2022.5.11 Lecture 5

10:25 – 11:55

Lecture on

Magnetic Properties of Materials

磁性 (Magnetism)

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Review of last four lectures

Chapter 1 Basic Notions of Magnetism

Classical pictures of magnetic moments in materials:

- Magnetic charges
- Circular currents

Breakdown of classical magnetism: cancellation of paramagnetic and diamagnetic terms (Bohr-van Leeuwen theorem)

Quest for the sources of magnetic dipoles in materials.

Spins and spin-orbit interactions

Chapter 2 Magnetism of Localized Electrons

Spherical potential, closed shell magnetization

Electronic states of magnetic ions

- LS (j-j) coupling, Hund’s rule
- Ligand field

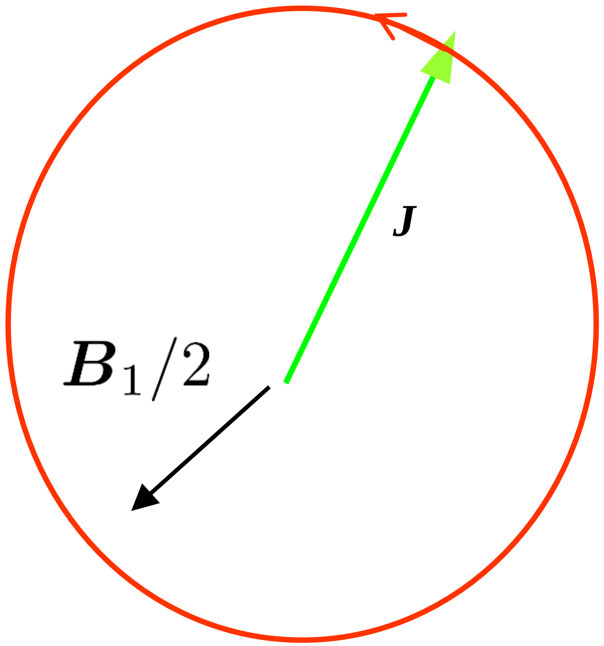
Representative experimental method: magnetic resonance

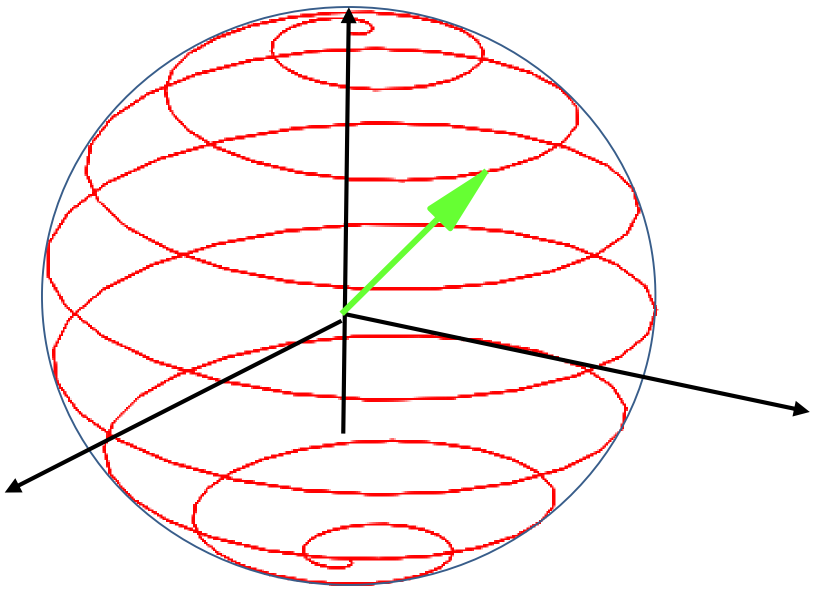
Outline

- Magnetic resonance (continued)
- Spin Hamiltonian
- Example of analyzing experimental data on electron paramagnetic resonance
- Application of paramagnetism: magnetic refrigeration

Chapter 3 Magnetism of conduction electrons

- Pauli paramagnetism
- Landau diamagnetism





High frequency magnetic field in *xy*-plane

$$\boldsymbol{B}(t) = \boldsymbol{B}_1 \cos(\omega t)$$
$$= \frac{\boldsymbol{B}_1}{2}[\exp(i\omega t) + \exp(-i\omega t)]$$

Two rotational magnetic fields

when $\omega \approx \omega_{\mathrm{L}}$

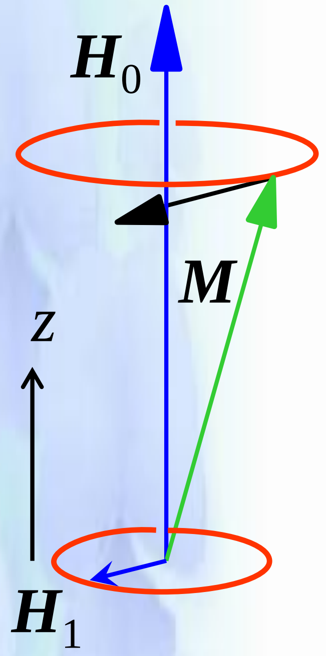
On the rotational coordinate: $\omega \approx 0, 2\omega_{\mathrm{L}}$

Ignore $2\omega_{\mathrm{L}}$ component: rotational wave approximation

Precession around $\boldsymbol{B_1}$ $\quad \omega_1 = g_j \frac{eB_1}{2m}$

Total motion: spiral

Magnetic resonance



Macroscopic magnetization $\boldsymbol{M}$

Classical equation of motion

Phenomenological introduction of relaxation time

$$\begin{cases} \dfrac{dM_z}{dt} = \gamma[\boldsymbol{M}\times\boldsymbol{H}]_z + \dfrac{M_0 - M_z}{T_1}, \\ \dfrac{dM_{x,y}}{dt} = \gamma[\boldsymbol{M}\times\boldsymbol{H}]_{x,y} - \dfrac{M_{x,y}}{T_2}. \end{cases}$$

T_1: energy relaxation time, T_2: phase relaxation time

$\boldsymbol{H}_0$: static field (z)

$\boldsymbol{H}_1$: rotating field with $-\omega$

$$\boldsymbol{H} = (\frac{H_1}{2}\cos\omega t, -\frac{H_1}{2}\sin\omega t, H_0)$$

