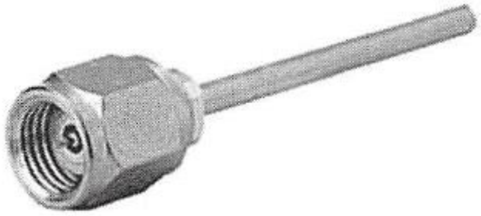
(b) プラグ

V-type

写真7 V型コネクタ



(a) ジャック



(b) プラグ

LEMO cables and connectors

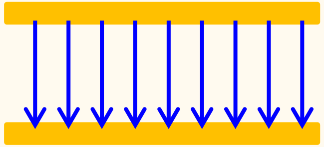
http://www.lemo.com/

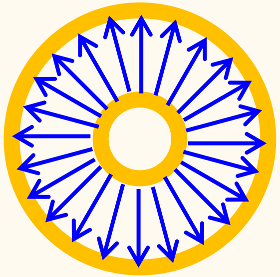
High-energy physics experiment, etc.

Transmission lines with TEM mode

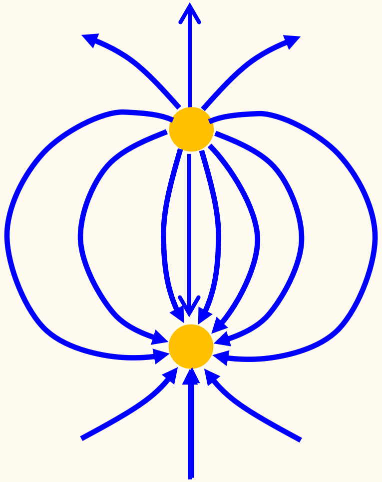
Transmission lines with two conductors are “families”.

Electromagnetic field confinement with parallel-plate capacitor



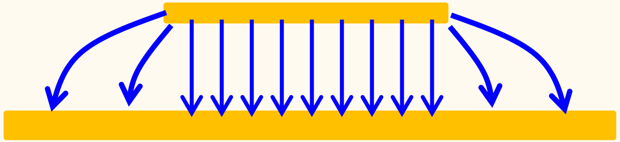


Roll up to coaxial cable



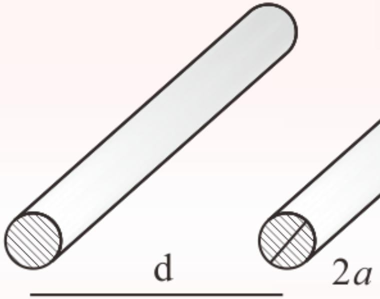
Shrink to dipole (Lecher line)

Open to micro-strip line

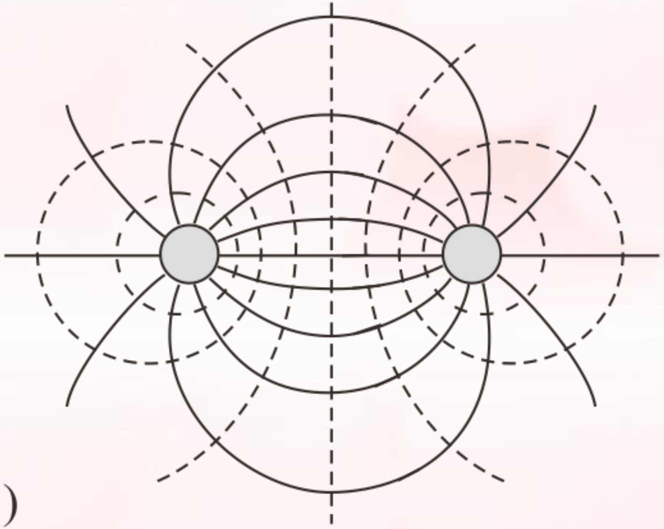


Commonly **TEM is the primary mode.**

Lecher line



(a)



