

The background image shows a vast, dimly lit industrial or scientific facility, likely a particle detector. It is filled with intricate machinery, including large metal structures, numerous blue and red cables, and various electronic components. A man wearing a red hard hat and a dark jacket stands in the center of the frame, looking towards the camera. The overall atmosphere is one of a high-tech, complex environment.

電子回路論第9回

Electric Circuits for Physicists #9

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Outline

Various transmission lines
Termination and connection
Smith chart
S-matrix (S-parameters)

An example FET power matching

NEC

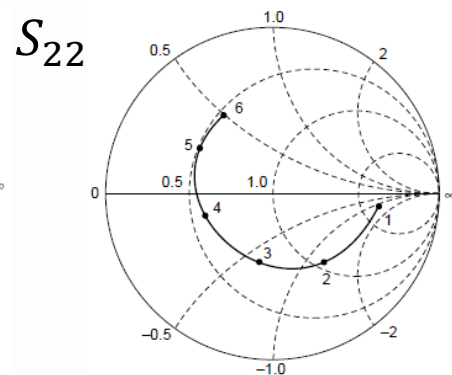
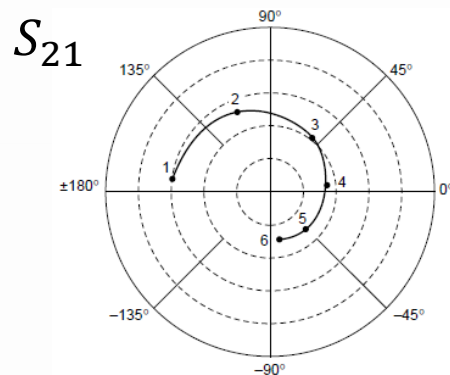
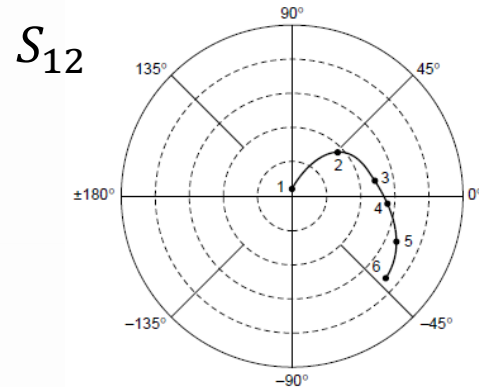
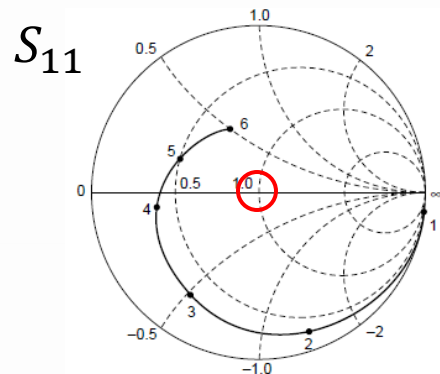
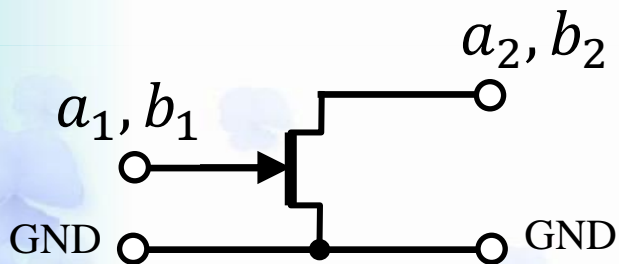
DATA SHEET

GaAs MES FET
NE76084

C to Ku BAND LOW NOISE AMPLIFIER
N-CHANNEL GaAs MES FET

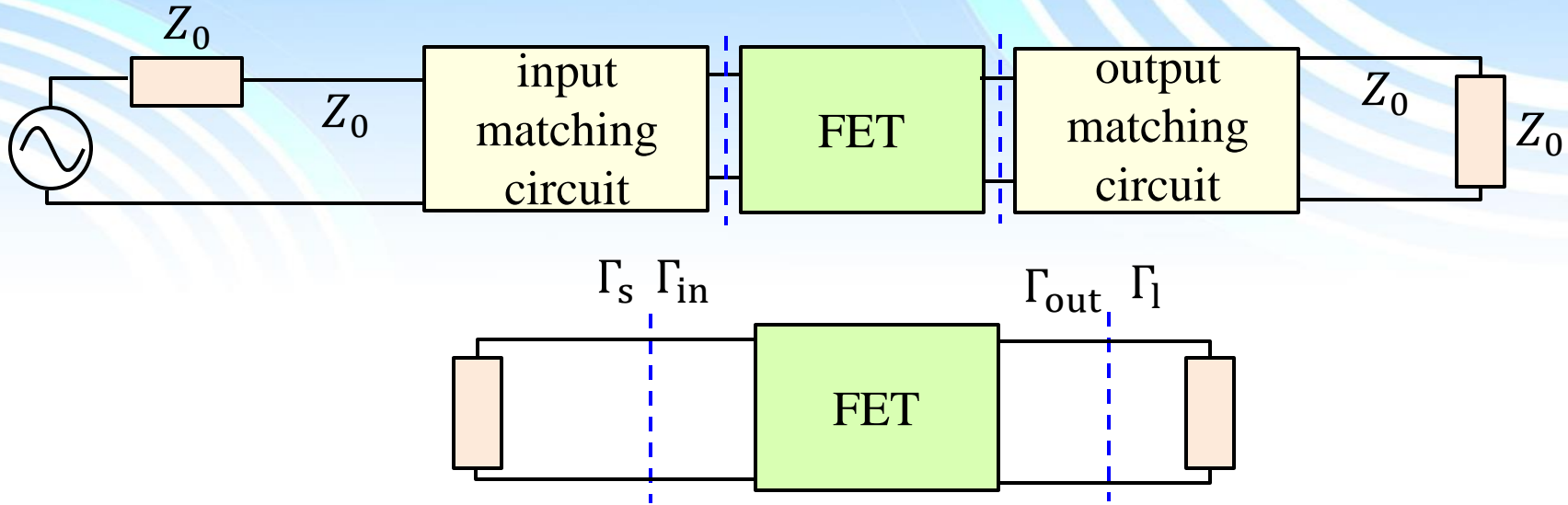


C band: 4 – 8 GHz
X band: 8 – 12 GHz
Ku band: 12 – 18 GHz



S_{12} and S_{21} are normalized with some constants

S parameter representation and matching circuits



$\Gamma_{s,l,in,out}$: reflection coefficients.

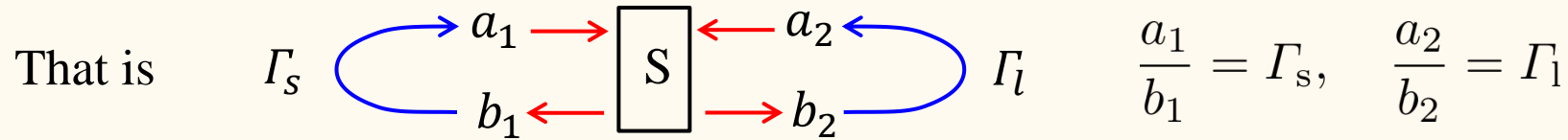
$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

FET S-matrix

The situation can be seen as:

Source with reflection coefficient Γ_s , load with reflection coefficient Γ_l are connected.

Power matching with S-parameters



Then the total coefficients are $\Gamma_{\text{in}} = S_{11} + \frac{S_{12}S_{21}\Gamma_l}{1 - S_{22}\Gamma_l}$, $\Gamma_{\text{out}} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}$

The power matching conditions are

$$\Gamma_l = \Gamma_{\text{out}}^*, \quad \Gamma_s = \Gamma_{\text{in}}^*$$

The quadratic equation gives

$$\Gamma_s = \frac{B_1 \pm \sqrt{B_1^2 - 4|M|^2}}{2M}, \quad \Gamma_l = \frac{B_2 \pm \sqrt{B_2^2 - 4|N|^2}}{2N}$$

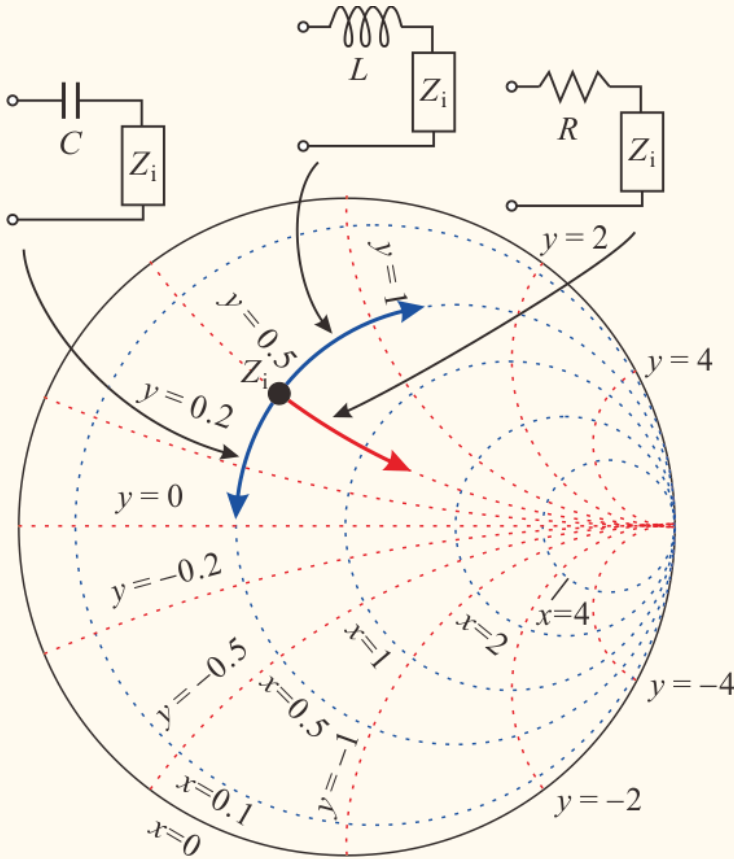
$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\det S|^2, \quad B_2 = 1 - |S_{11}|^2 + |S_{22}|^2 - |\det S|^2,$$

