電子回路論第8回

Electric Circuits for Physicists #8

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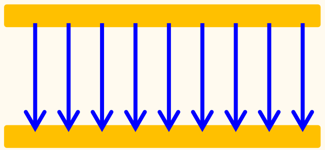
Outline

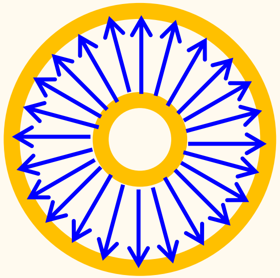
- Various transmission lines
- Termination and connection
- Smith chart
- S-matrix (S-parameters)

Transmission lines with TEM mode

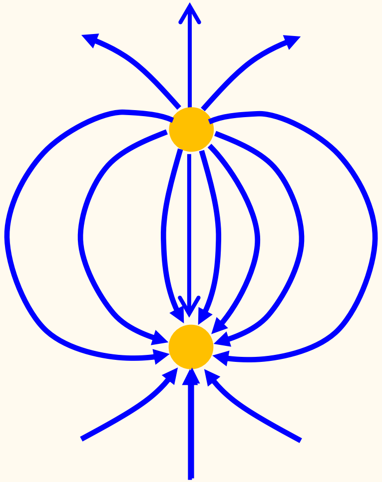
Transmission lines with two conductors are “families”.

Electromagnetic field confinement with parallel-plate capacitor



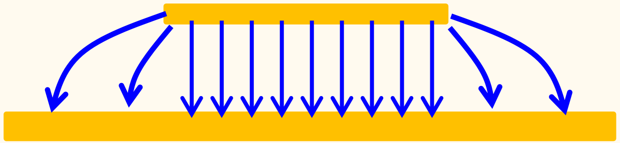


Roll up to coaxial cable



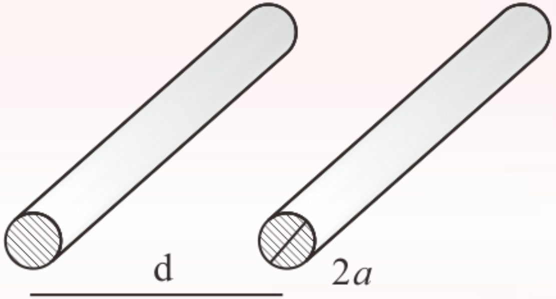
Shrink to dipole (Lecher line)

Open to micro-strip line

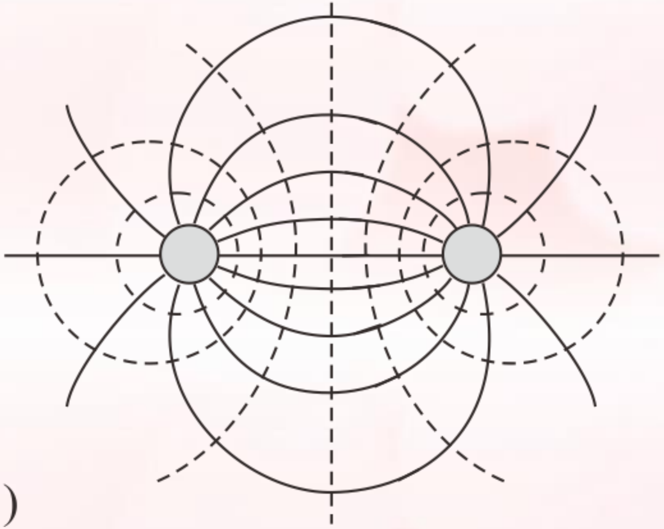


Commonly **TEM is the primary mode.**

Lecher line



(a)



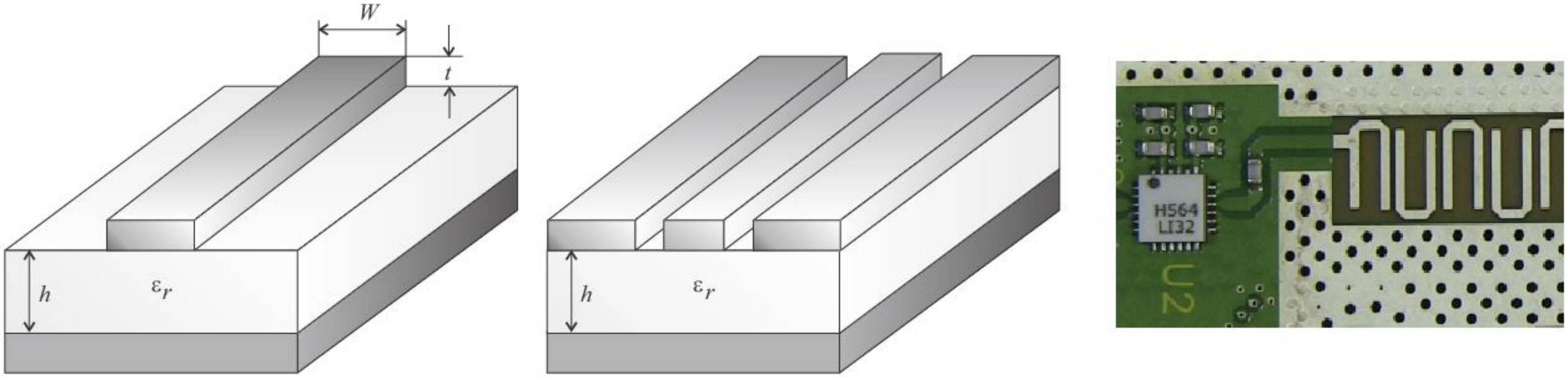
(b)

(c)

$$\phi_1 = -\phi_2 = \frac{J\sqrt{\mu}}{2\pi} \log \frac{d}{a}$$

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}} \frac{1}{\pi} \log \frac{d}{a}$$

Micro strip line



(a) (b) (c)

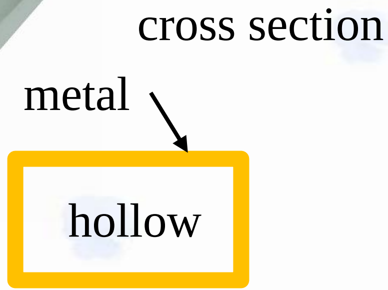
Wide (W/h>3.3) strip

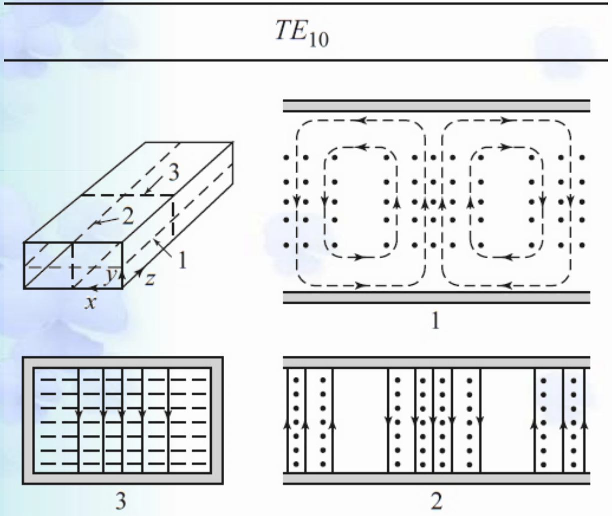
$$Z(W,h,\epsilon_r)=\frac{Z_{F0}}{2\sqrt{\epsilon_r}}\left\{\frac{W}{2h}+\frac{1}{\pi}\log 4+\frac{\epsilon_r+1}{2\pi\epsilon_r}\log\left[\frac{\pi e}{2}\left(\frac{W}{2h}+0.94\right)\right]\frac{\epsilon_r-1}{2\pi\epsilon_r^2}\log\frac{e\pi^2}{16}\right\}^{-1}$$

Narrow (W/h<3.3) strip

$$Z(W,h,\epsilon_r)=\frac{Z_{F0}}{\pi\sqrt{2(\epsilon_r+1)}}\left\{\log\left[\frac{4h}{W}+\sqrt{\left(\frac{4h}{W}\right)^2+2}\right]-\frac{1}{2}\frac{\epsilon_r-1}{\epsilon_r+1}\left(\log\frac{\pi}{2}+\frac{1}{\epsilon_r}\log\frac{4}{\pi}\right)\right\}$$

Waveguide





Electromagnetic field is confined into a simply-connected space.

⇩

TEM mode cannot exist.