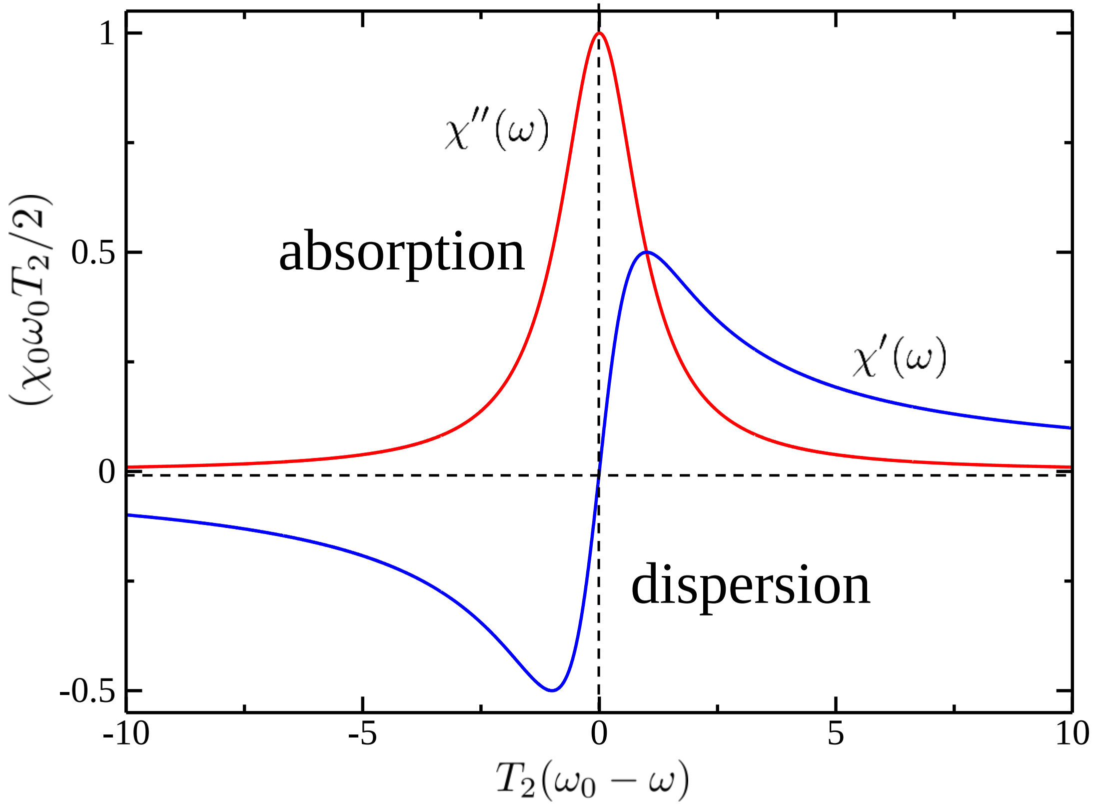
Then the equation of motion is given as

$$\begin{cases} \dfrac{dM_x}{dt} = \gamma[M_y H_0 + M_z \dfrac{H_1}{2}\sin\omega t] - \dfrac{M_x}{T_2}, \\ \dfrac{dM_y}{dt} = \gamma[M_z \dfrac{H_1}{2}\cos\omega t - M_x H_0] - \dfrac{M_y}{T_2}, \\ \dfrac{dM_z}{dt} = \gamma[-M_x H_1 \sin\omega t - M_y \dfrac{H_1}{2}\cos\omega t] + \dfrac{M_0 - M_z}{T_1} \end{cases}$$

Magnetic resonance (2)

We introduce the coordinate system (x', y', z') rotating around z-axis with freq. ω.

$$\begin{cases} M_{x'} = M_x \cos\omega t - M_y \sin\omega t, \\ M_{y'} = M_x \sin\omega t + M_y \cos\omega t \end{cases}$$



conditions

$$\begin{cases} \dfrac{dM_{x'}}{dt} = \dfrac{dM_{y'}}{dt} = 0 \quad \text{(stationary state)}, \\ M_z \simeq M_0 = \chi_0 H_0 \quad \text{(oblique angle is small)} \end{cases}$$

Solution

$$\begin{cases} M_{x'} = \chi_0\omega_0 T_2 \dfrac{(\omega_0-\omega)T_2 H_1/2}{1+(\omega_0-\omega)^2T_2^2+\gamma^2(H_1/2)^2T_1T_2} \\ M_{y'} = \chi_0\omega_0 T_2 \dfrac{H_1/2}{1+(\omega_0-\omega)^2T_2^2+\gamma^2(H_1/2)^2T_1T_2} \end{cases}$$

Original coordinate

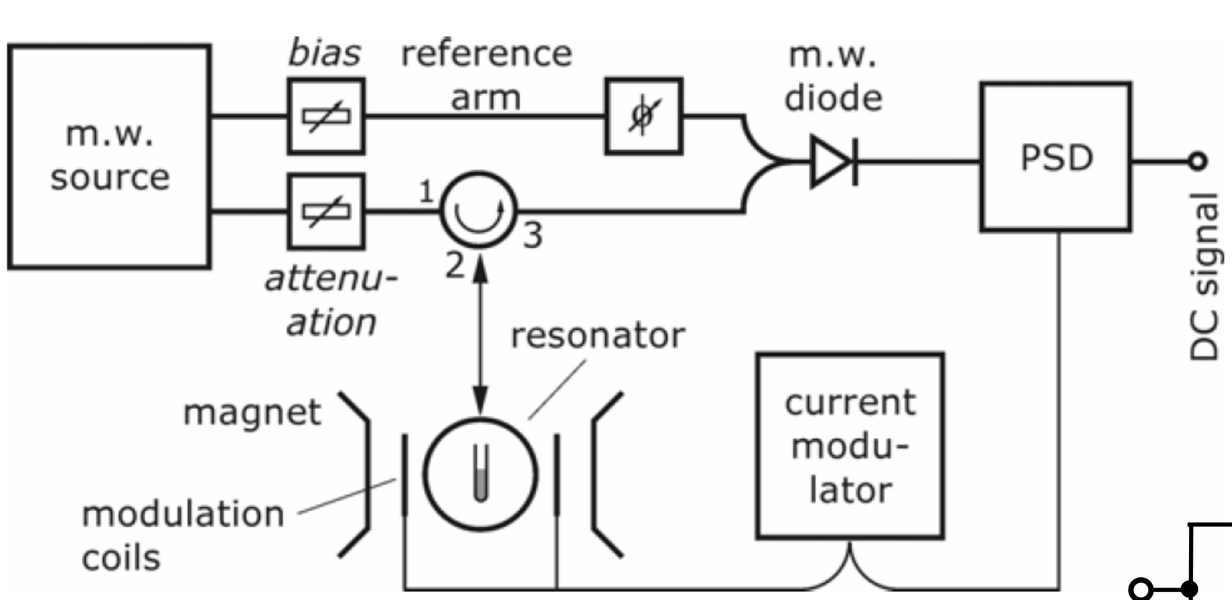
$$M_x = \chi'(\omega)H_1\cos\omega t + \chi''(\omega)H_1\sin\omega t,$$
$$M_y = -\chi'(\omega)H_1\sin\omega t + \chi''(\omega)H_1\cos\omega t$$