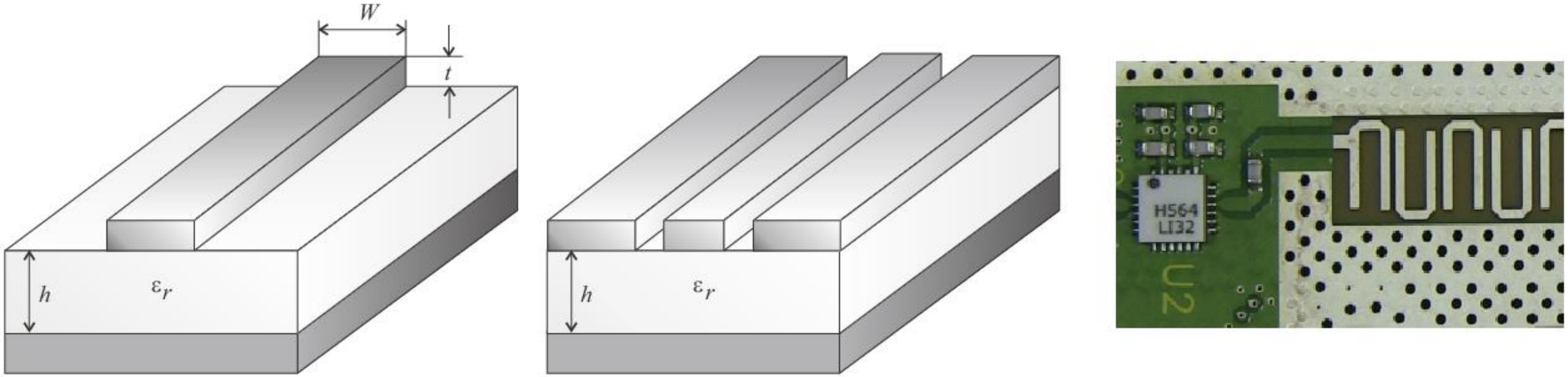
(b)

(c)

$$\phi_1 = -\phi_2 = \frac{J\sqrt{\mu}}{2\pi}\log\frac{d}{a}$$

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}}\frac{1}{\pi}\log\frac{d}{a}$$

Micro strip line



(a) (b) (c)

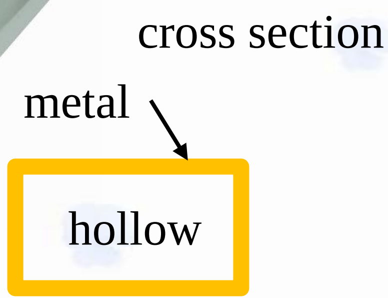
Wide (W/h>3.3) strip

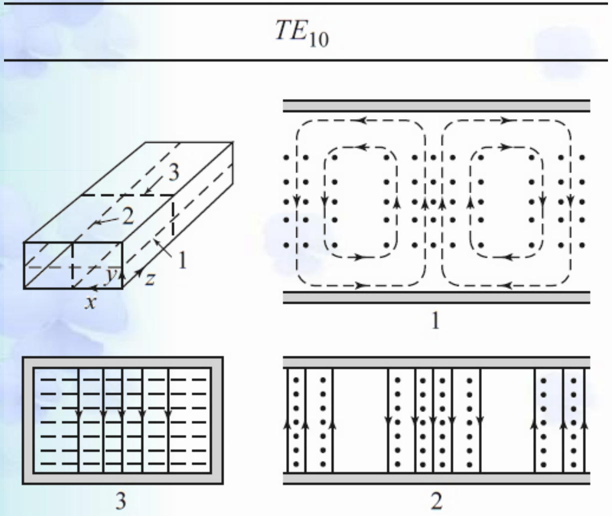
$$Z(W,h,\epsilon_r) = \frac{Z_{F0}}{2\sqrt{\epsilon_r}}\left\{\frac{W}{2h} + \frac{1}{\pi}\log 4 + \frac{\epsilon_r+1}{2\pi\epsilon_r}\log\left[\frac{\pi e}{2}\left(\frac{W}{2h}+0.94\right)\right]\frac{\epsilon_r-1}{2\pi\epsilon_r^2}\log\frac{e\pi^2}{16}\right\}^{-1}$$