Maxwell equations give

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]E_z=-(\omega^2\epsilon\mu+\gamma^2)E_z,$$

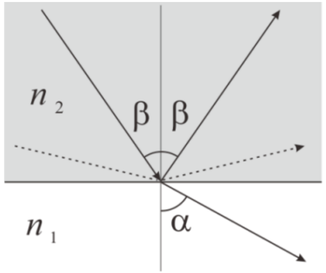
$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]H_z=-(\omega^2\epsilon\mu+\gamma^2)H_z.$$

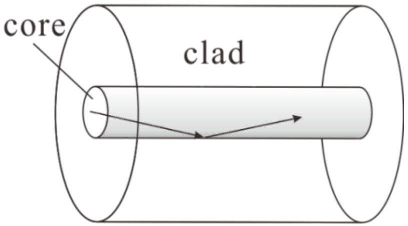
Helmholtz equation

$E_z=0$: TE mode,

$H_z=0$: TM mode

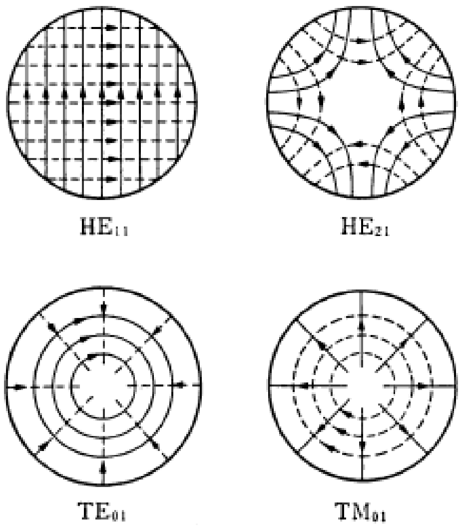
Optical fiber



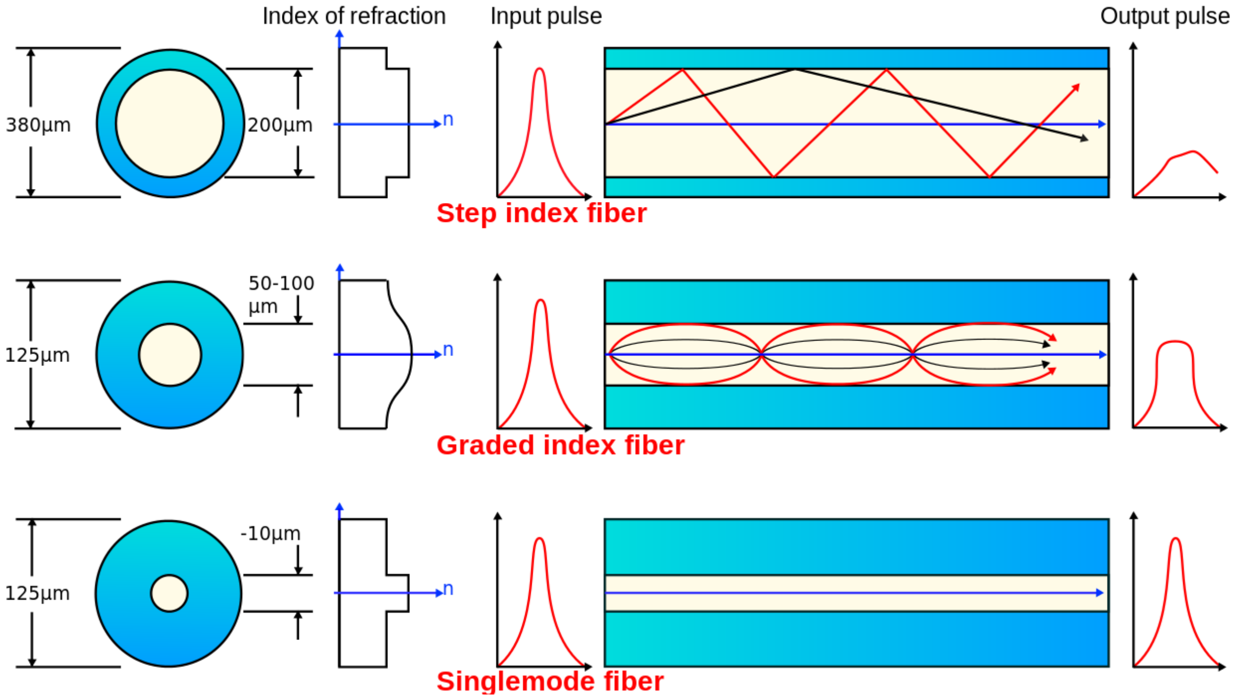


Difference in dielectric constant

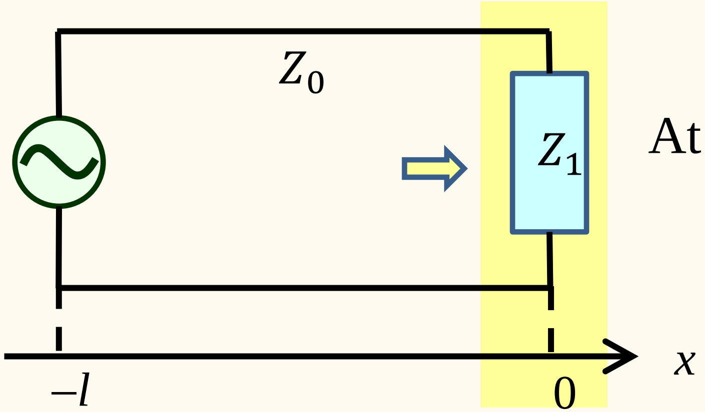
step-type optical fiber



no dispersion



Connection and termination



Termination of a transmission line with length l and characteristic impedance Z_0 at $x = 0$ with a resistor Z_1.

Comment: Sign of Ohm's law in transmission lines reflects direction of waves (depends on the definitions).

At $x = 0$:

$$J = J_+ + J_- \quad \text{(definition right positive)}$$

progressive (J_+) retrograde (J_-)

$$V = V_+ + V_- = Z_0(J_+ - J_-)$$

$$Z_1 = \frac{V}{J} = \frac{J_+ - J_-}{J_+ + J_-} Z_0$$

Reflection coefficient is

$$r = \frac{V_-}{V_+} = -\frac{J_-}{J_+} = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

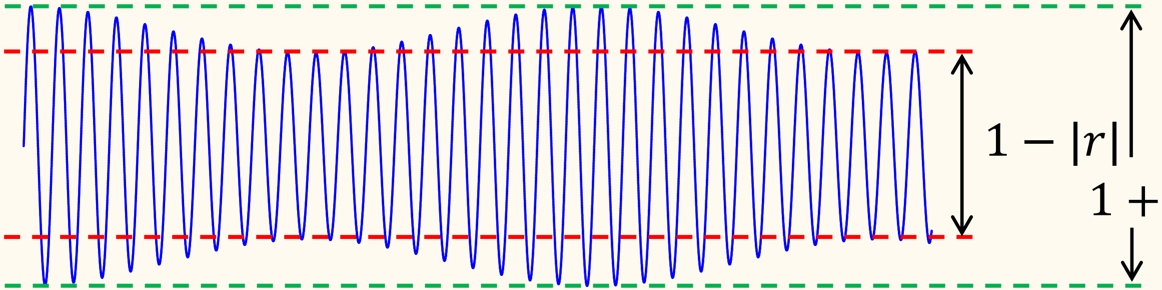
$Z_1 = Z_0$: no reflection, *i.e.*, impedance matching

$Z_1 = +\infty$ (open circuit end) : $r = 1$, *i.e.*, free end

$Z_1 = 0$ (short circuit end) : $r = -1$, *i.e.*, fixed end

Connection and termination

Finite reflection → Standing wave Voltage-Standing Wave Ratio (VSWR): $= \dfrac{1+|r|}{1-|r|}$



At $x = -l$

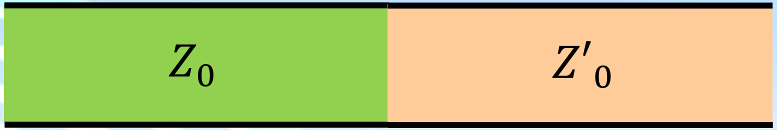
$$\left.\begin{aligned} V &= V_{+0}\exp(\kappa l) + V_{-0}\exp(-\kappa l) = [J_{+0}\exp(\kappa l) - J_{-0}\exp(-\kappa l)]Z_0 \\ J &= J_{+0}\exp(\kappa l) + J_{-0}\exp(-\kappa l) \end{aligned}\right\}$$

Then at $x = -l$ (at power source), the right hand side can be represented by

$$Z_l = \frac{V}{J} = \frac{J_{+0}e^{\kappa l} - J_{-0}e^{-\kappa l}}{J_{+0}e^{\kappa l} + J_{-0}e^{-\kappa l}} Z_0$$

Reflection coefficient: $$r_l = \frac{V_-}{V_+} = \frac{V_{0-}\exp(-\kappa l)}{V_{0+}\exp(\kappa l)} = r\exp(-2\kappa l)$$

Connection and termination

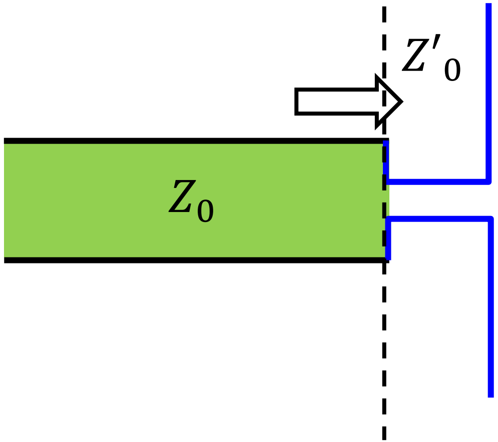


Transmission line connection.
Characteristic impedance Z_0, $Z_0{}'$

At the connection point, only the local relation between V and J affects the reflection coefficient.