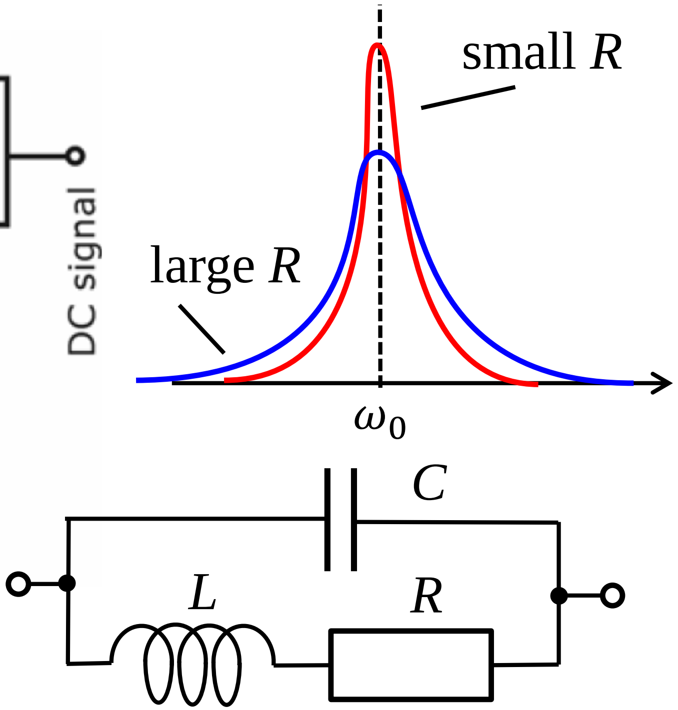
large relaxation

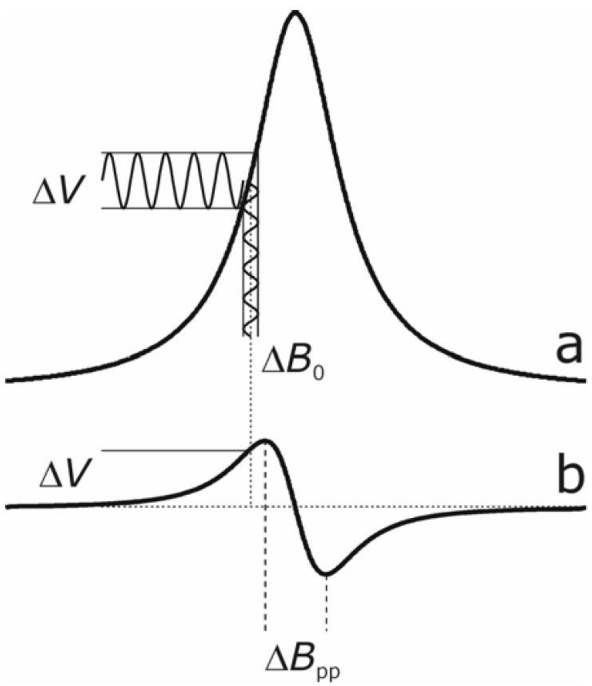
$\gamma^2 H_1^2 T_1 T_2 \ll 1$

$$\chi'(\omega) = \frac{\chi_0\omega_0}{2}T_2\frac{(\omega_0-\omega)T_2}{1+(\omega_0-\omega)^2T_2^2},$$
$$\chi''(\omega) = \frac{\chi_0\omega_0}{2}T_2\frac{1}{1+(\omega_0-\omega)^2T_2^2}$$

Electron paramagnetic resonance (EPR) experimental setup







Double resonator sample + cavity

Continuous wave (CW) measurement: detection of resonance dissipation

Pulse, Fourier transform measurement: detection of magnetic field due to the precession of magnetic moment

Spin Hamiltonian

For the comparison of the theory with EPR experiments we need to go a little further in approximation.

Effective spin Hamiltonian: (in case $\mathcal{H}_{\rm CF}$ is diagonalized)

Only contains spin operators, i.e. the orbital part is already integrated out.

{o}

Orbital basis: $\{\varphi_0, \varphi_1, \cdots, \}$ diagonalizes $\mathcal{H}_{\rm orb} = \mathcal{H}_0 + \mathcal{H}_{\rm CF}$

In ket form: $|\varphi_n\rangle = |n\rangle_{\rm o}$

Energy eigenstates: ${}_{\rm o}\langle n|\mathcal{H}_{\rm orb}|n'\rangle_{\rm o} = E_n \delta_{nn'}$

{s} Spin basis for total spin *S*: $\{\phi_{-2S}, \phi_{-2S+1}, \cdots, \phi_{2S}\}$

In ket form: $|\phi_m\rangle = |m\rangle_{\rm s}$

Perturbation Hamiltonian: $\mathcal{H}' = \lambda \boldsymbol{L} \cdot \boldsymbol{S} + \mu_{\rm B}(\boldsymbol{L} + g_{\rm e}\boldsymbol{S}) \cdot \boldsymbol{H}$

spin-orbit Zeeman

g_e : g-factor of electron

Expand the wavefunction with {o} and {s}as:

$$\Psi = \sum_{nm} a_{nm} \varphi_n \phi_m = \sum_{nm} a_{nm} |n\rangle_{\rm o} |m\rangle_{\rm s}$$

Spin Hamiltonian (2)

Eigenenergy equation: $\mathcal{H}\Psi = (\mathcal{H}_{\mathrm{orb}} + \mathcal{H}')\Psi = E\Psi.$

Second order perturbation in energy: $\tilde{\mathcal{H}} = {}_{\mathrm{o}}\langle 0|\mathcal{H}'|0\rangle_{\mathrm{o}} + \sum_{n\neq 0} \frac{{}_{\mathrm{o}}\langle 0|\mathcal{H}'|n\rangle_{\mathrm{o}}\,{}_{\mathrm{o}}\langle n|\mathcal{H}'|0\rangle_{\mathrm{o}}}{E_0 - E_n}$

Orbital angular moment is quenched: ${}_{\mathrm{o}}\langle 0|\boldsymbol{L}|0\rangle_{\mathrm{o}} = 0 \longrightarrow {}_{\mathrm{o}}\langle 0|\mathcal{H}'|0\rangle_{\mathrm{o}} = g_{\mathrm{e}}\mu_{\mathrm{B}}\boldsymbol{S}\cdot\boldsymbol{H}$

The second order term
→ reduced to second order in $\boldsymbol{L}$: ${}_{\mathrm{o}}\langle 0|\mathcal{H}'|n\rangle_{\mathrm{o}} = {}_{\mathrm{o}}\langle 0|\boldsymbol{L}|n\rangle_{\mathrm{o}} \cdot \underline{(\lambda\boldsymbol{S} + \mu_{\mathrm{B}}\boldsymbol{H})}$

effective magnetic field for $\boldsymbol{L}$

The effective spin Hamiltonian: $\tilde{\mathcal{H}} = g_{\mathrm{e}}\mu_{\mathrm{B}}\boldsymbol{S}\cdot\boldsymbol{H} - (\lambda\boldsymbol{S} + \mu_{\mathrm{B}}\boldsymbol{H})\Lambda(\lambda\boldsymbol{S} + \mu_{\mathrm{B}}\boldsymbol{H})$

where Λ is a tensor given by

$$\Lambda_{ij} = \sum_{n\neq 0} \frac{{}_{\mathrm{o}}\langle 0|L_i|n\rangle_{\mathrm{o}}\,{}_{\mathrm{o}}\langle n|L_j|0\rangle_{\mathrm{o}}}{E_n - E_0} \quad (i,j = x,y,z)$$

Expansion: $\tilde{\mathcal{H}} = \mu_{\mathrm{B}}\boldsymbol{S}g_{\mathrm{e}}(\mathbf{1} - \lambda\Lambda)\boldsymbol{H} - \lambda^2\boldsymbol{S}\Lambda\boldsymbol{S} - \mu_{\mathrm{B}}^2\boldsymbol{H}\Lambda\boldsymbol{H}$