Narrow (W/h<3.3) strip

$$Z(W,h,\epsilon_r) = \frac{Z_{F0}}{\pi\sqrt{2(\epsilon_r+1)}}\left\{\log\left[\frac{4h}{W}+\sqrt{\left(\frac{4h}{W}\right)^2+2}\right] - \frac{1}{2}\frac{\epsilon_r-1}{\epsilon_r+1}\left(\log\frac{\pi}{2}+\frac{1}{\epsilon_r}\log\frac{4}{\pi}\right)\right\}$$

Waveguide





Electromagnetic field is confined into a simply-connected space.

TEM mode cannot exist.

Maxwell equations give

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]E_z=-(\omega^2\epsilon\mu+\gamma^2)E_z,$$

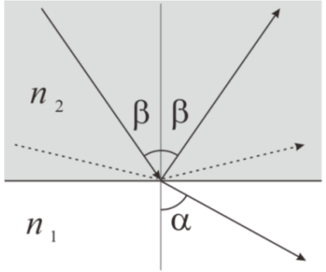
$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]H_z=-(\omega^2\epsilon\mu+\gamma^2)H_z.$$

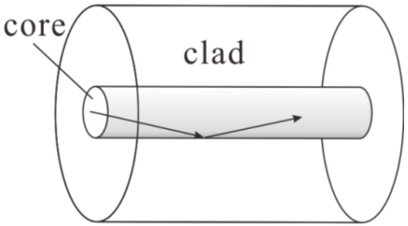
Helmholtz equation

$E_z=0$: TE mode,

$H_z=0$: TM mode

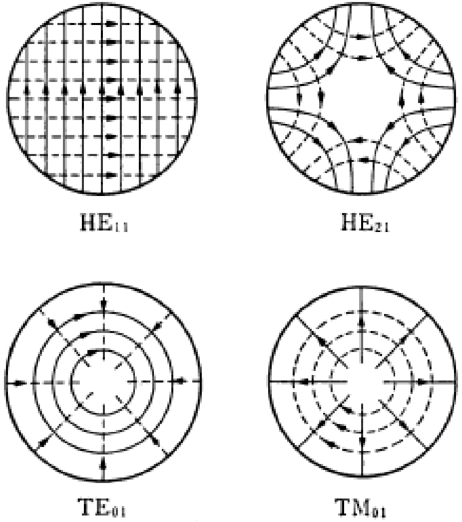
Optical fiber



