電子回路論　第3回 2015年10月22日

Electric Circuit No.3

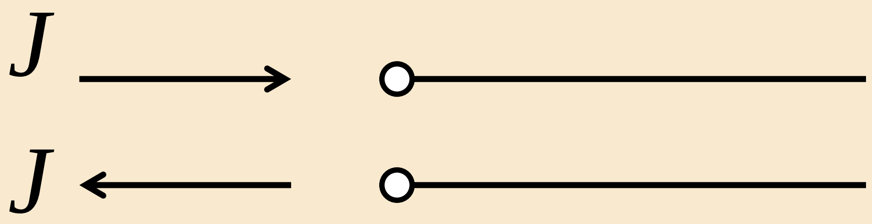
2015.10.22

Shingo Katumoto

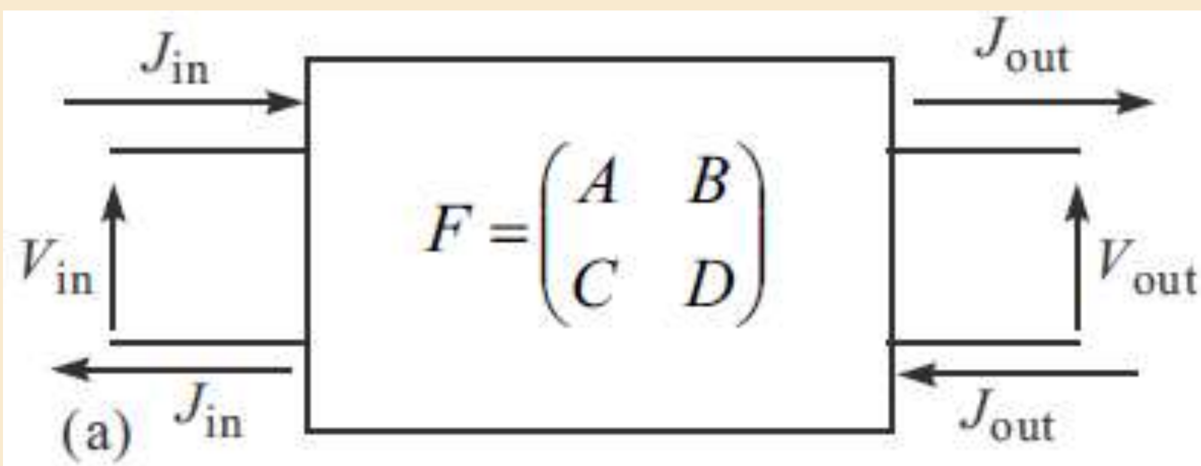
4-terminal (2-terminal pair) circuits 4端子回路

Terminal pair (端子対)

Current: circulation, no net current

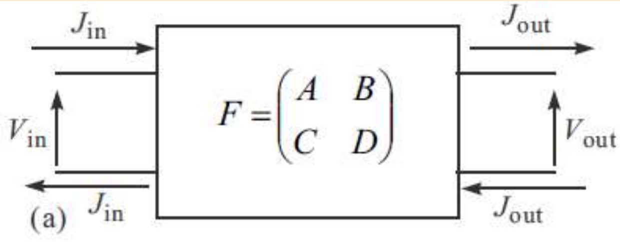


2-terminal pair (4-terminal) circuit



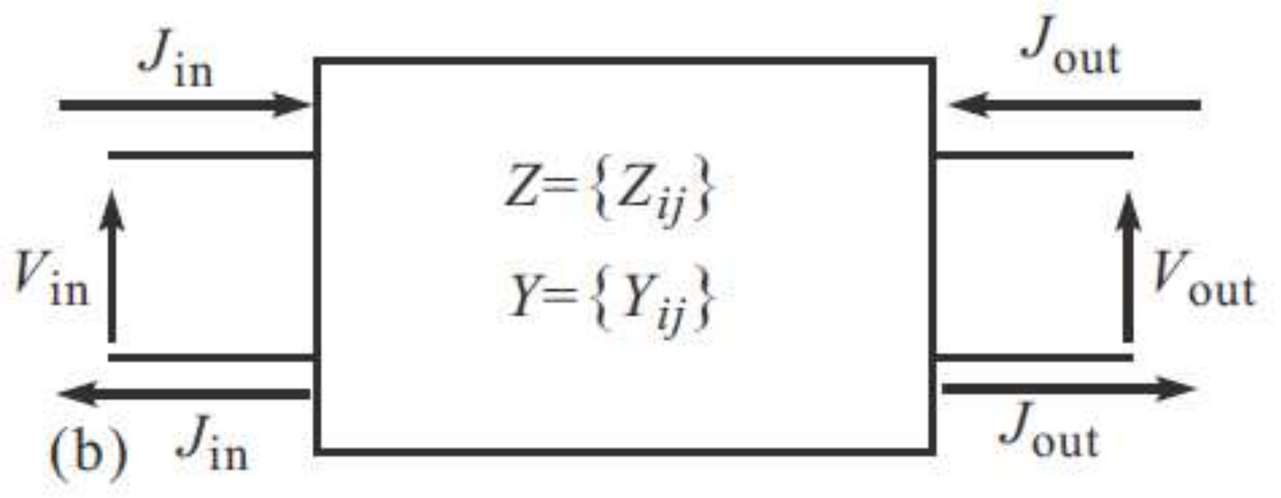
F-matrix of 4-terminal circuit

$$\begin{pmatrix} V_{\rm in} \\ J_{\rm in} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_{\rm out} \\ J_{\rm out} \end{pmatrix} \equiv F \begin{pmatrix} V_{\rm out} \\ J_{\rm out} \end{pmatrix}$$

