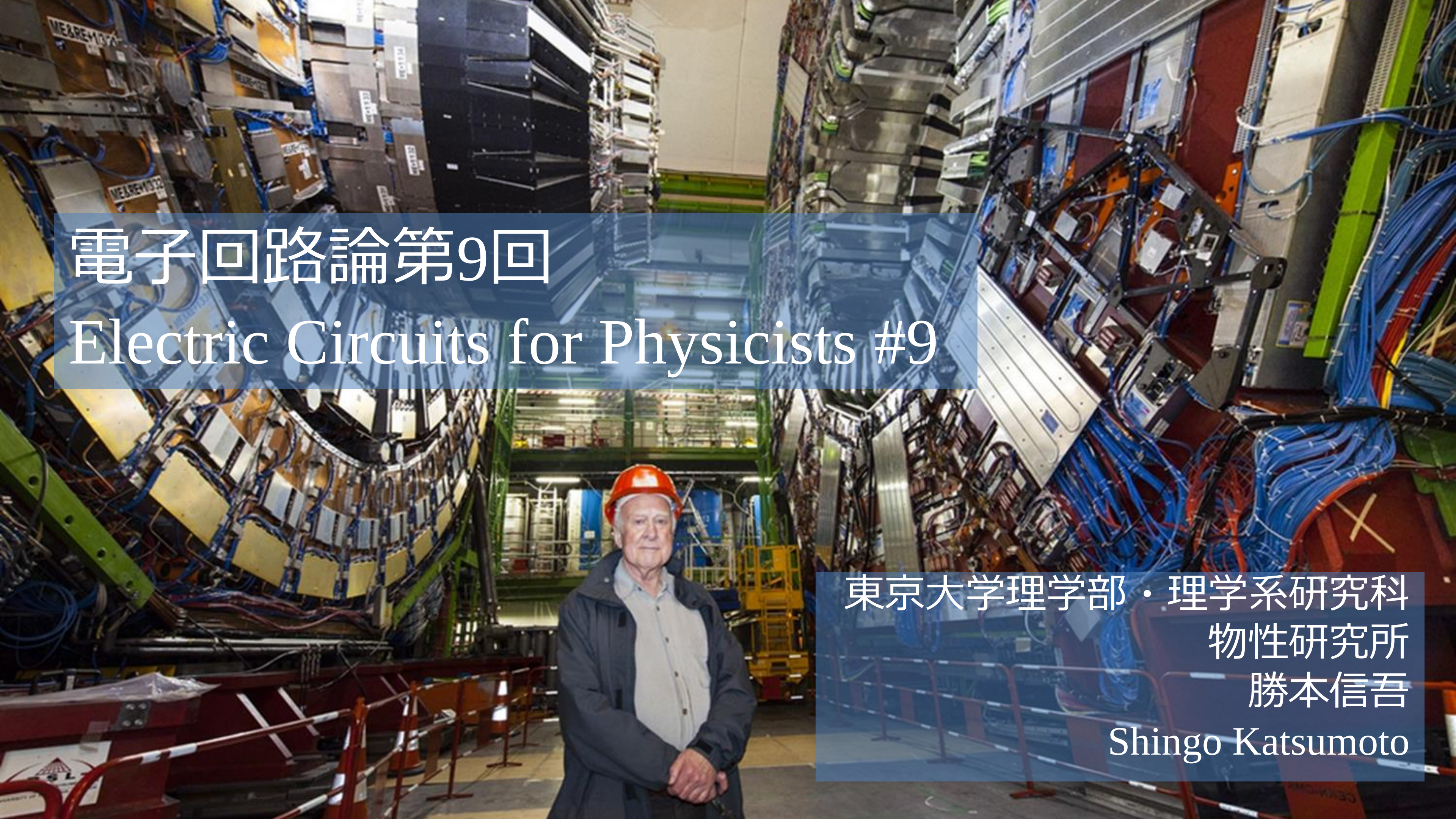

電子回路論第9回
Electric Circuits for Physicists #9

東京大学理学部・理学系研究科
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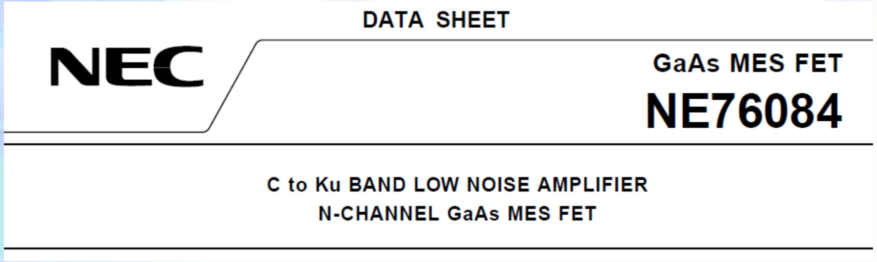
勝本信吾
Shingo Katsumoto

Outline

- Various transmission lines
- Termination and connection
- Smith chart
- S-matrix (S-parameters)

An example FET power matching



DATA SHEET

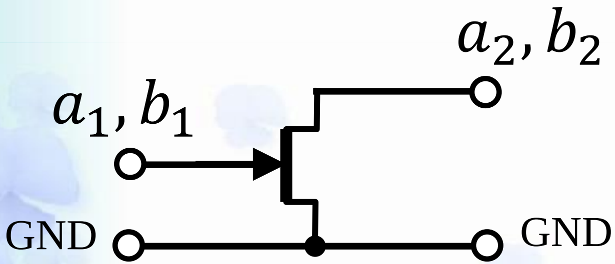
NEC

GaAs MES FET

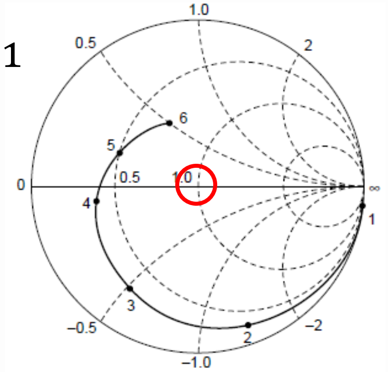
NE76084

C to Ku BAND LOW NOISE AMPLIFIER
N-CHANNEL GaAs MES FET

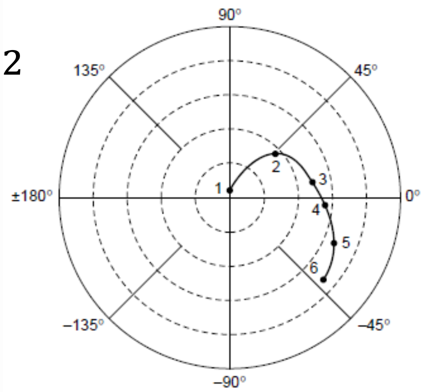
C band: 4 – 8 GHz
X band: 8 – 12 GHz
Ku band: 12 – 18 GHz



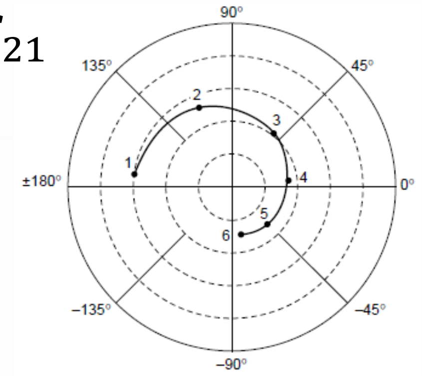
S_{11}



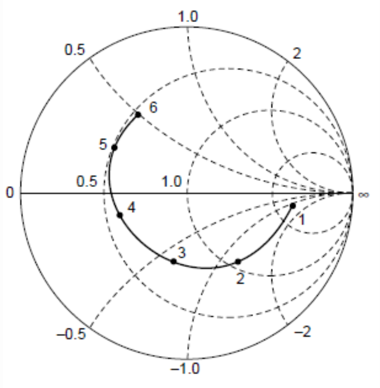
S_{12}



S_{21}

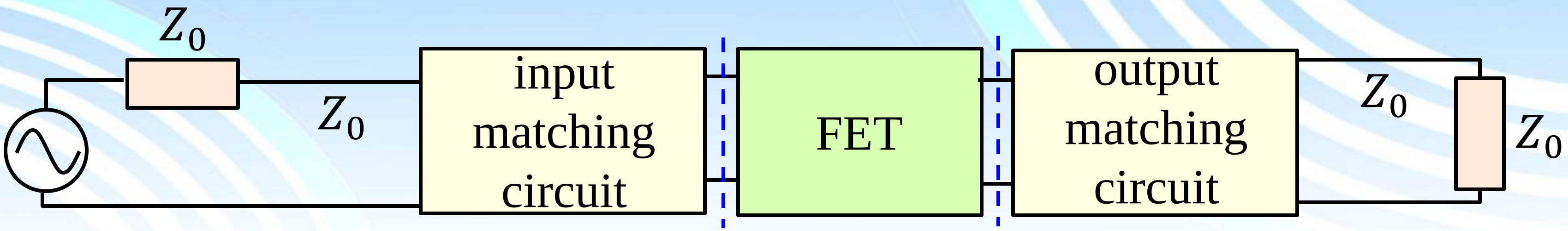


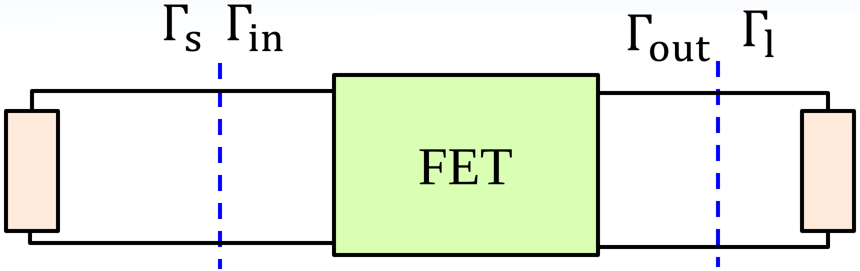
S_{22}



S_{12} and S_{21} are normalized with some constants

S parameter representation and matching circuits





$\Gamma_{s,l,in,out}$: reflection coefficients.

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

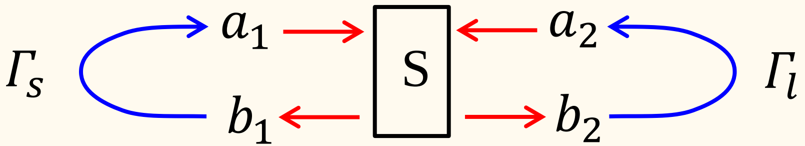
FET S-matrix

The situation can be seen as:

Source with reflection coefficient Γ_s, load with reflection coefficient Γ_l are connected.