$$N = S_{22} - S_{11}^* \det S, \quad M = S_{11} - S_{22}^* \det S$$

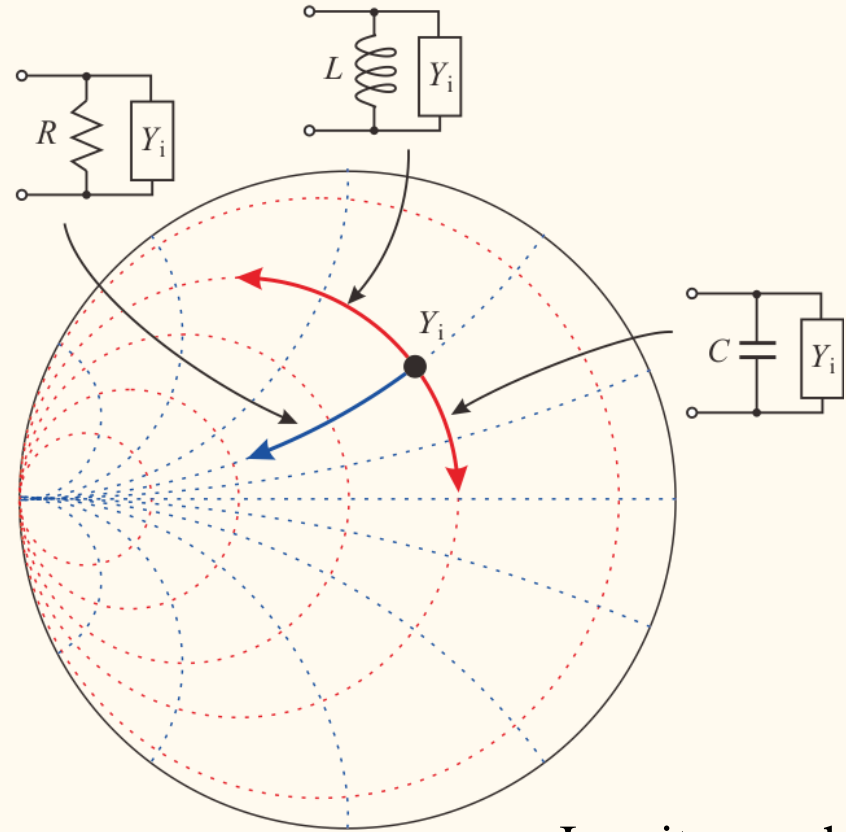
Then the problem is reduced to tune the passive circuits to Γ_s and Γ_l

A simplified way to make impedance matching

Series and parallel connection of passive elements and traces on charts



Smith chart



Immittance chart

6.1.2 Wiener-Khintchine theorem

Autocorrelation function $C(\tau) = \overline{\langle x(t)x(t + \tau) \rangle}$

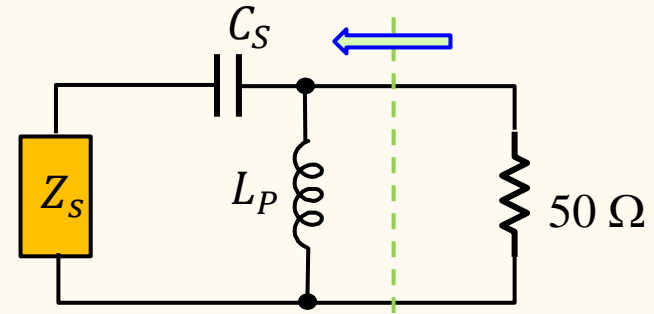
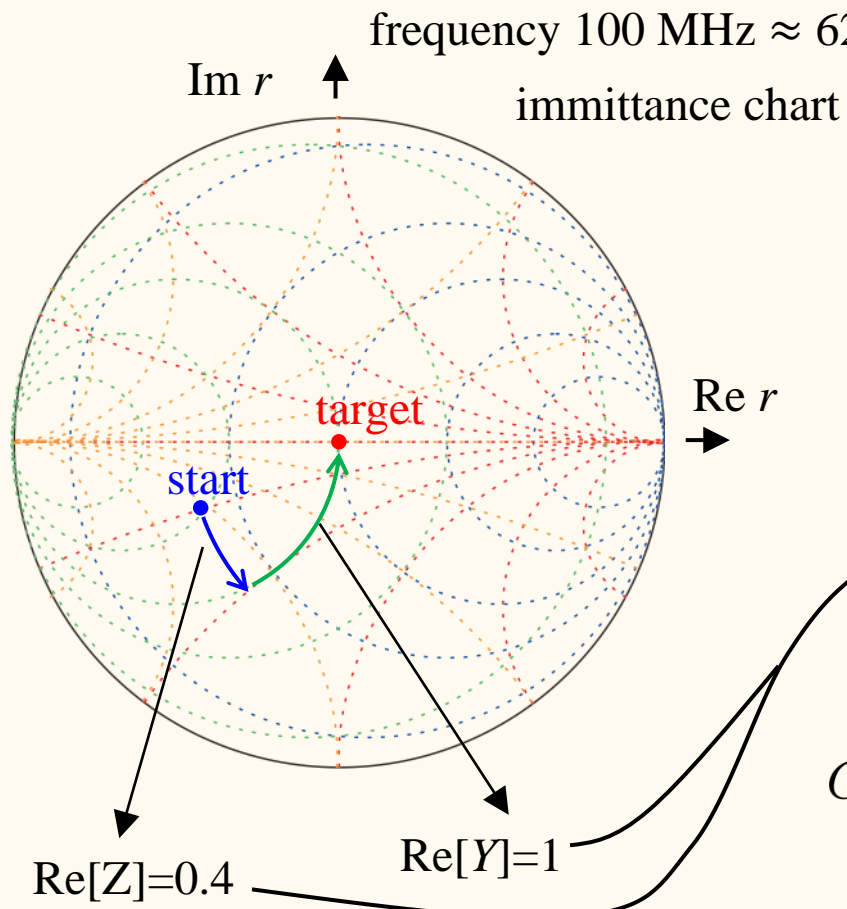
$$\begin{aligned} &= \overline{\sum_{n,m} \langle [a_n \cos \omega_n t + b_n \sin \omega_n t][a_m \cos \omega_m(t + \tau) + b_m \sin \omega_m(t + \tau)] \rangle} \\ &= \frac{1}{2} \sum_n \overline{\langle a_n^2 + b_n^2 \rangle} \cos \omega_n \tau = \sum_n \overline{\langle \mathcal{P}_n \rangle} \cos \omega_n \tau \\ &= \int_0^\infty G(\omega) \cos \omega \tau \frac{d\omega}{2\pi} \end{aligned}$$

$$C(\tau) = \int_0^\infty G(\omega) \cos \omega \tau \frac{d\omega}{2\pi}$$

$$G(\omega) = 4 \int_0^\infty C(\tau) \cos \omega \tau d\tau$$

Wiener-Khintchine theorem

An example of impedance matching



$$Z_S = 20 - 10i \quad (\Omega)$$

$$= 0.4 - 0.2i \quad (Z_0)$$

equalize: $0.4 + iy = \frac{1}{1 + iq}$

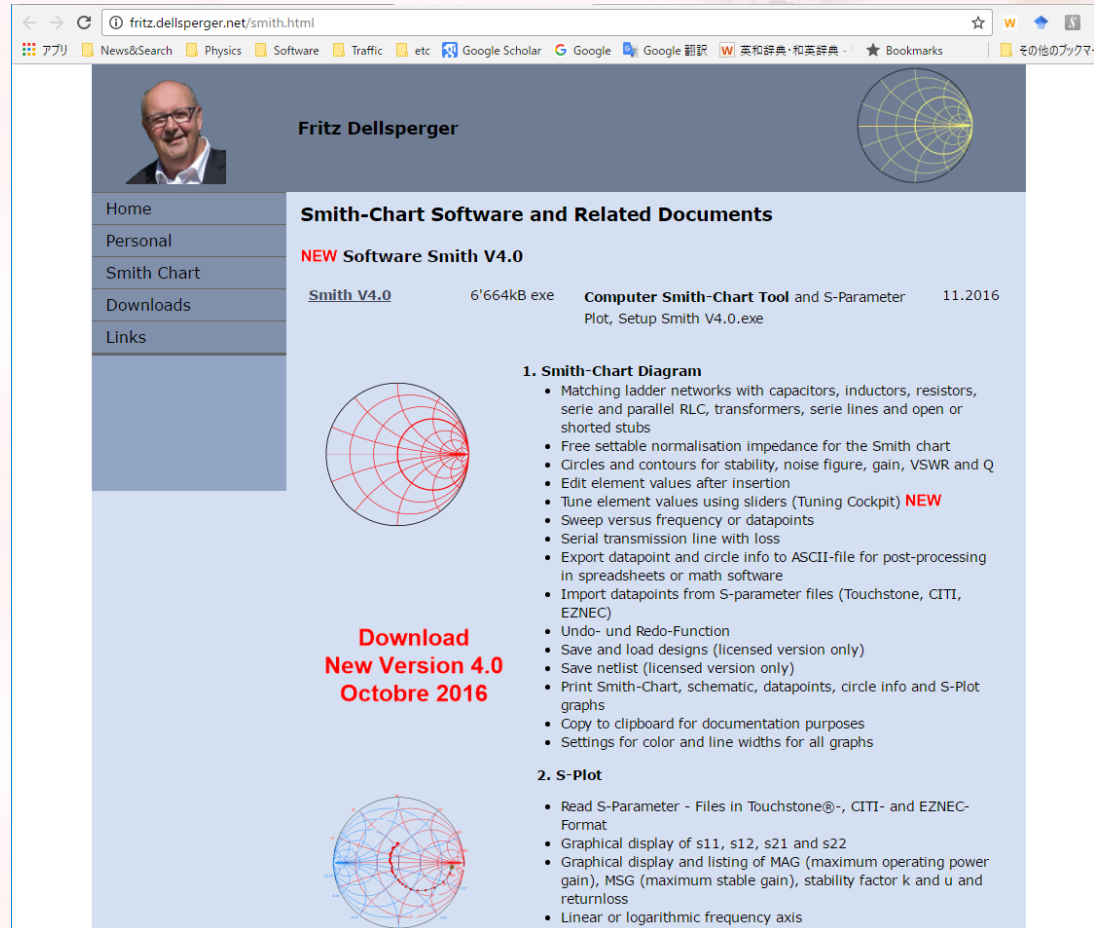
$$y = -\sqrt{0.24} \approx -0.49 = -0.2 - \frac{1}{\omega C_S Z_0}$$

$$C_S \approx \frac{1}{2\pi \times 10^6 \times 50 \times 0.29} \approx 110 \text{ pF}$$

Similarly $L_P \approx 65 \text{ nH}$

Introduction of useful freeware Smith v4.0 (present version 4.1)

<http://fritz.dellsperger.net/smith.html>



The screenshot shows a web browser displaying the website fritz.dellsperger.net/smith.html. The page features a navigation menu on the left with links for Home, Personal, Smith Chart, Downloads, and Links. The main content area is titled "Smith-Chart Software and Related Documents" and includes a "NEW Software Smith V4.0" announcement. Below this, there is a table listing the software version, file size, and download date. The page also contains two sections of bullet points: "1. Smith-Chart Diagram" and "2. S-Plot", each with a corresponding Smith chart diagram. The first diagram is a red Smith chart, and the second is a blue Smith chart.