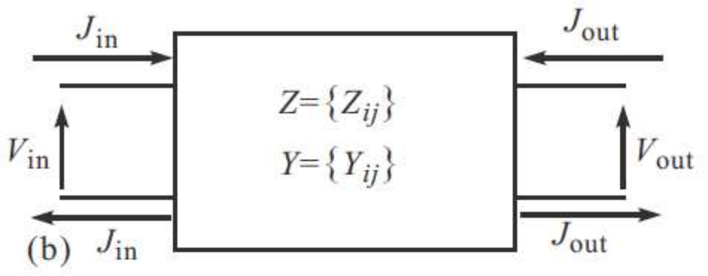


$$A = \left(\frac{V_{\rm in}}{V_{\rm out}}\right)_{J_{\rm out}=0}, \quad B = \left(\frac{V_{\rm in}}{J_{\rm out}}\right)_{V_{\rm out}=0}, \quad C = \left(\frac{J_{\rm in}}{V_{\rm out}}\right)_{J_{\rm out}=0}, \quad D = \left(\frac{J_{\rm in}}{J_{\rm out}}\right)_{V_{\rm out}=0}.$$

$$\begin{pmatrix} V_{\rm out} \\ J_{\rm out} \end{pmatrix} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} V_{\rm in} \\ J_{\rm in} \end{pmatrix} \equiv K \begin{pmatrix} V_{\rm in} \\ J_{\rm in} \end{pmatrix}$$



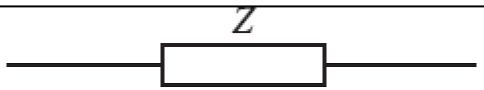
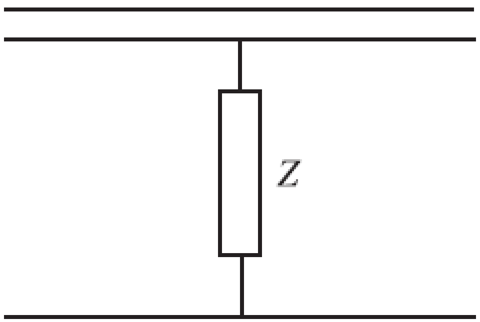
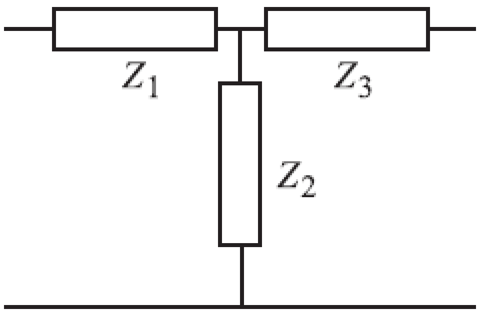
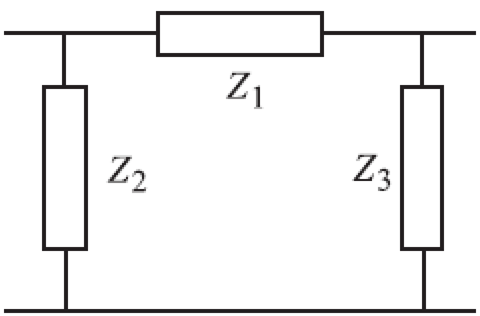
Impedance matrix, Admittance matrix



$$\begin{pmatrix} V_{\rm in} \\ V_{\rm out} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} J_{\rm in} \\ J_{\rm out} \end{pmatrix} \equiv Z \begin{pmatrix} J_{\rm in} \\ J_{\rm out} \end{pmatrix}$$

$$\begin{pmatrix} J_{\rm in} \\ J_{\rm out} \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_{\rm in} \\ V_{\rm out} \end{pmatrix} \equiv Y \begin{pmatrix} V_{\rm in} \\ V_{\rm out} \end{pmatrix}$$