Power matching with S-parameters

That is



$$\frac{a_1}{b_1} = \Gamma_\mathrm{s}, \quad \frac{a_2}{b_2} = \Gamma_\mathrm{l}$$

Then the total coefficients are $\Gamma_\mathrm{in} = S_{11} + \dfrac{S_{12}S_{21}\Gamma_\mathrm{l}}{1 - S_{22}\Gamma_\mathrm{l}}, \quad \Gamma_\mathrm{out} = S_{22} + \dfrac{S_{12}S_{21}\Gamma_\mathrm{s}}{1 - S_{11}\Gamma_\mathrm{s}}$

The power matching conditions are $\Gamma_\mathrm{l} = \Gamma_\mathrm{out}^*, \quad \Gamma_\mathrm{s} = \Gamma_\mathrm{in}^*$

The quadratic equation gives

$$\Gamma_\mathrm{s} = \frac{B_1 \pm \sqrt{B_1^2 - 4|M|^2}}{2M}, \quad \Gamma_l = \frac{B_2 \pm \sqrt{B_2^2 - 4|N|^2}}{2N}$$

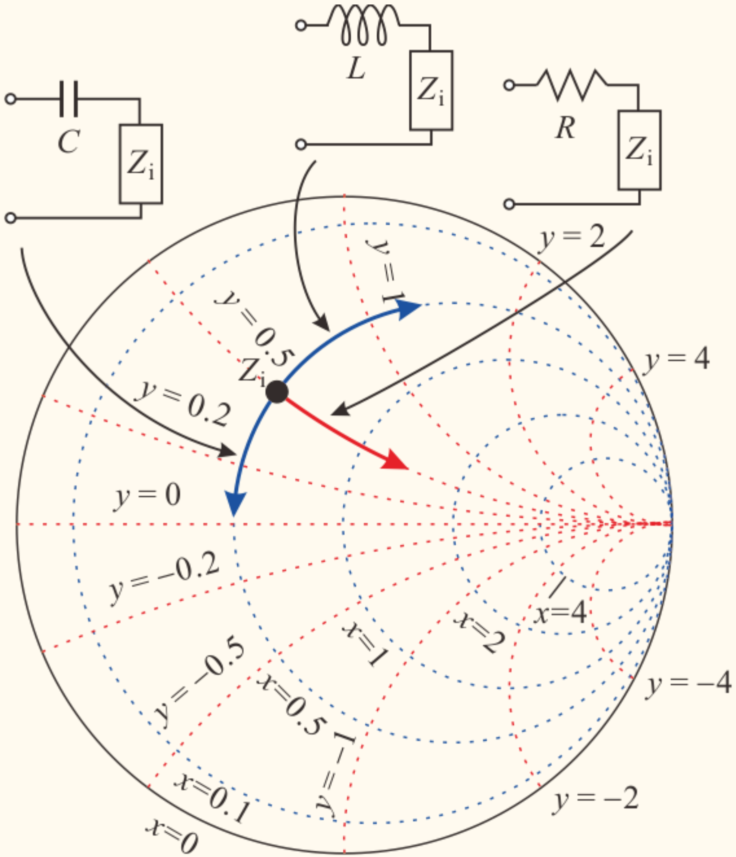
$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\mathrm{det}S|^2, \quad B_2 = 1 - |S_{11}|^2 + |S_{22}|^2 - |\mathrm{det}S|^2,$$

$$N = S_{22} - S_{11}^*\mathrm{det}S, \quad M = S_{11} - S_{22}^*\mathrm{det}S$$

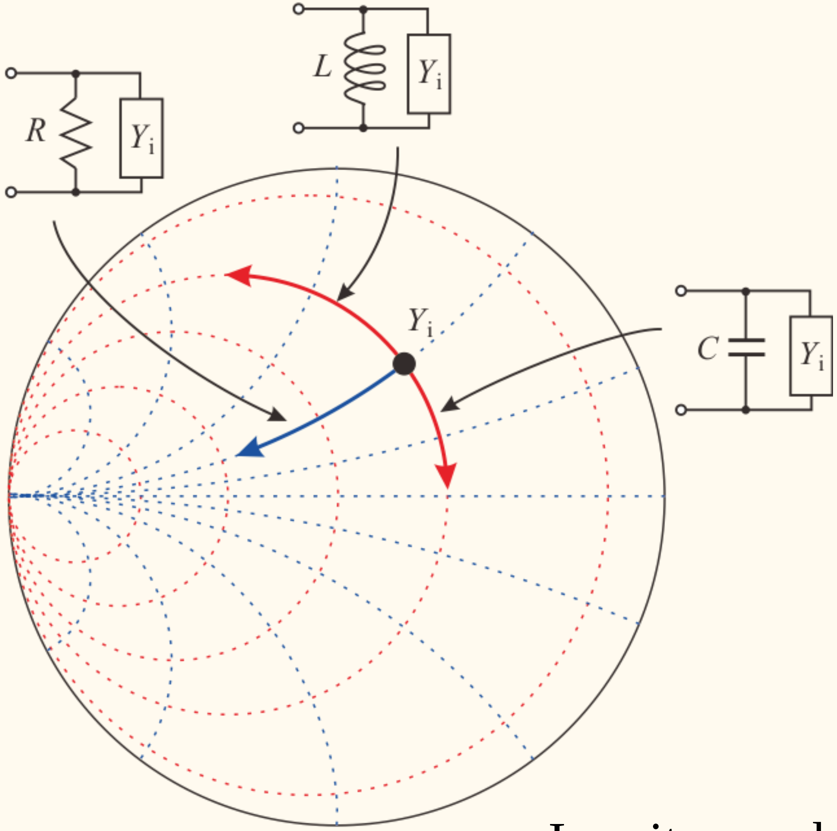
Then the problem is reduced to tune the passive circuits to Γ_s and Γ_l

A simplified way to make impedance matching

Series and parallel connection of passive elements and traces on charts



Smith chart



Immitance chart

6.1.2 Wiener-Khintchine theorem

Autocorrelation function $C(\tau) = \overline{\langle x(t)x(t+\tau)\rangle}$

$$= \overline{\sum_{n,m} \langle [a_n \cos\omega_n t + b_n \sin\omega_n t][a_m \cos\omega_m (t+\tau) + b_m \sin\omega_m (t+\tau)]\rangle}$$

$$= \frac{1}{2}\sum_n \overline{\langle a_n^2 + b_n^2\rangle} \cos\omega_n \tau = \sum_n \overline{\langle \mathscr{P}_n\rangle} \cos\omega_n \tau$$

$$= \int_0^\infty G(\omega)\cos\omega\tau \frac{d\omega}{2\pi}$$

$$C(\tau) = \int_0^\infty G(\omega)\cos\omega\tau \frac{d\omega}{2\pi}$$

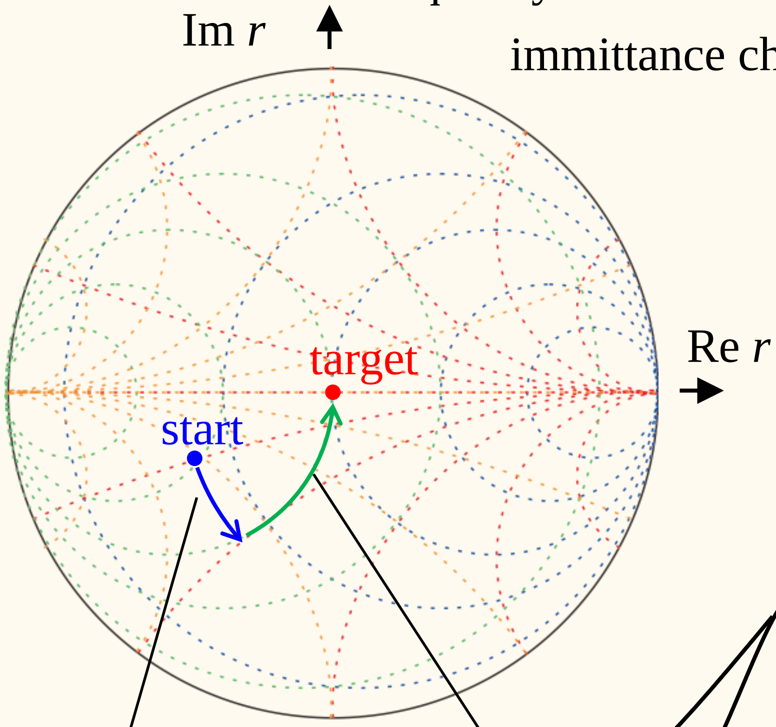
$$G(\omega) = 4\int_0^\infty C(\tau)\cos\omega\tau d\tau$$