The local impedance from the left hand side is $Z_0{}'$.

$$r = \frac{Z'_0 - Z_0}{Z'_0 + Z_0}$$



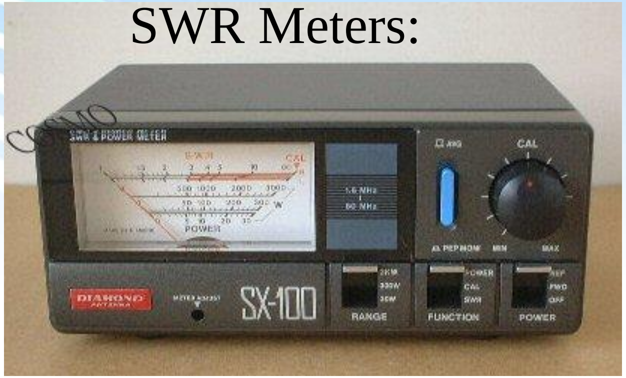
antenna

$$Z_0^{\mathrm{v}} = 120\pi \approx 377\ \Omega$$

Z'_0: input impedance of antenna

SWR measurement

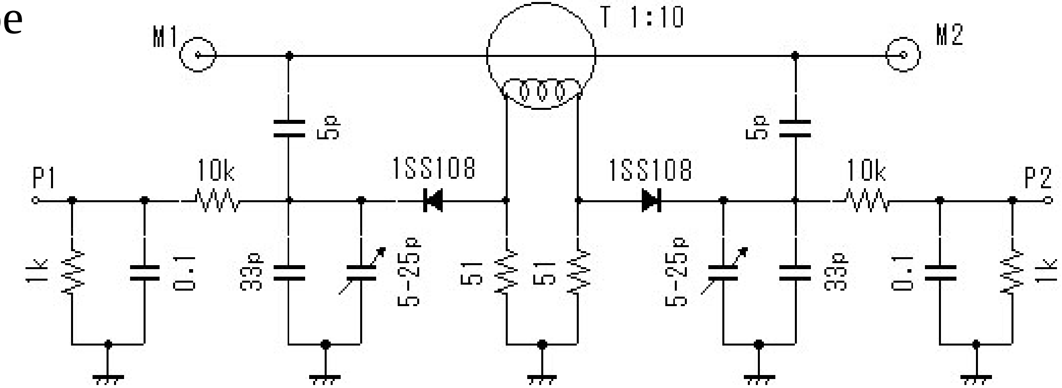
SWR Meters:



desktop types

cross-meter

handy type



directional coupler

5.2.3 Smith chart, immittance chart

End impedance Z_1: Normalized end impedance $Z_n \equiv Z_1/Z_0$

$$Z_n = x + iy, \quad r = u + iw \ \ (x, y, u, w \in \mathbb{R})$$

$$u + iw = r = \frac{Z_n - 1}{Z_n + 1} = \frac{(x-1) + iy}{(x+1) + iy}$$

real: $x - 1 = (x+1)u - yw$

imaginary: $y = yu + w(x+1)$

y: erase
x: constant

$$\left(u - \frac{x}{x+1}\right)^2 + w^2 = \frac{1}{(x+1)^2}$$

constant reactance circle

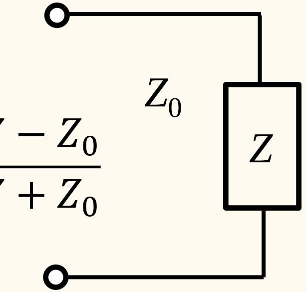
x: erase
y: constant

$$(u-1)^2 + \left(w - \frac{1}{y}\right)^2 = \frac{1}{y^2}$$

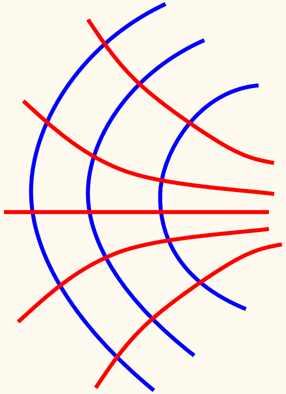
constant reactance circle

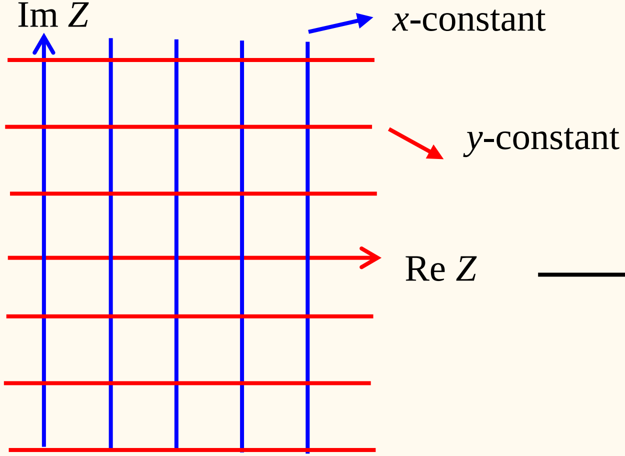
5.2.3 Smith chart, immittance chart

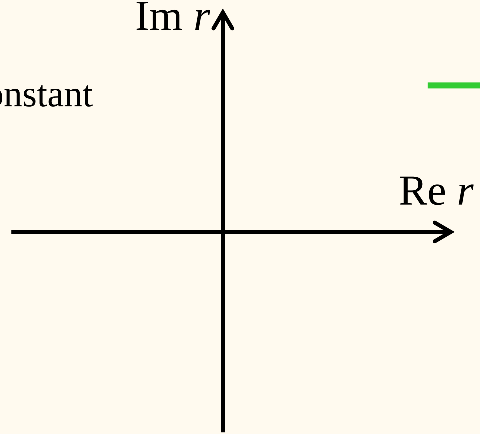
r : reflection coefficient

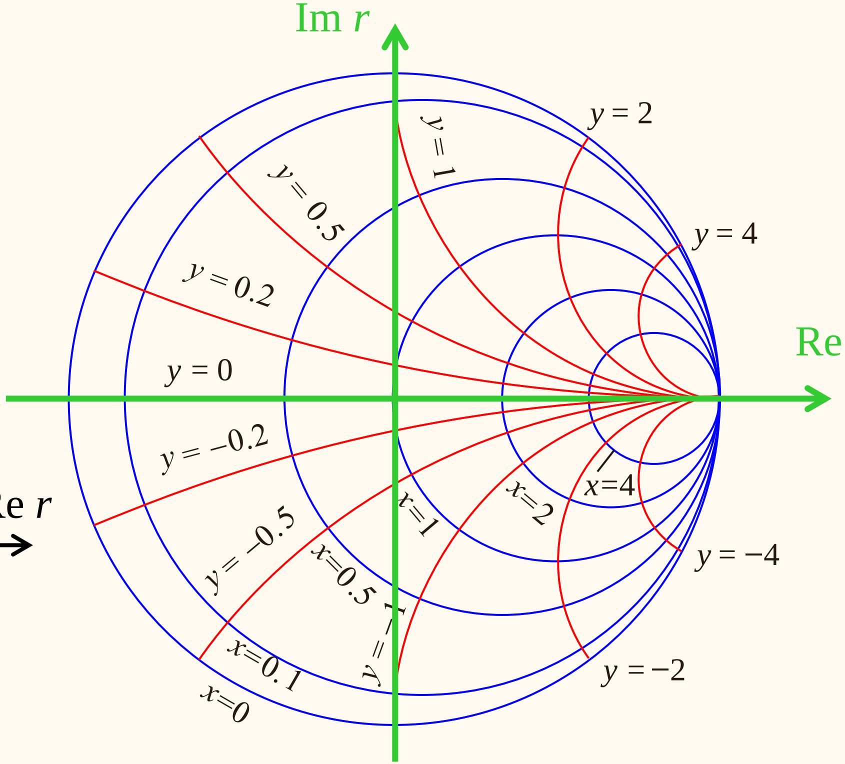


$$r = \frac{V_-}{V_+} = \frac{Z - Z_0}{Z + Z_0}$$









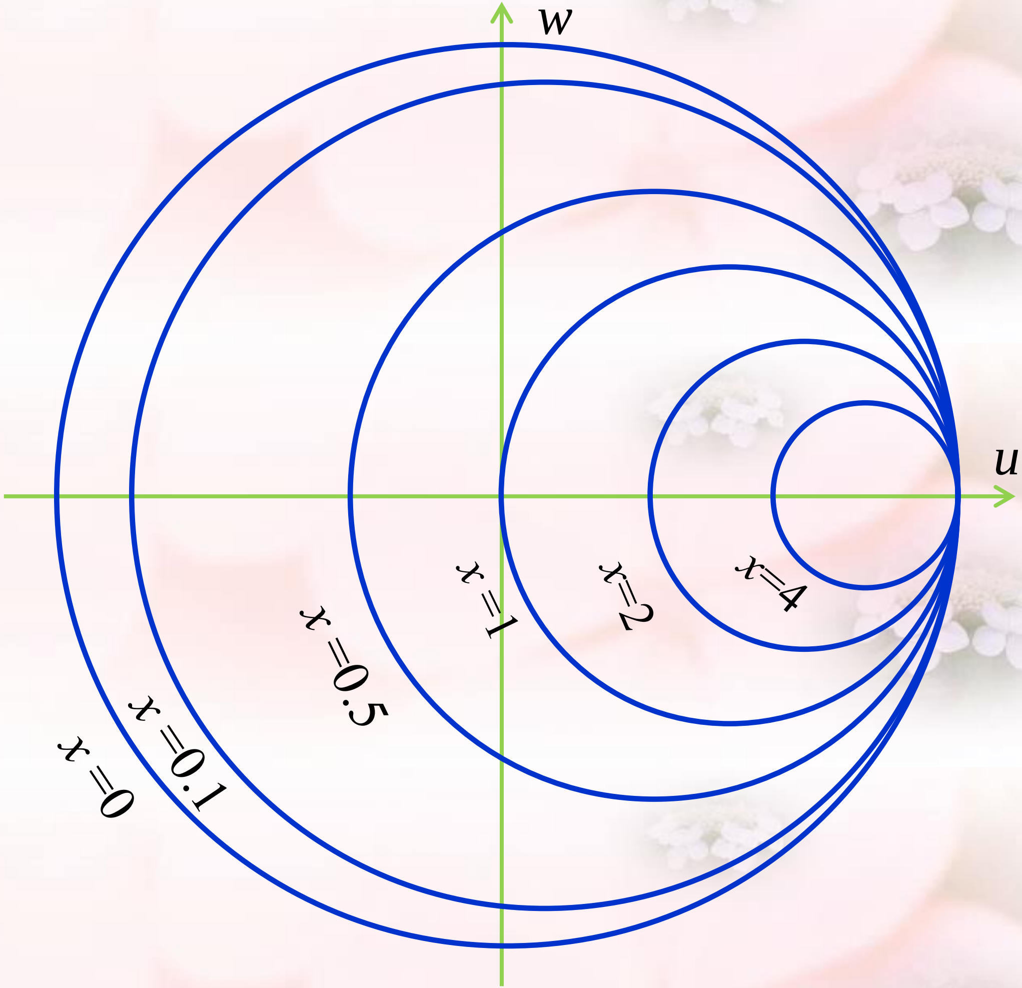
Circles with constant x

$$r = u + iw$$

$$|r| \leq 1$$

$$\left(u - \frac{x}{x+1}\right)^2 + w^2 = \frac{1}{(x+1)^2}$$

Common point: $(u, w) = (1, 0)$



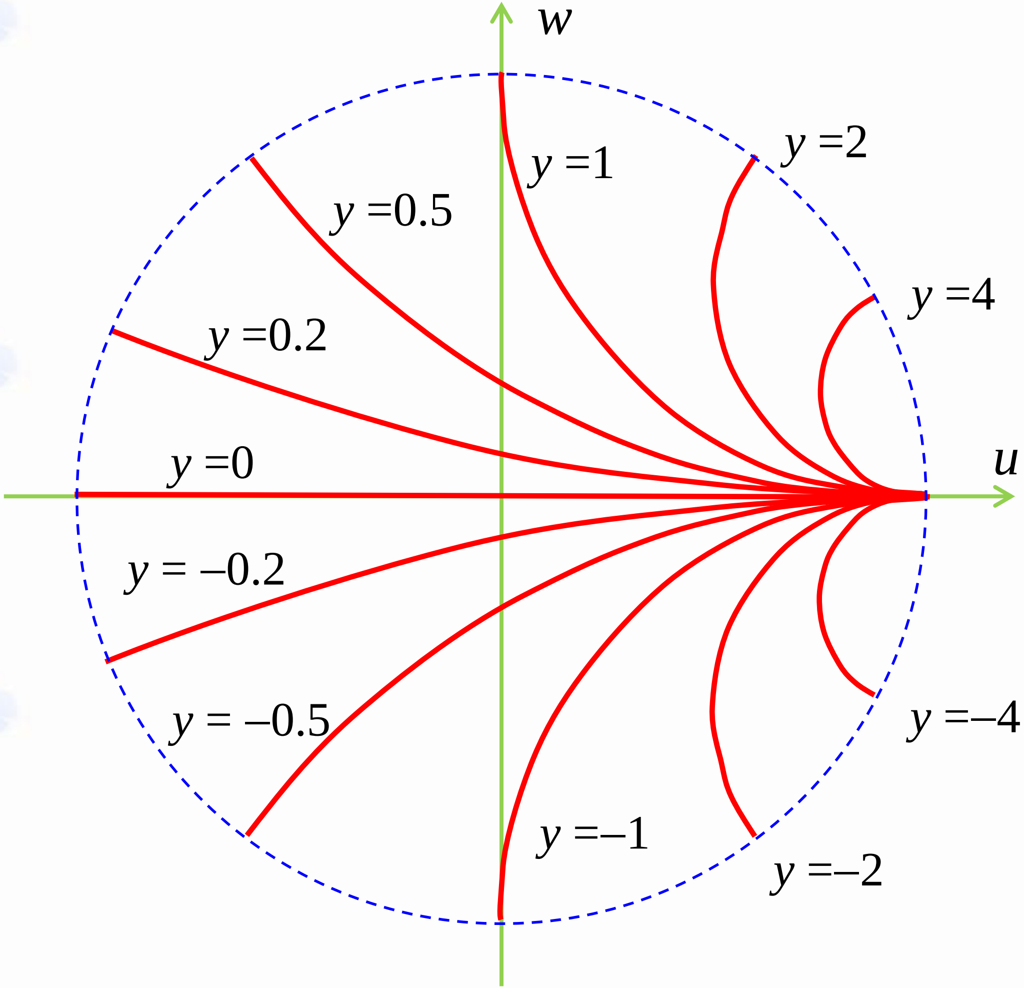
Circles with constant y

$$r = u + iw$$

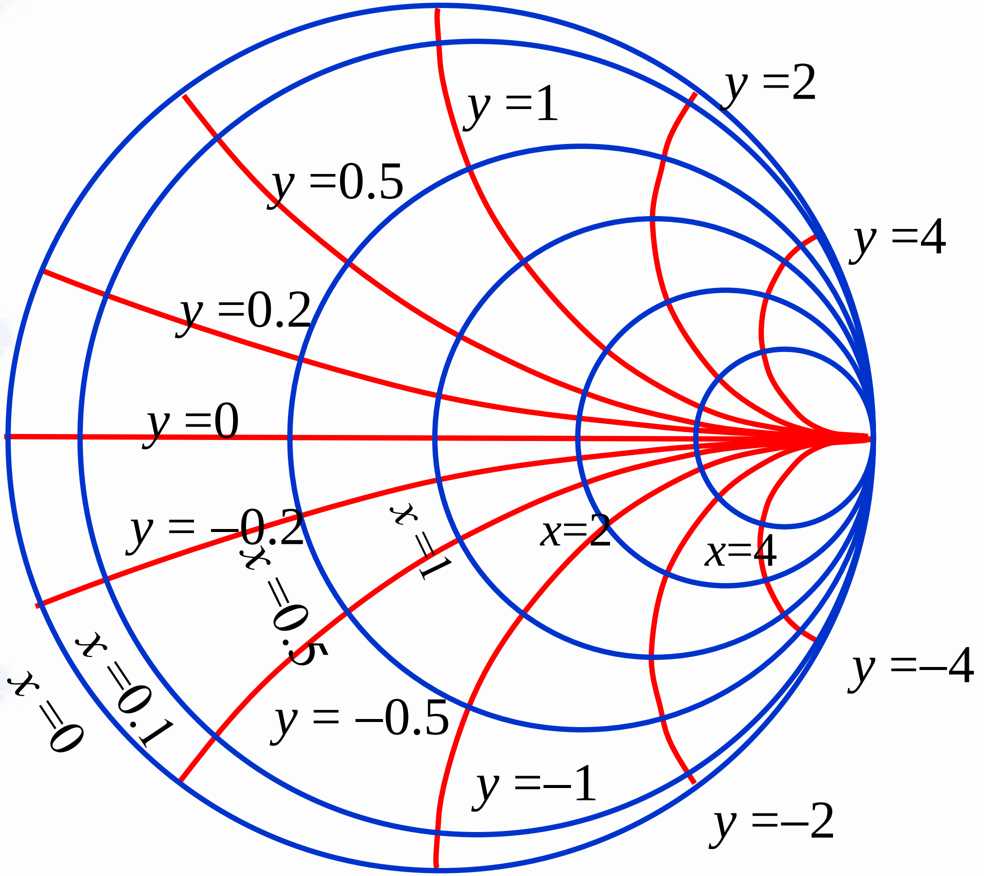
$$|r| \le 1$$

$$(u-1)^2 + \left(w - \frac{1}{y}\right)^2 = \frac{1}{y^2}$$

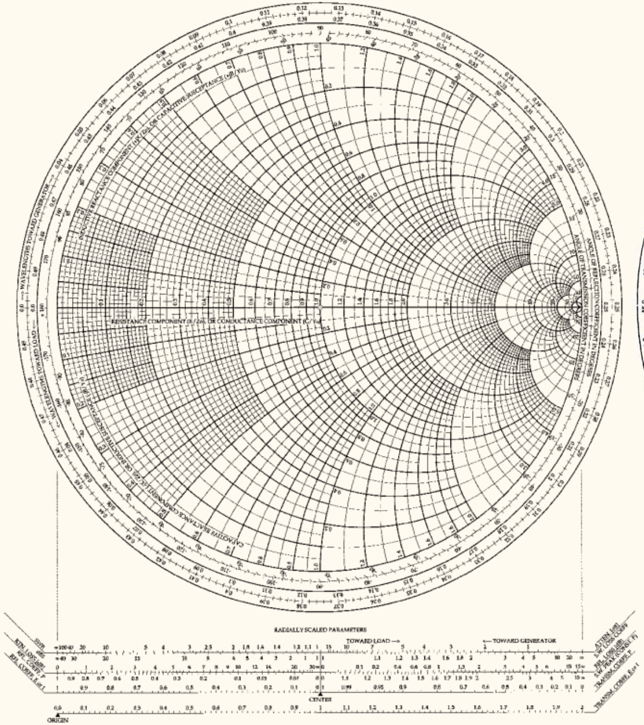
Common point: $(u, w) = (1, 0)$