Examples with impedances

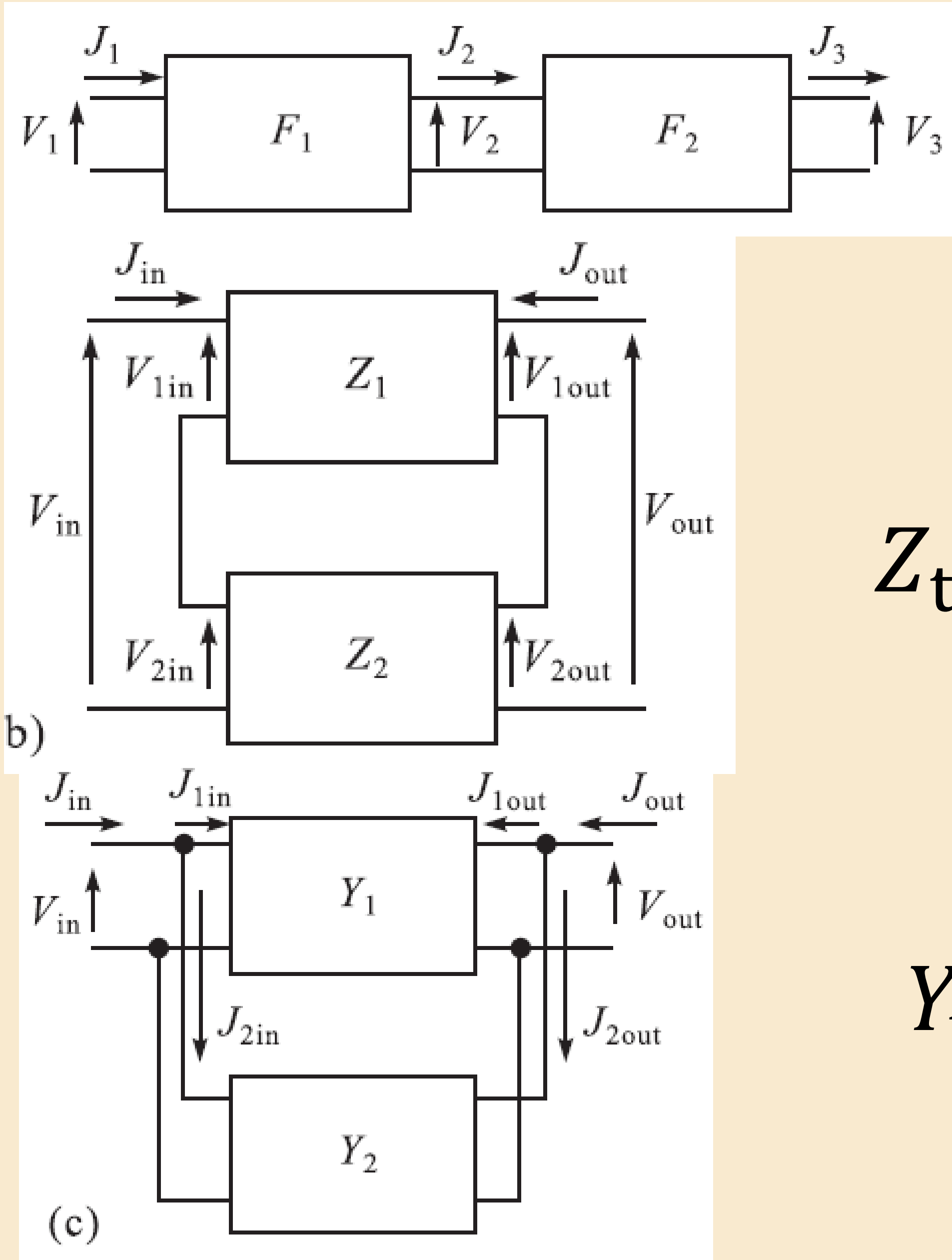
	A	B	C	D
	1	Z	0	1
	1	0	$\frac{1}{Z}$	1
	$1+\frac{Z_1}{Z_2}$	$\frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$	$\frac{1}{Z_2}$	$1+\frac{Z_3}{Z_2}$
	$1+\frac{Z_1}{Z_3}$	Z_1	$\frac{Z_1+Z_2+Z_3}{Z_2 Z_3}$	$1+\frac{Z_1}{Z_2}$

Connections of 4-terminal circuits

Cascade

Series

Parallel



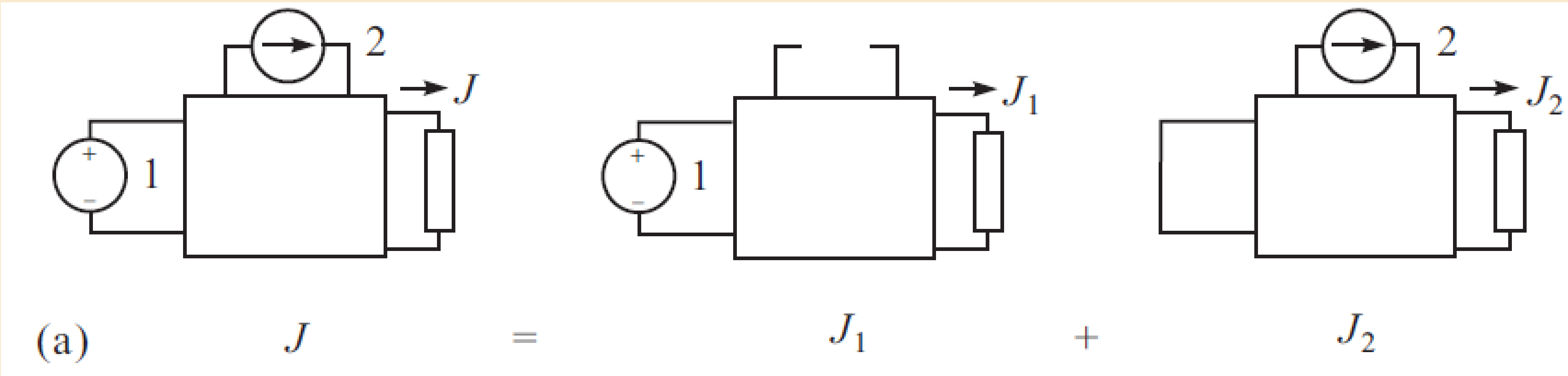
$$F_{\mathrm{tot}} = \prod_{i=1}^{N} F_i$$