Wiener-Khintchine theorem

An example of impedance matching

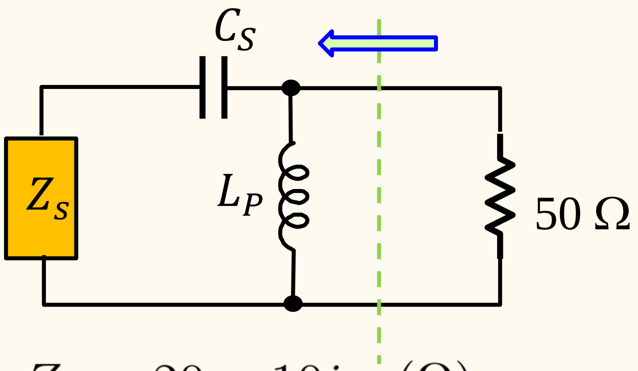
frequency 100 MHz ≈ 628 Mrad/s

immittance chart



Re[Z]=0.4

Re[Y]=1



$$Z_s = 20 - 10i \quad (\Omega)$$
$$= 0.4 - 0.2i \quad (Z_0)$$

equalize: $0.4 + iy = \dfrac{1}{1+iq}$

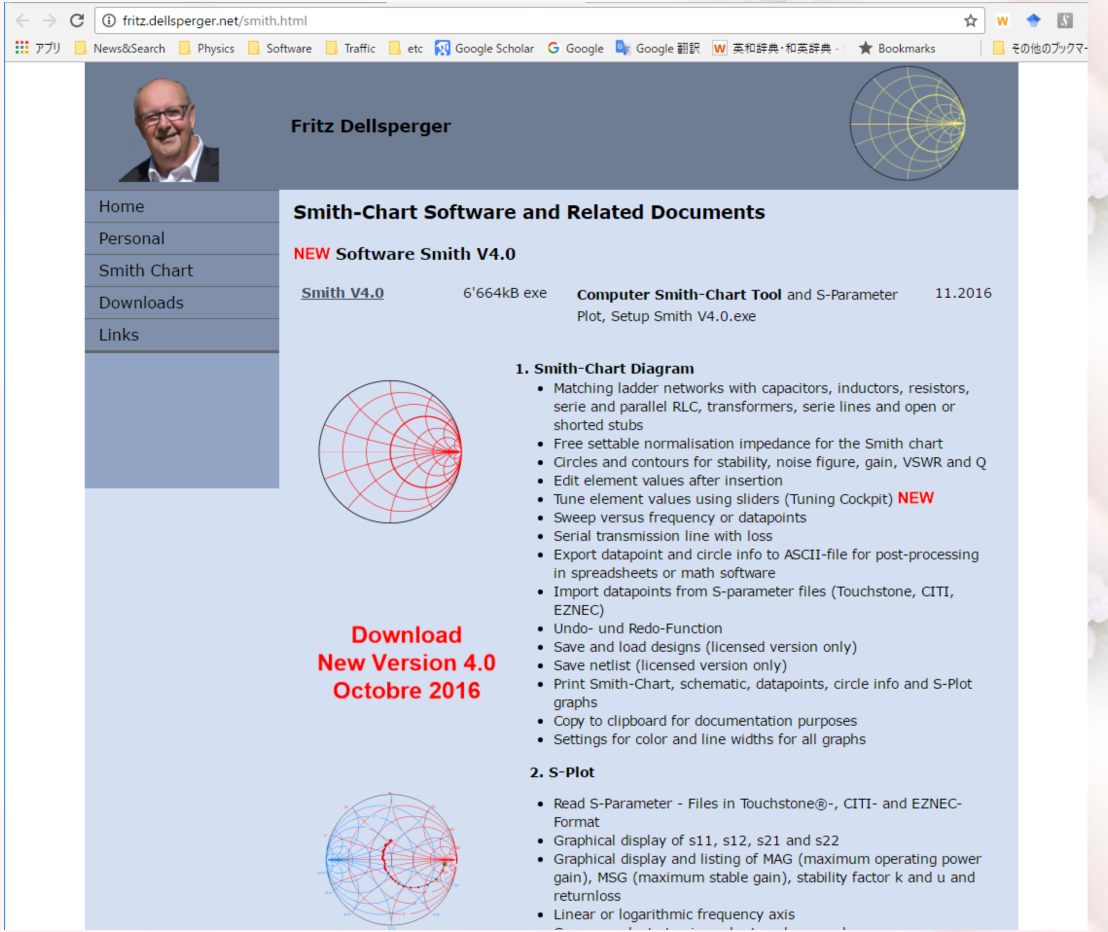
$$y = -\sqrt{0.24} \approx -0.49 = -0.2 - \frac{1}{\omega C_S Z_0}$$

$$C_S \approx \frac{1}{2\pi \times 10^6 \times 50 \times 0.29} \approx 110 \text{ pF}$$

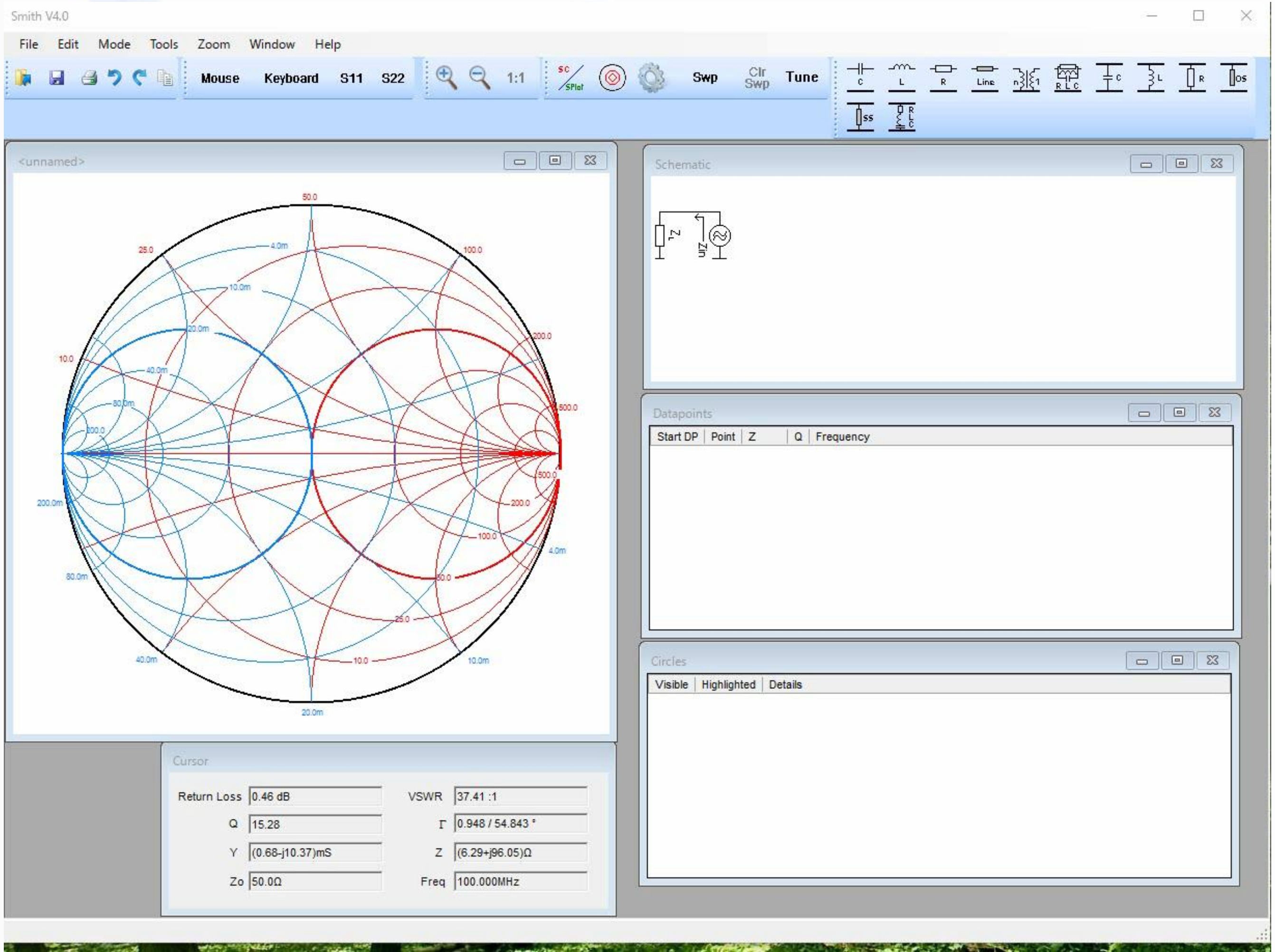
Similarly $L_P \approx 65$ nH

Introduction of useful freeware Smith v4.0 (present version 4.1)

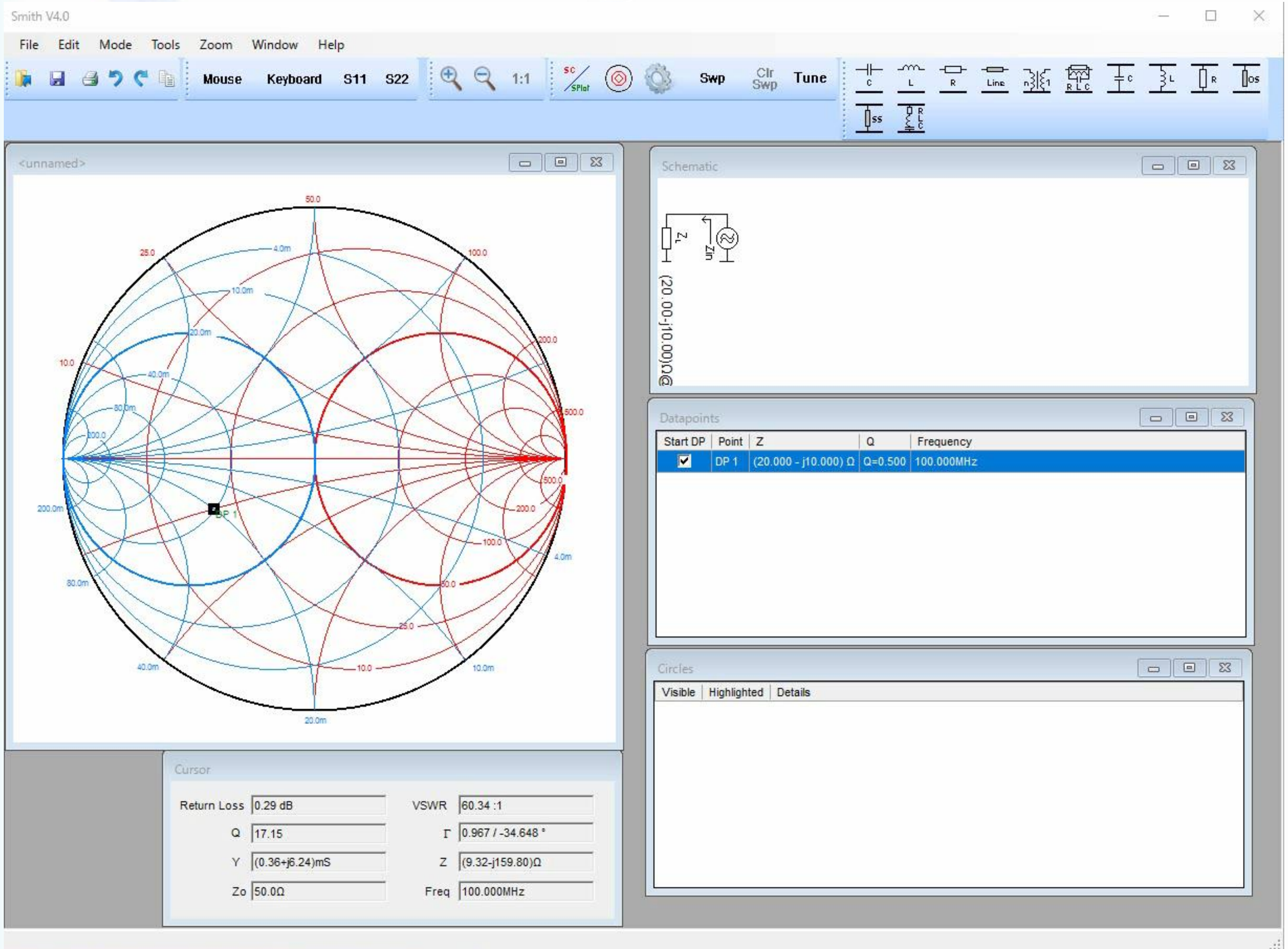
http://fritz.dellsperger.net/smith.html



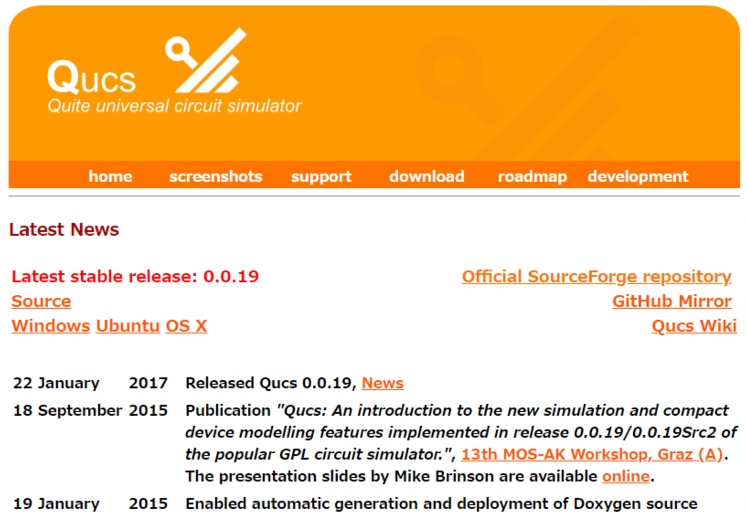
Impedance matching with Smith v4 (1)