Navigation menu:

- Home
- Personal
- Smith Chart
- Downloads
- Links

Header: Fritz Dellsperger

Smith-Chart Software and Related Documents

NEW Software Smith V4.0

Software	File Size	Software Name	Release Date
Smith V4.0	6'664kB exe	Computer Smith-Chart Tool and S-Parameter Plot, Setup Smith V4.0.exe	11.2016

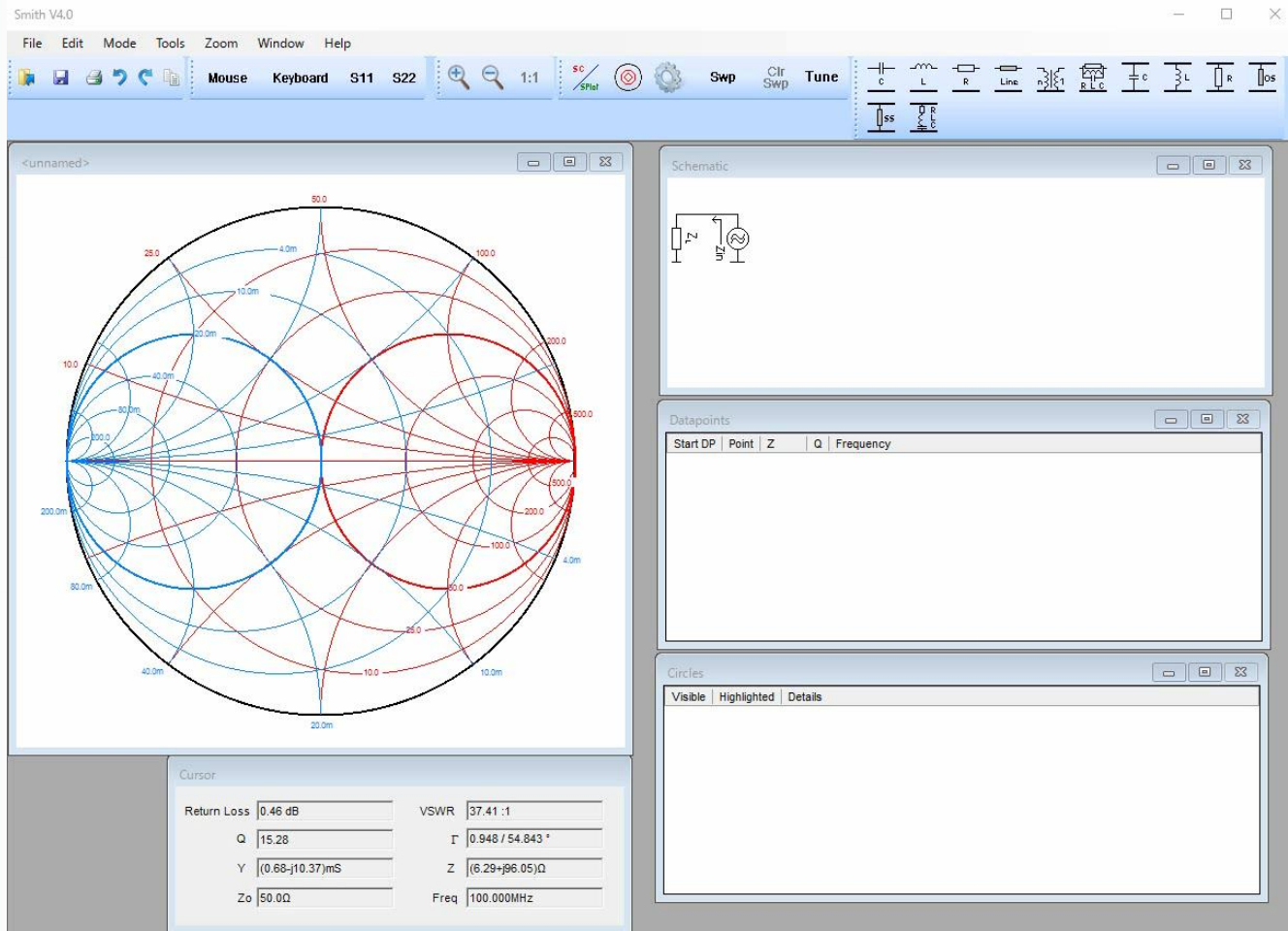
1. Smith-Chart Diagram

- Matching ladder networks with capacitors, inductors, resistors, serie and parallel RLC, transformers, serie lines and open or shorted stubs
- Free settable normalisation impedance for the Smith chart
- Circles and contours for stability, noise figure, gain, VSWR and Q
- Edit element values after insertion
- Tune element values using sliders (Tuning Cockpit) **NEW**
- Sweep versus frequency or datapoints
- Serial transmission line with loss
- Export datapoint and circle info to ASCII-file for post-processing in spreadsheets or math software
- Import datapoints from S-parameter files (Touchstone, CITI, EZNEC)
- Undo- und Redo-Funktion
- Save and load designs (licensed version only)
- Save netlist (licensed version only)
- Print Smith-Chart, schematic, datapoints, circle info and S-Plot graphs
- Copy to clipboard for documentation purposes
- Settings for color and line widths for all graphs

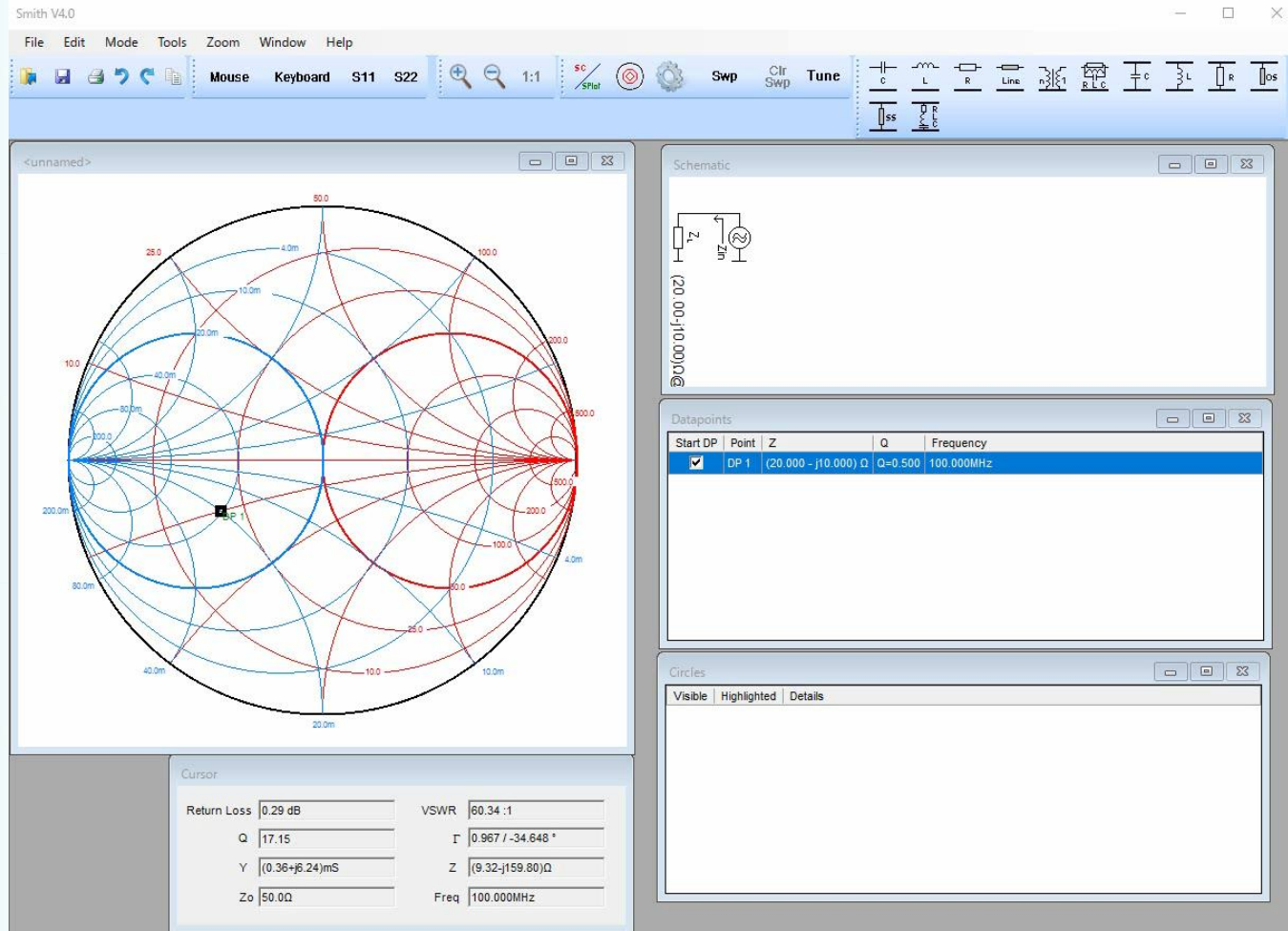
2. S-Plot

- Read S-Parameter - Files in Touchstone@-, CITI- and EZNEC-Format
- Graphical display of s11, s12, s21 and s22
- Graphical display and listing of MAG (maximum operating power gain), MSG (maximum stable gain), stability factor k and u and returnloss
- Linear or logarithmic frequency axis

Impedance matching with Smith v4 (1)



Impedance matching with Smith V4 (2)

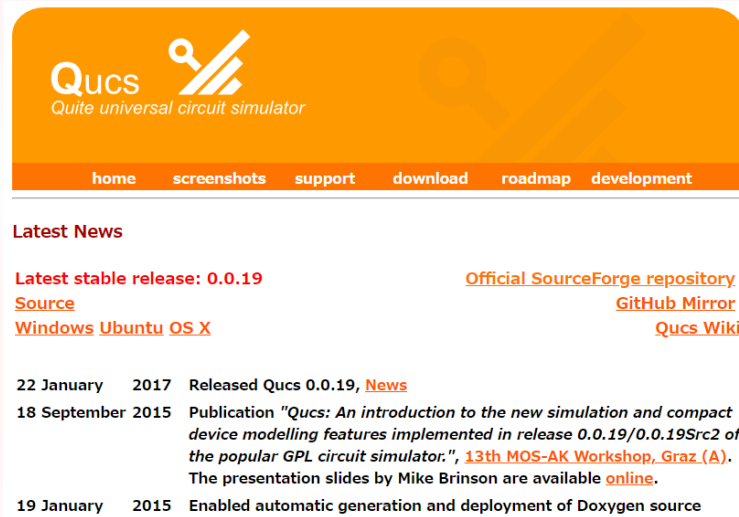


Introduction of useful freeware Qucs 0.0.19

Qucs: Quite Universal Circuit Simulator

<http://qucs.sourceforge.net/>

The Qucs project was begun by Michael Margraf in Germany in 2004.