Spin Hamiltonian (3)

$$\tilde{\mathcal{H}} = \mu_B \boldsymbol{S} g_e(\boldsymbol{1} - \lambda\Lambda)\boldsymbol{H} - \lambda^2 \boldsymbol{S}\Lambda\boldsymbol{S} - \mu_B^2 \boldsymbol{H}\Lambda\boldsymbol{H}$$

The third term is small, does not contribute to level splitting → Drop

The first term: extension of Zeeman energy with effective g-tensor: $\tilde{\boldsymbol{g}} = g_e(\boldsymbol{1} - \lambda\Lambda)$

The second term is written as $-\lambda^2 \boldsymbol{S}\Lambda\boldsymbol{S} = D\left[S_z^2 - \frac{S(S+1)}{3}\right] + E(S_x^2 - S_y^2)$

principal axes: *x*, *y*, *z*

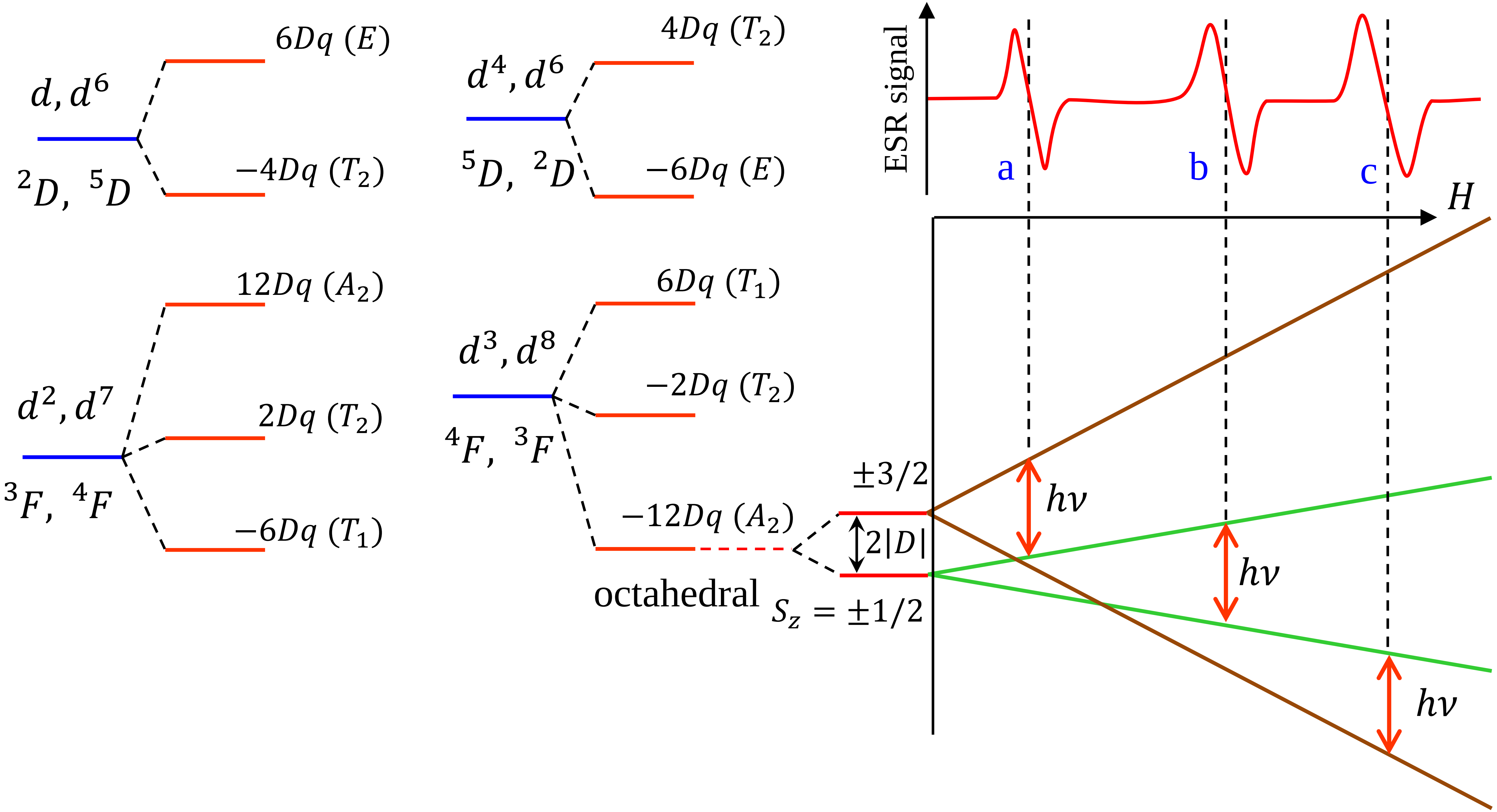
D: axial fine structure parameter

E: rhombic fine structure parameter

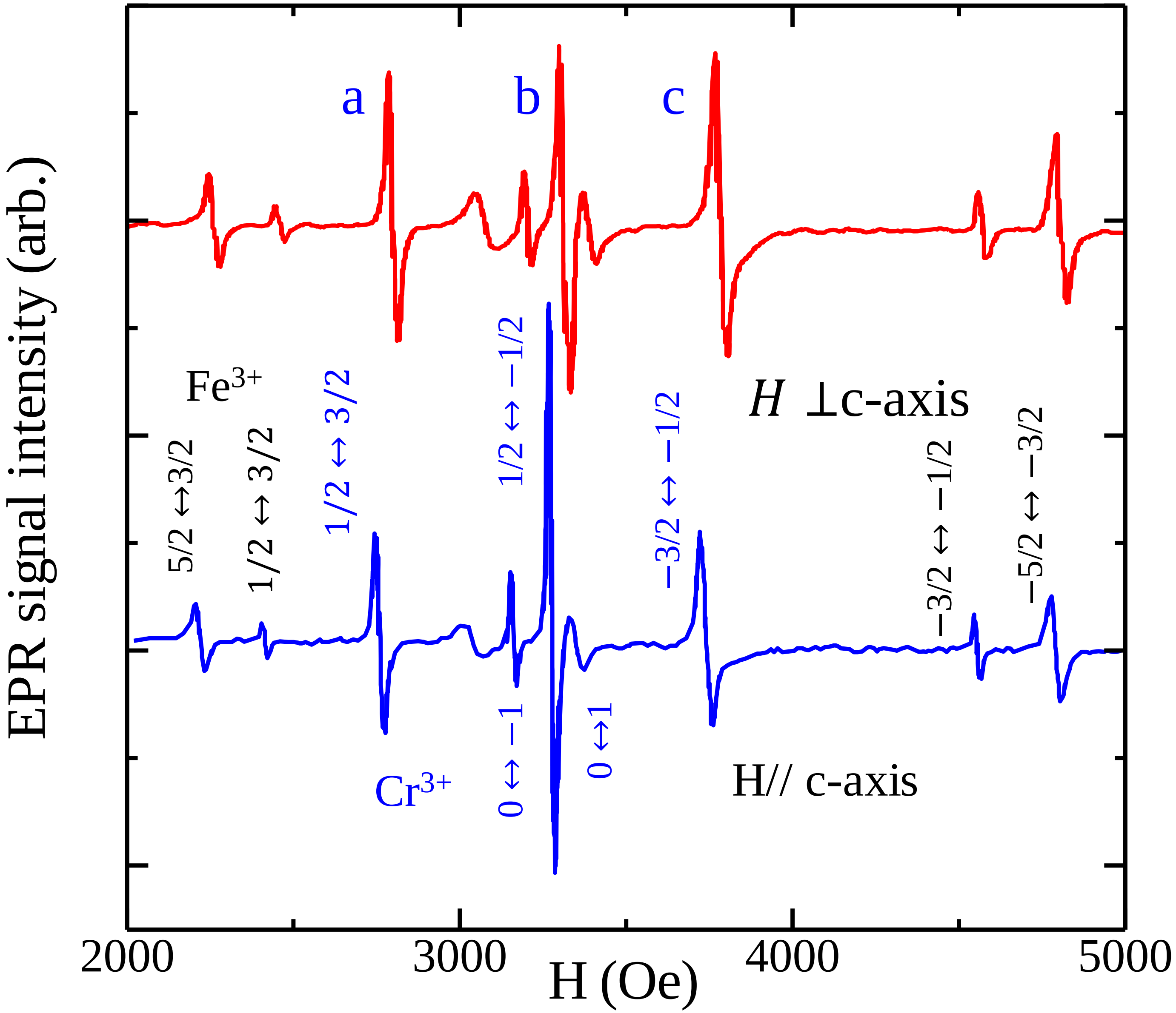
The form frequently used for the analysis of experiments