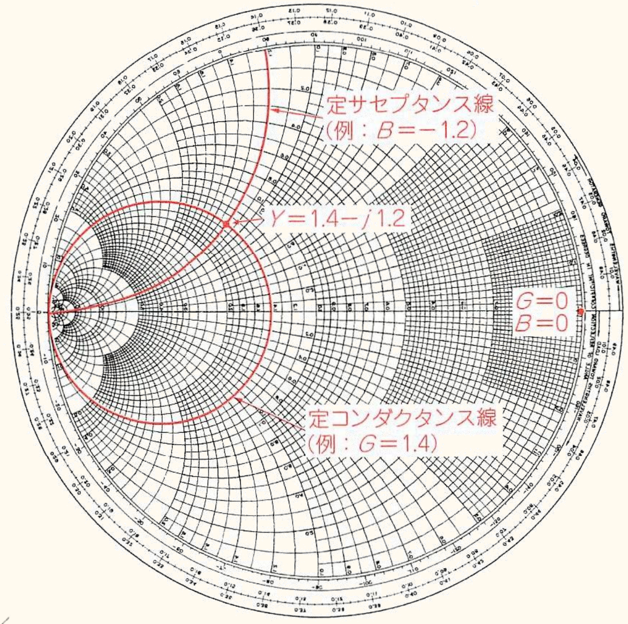


Smith chart

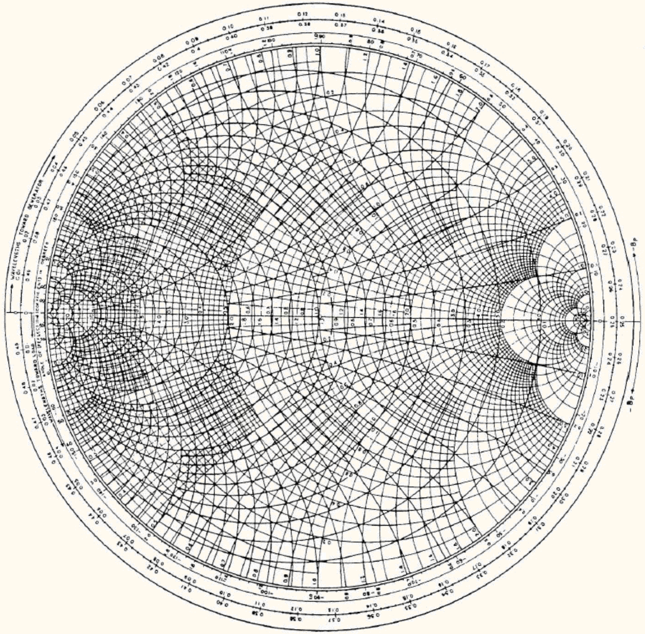


Smith chart, immittance chart





Admittance chart



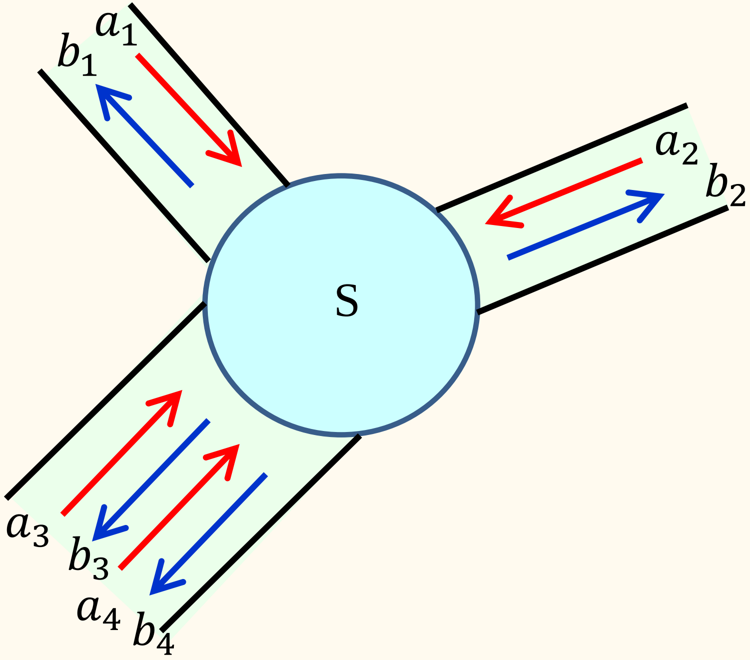
Immittance chart

5.3 Scattering (S) matrix (S parameters)

How to treat multipoint (crossing point) in transmission lines systematically?

Transmission lines: wave propagating modes → Channels

Take $|a_i|^2$, $|b_i|^2$ to be powers (energy flow).



output S-matrix input

$$\begin{pmatrix} b_1 \\ \vdots \\ b_i \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} S_{11} & \cdots & S_{1i} & \cdots & S_{1n} \\ \vdots & \ddots & & & \vdots \\ S_{i1} & & S_{ii} & & S_{in} \\ \vdots & & & \ddots & \vdots \\ S_{n1} & \cdots & S_{ni} & \cdots & S_{nn} \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{pmatrix}$$

$$\boldsymbol{b} = \mathrm{S}\boldsymbol{a}$$

S-matrix (S parameters)

S-matrix symmetries

(In case no dissipation, no amplification)

Reciprocity $S_{ij} = S_{ji}$

Unitarity $\sum_j S_{ji} S_{jk}^* = \delta_{ik}$

Propagation
with no dissipation

$$\begin{cases} a_n = \dfrac{V_{n+}}{\sqrt{Z_{0n}}} = J_{n+}\sqrt{Z_{0n}}, \\ b_n = \dfrac{V_{n-}}{\sqrt{Z_{0n}}} = J_{n-}\sqrt{Z_{0n}} \end{cases}$$

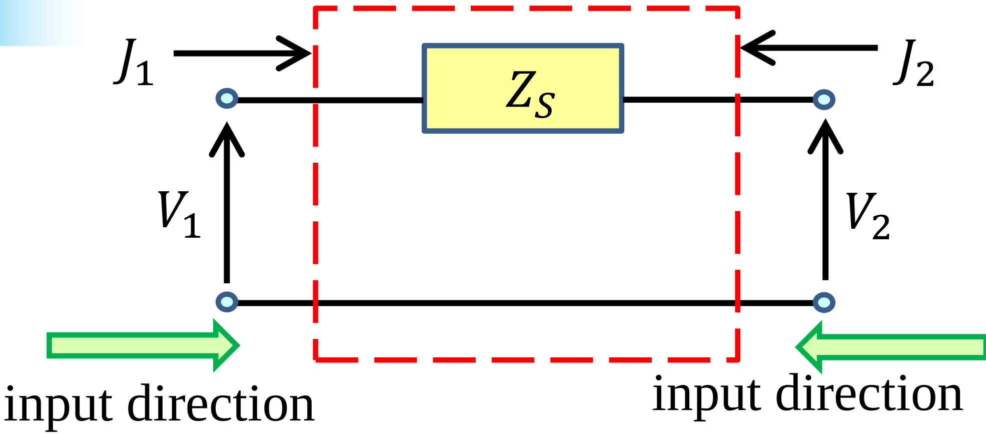
Incident power wave

Reflected (transmitted)
power wave

$$|a_n|^2 = \frac{|V_{n+}|^2}{Z_{0n}} = |J_{n+}|^2 Z_{0n}$$

S matrix (S parameters)