The screenshot shows the Qucs website homepage. At the top, there is a logo for Qucs (Quite Universal Circuit Simulator) with the text "Qucs" and "Quite universal circuit simulator". Below the logo is a navigation menu with links for "home", "screenshots", "support", "download", "roadmap", and "development". Underneath the navigation menu is a "Latest News" section. It features a "Latest stable release: 0.0.19" with links to the "Official SourceForge repository", "Source", "Windows", "Ubuntu", and "OS X". There are also links for "GitHub Mirror" and "Qucs Wiki". Below this, there is a list of news items with dates and descriptions.

Latest News

Latest stable release: 0.0.19 [Official SourceForge repository](#)
[Source](#) [GitHub Mirror](#)
[Windows](#) [Ubuntu](#) [OS X](#) [Qucs Wiki](#)

22 January 2017 Released Qucs 0.0.19, [News](#)

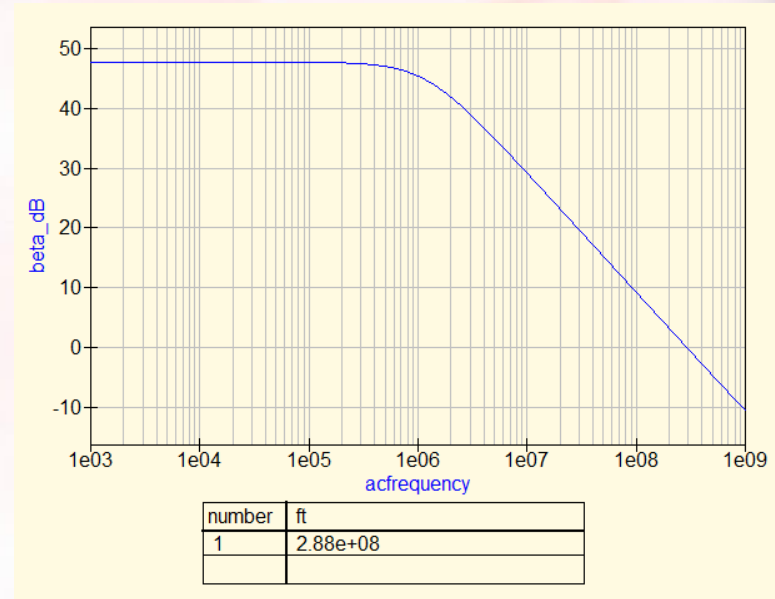
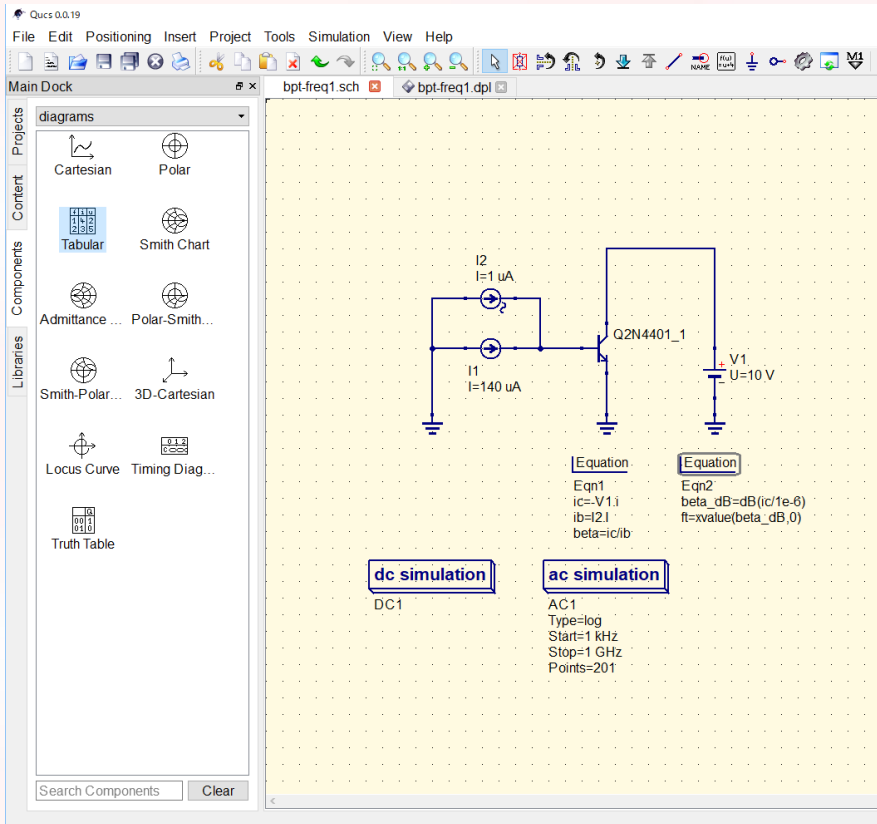
18 September 2015 Publication "Qucs: An introduction to the new simulation and compact device modelling features implemented in release 0.0.19/0.0.19Src2 of the popular GPL circuit simulator.", [13th MOS-AK Workshop, Graz \(A\)](#). The presentation slides by Mike Brinson are available [online](#).

19 January 2015 Enabled automatic generation and deployment of Doxygen source

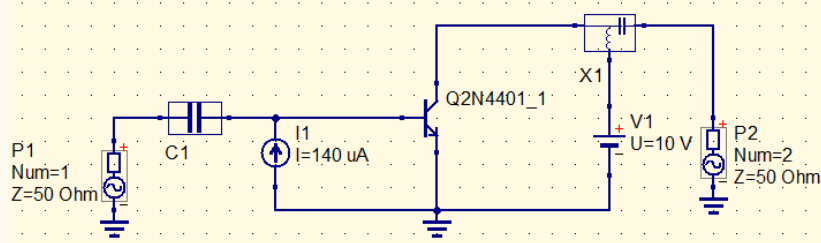
- Based on SPICE simulation language.
- Free but no restriction in the number of nodes, etc.
- Can read S-parameter files. Have S-parameter analysis options.
- In that sense, better than LTSpice.

Introduction of useful freeware Qucs 0.0.19

Example: Frequency characteristics of a bipolar transistor



S-parameter simulation of a bipolar transistor with Qucs



dc simulation

DC1:

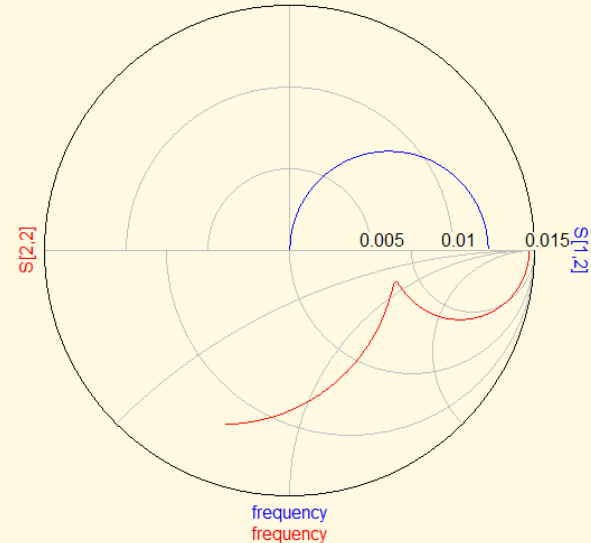
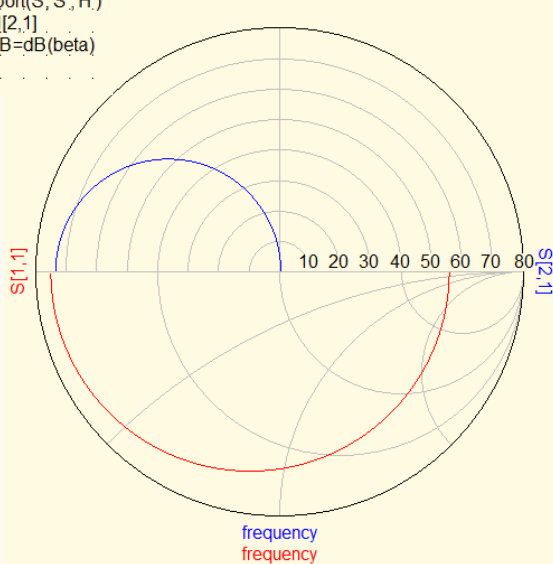
S parameter simulation