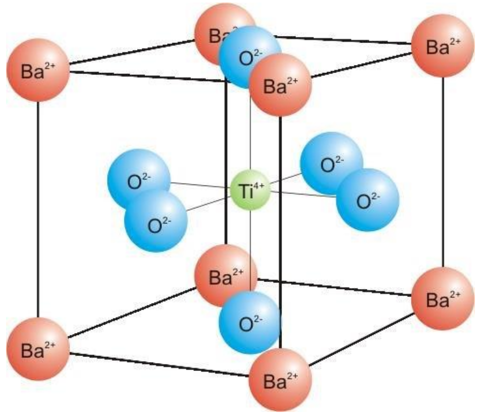
$$\tilde{\mathcal{H}} = \mu_B \boldsymbol{S}\tilde{\boldsymbol{g}}\boldsymbol{H} + D\left[S_z^2 - \frac{S(S+1)}{3}\right] + E(S_x^2 - S_y^2)$$

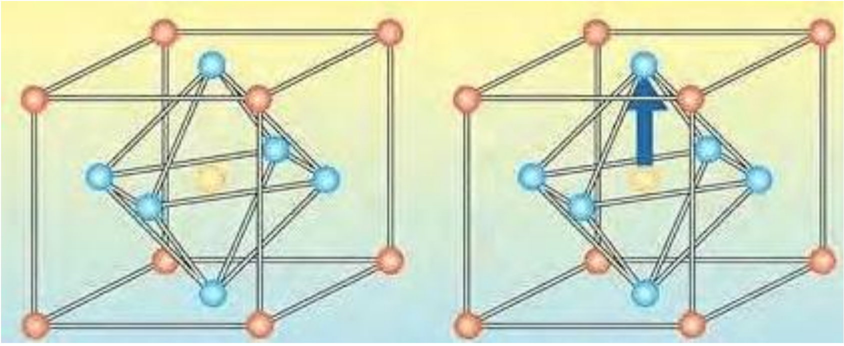
Weak crystal field approximation



EPR signal from Cr^{3+} and Fe^{3+} ions in $BaTiO_3$







hexagonal

$E = 0$

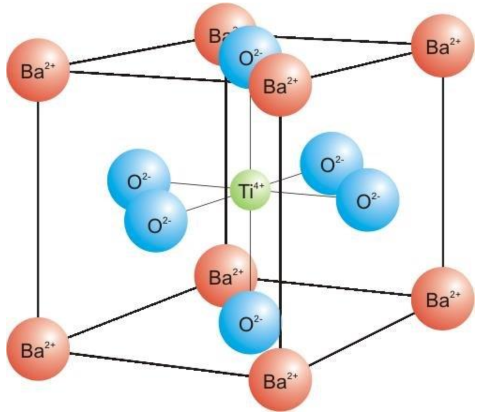
tetragonal

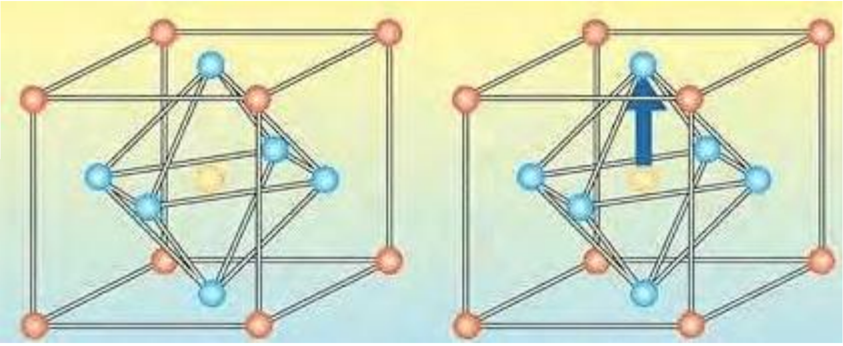
$E \neq 0$

Bairavarasu et al. SPIE Proc. 6698-05

EPR signal from Cr^{3+} and Fe^{3+} ions in $BaTiO_3$

Ion	Crystal	g	$\|D\|$ (cm^{-1})	$\|E\|$ (cm^{-1})
Fe^{3+}	$BaTiO_3$	2.000	0.022	0.0079
	another report	2.003	0.0987	
Cr^{3+}	$BaTiO_3$	1.975	0.046	0.0055
	h-$BaTiO_3$	H1 g_z= 1.9797	0.105	
		H1 $g_{x,y}$= 1.9857		
		H2 g_z= 1.9736	0.3220	
		H2 $g_{x,y}$= 1.9756		