Impedance matching with Smith V4 (2)



Introduction of useful freeware Qucs 0.0.19



Qucs: Quite Universal Circuit Simulator

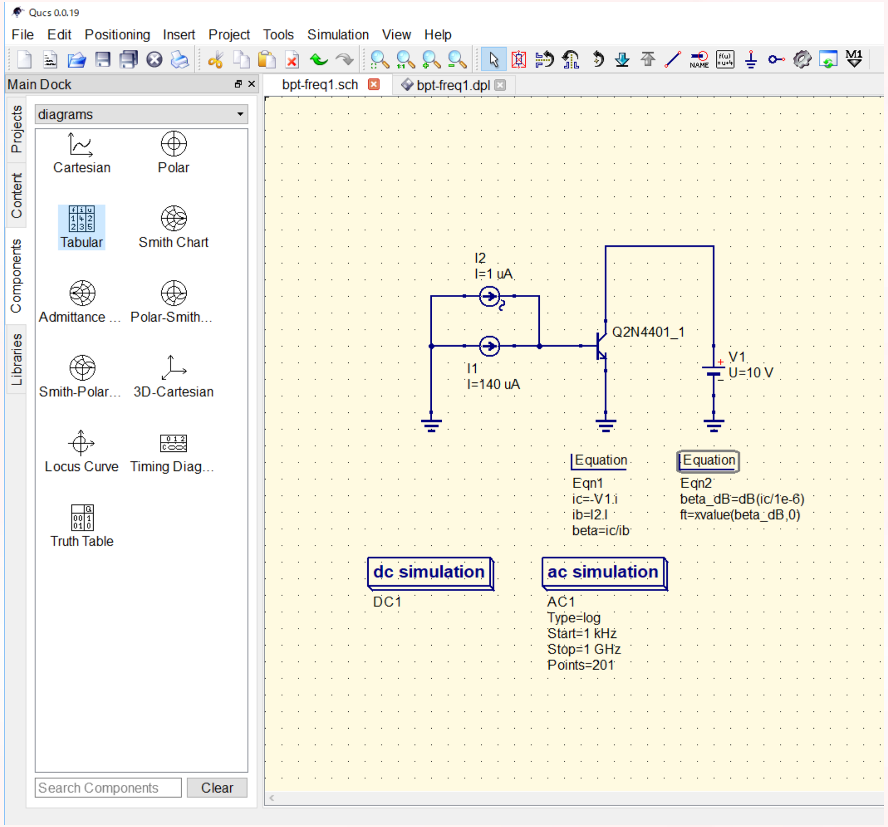
http://qucs.sourceforge.net/

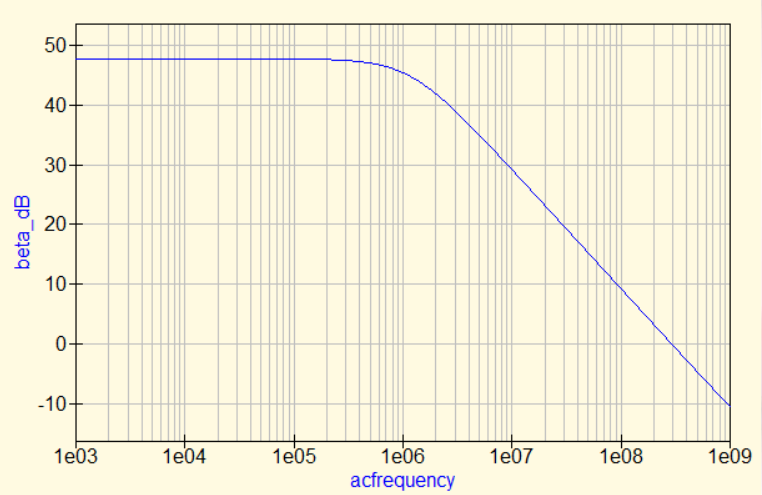
The Qucs project was begun by Michael Margraf in Germany in 2004.

- Based on SPICE simulation language.
- Free but no restriction in the number of nodes, etc.
- Can read S-parameter files. Have S-parameter analysis options.
- In that sense, better than LTSpice.

Introduction of useful freeware Qucs 0.0.19

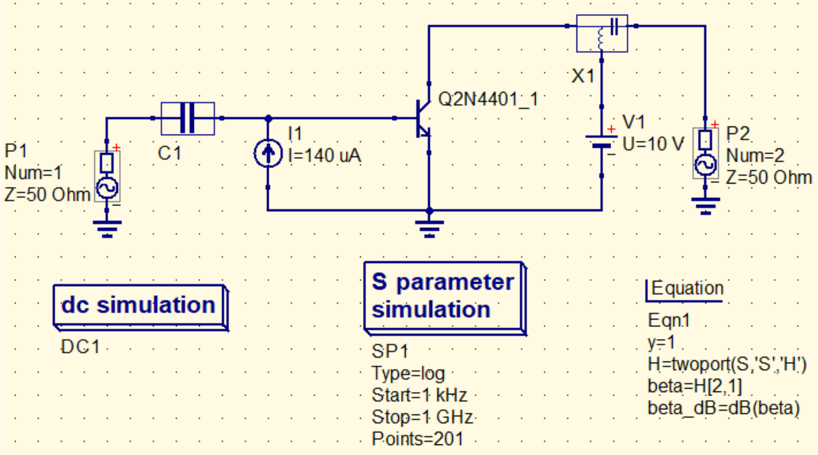
Example: Frequency characteristics of a bipolar transistor

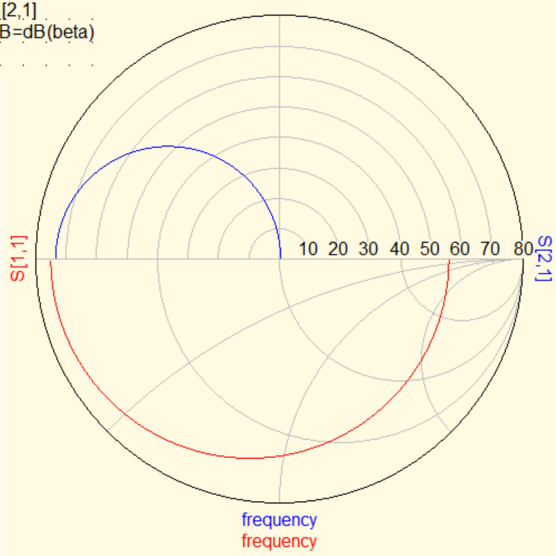


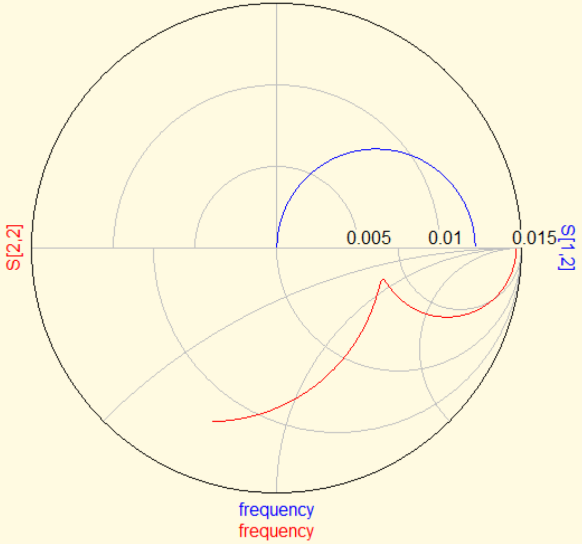


number	ft
1	2.88e+08

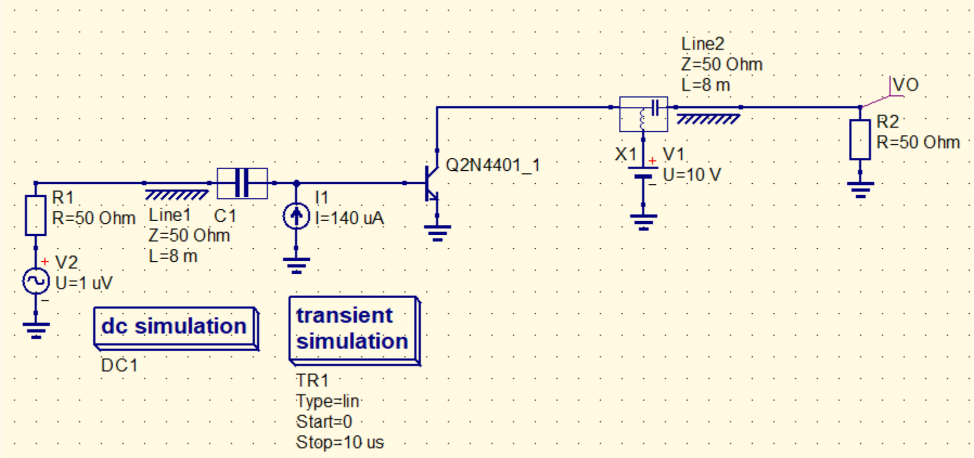
S-parameter simulation of a bipolar transistor with Qucs

