# Answer for the question in the lecture

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Sorry for some confusion in the lecture. And the statement "Average of a sinusoidal function etc." is wrong. Please just forget it (that kind of factor only should appear next to the average of coefficients for sinusoidal functions). We begin with repeating the description of Nyquist theorem. Though I believe I finally gave the right explanation, because we here use a bit tricky logic and I myself was a bit confused in the first place, I would like to begin with a bit revised explanation. If you just wish to know the (very simple) answer, skip the following and jump to **Answer**.

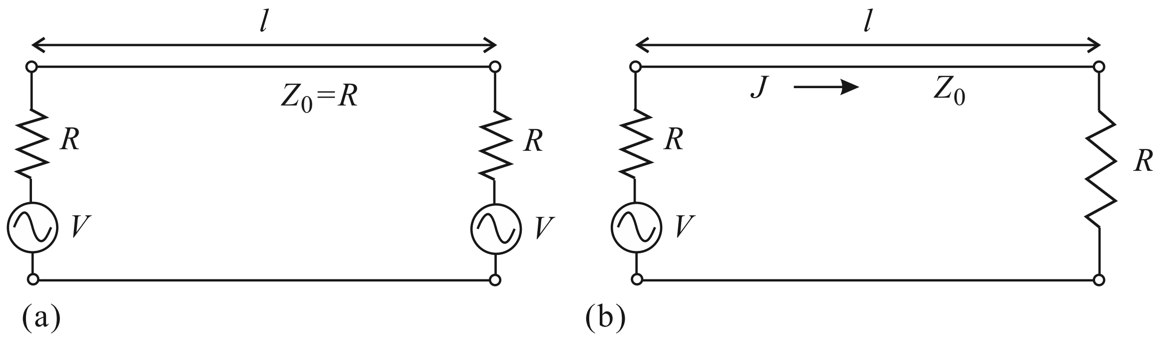


Figure 1: Setup (Gedankenexperiment) to derive Nyquist theorem. (a) Two Resistances with the value $R$ and the noise sources are connected with an impedance matched transmission line. (b) The noise source at the right end is removed.

We consider the circuit diagram shown in Fig. 1 to consider the thermal noise in resistances. As was explained, an approach from electric circuit is to model a resistance with noise as a series of a noiseless resistance and an electric power source which produces noise. Hence, the both resistances are associated with noise power sources in Fig. 1(a). When they are connected to the transmission line, two noise sources should be in thermal equilibrium with the line. We can treat the transmission line as an electromagnetic cavity and calculate the thermal fluctuation in the photon system. In the approximation for $k_{\rm B}T \gg \hbar\omega$, the energy density per mode is simply $k_{\rm B}T$.

In the situation shown in Fig. 1(a), the whole system is under equilibrium, *i.e.*, at a uniform temperature. And all the photons reached to the ends of the transmission line should be absorbed at the two resistors due to the impedance matching. Hence the absorbed photons are in balance with those emitted from the noise sources. In the transmission line, left-going and right-going photons are in balance and there is no total energy flow through the line.

Now in the Gedankenexperiment, we drop the noise source at the right end (Fig. 1(b)). We may think this can be accomplished with cooling the right resistor to absolute zero (anyway this is a Gedankenexperiment (thought experiment)). The balance is broken and there should be a total energy flow to the right. Presently the source of the imbalance is the noise power source associated with the left resistor. Let $V$ the voltage of the source and the current $J$ is $J = V/2R$. This causes the energy consumption at the right terminating resistor $(V/2) \times (V/2R) = V^2/4R$.

Then we can easily derive

$$\overline{(V/2) \times V/2R} = k_{\rm B}T.$$

## Question

The product of current and voltage is $V^2/2R$ and this should be equal to $k_\mathrm{B}T$. Why we need to use $V^2/4R$ instead?

## Answer

As mentioned above, the enegy from the fluctuation voltage is consumed in the left (*i.e.*, own) resistor by half. What we calculated as $k_\mathrm{B}T$ is the energy flowing to the right resistor and thus it is equal to $V^2/4R$.