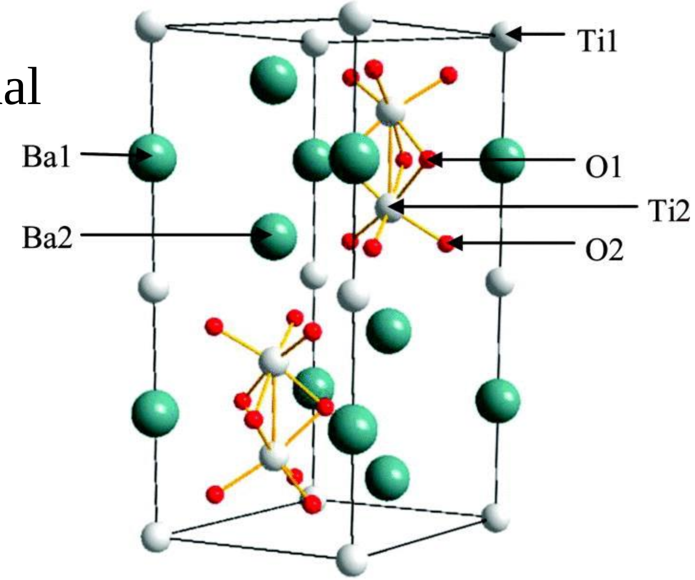
cubic

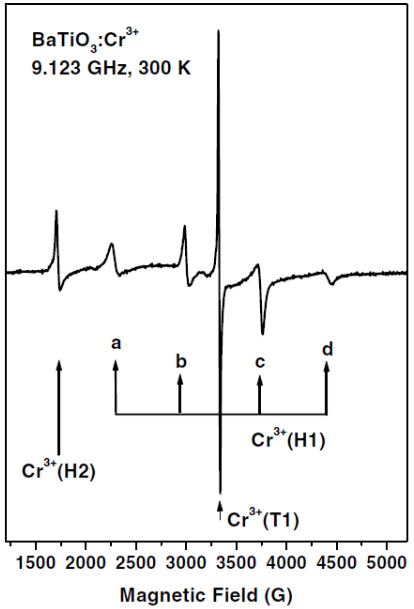
$E = 0$

tetragonal

$E \neq 0$

hexagonal

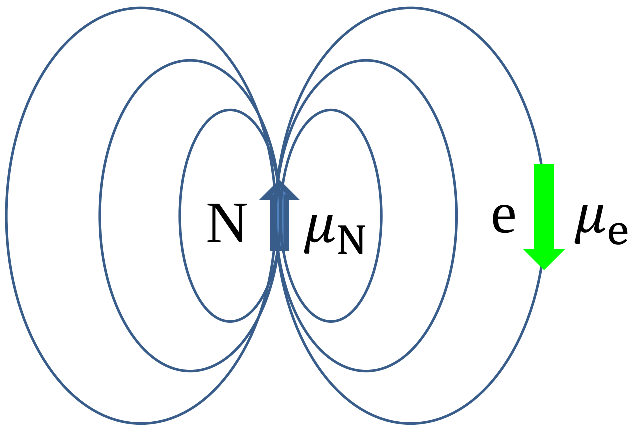




Boettcher et al. JPCM **17**, 2763 (2005)

Bairavarasu et al. SPIE Proc. 6698-05

electron-nuclear spin exchange interaction:



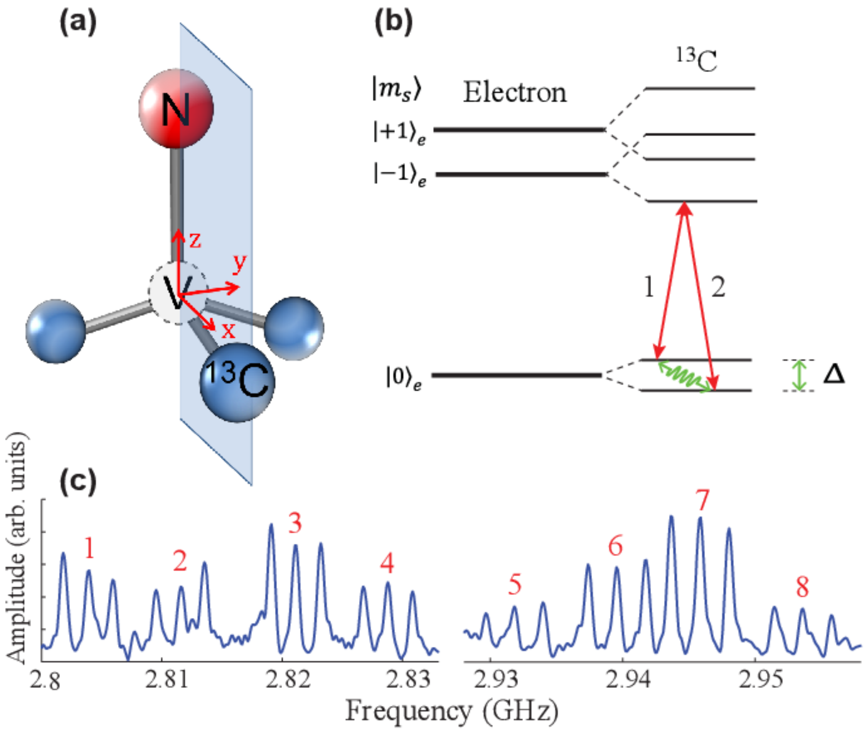
$$\mathcal{H}_{\mathrm{HF}} = A\boldsymbol{I}\cdot\boldsymbol{J}$$

the same form as spin-orbit interaction

$$\boldsymbol{F} = \boldsymbol{I} + \boldsymbol{J}$$

$$\mathcal{H}_{\mathrm{HF}}\left|F, M_F\right\rangle = A\frac{\boldsymbol{F}^2 - \boldsymbol{I}^2 - \boldsymbol{J}^2}{2}\left|F, M_F\right\rangle$$

$$= A\frac{F(F+1) - I(I+1) - J(J+1)}{2}\left|F, M_F\right\rangle$$



NV center

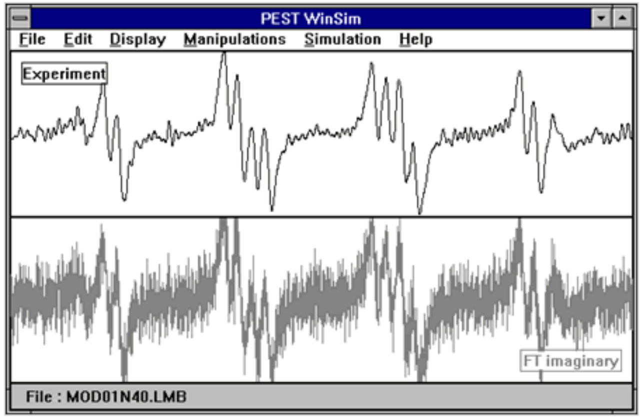
Rama et al., PRB 94, 060101 (`16).

ESR detector/analyzer

EPR public software

EPR-WinSim

https://www.niehs.nih.gov/research/resources/software/tox-pharm/tools/

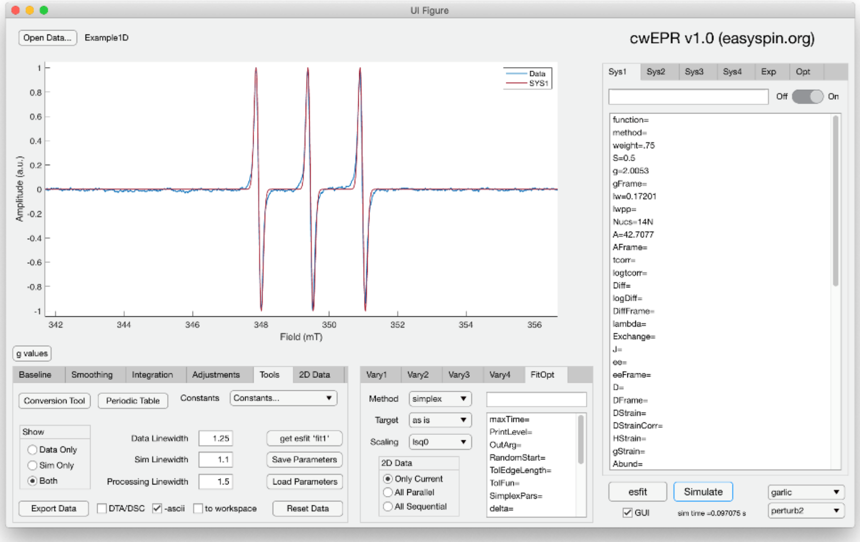


Easyspin

https://www.easyspin.org/

Works on MATLAB

GUI front: cwEPR etc.