SP1

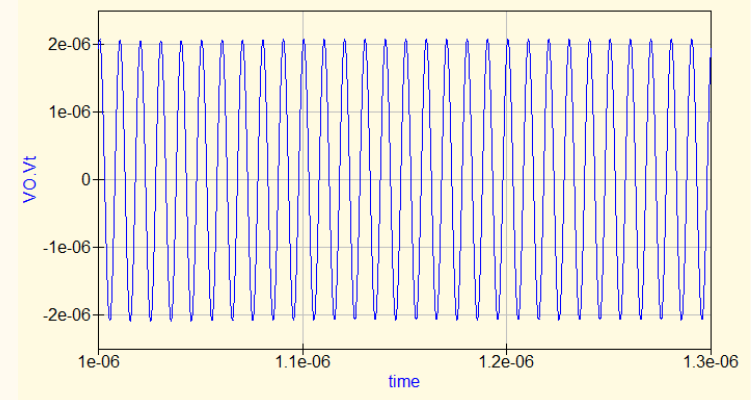
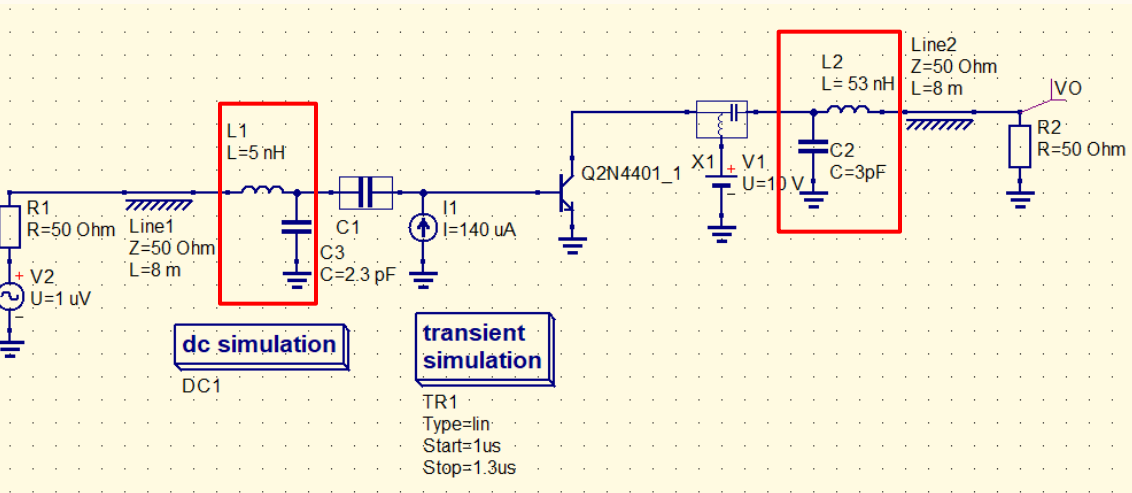
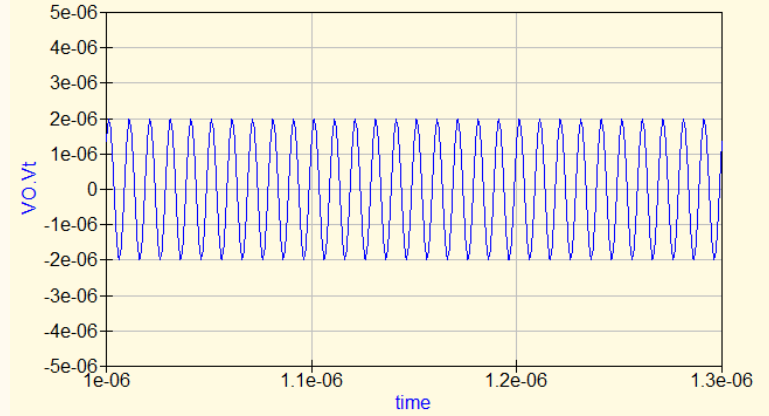
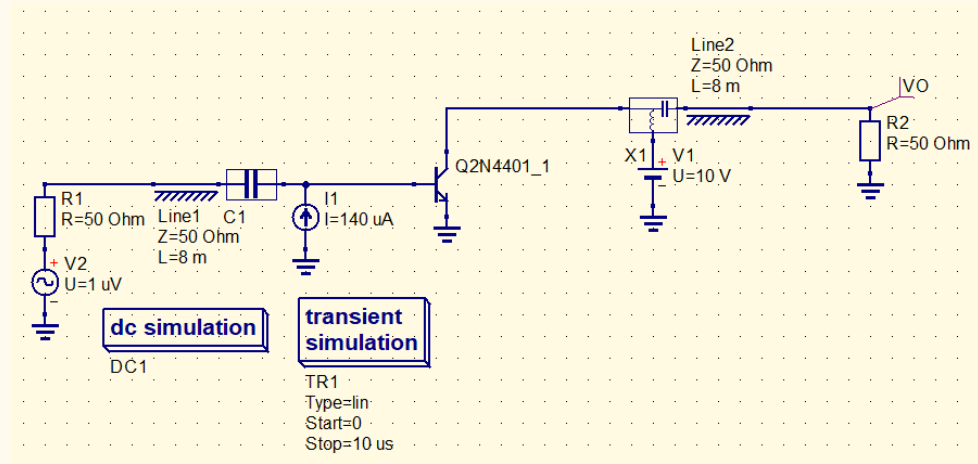
Type=log
Start=1 kHz
Stop=1 GHz
Points=201

Equation

Eqn1
y=1
H=twoport(S,'S','H')
beta=H[2,1]
beta_dB=dB(beta)



Simulation of matching circuit



Comments: Impedance match

Propagation of a wave: Impedance match: complete absorption (propagation without reflection)
Mismatch: wave reflection

Impedance match/mismatch is an important concept applicable to a broad area of physics.

➤ Antenna: should be matched to the vacuum.

EM wave propagation simulation: boundary is shunted
with the characteristic impedance of vacuum.

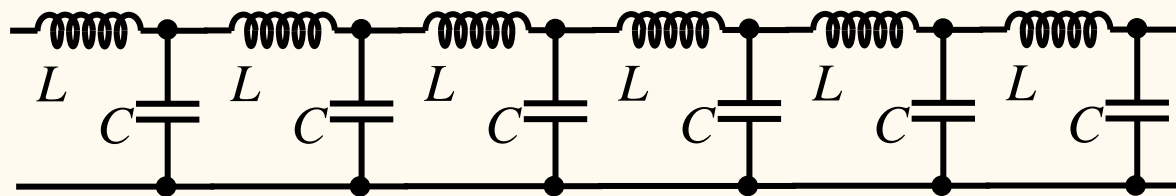
➤ Optics: impedance mismatch → disagreement in refractive index

➤ Plasma: should be matched to electrodes for excitation.

➤ Phonon impedance mismatch at low temperatures: Kapitza resistance

➤ Sound insulated booth: should have sound impedance mismatch.

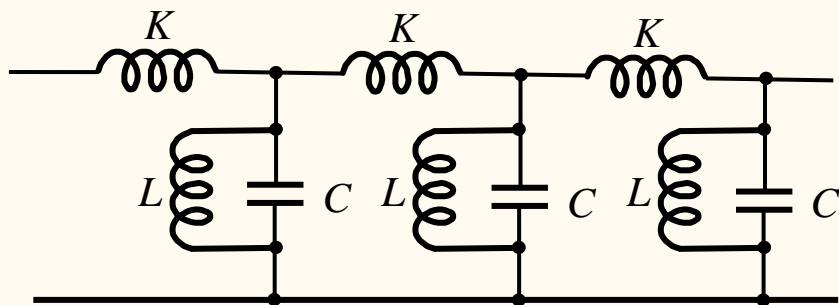
5.4 Non TEM mode transmission lines



... LC model: $Z = i\omega L$,
 $Y = i\omega C$

The inductance represents magnetic fields circulating the core and the capacitance electric fields directing from the core to the shield.

$$Z_0 = \sqrt{\frac{L}{C}} \quad : \text{real, dispersionless (linear } \omega\text{-}k \text{ relation)}$$



Non-linear ω -term in Z or $Y \rightarrow$ dispersion
(longitudinal components)

$$Y = i\omega C + \frac{1}{i\omega L}$$

C : capacitance per unit length

L : inductance per inverse unit length

K : inductance per unit length

5.4 Non TEM mode gives mass to the transmission mode

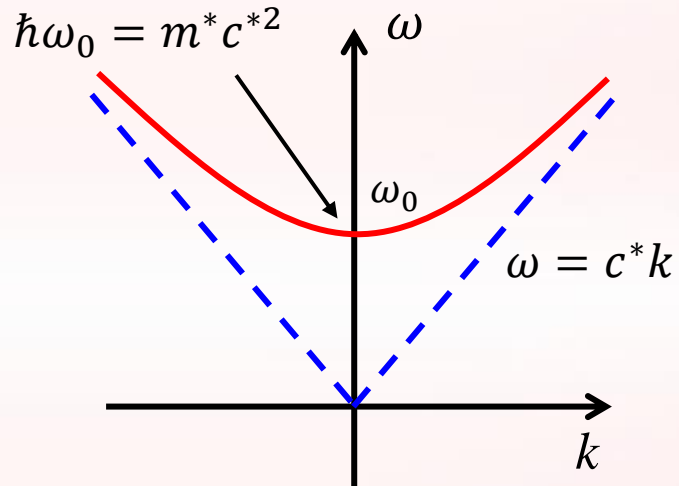
$$-k^2 = YZ = \left(i\omega C + \frac{1}{i\omega L} \right) i\omega K = -CK\omega^2 + \frac{K}{L}$$

Constant finite mass: $E = \hbar\omega \propto k^2$ (Schrodinger eq.: Parabolic partial differential equation)

Coupling between linear dispersions: mass mechanism (cf. Higgs)

$$\frac{1}{\sqrt{LC}} = \omega_0 \text{ unchanged with } dx \rightarrow 0$$

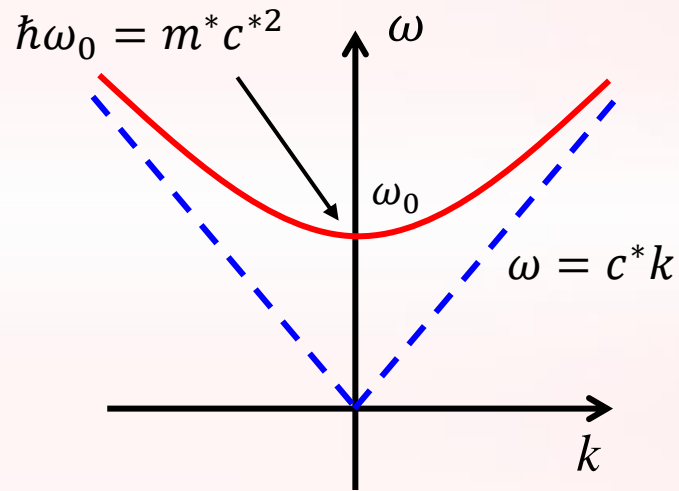
$$Z = i\omega K, \quad Y = \frac{1 - (\omega/\omega_0)^2}{i\omega L} \quad \text{then}$$



$$ik = \kappa = \sqrt{YZ} = i \sqrt{\frac{K}{L} \left[\left(\frac{\omega}{\omega_0} \right)^2 - 1 \right]}$$

$$k = \eta \sqrt{(\omega/\omega_0)^2 - 1}, \quad \eta^2 \equiv (K/L)$$

5.4 Non TEM mode gives mass to the transmission mode



$$\omega \gg \omega_0 \rightarrow k \sim \eta \frac{\omega}{\omega_0} \quad \text{No dispersion}$$

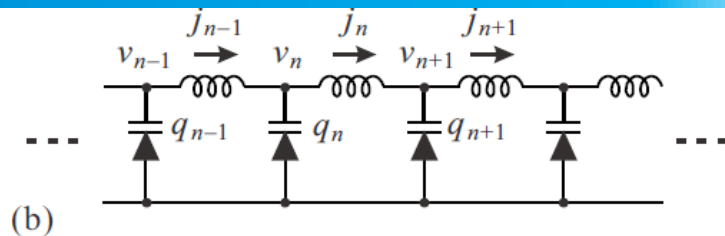
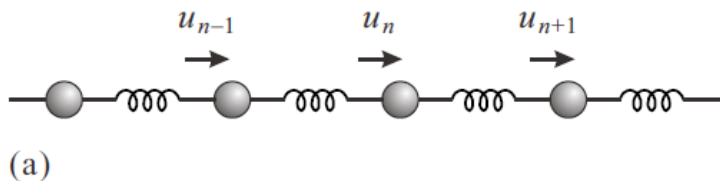
$$\text{velocity} \quad c^* = \frac{\omega}{k} = \frac{\omega_0}{\eta} = \frac{1}{\sqrt{KC}}$$

$$\omega \sim \omega_0 \quad \omega = \omega_0 + \delta\omega \quad \rightarrow k^2 \approx 2\eta^2 \frac{\delta\omega}{\omega_0}$$

$$\therefore \epsilon \equiv \hbar\delta\omega = \frac{\hbar k^2}{2(\eta^2/\omega_0)} = \frac{\hbar^2 k^2}{2m^*} \quad \left(m^* \equiv \frac{\hbar\eta^2}{\omega_0} \right)$$

$$E_0 = \hbar\omega_0 = \frac{\hbar\eta^2}{\omega_0} \cdot \left(\frac{\omega_0}{\eta} \right)^2 = m^* c^{*2}$$

5.5 Non-linear LC transmission line and Toda lattice



Toda lattice is a typical non-linear system with exact (soliton) solutions.