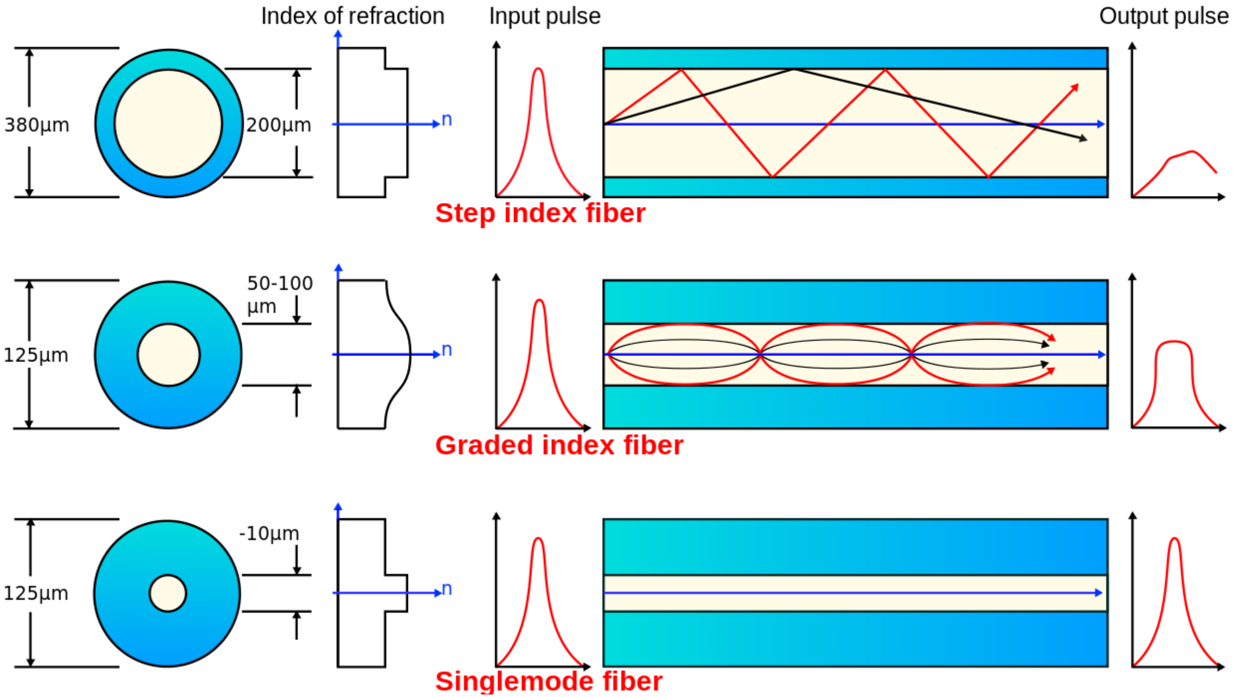


Difference in dielectric constant

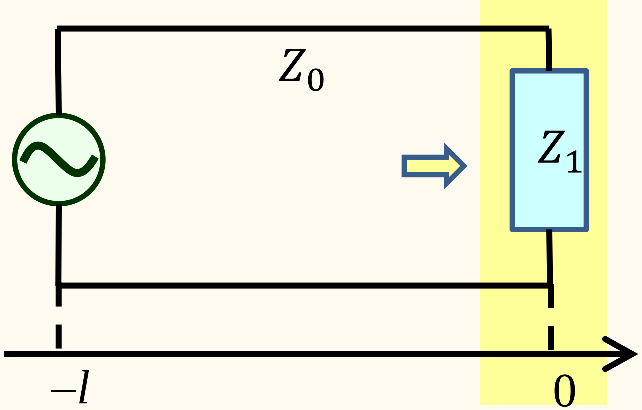
step-type optical fiber



no dispersion



Connection and termination



Termination of a transmission line with length l and characteristic impedance Z_0 at $x = 0$ with a resistor Z_1.

At $x = 0$: $\begin{cases} J = \underbrace{J_+}_{\text{progressive}} + \underbrace{J_-}_{\text{retrograde}} \quad \text{(definition right positive)} \\ V = V_+ + V_- = Z_0(J_+ - J_-) \end{cases}$

$$Z_1 = \frac{V}{J} = \frac{J_+ - J_-}{J_+ + J_-} Z_0$$

Reflection coefficient is $\boxed{r = \frac{V_-}{V_+} = -\frac{J_-}{J_+} = \frac{Z_1 - Z_0}{Z_1 + Z_0}}$

$Z_1 = Z_0$: no reflection, *i.e.*, impedance matching

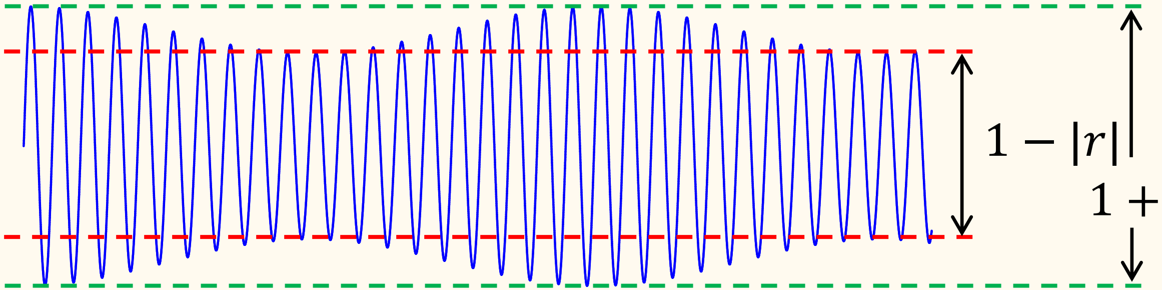
$Z_1 = +\infty$ (open circuit end) : $r = 1$, *i.e.*, free end