Announcement of Easyspin for Octave

https://octave.discourse.group/t/easyspin-for-octave/1177

Commercial machines

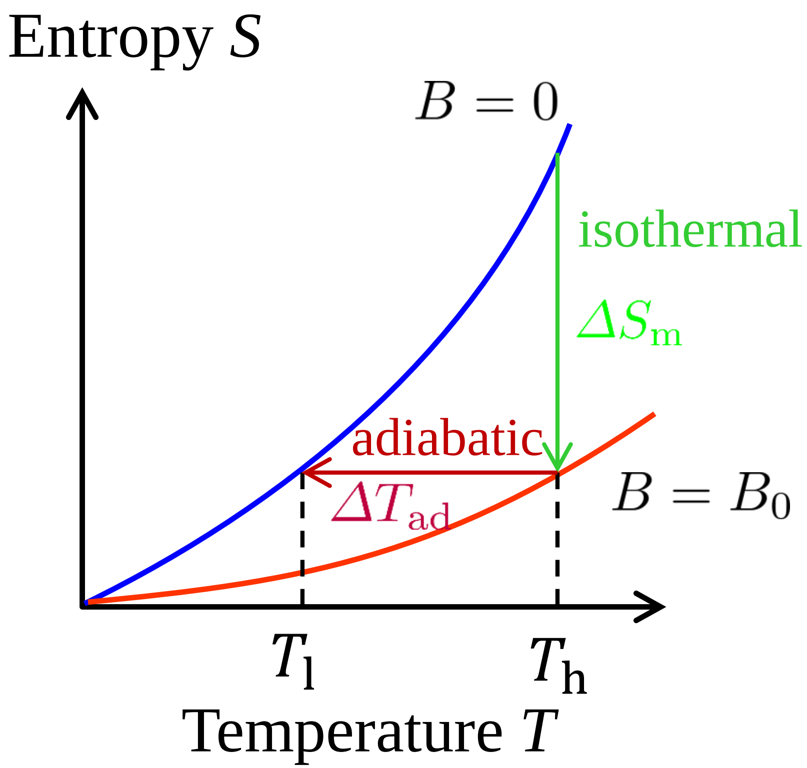
Many attempts for commercial use

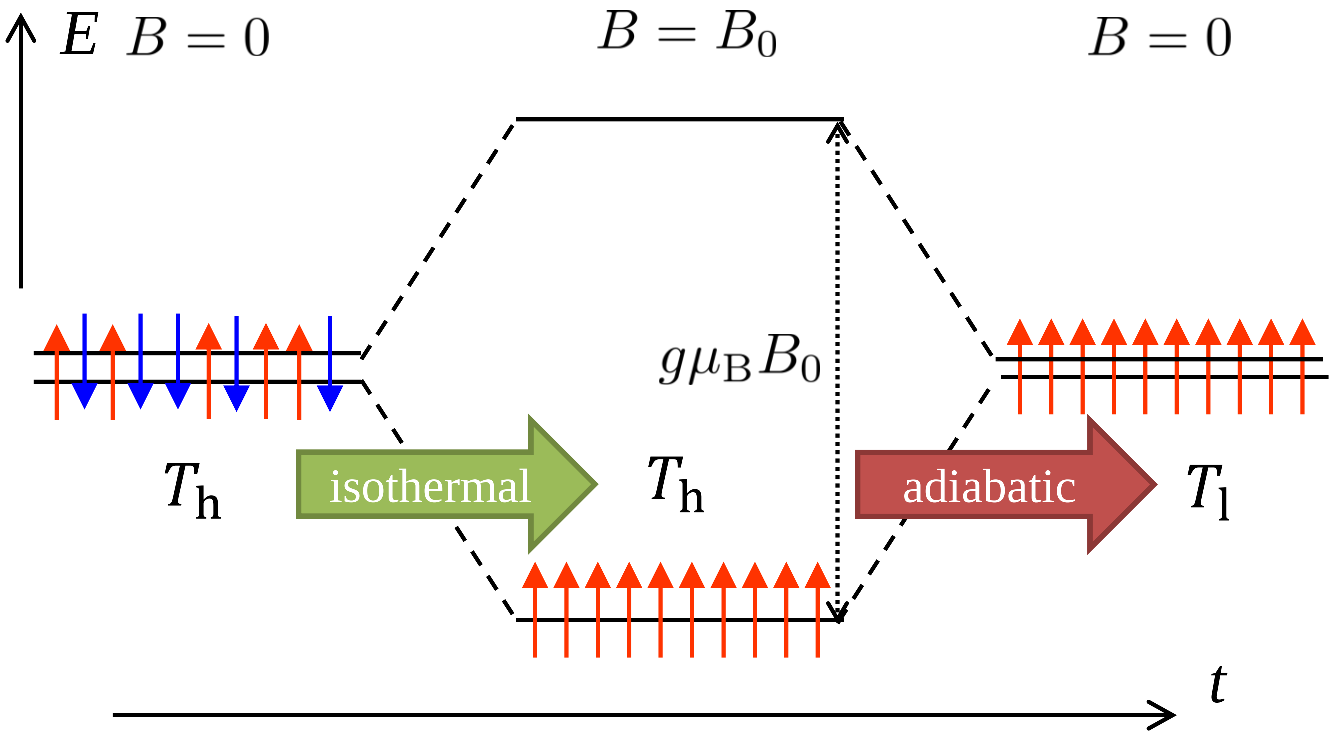
Air Conditioning with Magnetic Refrigeration