It is defined as follows:

The springs in (a) have Toda-potential:
$$\phi(r) = \frac{a}{b} e^{-br} + ar \quad (ab > 0)$$

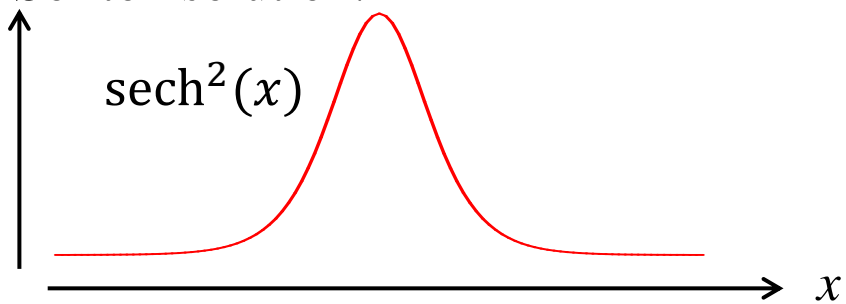
Equation of motion:
$$m \frac{d^2 u_n}{dt^2} = -a \exp[-b(u_{n+1} - u_n)] + a \exp[-b(u_n - u_{n-1})]$$

For relative shift $r_n = u_{n+1} - u_n$
$$m \frac{d^2 r_n}{dt^2} = a(2e^{-br_n} - e^{-br_{n+1}} - e^{-br_{n-1}})$$

Force of a spring:
$$f = -\phi'(r) = a(e^{-br} - 1)$$

Solitons in Toda lattice

Soliton solution:

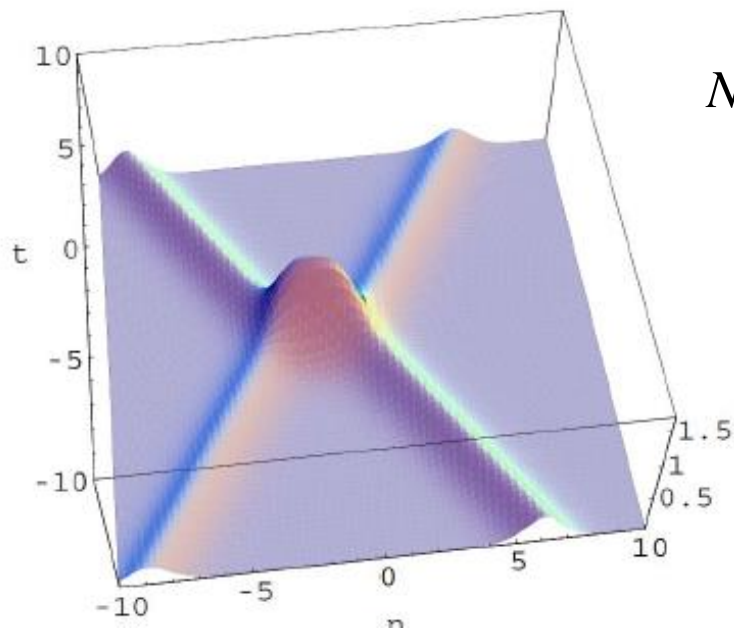


$$\frac{d^2}{dt^2} \log \left(1 + \frac{f_n}{a} \right) = \frac{b}{m} (f_{n+1} + f_{n-1} - 2f_n)$$

$$u_n = \omega^2 \operatorname{sech}^2(\kappa n + \sigma \omega t + \delta),$$

$$\sigma = \pm 1, \quad \omega = \sinh \kappa,$$

κ, δ : constants



$N = 2$ soliton solution:

$$u_n = \frac{\tau_{n+1}\tau_{n-1}}{\tau_n^2} - 1,$$

$$\tau_n = 1 + e^{2\eta_1} + e^{2\eta_2} + A_{12}e^{2(\eta_1+\eta_2)},$$

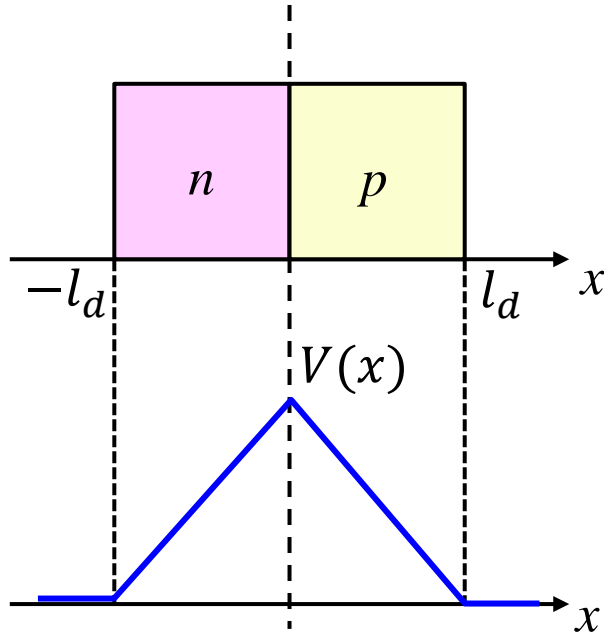
$$\eta_i = \kappa_i n + \sigma_i \omega_i t + \delta_i, \quad \sigma_i = \pm 1, \quad \omega_i = \sinh \kappa_i,$$

$$A_{12} = \frac{ab \sinh^2(\kappa_1 - \kappa_2) - m(\sigma_1 \omega_1 - \sigma_2 \omega_2)^2}{m(\sigma_1 \omega_1 + \sigma_2 \omega_2)^2 - ab \sinh^2(\kappa_1 + \kappa_2)}$$

Non-linear capacitance: Varicap



Varicap BB505



$$V_b = \frac{en}{\epsilon} \int_{-l_d}^0 2(x + l_d) dx + \frac{en}{\epsilon} \int_0^{l_d} 2(l_d - x) dx = \frac{2enl_d^2}{\epsilon}$$

$$V + V_b = \frac{2en}{\epsilon} \left(l_d + \frac{Q}{nS} \right)^2$$

$$\therefore C = \frac{dQ}{dV} = \sqrt{\frac{\epsilon}{2en}} \frac{nS}{\sqrt{V + V_b}}$$

$$V + V_b = V_0 + \delta V \quad \delta V \rightarrow V$$

L-Varicap transmission line

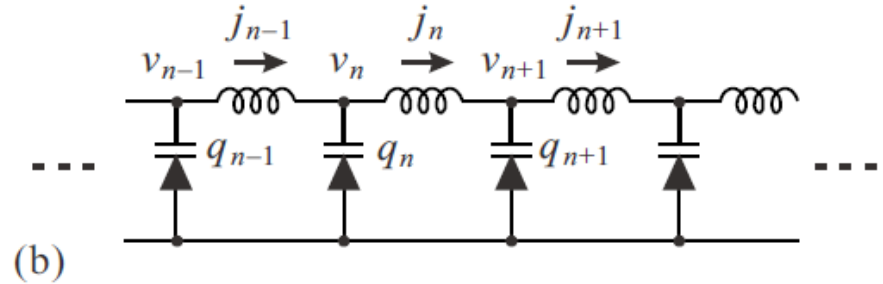
$$L \frac{dJ_n}{dt} = v_n - v_{n-1},$$

$$\frac{dq_n}{dt} = J_{n-1} - J_n,$$

$$q_n = \int_0^{v_n} C(V) dV, \quad C(V) = \frac{Q(V_0)}{F(V_0) + V - V_0}$$

$$q_n = Q(V_0) \log \left[1 + \frac{V_n}{F(V_0)} \right] + \text{const.}$$

$$\frac{d^2}{dt^2} \log \left[1 + \frac{V_n}{F(V_0)} \right] = \frac{1}{LQ(V_0)} (V_{n-1} + V_{n+1} - 2V_n)$$



Solitons in non-linear circuit

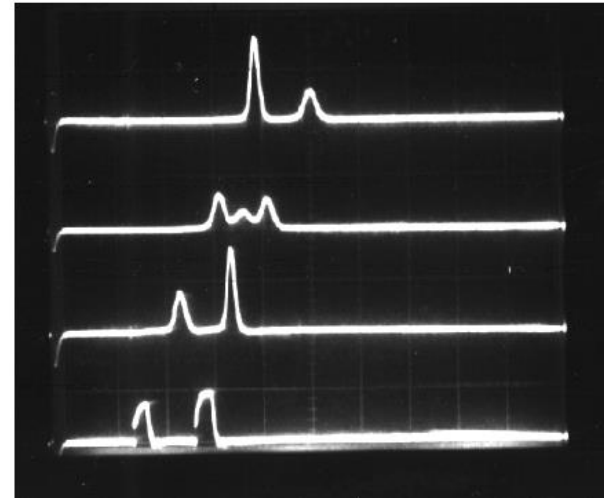
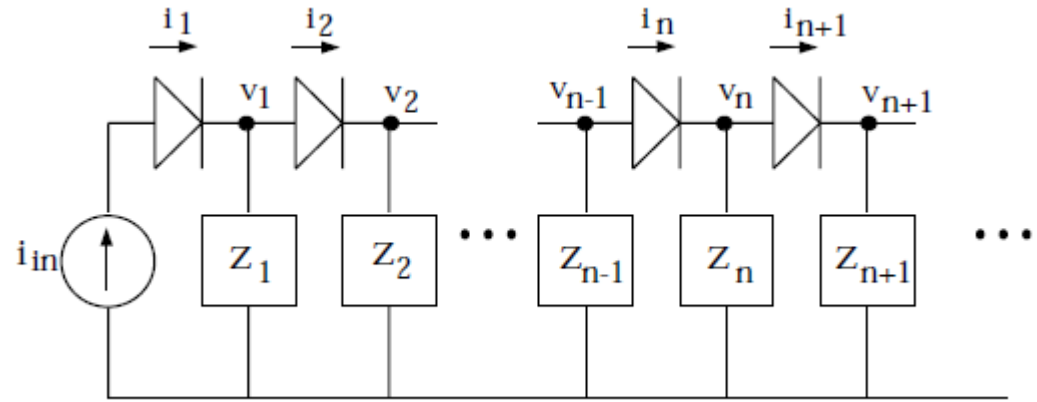
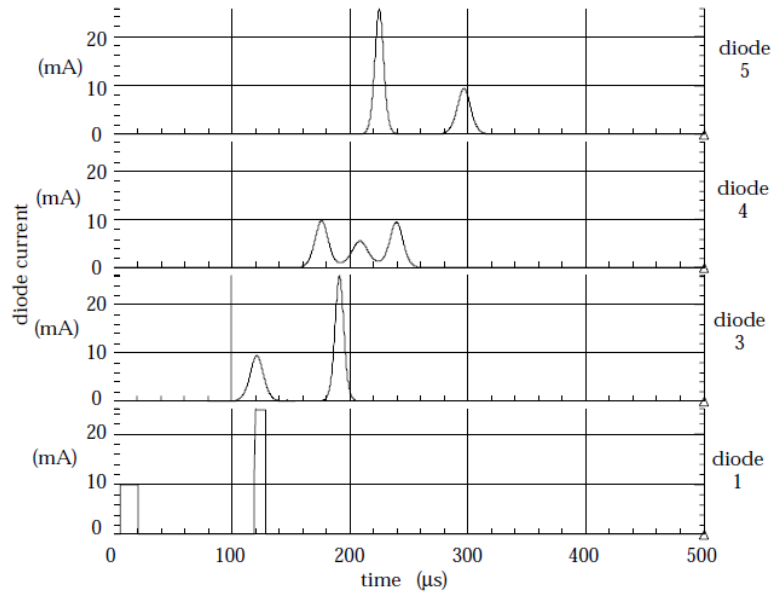
International Journal of Bifurcation and Chaos, Vol. 9, No. 4 (1999) 571-590
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CIRCUIT IMPLEMENTATIONS OF SOLITON SYSTEMS*