$Z_1 = 0$ (short circuit end) : $r = -1$, *i.e.*, fixed end

Comment: Sign of Ohm's law in transmission lines
reflects direction of waves
(depends on the definitions).

Connection and termination

Finite reflection → Standing wave   Voltage-Standing Wave Ratio (VSWR): $=\boxed{\dfrac{1+|r|}{1-|r|}}$



At $x=-l$

$$\left.\begin{aligned} V &= V_{+0}\exp(\kappa l) + V_{-0}\exp(-\kappa l) = [J_{+0}\exp(\kappa l) - J_{-0}\exp(-\kappa l)]Z_0 \\ J &= J_{+0}\exp(\kappa l) + J_{-0}\exp(-\kappa l) \end{aligned}\right\}$$

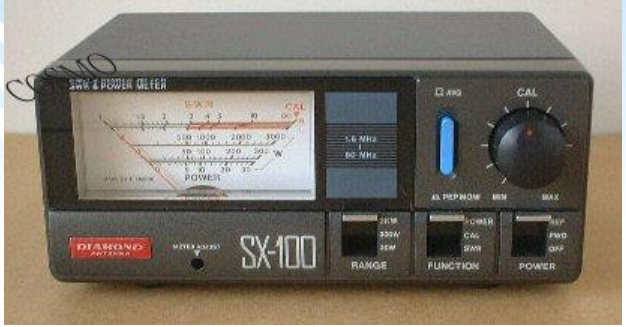
Then at $x=-l$ (at power source), the right hand side can be represented by

$$Z_l = \frac{V}{J} = \frac{J_{+0}e^{\kappa l} - J_{-0}e^{-\kappa l}}{J_{+0}e^{\kappa l} + J_{-0}e^{-\kappa l}} Z_0$$

Reflection coefficient: $$r_l = \frac{V_-}{V_+} = \frac{V_{0-}\exp(-\kappa l)}{V_{0+}\exp(\kappa l)} = r\exp(-2\kappa l)$$

SWR measurement

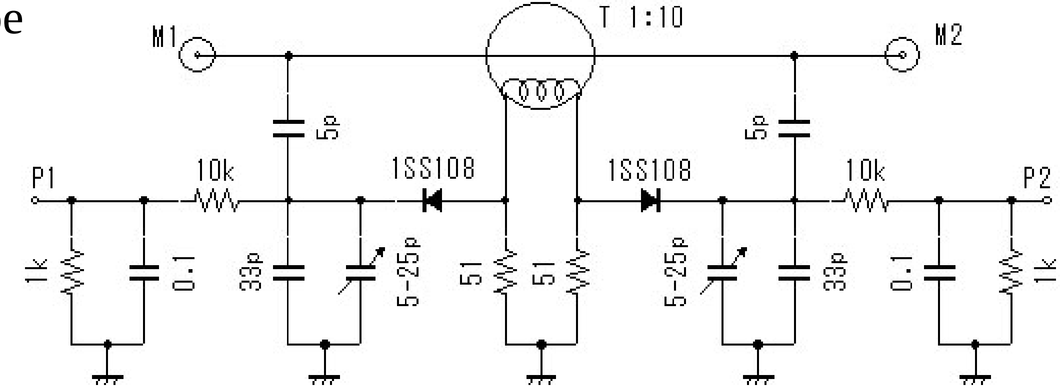
SWR Meters:



desktop types

cross-meter

handy type



directional coupler