Simplest example: series impedance Z_S

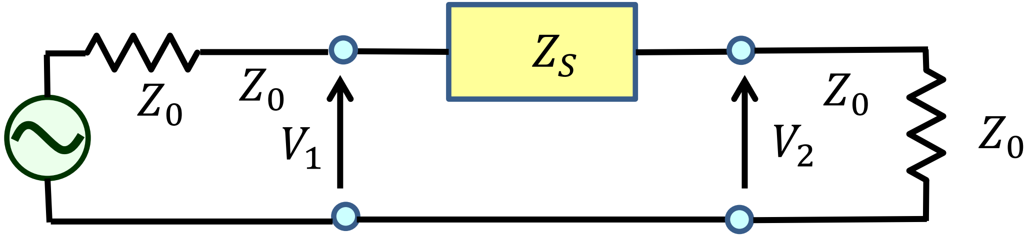


Take voltage as the "flow" quantity.
(assume common characteristic impedance)

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \qquad \begin{pmatrix} V_{1-} \\ V_{2-} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_{1+} \\ V_{2+} \end{pmatrix}$$

Measurement (calculation) of the S-matrix in the case the above is placed in transmission line with characteristic impedance Z_0.

Consider the case that the ends of the line is terminated with Z_0 and the source is connected at the left end.



Simple example of S-matrix

Terminate 2 with $Z_0 \rightarrow\ a_2 = 0$ (no reflection, no wave source at the right end)

$$S_{11} = \frac{V_{1-}}{V_{1+}} = \frac{V_1 - Z_0 J_1}{V_1 + Z_0 J_1} = \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{(Z_S + Z_0) - Z_0}{(Z_S + Z_0) + Z_0} = \frac{Z_S}{Z_S + 2Z_0}$$

$$S_{21} = \frac{V_{2-}}{V_{1+}} = \frac{V_2 - Z_0 J_2}{V_1 + Z_0 J_1} = \frac{Z_0 J_1 + Z_0 J_1}{(Z_S + Z_0) J_1 + Z_0 J_1} = \frac{2Z_0}{Z_S + 2Z_0} \quad (J_2 = -J_1)$$

From symmetry of S-matrix: $S_{22} = S_{11} = \dfrac{Z_S}{Z_S + 2Z_0}, \quad S_{12} = S_{21} = \dfrac{2Z_0}{Z_S + 2Z_0}$

When the impedance matrix $Z = \{Z_{ij}\}$ is given, the S-matrix is obtained as

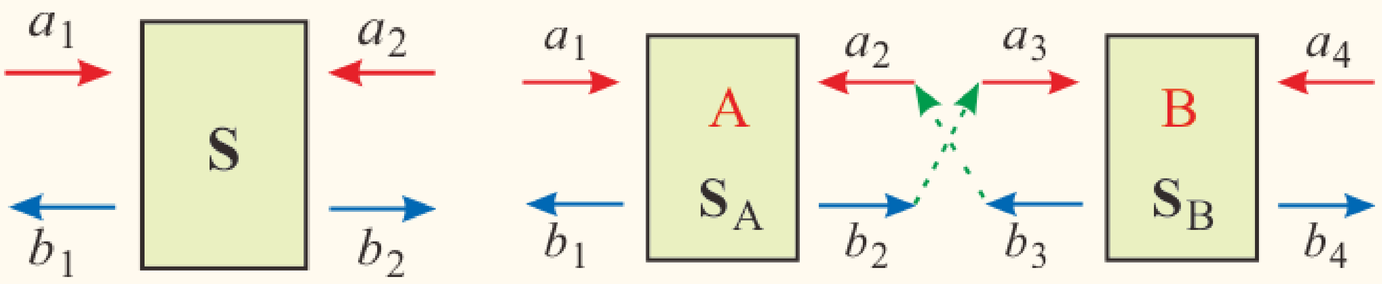
$$S = \frac{1}{\det Z} \begin{pmatrix} (Z_{11} - Z_0)(Z_{22} + Z_0) - Z_{12} Z_{21} & 2Z_0 Z_{12} \\ 2Z_0 Z_{21} & (Z_{11} - Z_0)(Z_{22} + Z_0) - Z_{12} Z_{21} \end{pmatrix}$$

Cascade connection of S-matrices

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \mathbf{S} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} r_\mathrm{L} & t_\mathrm{R} \\ t_\mathrm{L} & r_\mathrm{R} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$r_{L,R}$, $t_{L,R}$: complex reflection, transmission coefficients satisfying

$$T_\mathrm{L,R} = |t_\mathrm{L,r}|^2 = 1 - R_\mathrm{L,R} = 1 - |r_\mathrm{L,R}|^2$$

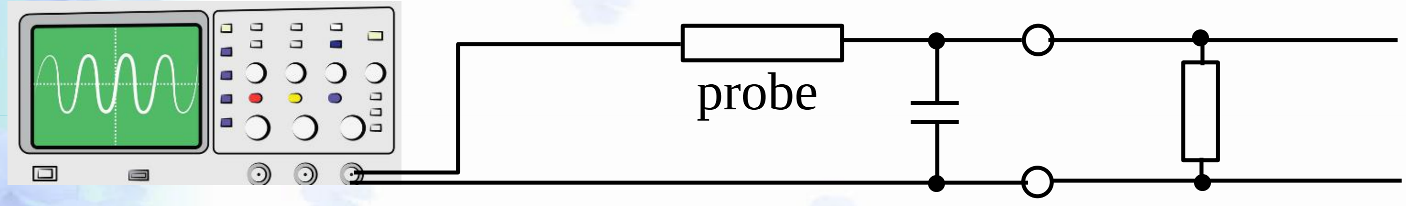


$$\mathbf{S}_\mathrm{AB} = \begin{pmatrix} r_\mathrm{L}^\mathrm{AB} & t_\mathrm{R}^\mathrm{AB} \\ t_\mathrm{L}^\mathrm{AB} & r_\mathrm{R}^\mathrm{AB} \end{pmatrix} = \begin{pmatrix} r_\mathrm{L}^\mathrm{A} + t_\mathrm{R}^\mathrm{A} r_\mathrm{L}^\mathrm{B} \left(I - r_\mathrm{R}^\mathrm{A} r_\mathrm{L}^\mathrm{B}\right)^{-1} t_\mathrm{L}^\mathrm{A} & t_\mathrm{R}^\mathrm{A} \left(I - r_\mathrm{L}^\mathrm{B} r_\mathrm{R}^\mathrm{A}\right)^{-1} t_\mathrm{R}^\mathrm{B} \\ t_\mathrm{L}^\mathrm{B} \left(I - r_\mathrm{R}^\mathrm{A} r_\mathrm{L}^\mathrm{B}\right)^{-1} t_\mathrm{L}^\mathrm{A} & r_\mathrm{R}^\mathrm{B} + t_\mathrm{L}^\mathrm{B} \left(I - r_\mathrm{R}^\mathrm{A} r_\mathrm{L}^\mathrm{B}\right)^{-1} r_\mathrm{R}^\mathrm{A} t_\mathrm{R}^\mathrm{B} \end{pmatrix}$$

$$(I - r_R^A r_L^B)^{-1} = I + r_R^A r_L^B + (r_R^A r_L^B)^2 + \cdots$$

Why we need S-matrix (S-parameters) for high frequency lines?

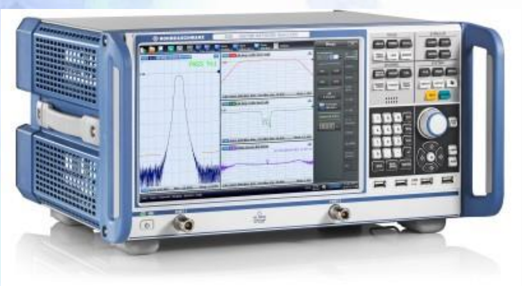
Difficulty in measurement of voltage at high frequencies.



oscilloscope

1. Cannot ignore distributed capacitance
2. Probe line should also be treated as a transmission line

Vector network analyzer



On the other hand, S-parameters can be obtained from power measurements.

S-parameters give enough information for designing high frequency circuits.