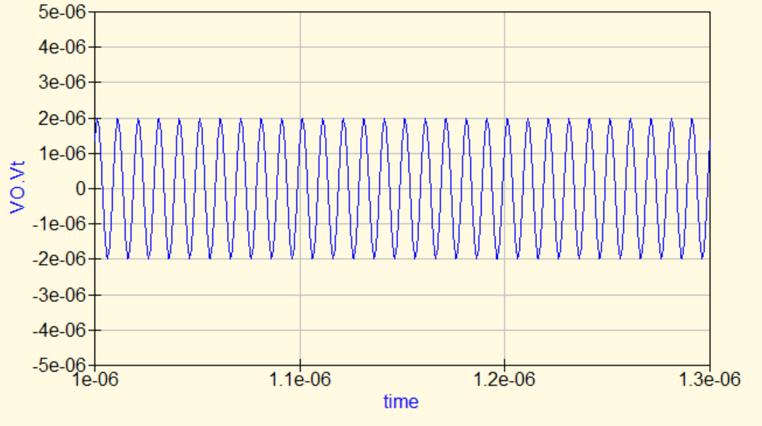


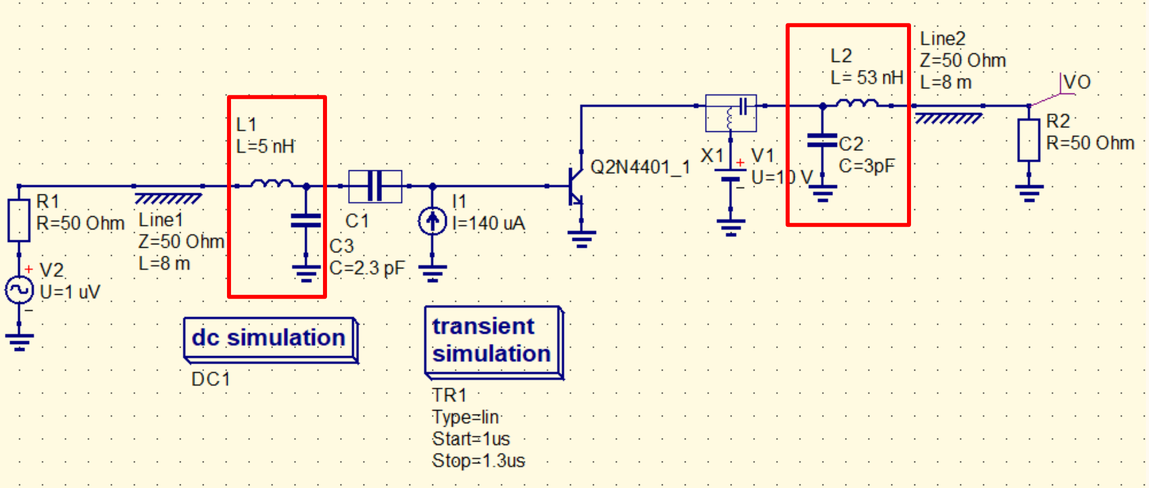


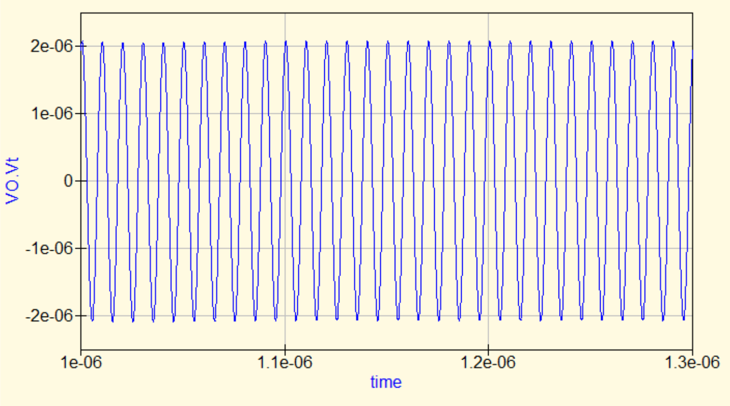


Simulation of matching circuit









Comments: Impedance match

Propagation of a wave:
- Impedance match: complete absorption (propagation without reflection)
- Mismatch: wave reflection

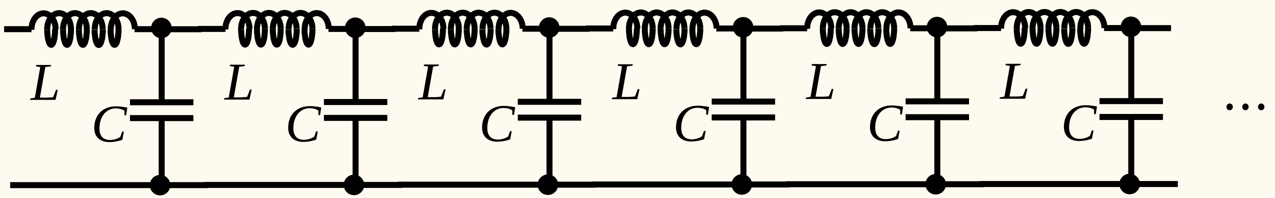
Impedance match/mismatch is an important concept applicable to a broad area of physics.

- Antenna: should be matched to the vacuum.

  EM wave propagation simulation: boundary is shunted

  with the characteristic impedance of vacuum.
- Optics: impedance mismatch → disagreement in refractive index
- Plasma: should be matched to electrodes for excitation.
- Phonon impedance mismatch at low temperatures: Kapitza resistance
- Sound insulated booth: should have sound impedance mismatch.

5.4 Non TEM mode transmission lines

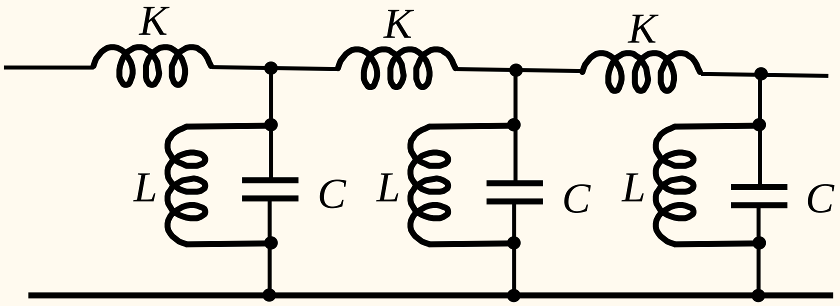


LC model: $Z = i\omega L$, $Y = i\omega C$

The inductance represents magnetic fields circulating the core and the capacitance electric fields directing from the core to the shield.

$$Z_0 = \sqrt{\frac{L}{C}}$$

: real, dispersionless (linear ω-*k* relation)



Non-linear ω-term in Z or Y → dispersion (longitudinal components)

$$Y = i\omega C + \frac{1}{i\omega L}$$

C: capacitance per unit length

L: inductance per inverse unit length

K: inductance per unit length

5.4 Non TEM mode gives mass to the transmission mode

$$-k^2 = YZ = \left(i\omega C + \frac{1}{i\omega L}\right) i\omega K = -CK\omega^2 + \frac{K}{L}$$

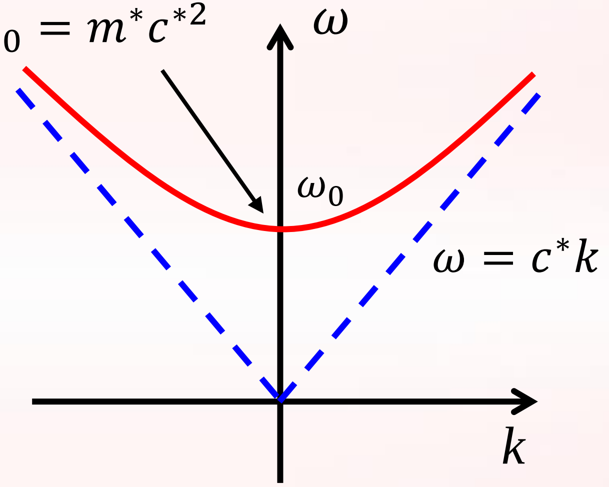
Constant finite mass: $E = \hbar\omega \propto k^2$ (Schrodinger eq.: Parabolic partial differential equation)

Coupling between linear dispersions: mass mechanism (*cf*. Higgs)

$\frac{1}{\sqrt{LC}} = \omega_0$ unchanged with $dx \to 0$

$$Z = i\omega K, \quad Y = \frac{1 - (\omega/\omega_0)^2}{i\omega L}$$ then

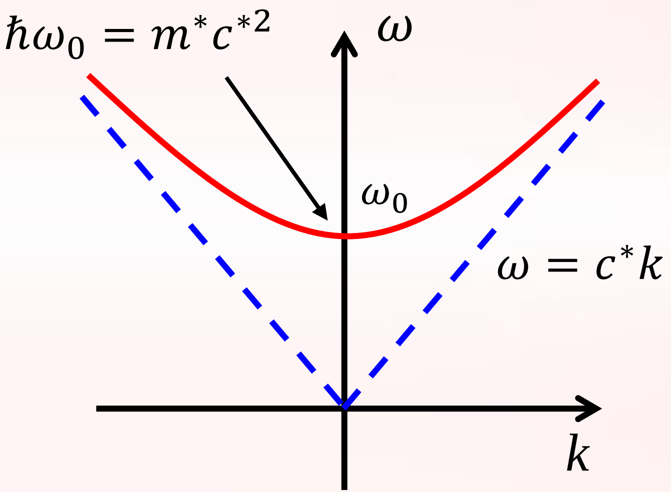
$\hbar\omega_0 = m^* c^{*2}$



$$ik = \kappa = \sqrt{YZ} = i\sqrt{\frac{K}{L}\left[\left(\frac{\omega}{\omega_0}\right)^2 - 1\right]}$$

$$k = \eta\sqrt{(\omega/\omega_0)^2 - 1}, \quad \eta^2 \equiv (K/L)$$

5.4 Non TEM mode gives mass to the transmission mode



$$\omega \gg \omega_0 \rightarrow k \sim \eta \frac{\omega}{\omega_0}$$ No dispersion

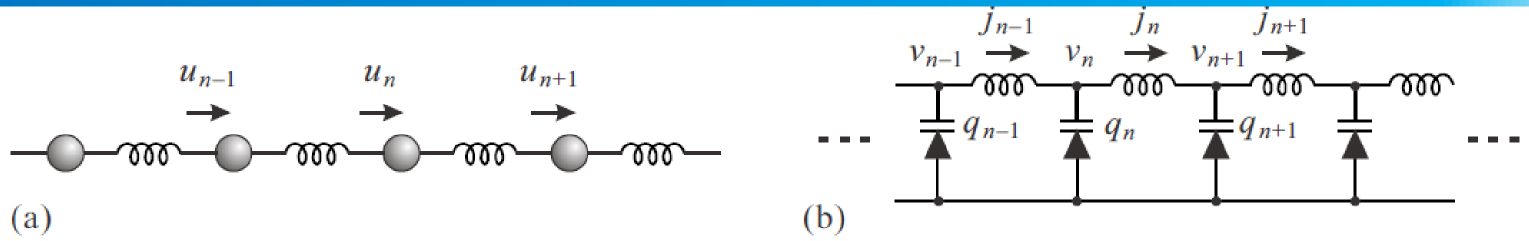
velocity $$c^* = \frac{\omega}{k} = \frac{\omega_0}{\eta} = \frac{1}{\sqrt{KC}}$$

$$\omega \sim \omega_0 \quad \omega = \omega_0 + \delta\omega \quad \rightarrow k^2 \approx 2\eta^2 \frac{\delta\omega}{\omega_0}$$

$$\therefore \epsilon \equiv \hbar\delta\omega = \frac{\hbar k^2}{2(\eta^2/\omega_0)} = \frac{\hbar^2 k^2}{2m^*} \quad \left(m^* \equiv \frac{\hbar\eta^2}{\omega_0} \right)$$

$$E_0 = \hbar\omega_0 = \frac{\hbar\eta^2}{\omega_0} \cdot \left(\frac{\omega_0}{\eta} \right)^2 = m^* c^{*2}$$