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Toda lattice circuit, Soliton circuit

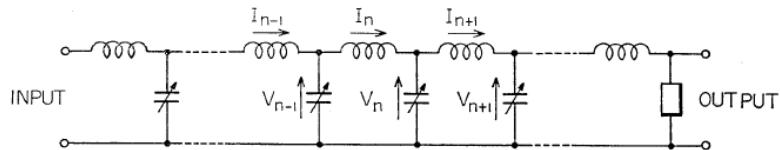


Fig. 1. A nonlinear network equivalent to a one-dimensional anharmonic lattice. The circuit elements have an inductance $L=22 \mu\text{H}$ or capacitance $C(V)=27 V^{-0.48} \text{ pF}$.

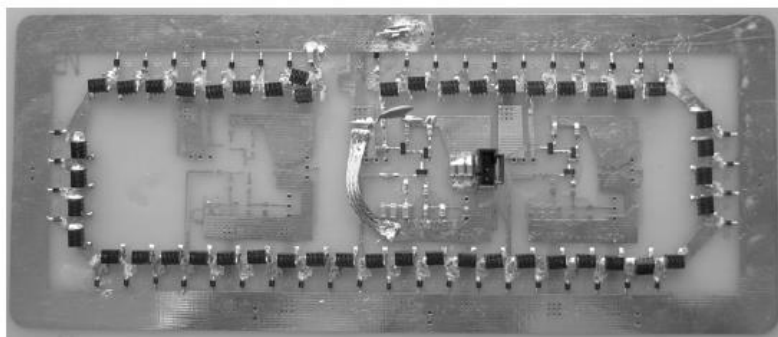
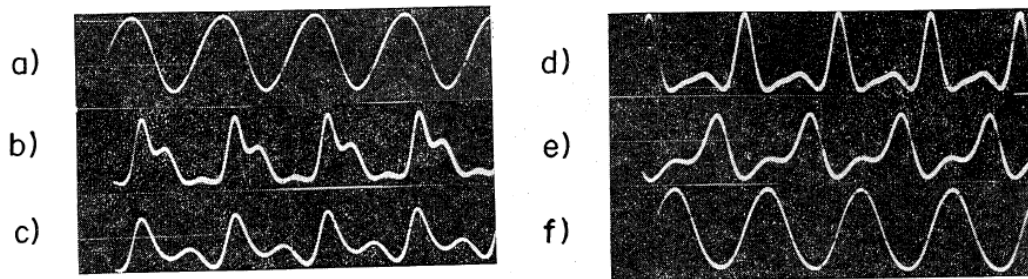
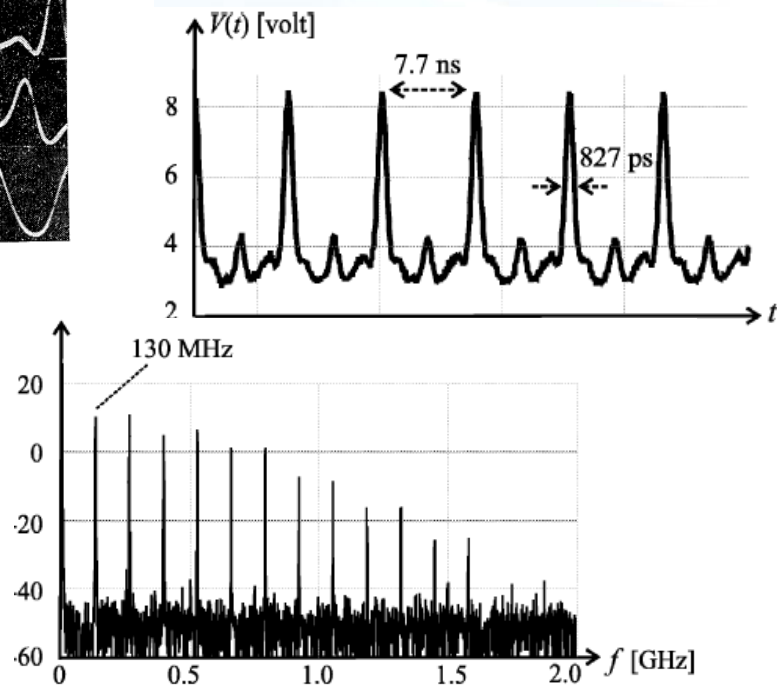


Fig. 16. Microwave soliton oscillator prototype.

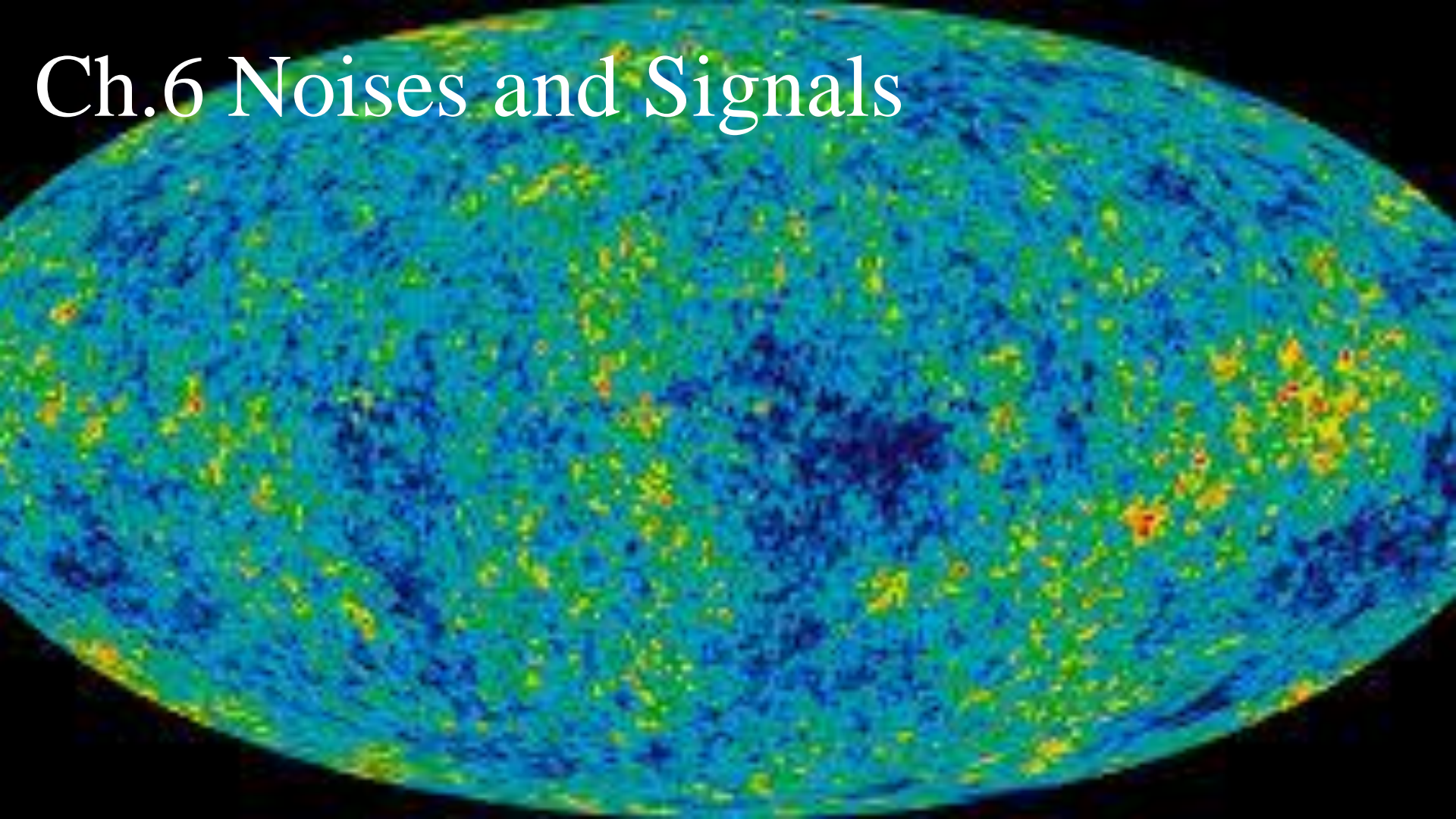
J. PHYS. SOC. JAPAN 28 (1970) 1366~1367

Studies on Lattice Solitons by Using Electrical Networks

Ryogo HIROTA and Kimio SUZUKI



Ch.6 Noises and Signals



Outline

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Noises

Electric circuits transport 1) Information; 2) Electromagnetic power, on some physical quantities like voltages, current, ...

Noises: stochastic (uncontrollable, unpredictable by human) variation in other words, fluctuation in such a quantity.

Internal noise: {

- Intrinsic noise: Thermal noise (Johnson-Nyquist noise), Shot noise
- Noise related to a specific physical phenomenon
Avalanche, Popcorn, Barkhausen, etc.
- 1/f noise: Name for a group of noises with spectra $1/f$.

External noise: EMI, microphone noise, etc.