$$Z_{\mathrm{tot}} = \sum_{i=1}^{N} Z_i$$

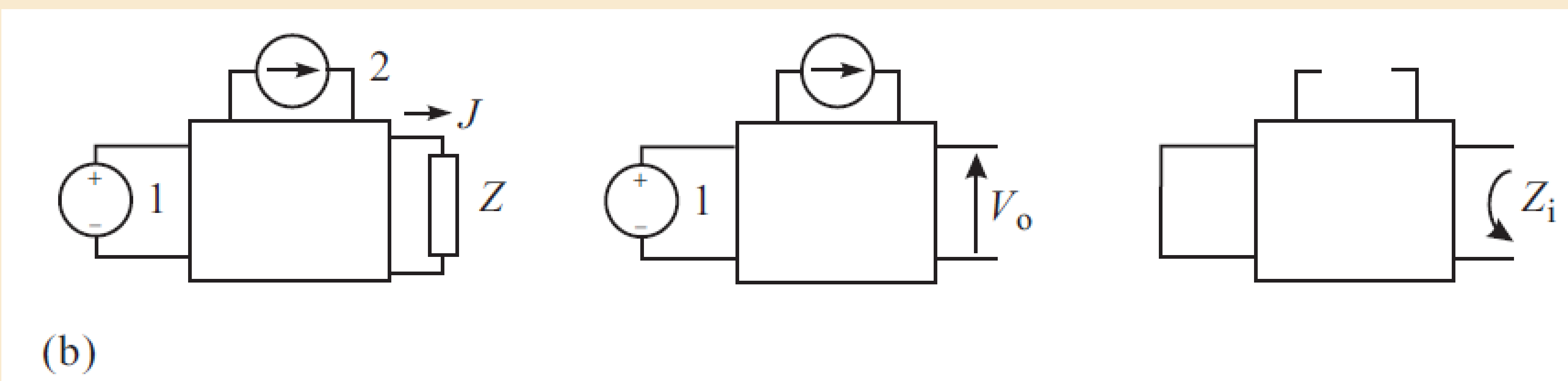
$$Y_{\mathrm{tot}} = \sum_{i=1}^{N} Y_i$$

Theorems for terminal-pair circuits

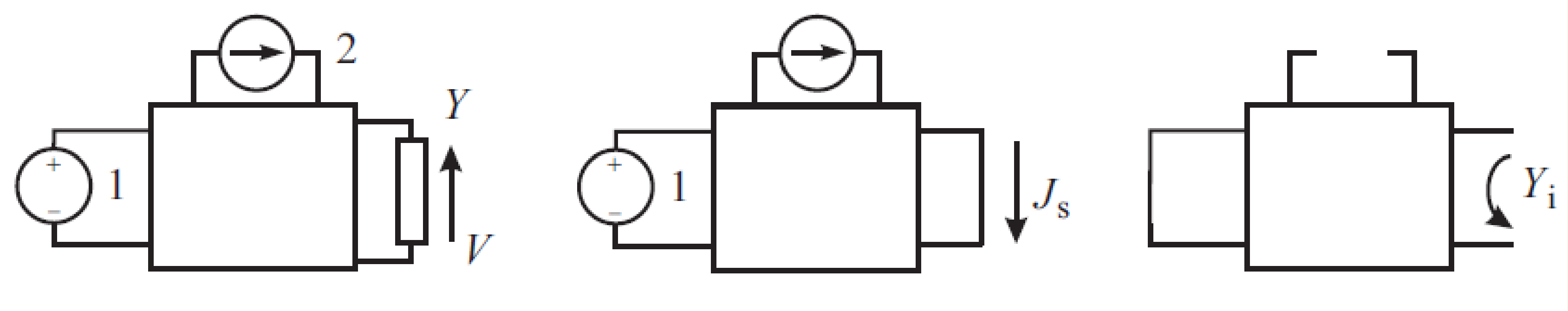
Superposition theorem



Ho-Thevenin's theorem



Norton's theorem



$$V = \frac{J_s}{Y + Y_i}$$

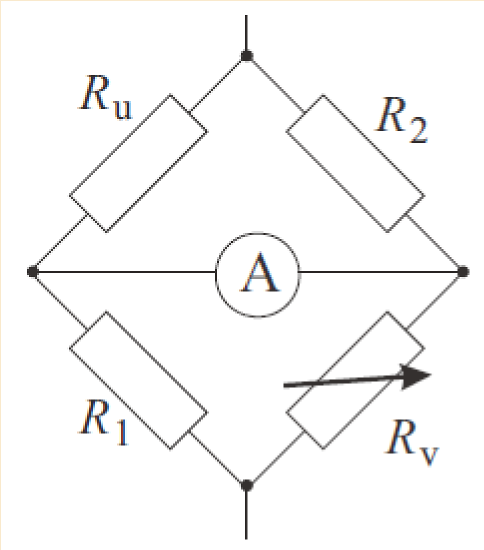
Duality 双対性

直列接続	並列接続
開放	短絡
電場	磁場
キルヒホッフの第 2 法則	キルヒホッフの第 1 法則
電圧	電流
インピーダンス	アドミッタンス
抵抗	コンダクタンス
静電容量	インダクタンス
鳳-テブナンの定理	ノートンの定理

Duality

Series	Parallel
Open	Short
Voltage	Current
Impedance	Admittance
Capacitance	Inductance
Electric field	Magnetic field
Resistance	Conductance
Ho-Thevenin	Norton
Kirchhoff's 2nd law	Kirchhoff's 1st law