5.5 Non-linear LC transmission line and Toda lattice



Toda lattice is a typical non-linear system with exact (soliton) solutions.
It is defined as follows:

The springs in (a) have Toda-potential: $$\phi(r) = \frac{a}{b}e^{-br} + ar \quad (ab > 0)$$

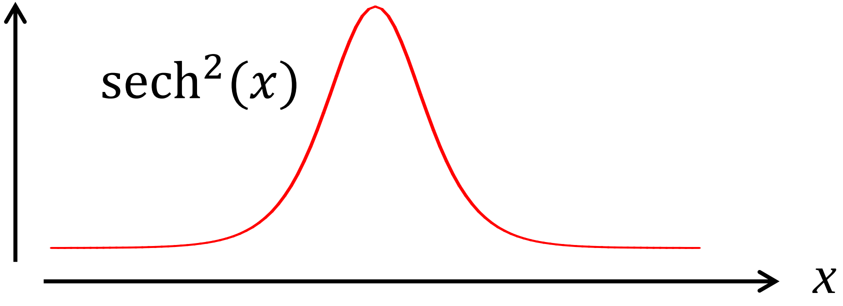
Equation of motion: $$m\frac{d^2u_n}{dt^2} = -a\exp[-b(u_{n+1} - u_n)] + a\exp[-b(u_n - u_{n-1})]$$

For relative shift $r_n = u_{n+1} - u_n$ $$m\frac{d^2r_n}{dt^2} = a(2e^{-br_n} - e^{-br_{n+1}} - e^{-br_{n-1}})$$

Force of a spring: $$f = -\phi'(r) = a(e^{-br} - 1)$$

Solitons in Toda lattice

Soliton solution:

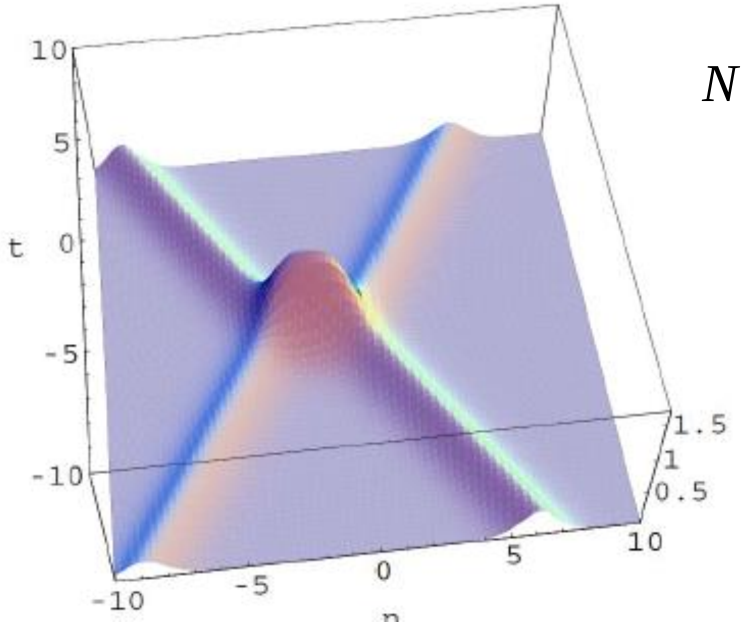


$$\frac{d^2}{dt^2}\log\left(1+\frac{f_n}{a}\right)=\frac{b}{m}(f_{n+1}+f_{n-1}-2f_n)$$

$$u_n=\omega^2\mathrm{sech}^2(\kappa n+\sigma\omega t+\delta),$$

$$\sigma=\pm 1,\quad \omega=\sinh\kappa,$$

$$\kappa,\quad \delta:\text{constants}$$



$N = 2$ soliton solution:

$$u_n=\frac{\tau_{n+1}\tau_{n-1}}{\tau_n^2}-1,$$

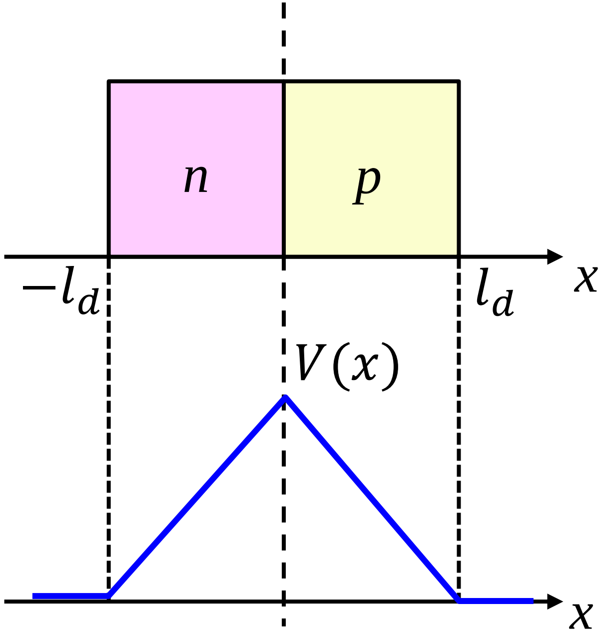
$$\tau_n=1+e^{2\eta_1}+e^{2\eta_2}+A_{12}e^{2(\eta_1+\eta_2)},$$

$$\eta_i=\kappa_i n+\sigma_i\omega_i t+\delta_i,\quad \sigma_i=\pm 1,\ \omega_i=\sinh\kappa_i,$$

$$A_{12}=\frac{ab\sinh^2(\kappa_1-\kappa_2)-m(\sigma_1\omega_1-\sigma_2\omega_2)^2}{m(\sigma_1\omega_1+\sigma_2\omega_2)^2-ab\sinh^2(\kappa_1+\kappa_2)}$$

Non-linear capacitance: Varicap

Varicap BB505



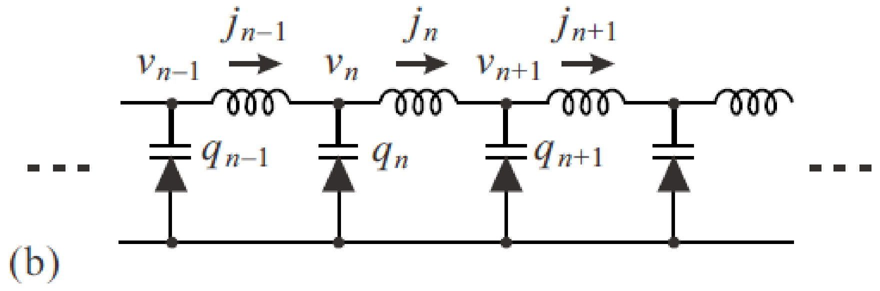
$$V_\mathrm{b} = \frac{en}{\epsilon}\int_{-l_d}^{0} 2(x+l_d)dx + \frac{en}{\epsilon}\int_{0}^{l_d} 2(l_d - x)dx = \frac{2enl_d^2}{\epsilon}$$

$$V + V_\mathrm{b} = \frac{2en}{\epsilon}\left(l_d + \frac{Q}{nS}\right)^2$$

$$\therefore C = \frac{dQ}{dV} = \sqrt{\frac{\epsilon}{2en}}\frac{nS}{\sqrt{V+V_\mathrm{b}}}$$

$$V + V_\mathrm{b} = V_0 + \delta V \quad \delta V \to V$$

L-Varicap transmission line



$$L\frac{dJ_n}{dt} = v_n - v_{n-1},$$

$$\frac{dq_n}{dt} = J_{n-1} - J_n,$$

$$q_n = \int_0^{v_n} C(V)dV, \quad C(V) = \frac{Q(V_0)}{F(V_0) + V - V_0}$$

$$q_n = Q(V_0)\log\left[1 + \frac{V_n}{F(V_0)}\right] + \text{const.}$$

$$\frac{d^2}{dt^2}\log\left[1 + \frac{V_n}{F(V_0)}\right] = \frac{1}{LQ(V_0)}(V_{n-1} + V_{n+1} - 2V_n)$$

Solitons in non-linear circuit

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CIRCUIT IMPLEMENTATIONS OF SOLITON SYSTEMS*

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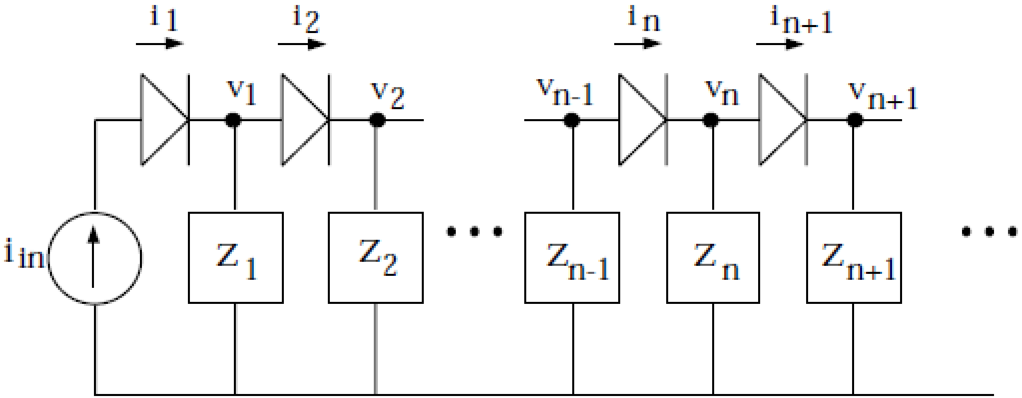
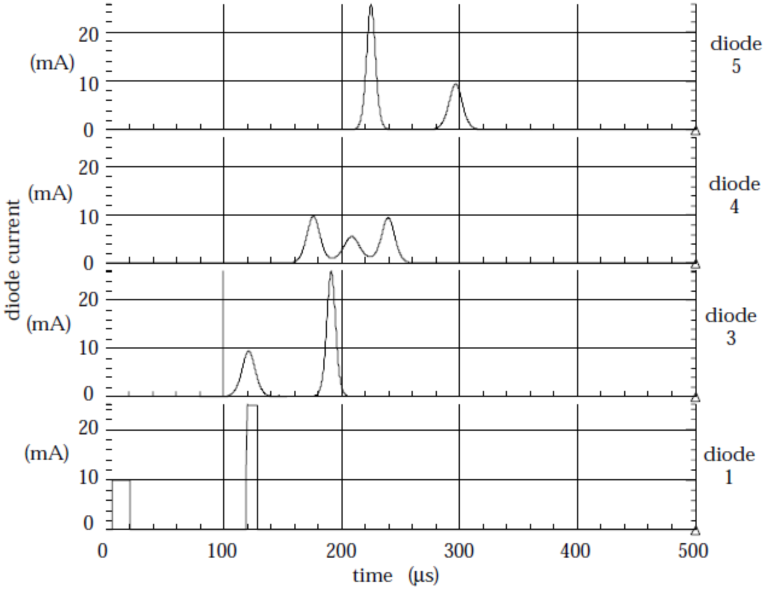
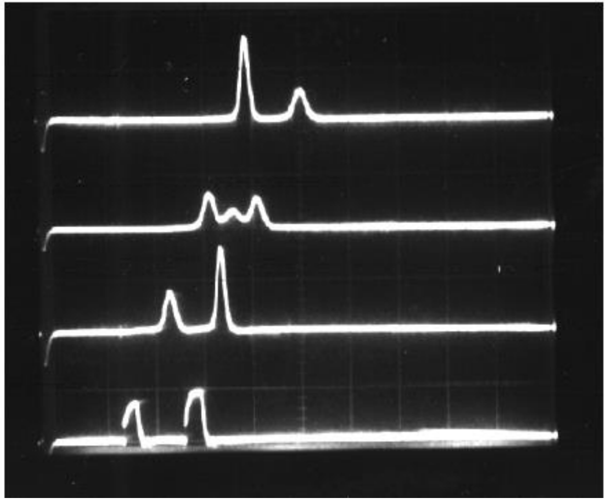