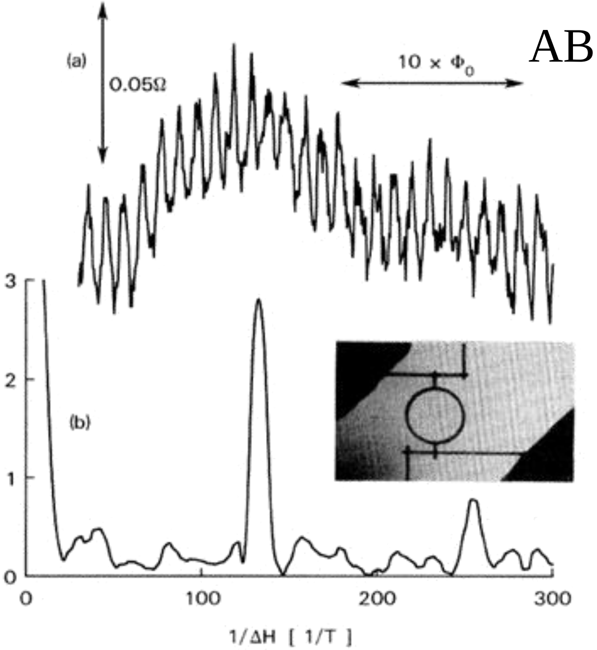
Application of S-matrix to quantum transport

Electron (quantum mechanical) waves also have propagating modes in solids.

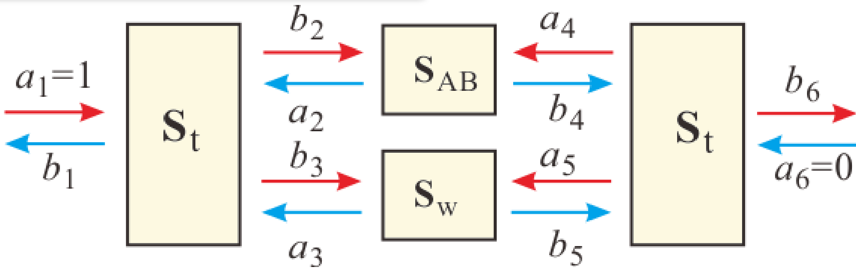
$\rightarrow$ Conduction channel

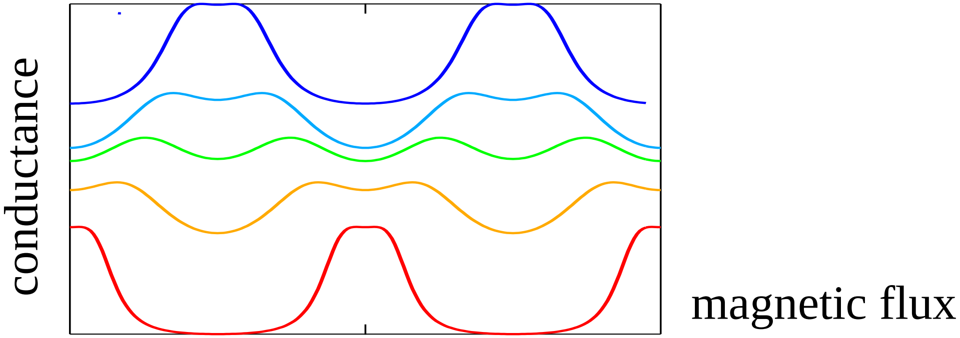
Landauer equation: the conductance of a single perfect quantum channel is $\dfrac{e^2}{h}$