Resistance bridge 抵抗ブリッジ

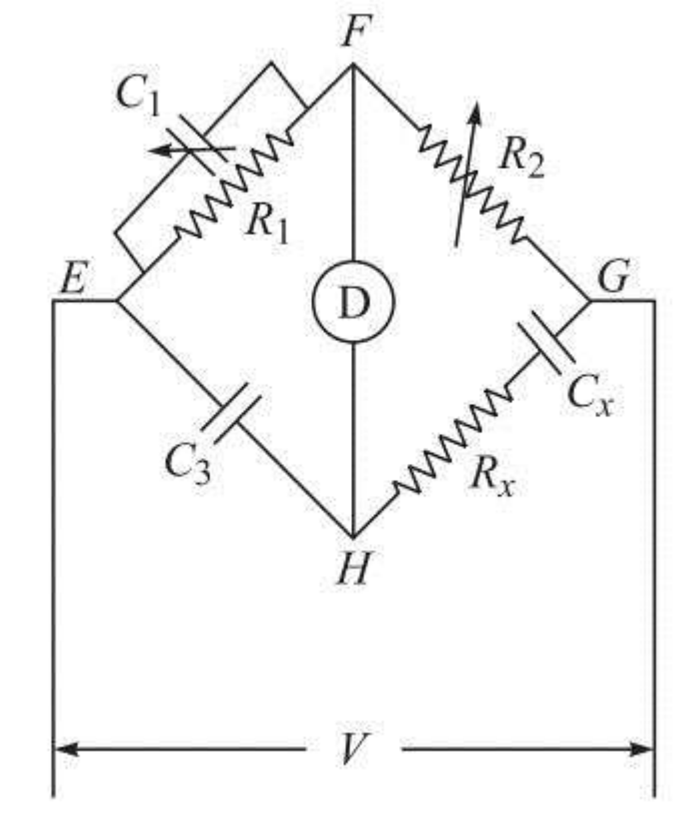


Wheatstone bridge

AVS-47 Resistance bridge

Not a “bridge” circuit!

Schering Bridge



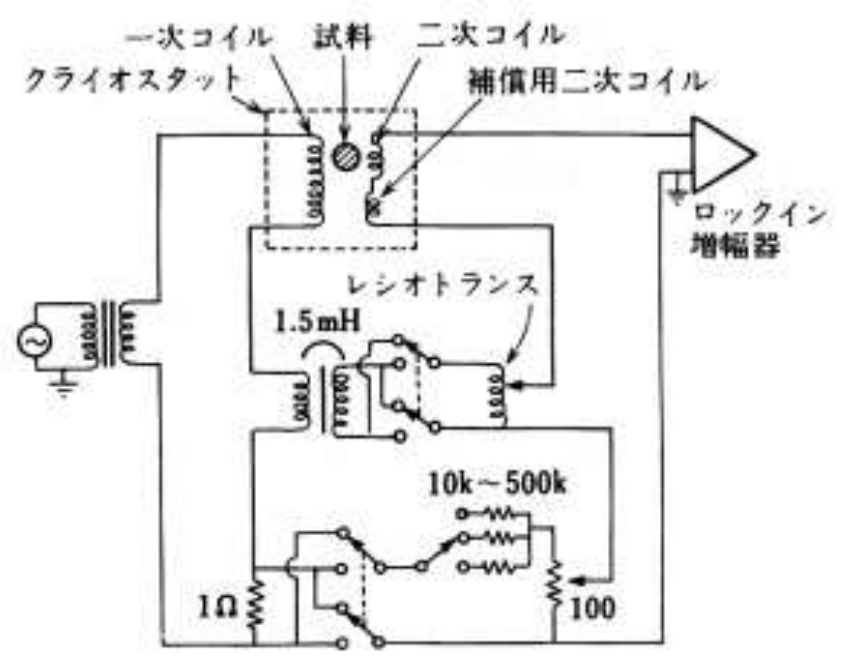
$$Z_1 Z_x = Z_2 Z_3, \quad Z_x = Z_2 Z_3 Y_1$$

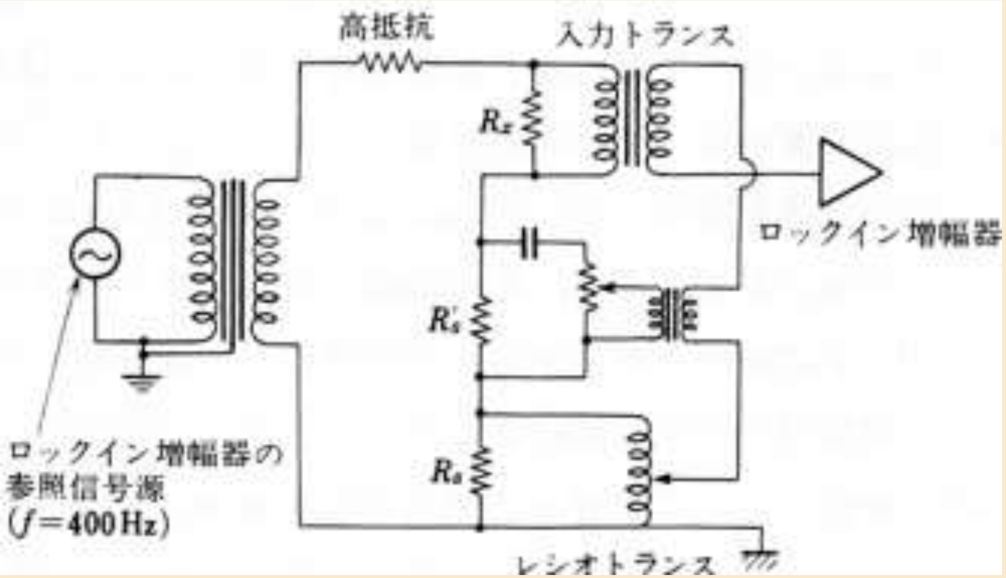
$$Z_x = R_x + \frac{1}{i\omega C_x}, \quad Z_2 = R_2, \quad Z_3 = \frac{1}{i\omega C_3}, \quad Y_1 = \frac{1}{R_1} + i\omega C_1$$