Toda lattice circuit, Soliton circuit

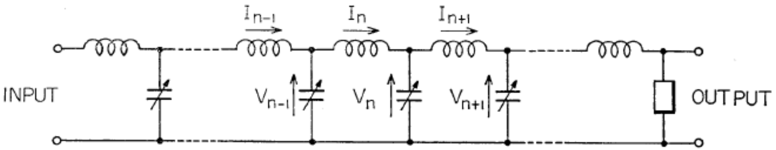
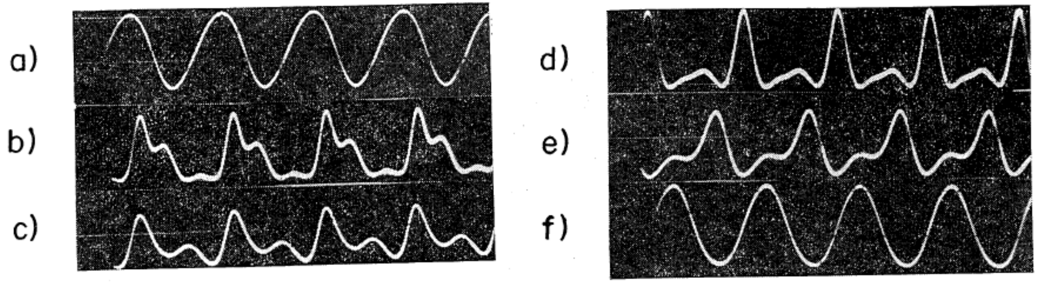


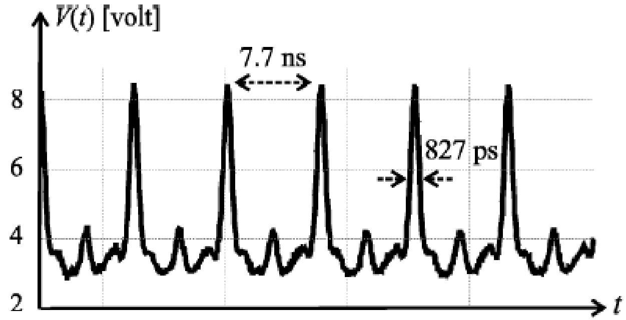
Fig. 1. A nonlinear network equivalent to a one-dimensional anharmonic lattice. The circuit elements have an inductance $L=22\ \mu$H or capacitance $C(V)=27\ V^{-0.48}$ pF.

J. Phys. Soc. Japan **28** (1970) 1366~1367

Studies on Lattice Solitons by Using Electrical Networks

Ryogo Hirota and Kimio Suzuki





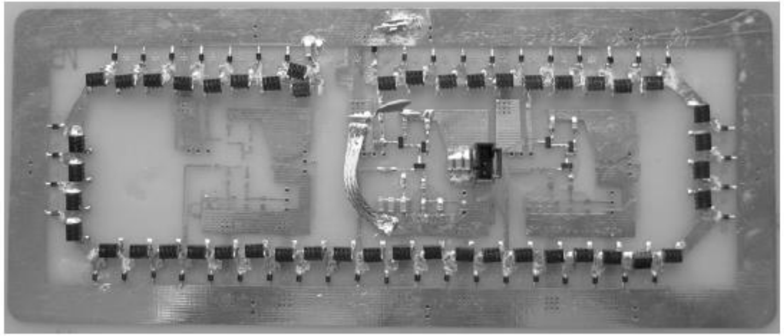
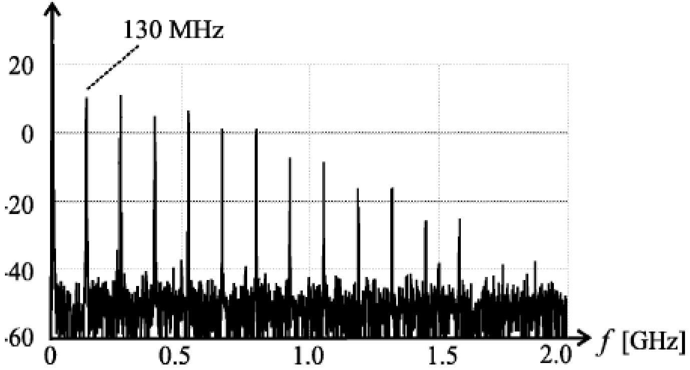
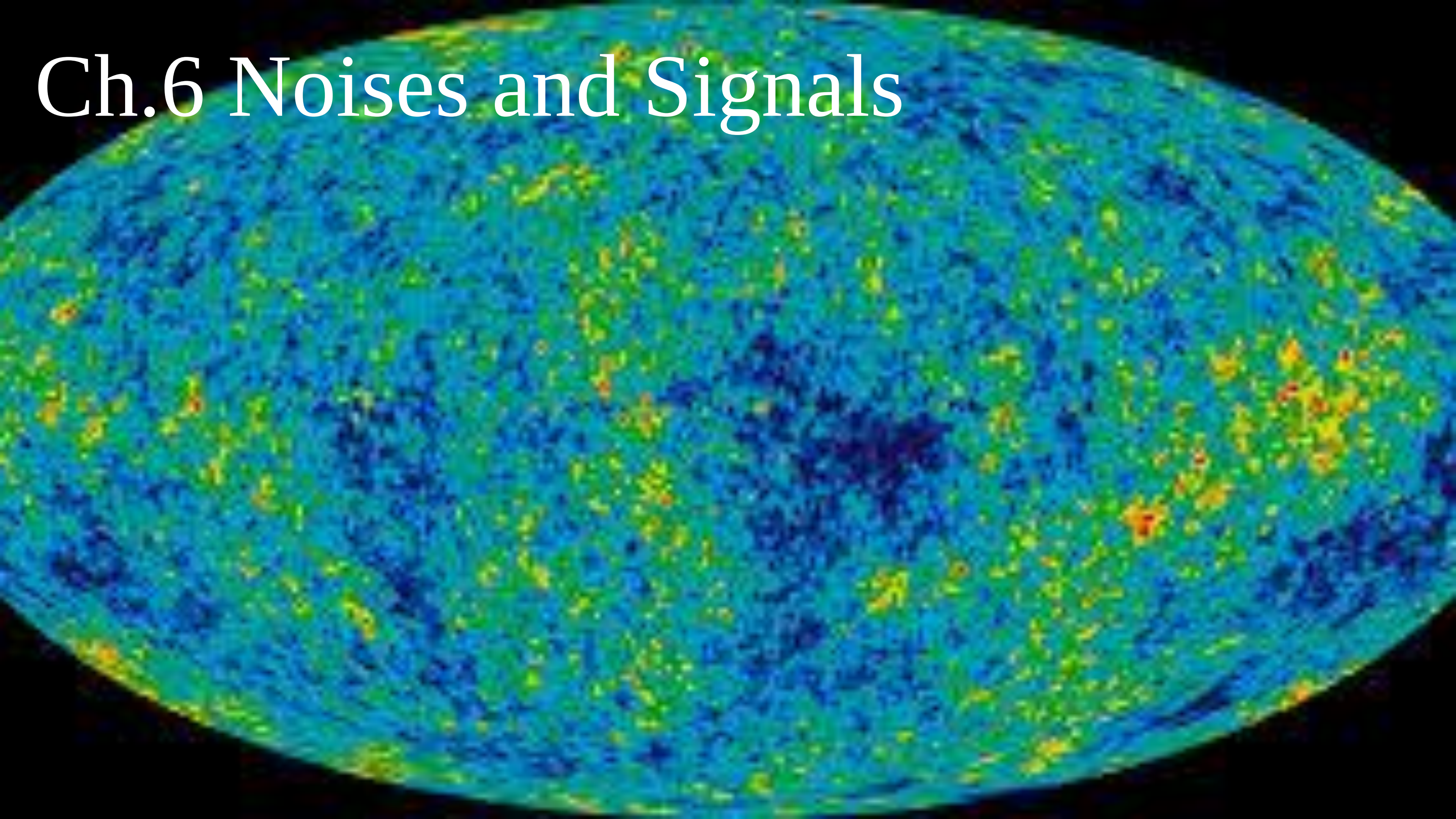


Fig. 16. Microwave soliton oscillator prototype.



Ch.6 Noises and Signals



Outline

Noises

Electric circuits transport 1) Information; 2) Electromagnetic power, on some physical quantities like voltages, current, …

Noises: stochastic (uncontrollable, unpredictable by human) variation in other words, fluctuation in such a quantity.

Internal noise:

- Intrinsic noise: Thermal noise (Johnson-Nyquist noise), Shot noise
- Noise related to a specific physical phenomenon: Avalanche, Popcorn, Barkhausen, etc.
- 1/f noise: Name for a group of noises with spectra 1/f.

External noise: EMI, microphone noise, etc.