Rolf Landauer

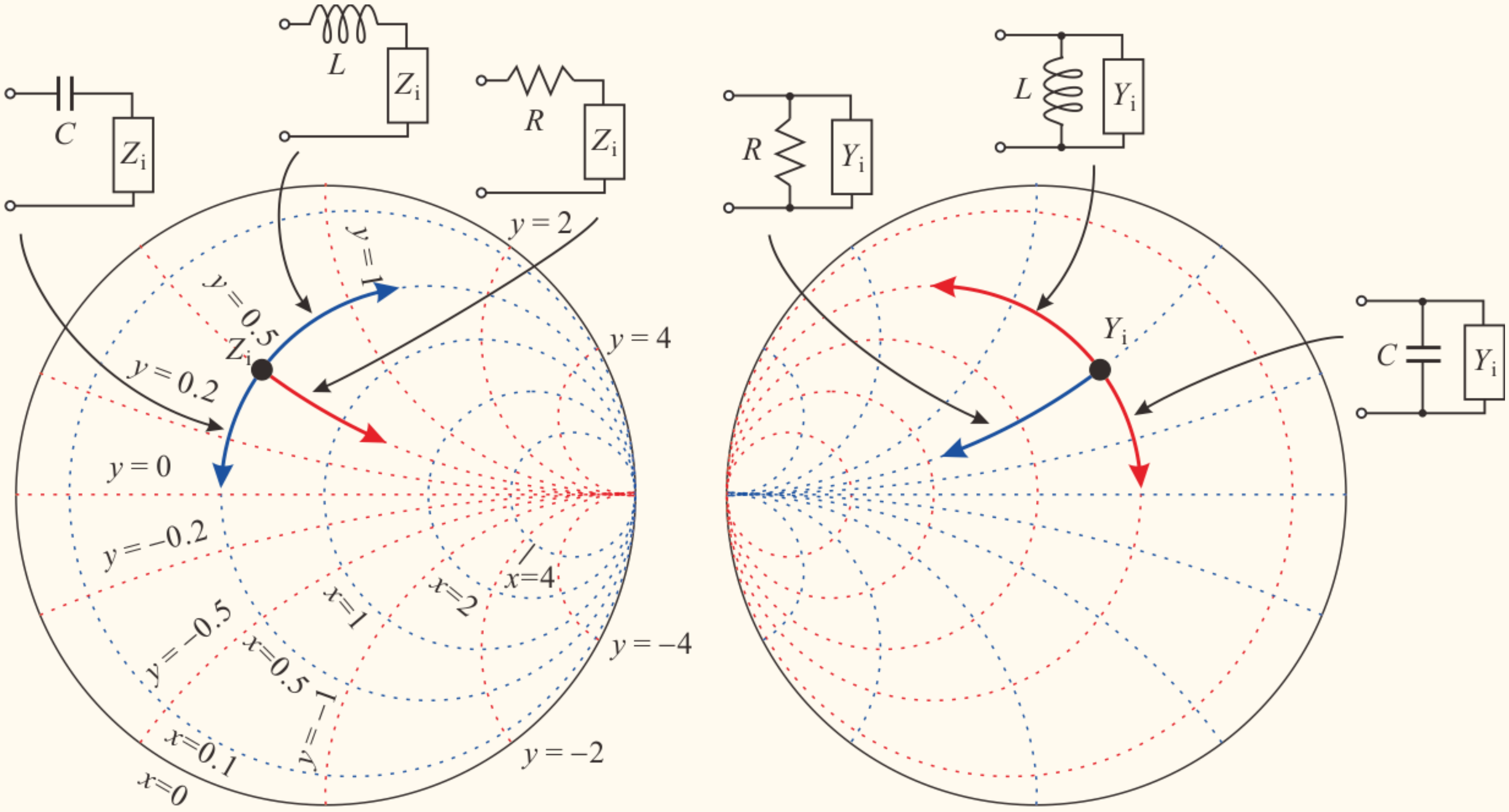


AB ring S-matrix model



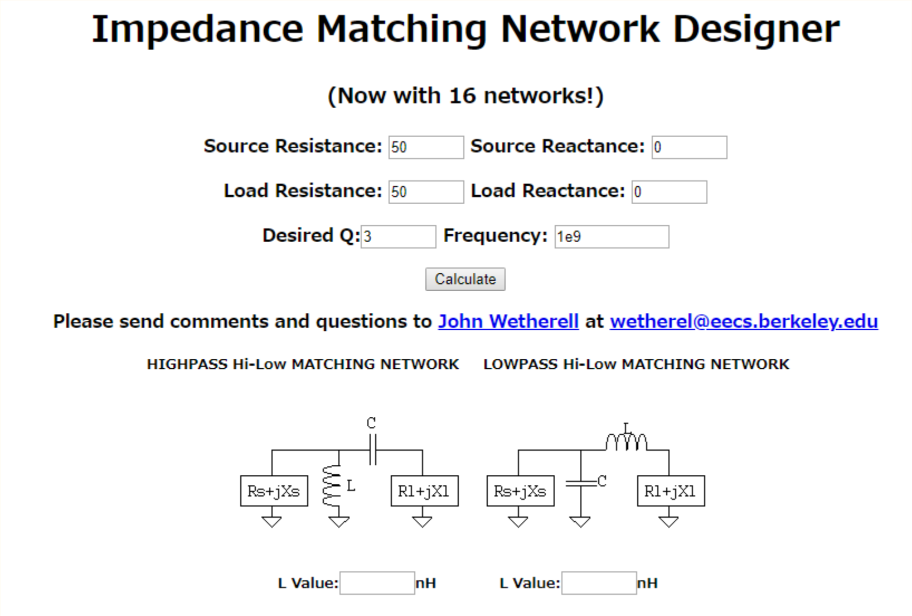


Practical impedance matching with Smith chart

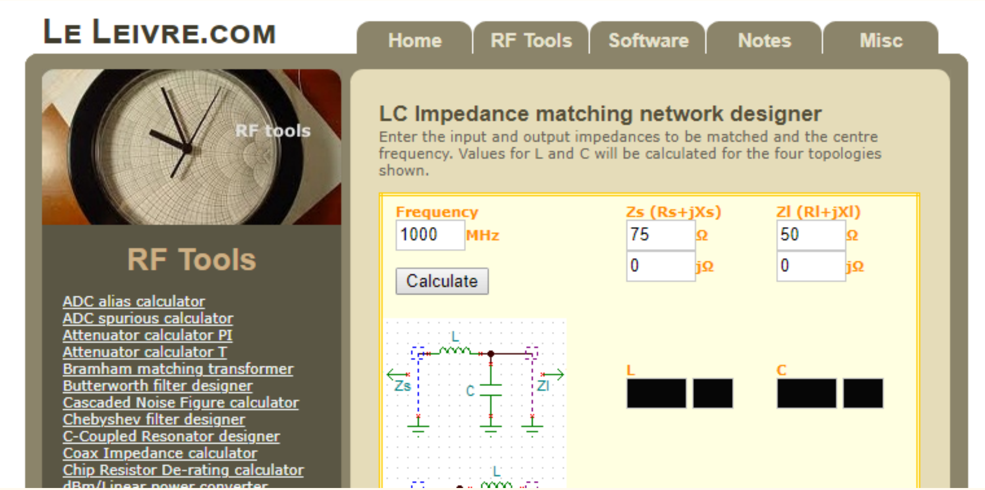


Impedance matching designer

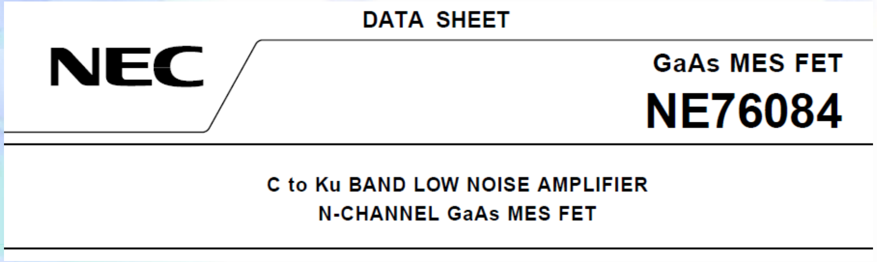
http://home.sandiego.edu/~ekim/e194rfs01/jwmatcher/matcher2.html



http://leleivre.com/rf_lcmatch.html



An example FET power matching


DATA SHEET

NEC

GaAs MES FET

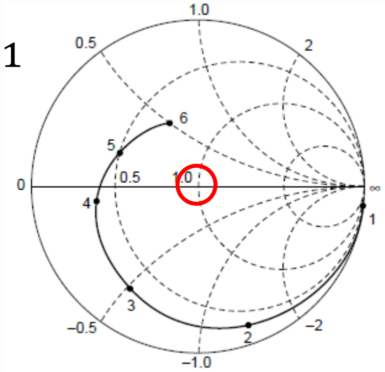
NE76084

C to Ku BAND LOW NOISE AMPLIFIER
N-CHANNEL GaAs MES FET

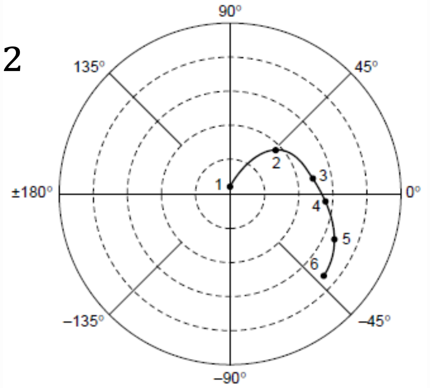
C band: 4 – 8 GHz
X band: 8 – 12 GHz
Ku band: 12 – 18 GHz

The datasheet tells that we need impedance matching circuits with transmission lines.

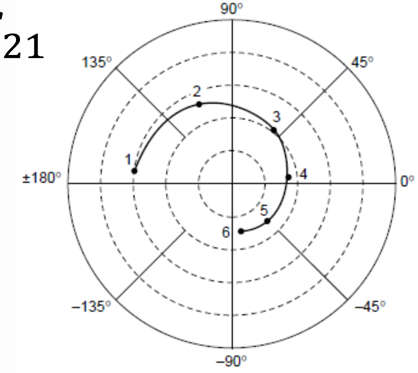
S_{11}



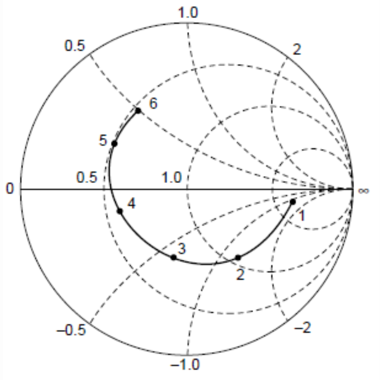
S_{12}



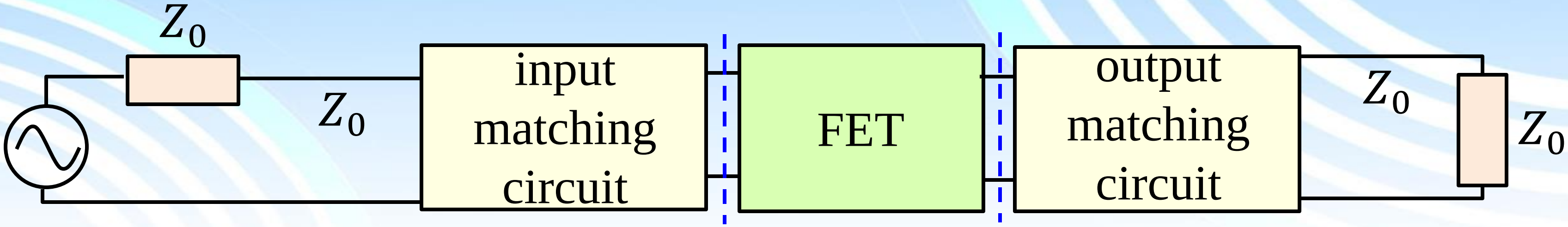
S_{21}

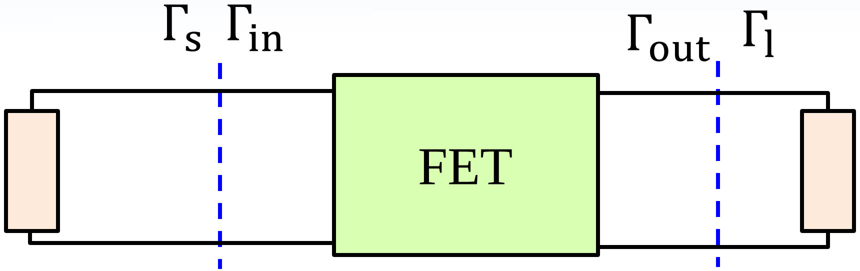


S_{22}



S parameter representation and matching circuits





$\Gamma_{\mathrm{s,l,in,out}}$: reflection coefficients.

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

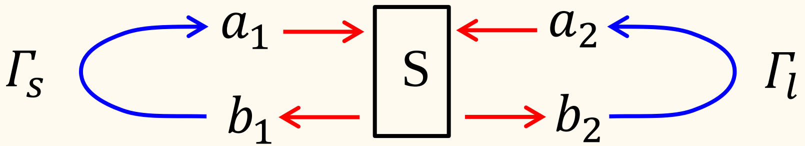
FET S-matrix

The situation can be seen as:

Source with reflection coefficient Γ_s, load with reflection coefficient Γ_l are connected.

Power matching with S-parameters

That is



$$\frac{a_1}{b_1} = \Gamma_{\mathrm{s}}, \quad \frac{a_2}{b_2} = \Gamma_{\mathrm{l}}$$

Then the total coefficients are $\Gamma_{\mathrm{in}} = S_{11} + \dfrac{S_{12}S_{21}\Gamma_{\mathrm{l}}}{1 - S_{22}\Gamma_{\mathrm{l}}}, \quad \Gamma_{\mathrm{out}} = S_{22} + \dfrac{S_{12}S_{21}\Gamma_{\mathrm{s}}}{1 - S_{22}\Gamma_{\mathrm{s}}}$

The power matching conditions are $\Gamma_{\mathrm{l}} = \Gamma_{\mathrm{out}}^*, \quad \Gamma_{\mathrm{s}} = \Gamma_{\mathrm{in}}^*$

The quadratic equation gives

$$\Gamma_{\mathrm{s}} = \frac{B_1 \pm \sqrt{B_1^2 - 4|M|^2}}{2M}, \quad \Gamma_l = \frac{B_2 \pm \sqrt{B_2^2 - 4|N|^2}}{2N}$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\mathrm{det}S|^2, \quad B_2 = 1 - |S_{11}|^2 + |S_{22}|^2 - |\mathrm{det}S|^2,$$

$$N = S_{22} - S_{11}^*\mathrm{det}S, \quad M = S_{11} - S_{22}^*\mathrm{det}S$$

Then the problem is reduced to tune the passive circuits to Γ_{s} and Γ_{l}