$$R_x + \frac{1}{i\omega C_x} = R_2 \frac{1}{i\omega C_3}\left(\frac{1}{R_1} + i\omega C_1\right)$$

$$R_x = \frac{R_2 C_1}{C_3}, \quad C_x = \frac{R_1}{R_2} C_3$$

Hartshorn bridge





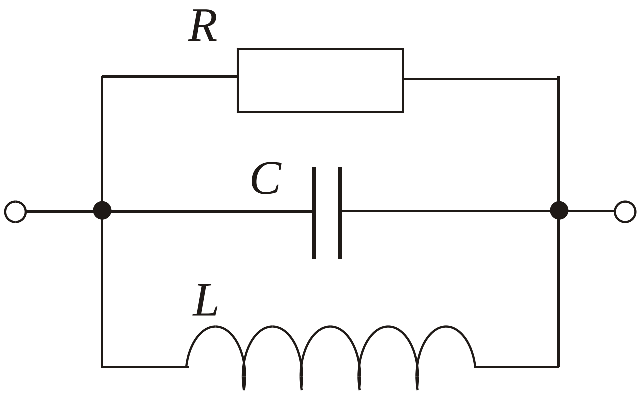
Capacitance bridge キャパシタンス ブリッジ

General Radio
3-terminal
Capacitance bridge

Agilent E4981A

2.4 General Properties of Resonance and Resonance Circuits

2.4.1 Resonance Phenomena



Harmonic oscillator: $$\frac{dq}{dt} = -\omega_0^2 q$$

Kirchhoff's law

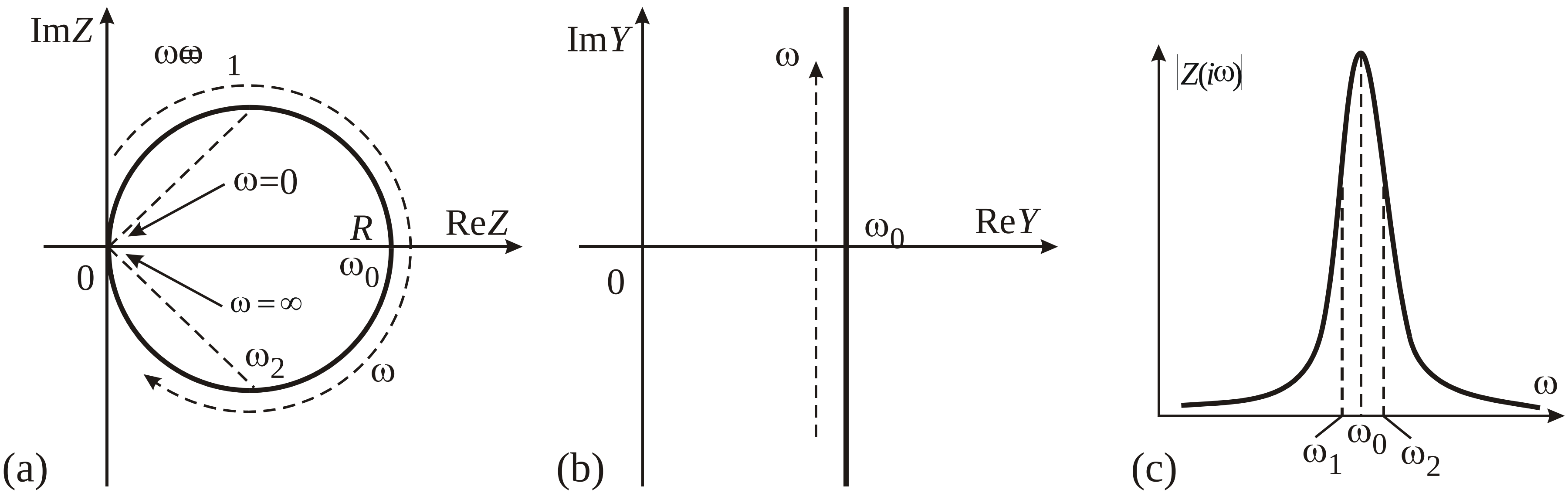
$$L\frac{dJ_\mathrm{L}}{dt} = -L\frac{d^2 q_\mathrm{L}}{dt^2} = \frac{q}{C} = RJ_\mathrm{R} = R\frac{dq_\mathrm{R}}{dt}$$

$$dq_\mathrm{L} + dq_\mathrm{R} + dq = 0$$

$$\frac{d^2 q}{dt^2} + \frac{1}{CR}\frac{dq}{dt} + \frac{1}{LC}q = \frac{d^2 q}{dt^2} + \frac{1}{\tau}\frac{dq}{dt} + \omega_0^2 q = 0$$

$$q = \exp(\lambda t) \qquad \lambda = \frac{1}{2\tau}\left[-1 \pm \sqrt{1 - 4(\omega_0 \tau)^2}\right] \approx -\frac{1}{2\tau} \pm i\omega_0 \quad (\omega_0 \tau \gg 1)$$