6.1 Fluctuation

Quantity x, fluctuation $\delta x = x - \bar{x}$ $\qquad \overline{(\delta x)^2} = \overline{(x-\bar{x})^2} = \overline{x^2} - \bar{x}^2 \quad (\overline{\delta x} = 0)$

$g(x)$: distribution function of x

Fourier transform: $\quad u(q) = \mathscr{F}\{g(x)\} = \int_{-\infty}^{\infty} g(x) e^{ixq} \frac{dx}{\sqrt{2\pi}}$

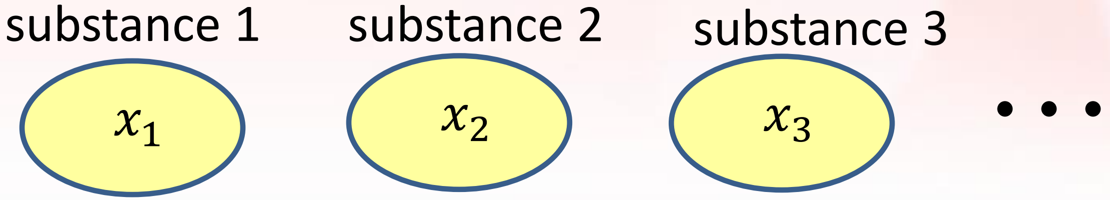
$u(q)$: **characteristic function** of the distribution

From Taylor expansion, n-th order moment can be obtained as

$$\overline{x^n} = \frac{\sqrt{2\pi}}{i^n} \left[\frac{d^n}{dq^n} u(q) \right]_{q=0}$$

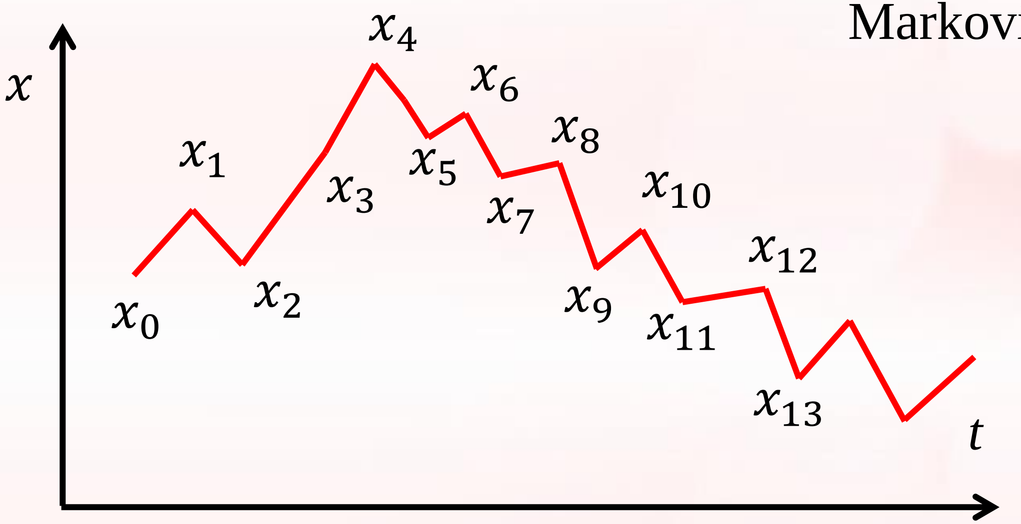
Moments to high orders → reconstruction of $g(x)$

In electric circuits we need to consider two kinds of averages:



x_j: independent

Ensemble average: $\bar{x}$

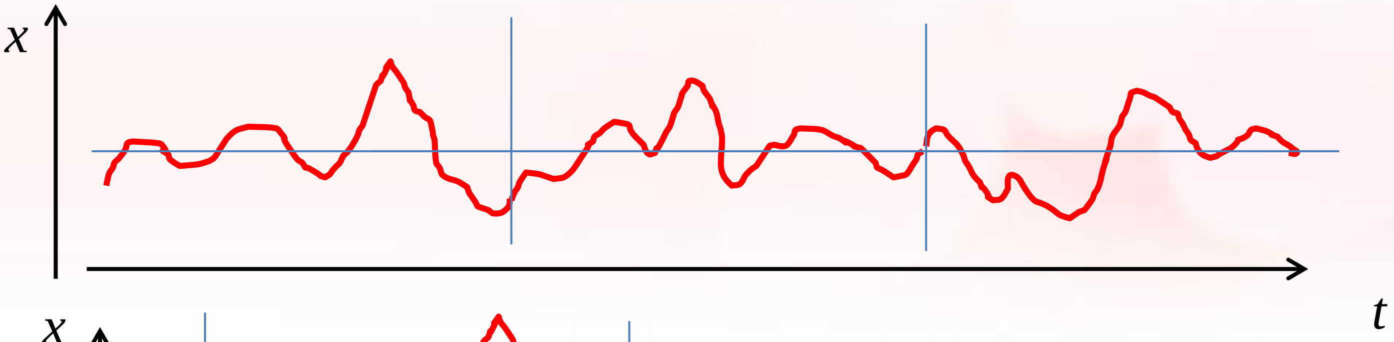


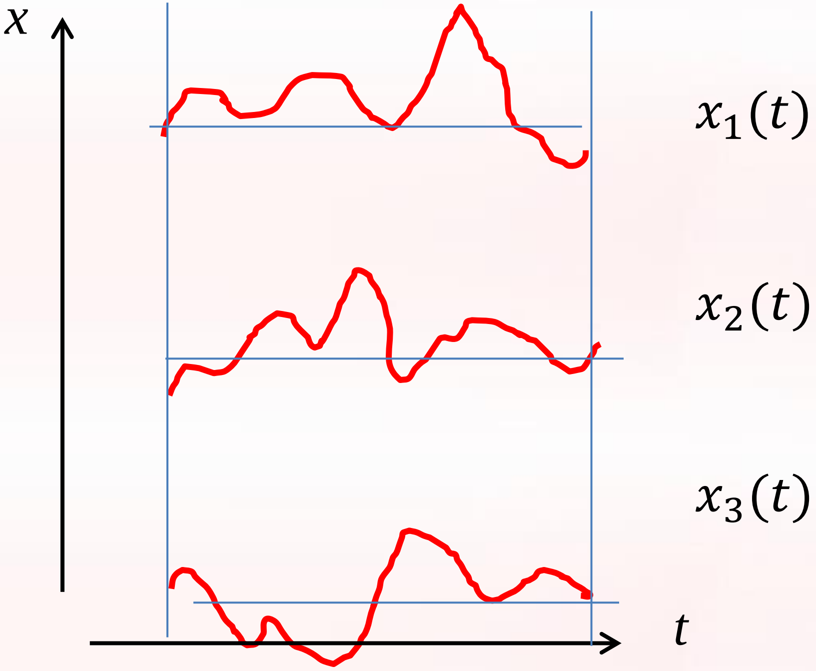
Markovian $x_1, \cdots, x_m \xrightarrow{\text{affect}} x_{m+1}$

m-th order Markovian

Time average of fluctuating variable: $\langle x \rangle$

Random process to distribution





The averaging interval should be longer than *m* in *m*-th order Markovian.

Power spectrum

Consider probability sets in the interval [0,T) with set index j .

$$x_j(t) = \sum_{n=1}^{\infty}(a_{jn}\cos\omega_n t + b_{jn}\sin\omega_n t), \quad \omega_n = \frac{2n\pi}{T}$$

$$\mathscr{P}_{jn} = (a_{nj}\cos\omega_n t + b_{nj}\sin\omega_n t)^2$$

(Power)

$$\langle\mathscr{P}_n\rangle = \frac{1}{2}\langle a_n^2 + b_n^2\rangle$$

∵ cross product terms are averaged out

Random process:
Gaussian distribution in time

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{x^2}{2\sigma^2}\right]$$

$$\overline{(\delta x)^2} = \sigma^2, \quad \overline{\left(\sum_{j=1}^{m}\delta x_j\right)^2} = m\sigma^2$$

Then $\overline{\langle\mathscr{P}_n\rangle} = \sigma_n^2$

Power spectrum $G(\omega)$

Frequency band width $\delta\omega$: separation between two adjacent frequencies

Power spectrum $G(\omega)$

Frequency band width $\delta\omega$: separation between two adjacent frequencies

$$\delta\omega = \omega_{n+1} - \omega_n = \frac{2(n+1)\pi}{T} - \frac{2n\pi}{T} = \frac{2\pi}{T}$$

$$G(\omega_n)\frac{\delta\omega}{2\pi} = \overline{\langle \mathscr{P}_n \rangle} \; (= \sigma_n^2)$$

$$\overline{\langle x^2(t) \rangle} = \sum_{n=1}^{\infty} \overline{\langle \mathscr{P}_n \rangle} \quad (\overline{x(t)} = 0)$$

$$= \sum_n G(\omega_n)\frac{\delta\omega}{2\pi} \rightarrow \int_0^{\infty} G(\omega)\frac{d\omega}{2\pi}$$

6.1.1 Fluctuation-Dissipation theorem

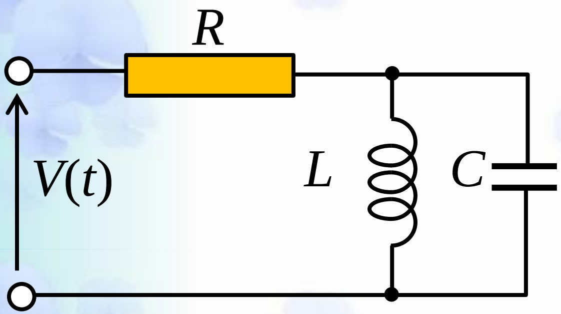
久保亮五
Ryogo Kubo
1920-1995

Harry Nyquist
1889-1976

Nobert Wiener
1894-1964

Aleksandr Khinchin
1894-1959

Fluctuation-dissipation theorem in the language of circuit



$$\omega_0 \equiv 1/\sqrt{LC}$$

$$Z(i\omega) = \frac{R(\omega_0^2 - \omega^2) + i\omega_0^2\omega L}{\omega_0^2 - \omega^2},$$

$$Y(i\omega) = \frac{\omega_0^2 - \omega^2}{R(\omega_0^2 - \omega^2) + i\omega_0^2\omega L}$$

$V(t)$ noise power spectrum → $G_v(\omega)$

$$G_{\mathrm{v}}(\omega) = 4k_{\mathrm{B}}T\mathrm{Re}[Z(i\omega)]$$
$$= 4k_{\mathrm{B}}TR$$

Johnson-Nyquist noise
Thermal noise

White noise (noise with no frequency dependence) in the case of frequency independent resistance

One of the representations for the fluctuation-dissipation theorem

6.1.2 Wiener-Khintchine theorem

Autocorrelation function

$$C(\tau) = \overline{\langle x(t)x(t+\tau)\rangle}$$

$$= \overline{\sum_{n,m}\langle [a_n \cos\omega_n t + b_n \sin\omega_n t][a_m \cos\omega_m (t+\tau) + b_m \sin\omega_m (t+\tau)]\rangle}$$

$$= \frac{1}{2}\sum_n \overline{\langle a_n^2 + b_n^2\rangle} \cos\omega_n \tau = \sum_n \overline{\langle \mathscr{P}_n\rangle} \cos\omega_n \tau$$

$$= \int_0^\infty G(\omega)\cos\omega\tau \frac{d\omega}{2\pi}$$

$$C(\tau) = \int_0^\infty G(\omega)\cos\omega\tau \frac{d\omega}{2\pi}, \quad G(\omega) = 4\int_0^\infty C(\tau)\cos\omega\tau d\tau$$

Wiener-Khintchine theorem