6.1 Fluctuation

Quantity x , fluctuation $\delta x = x - \bar{x}$ $\overline{(\delta x)^2} = \overline{(x - \bar{x})^2} = \overline{x^2} - \bar{x}^2$ ($\overline{\delta x} = 0$)

$g(x)$: distribution function of x

Fourier transform: $u(q) = \mathcal{F}\{g(x)\} = \int_{-\infty}^{\infty} g(x) e^{ixq} \frac{dx}{\sqrt{2\pi}}$

$u(q)$: **characteristic function** of the distribution

From Taylor expansion, n -th order moment can be obtained as

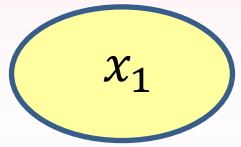
$$\overline{x^n} = \frac{\sqrt{2\pi}}{i^n} \left[\frac{d^n}{dq^n} u(q) \right]_{q=0}$$

Moments to high orders \rightarrow reconstruction of $g(x)$

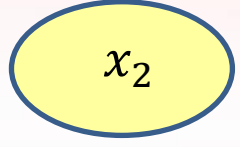
6.1 Fluctuation

In electric circuits we need to consider two kinds of averages:

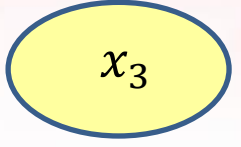
substance 1



substance 2



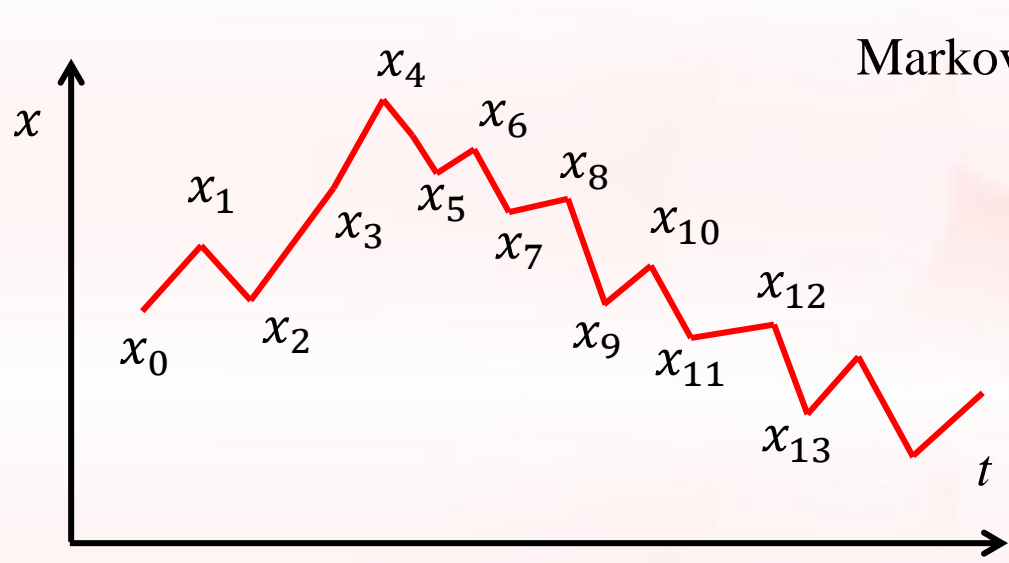
substance 3



...

x_j : independent

Ensemble average: \bar{x}

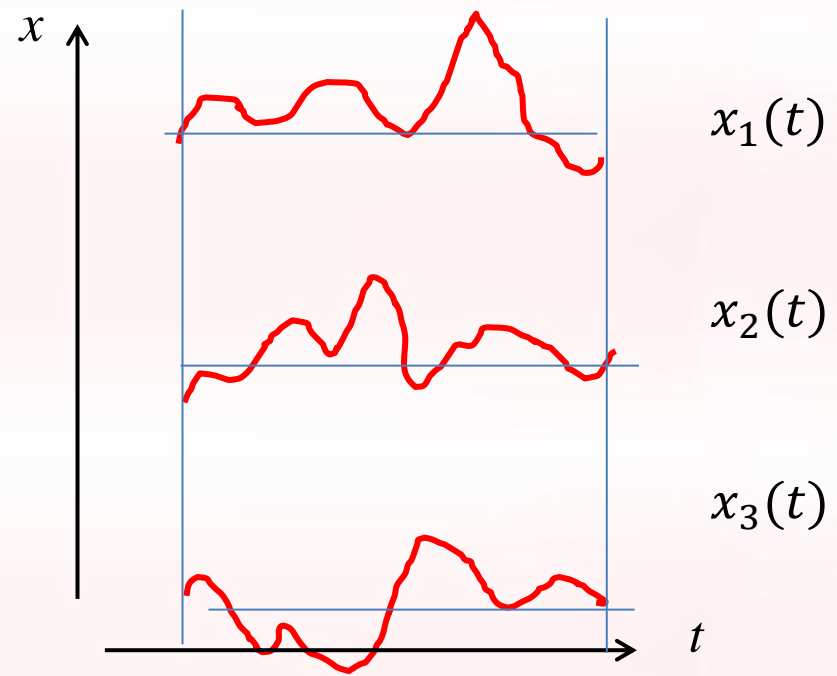
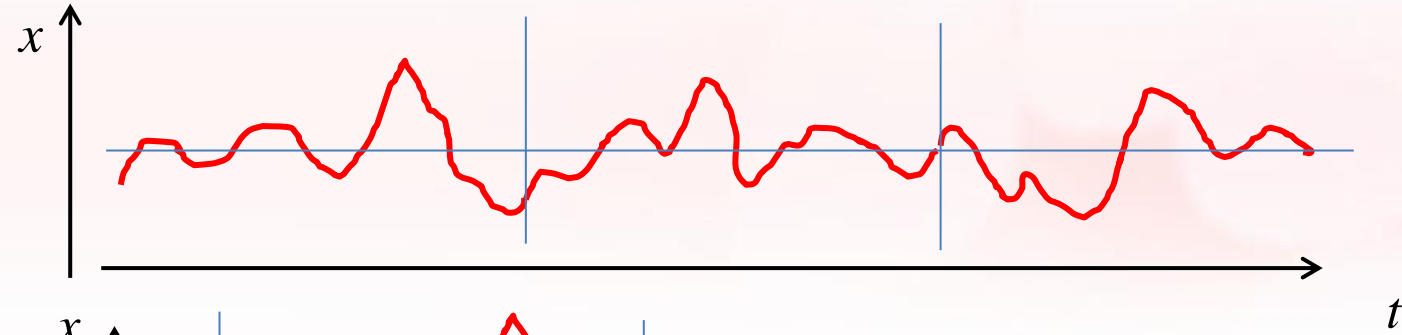


Markovian $x_1, \dots, x_m \xrightarrow{\text{affect}} x_{m+1}$

m -th order Markovian

Time average of fluctuating variable: $\langle x \rangle$

Random process to distribution



The averaging interval should be longer than m in m -th order Markovian.

Power spectrum

Consider probability sets in the interval $[0, T)$ with set index j .

$$x_j(t) = \sum_{n=1}^{\infty} (a_{jn} \cos \omega_n t + b_{jn} \sin \omega_n t), \quad \omega_n = \frac{2n\pi}{T}$$

$$\mathcal{P}_{jn} = (a_{nj} \cos \omega_n t + b_{nj} \sin \omega_n t)^2$$

(Power)

$$\langle \mathcal{P}_n \rangle = \frac{1}{2} \langle a_n^2 + b_n^2 \rangle$$

\therefore cross product terms are averaged out

Random process:
Gaussian distribution in time

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{x^2}{2\sigma^2}\right]$$

$$\overline{(\delta x)^2} = \sigma^2, \quad \overline{\left(\sum_{j=1}^m \delta x_j\right)^2} = m\sigma^2 \quad \text{Then} \quad \overline{\langle \mathcal{P}_n \rangle} = \sigma_n^2$$

Power spectrum **$G(\omega)$**

Frequency band width $\delta\omega$: separation between two adjacent frequencies

Power spectrum

Power spectrum $G(\omega)$

Frequency band width $\delta\omega$: separation between two adjacent frequencies

$$\delta\omega = \omega_{n+1} - \omega_n = \frac{2(n+1)\pi}{T} - \frac{2n\pi}{T} = \frac{2\pi}{T}$$

$$G(\omega_n) \frac{\delta\omega}{2\pi} = \overline{\langle \mathcal{P}_n \rangle} (= \sigma_n^2)$$

$$\overline{\langle x^2(t) \rangle} = \sum_{n=1}^{\infty} \overline{\langle \mathcal{P}_n \rangle} \quad (\overline{\langle x(t) \rangle} = 0)$$

$$= \sum_n G(\omega_n) \frac{\delta\omega}{2\pi} \rightarrow \int_0^{\infty} G(\omega) \frac{d\omega}{2\pi}$$

6.1.1 Fluctuation-Dissipation theorem



久保亮五
Ryogo Kubo
1920-1995



Harry Nyquist
1889-1976

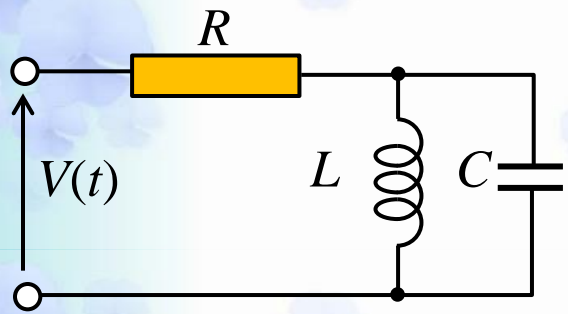


Nobert Wiener
1894-1964



Aleksandr Khinchin
1894-1959

Fluctuation-dissipation theorem in the language of circuit



$V(t)$ noise power spectrum $\rightarrow G_v(\omega)$

$$\omega_0 \equiv 1/\sqrt{LC}$$

$$Z(i\omega) = \frac{R(\omega_0^2 - \omega^2) + i\omega_0^2\omega L}{\omega_0^2 - \omega^2},$$

$$Y(i\omega) = \frac{\omega_0^2 - \omega^2}{R(\omega_0^2 - \omega^2) + i\omega_0^2\omega L}$$

$$G_v(\omega) = 4k_B T \operatorname{Re}[Z(i\omega)]$$
$$= 4k_B T R$$

Johnson-Nyquist noise
Thermal noise

White noise (noise with no frequency dependence) in the case of frequency independent resistance

One of the representations for the fluctuation-dissipation theorem

6.1.2 Wiener-Khintchine theorem

Autocorrelation function

$$\begin{aligned}C(\tau) &= \overline{\langle x(t)x(t+\tau) \rangle} \\&= \overline{\sum_{n,m} \langle [a_n \cos \omega_n t + b_n \sin \omega_n t][a_m \cos \omega_m(t+\tau) + b_m \sin \omega_m(t+\tau)] \rangle} \\&= \frac{1}{2} \sum_n \overline{\langle a_n^2 + b_n^2 \rangle} \cos \omega_n \tau = \sum_n \overline{\langle \mathcal{P}_n \rangle} \cos \omega_n \tau \\&= \int_0^\infty G(\omega) \cos \omega \tau \frac{d\omega}{2\pi}\end{aligned}$$

$$C(\tau) = \int_0^\infty G(\omega) \cos \omega \tau \frac{d\omega}{2\pi}, \quad G(\omega) = 4 \int_0^\infty C(\tau) \cos \omega \tau d\tau$$

Wiener-Khintchine theorem