2.4.2 Resonance and Phase shift



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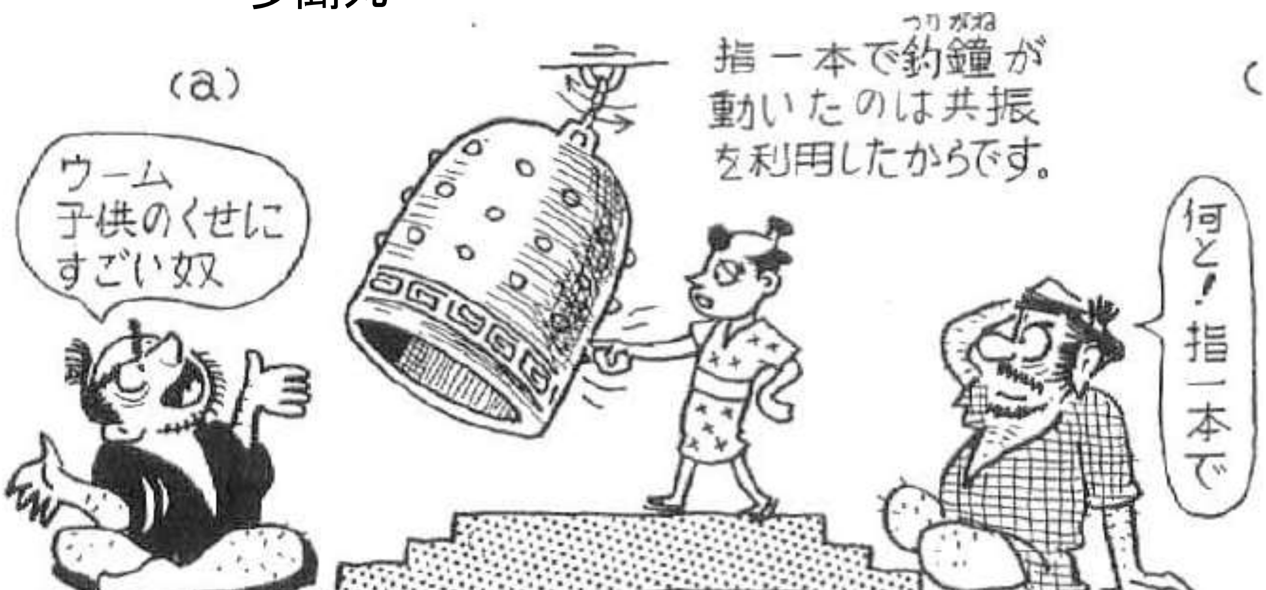


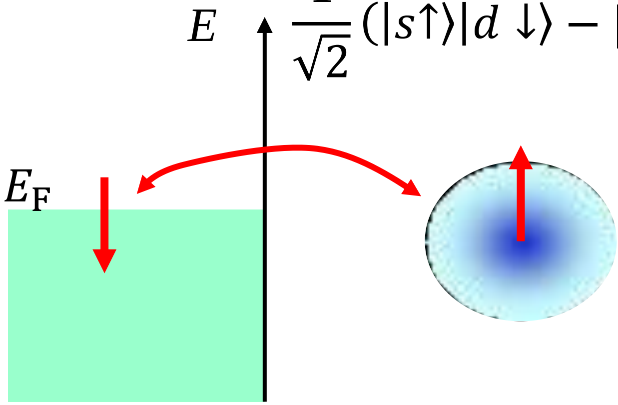
Resonance:
Reactance =0

Total Phase Shift
Change: π

Kondo Resonance and Phase shift

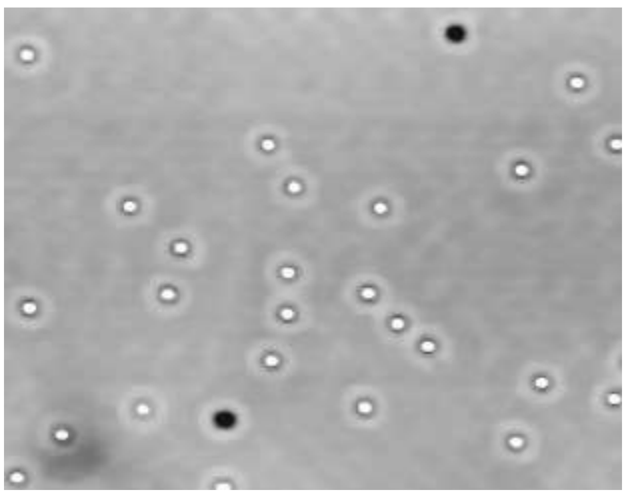
Jun Kondo

$$\frac{1}{\sqrt{2}}\left(|s\uparrow\rangle|d\downarrow\rangle - |s\downarrow\rangle|d\uparrow\rangle\right)$$

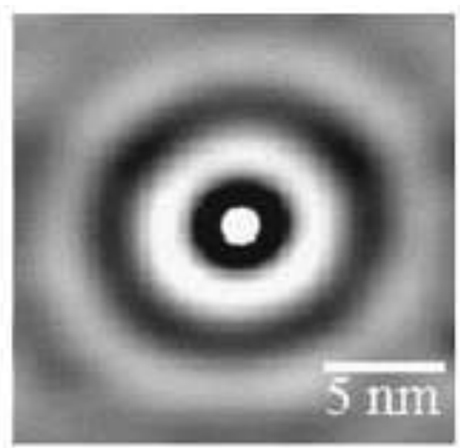


Many body resonance.
But still has the phase shift of $\pi/2$!

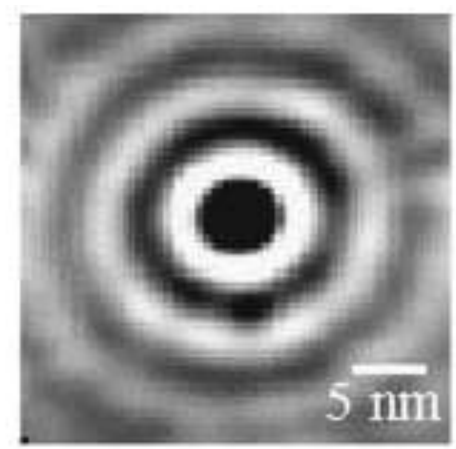
Co atoms on Ag (111) surface



Co (magnetic)



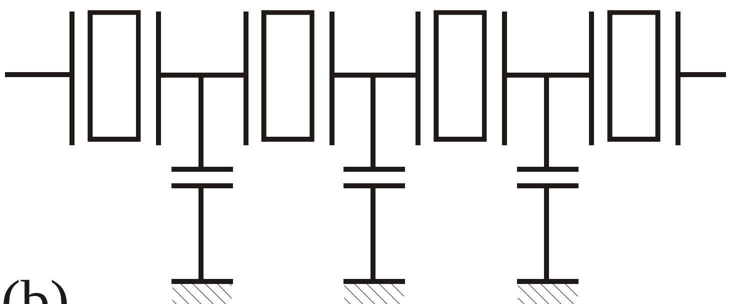
Defect (non-magnetic)



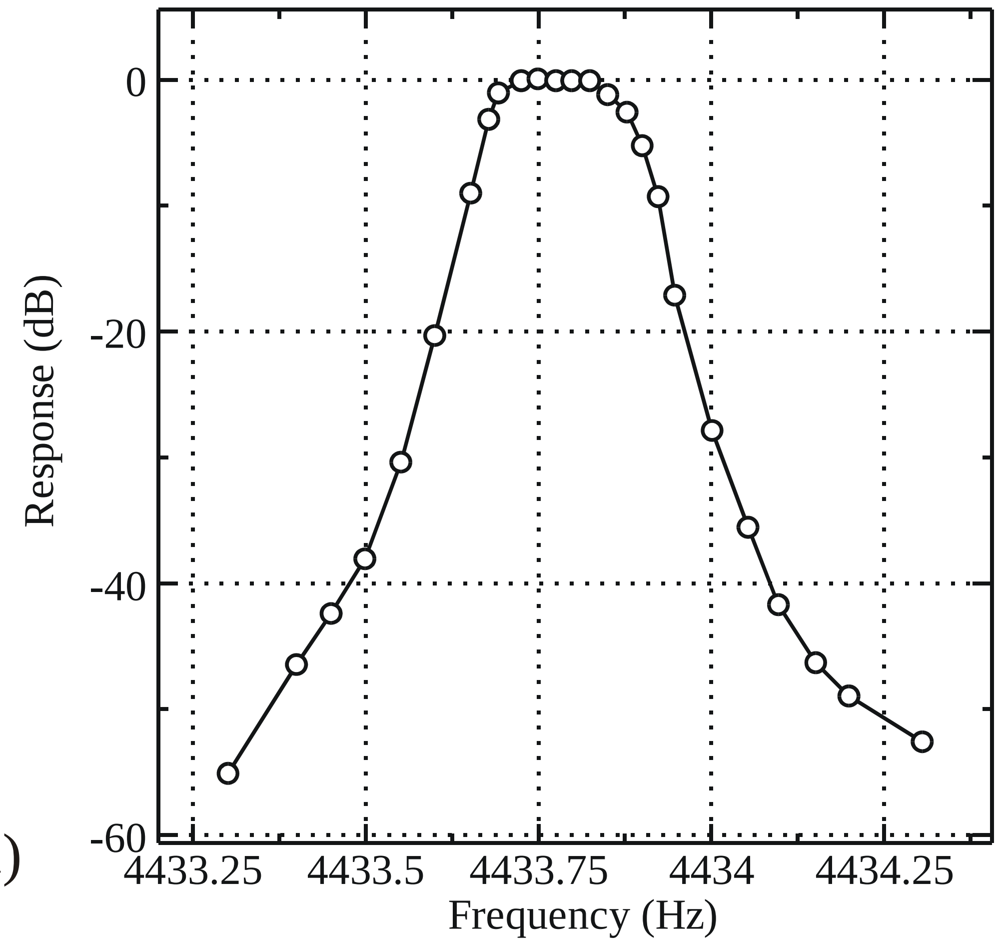
Schneider et al., Phys. Rev. B65, 121406 (2002).

Quartz crystal filter

(a)



(b)



(c)