- Building Efficiency
- Efficiency

PRINT

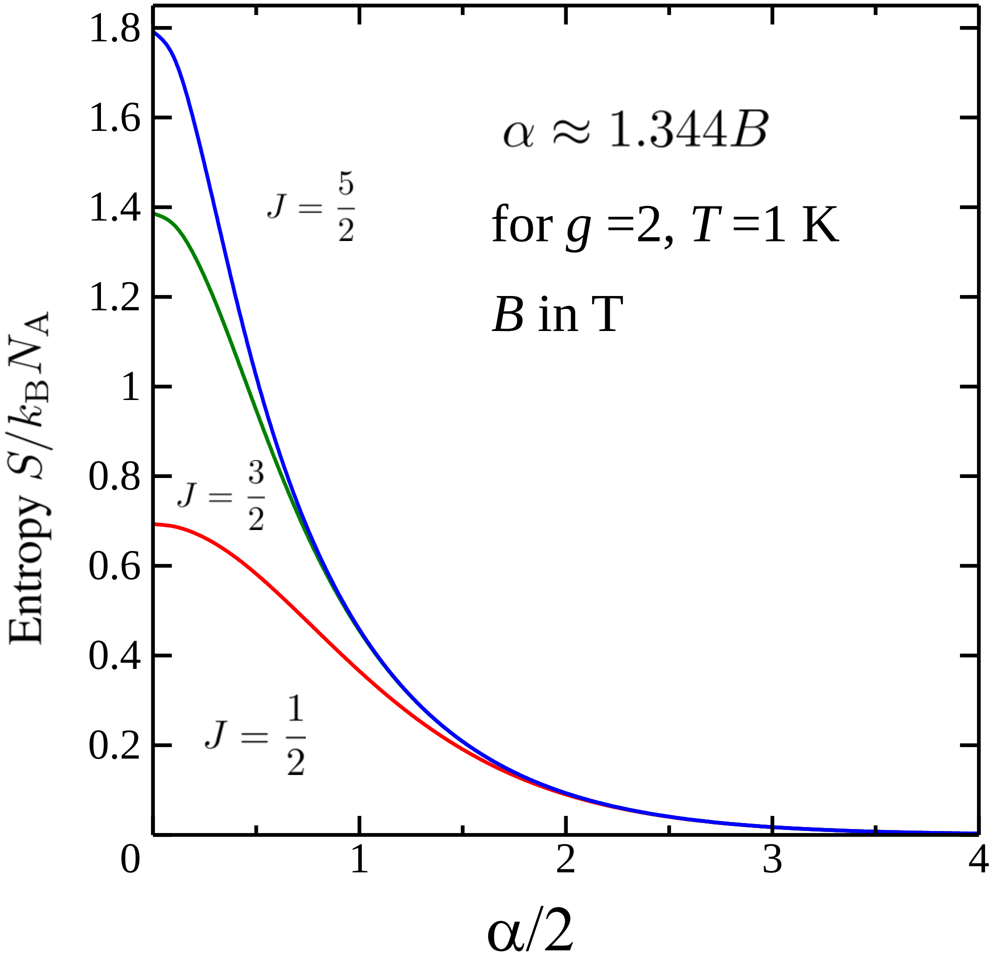
arpa·e PROJECT





Magnetic refrigeration (2)

Entropy of a free spin system



$$\Delta S(B,T_\mathrm{i}) = S(0,T_\mathrm{i}) - S(B,T_\mathrm{i}) = \int_{T_\mathrm{f}}^{T_\mathrm{i}} \frac{C_\mathrm{m}}{T} dT,$$

$$C_\mathrm{m} = T\left(\frac{\partial S}{\partial T}\right)_{B=0}$$

$$M = N_\mathrm{A} g \mu_\mathrm{B} \left[\frac{2J+1}{2}\coth\left(\frac{2J+1}{2}\alpha\right) - \frac{1}{2}\coth\frac{\alpha}{2}\right]$$

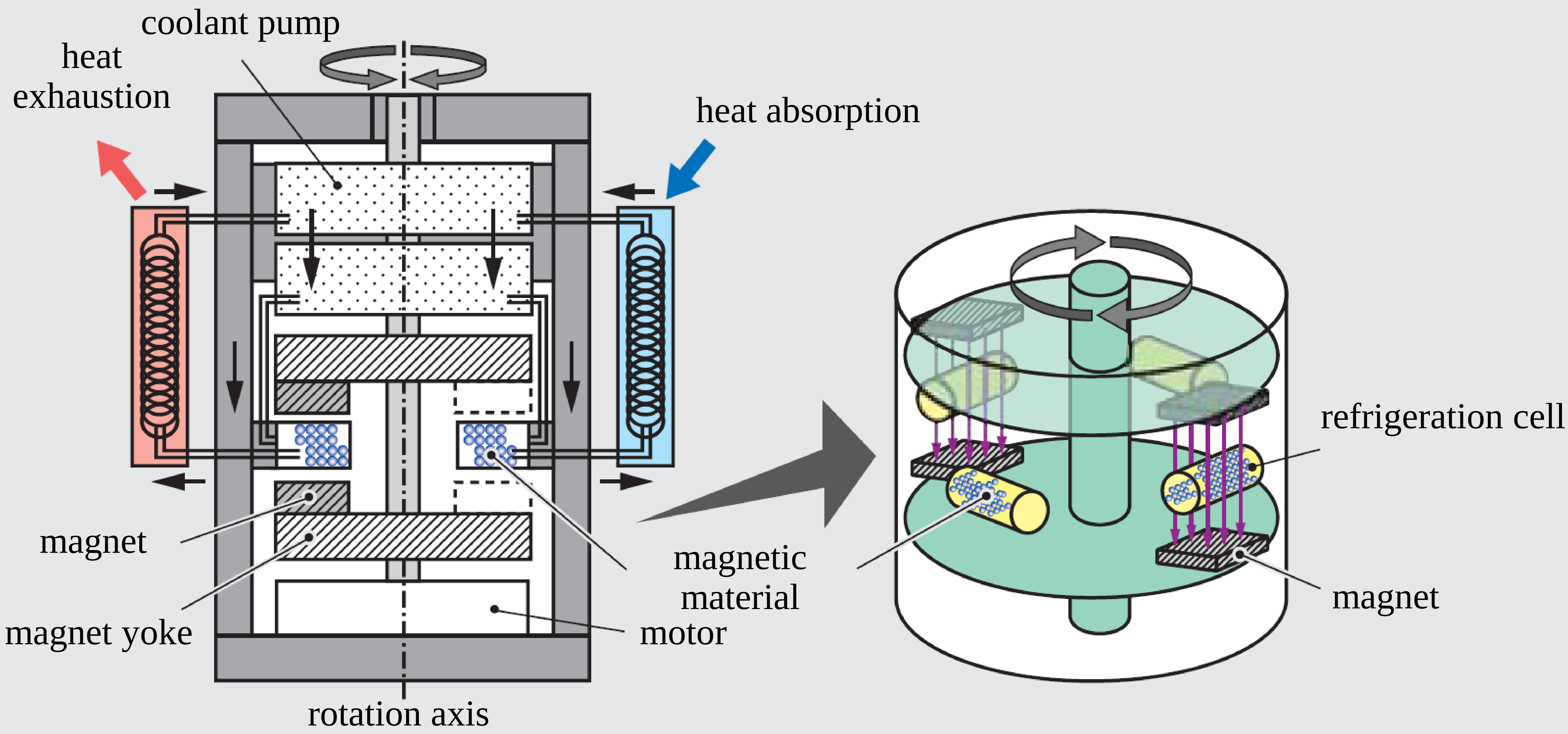
$$\alpha \equiv \frac{g\mu_\mathrm{B}B}{k_\mathrm{B}T}$$

$$\frac{S}{N_\mathrm{A}k_\mathrm{B}} = \frac{\alpha}{2}\coth\frac{\alpha}{2} - \frac{2J+1}{2}\alpha\coth\left[\frac{2J+1}{2}\alpha\right] + \ln\left[\frac{\sinh[(2J+1)\alpha/2]}{\sinh\alpha/2}\right].$$

Cooling material

Maxwell relation: $\left(\frac{\partial S}{\partial B}\right)_T = \left(\frac{\partial M}{\partial T}\right)_B$

Active magnetic refrigeration



Chapter 3 Magnetism of Conduction Electrons

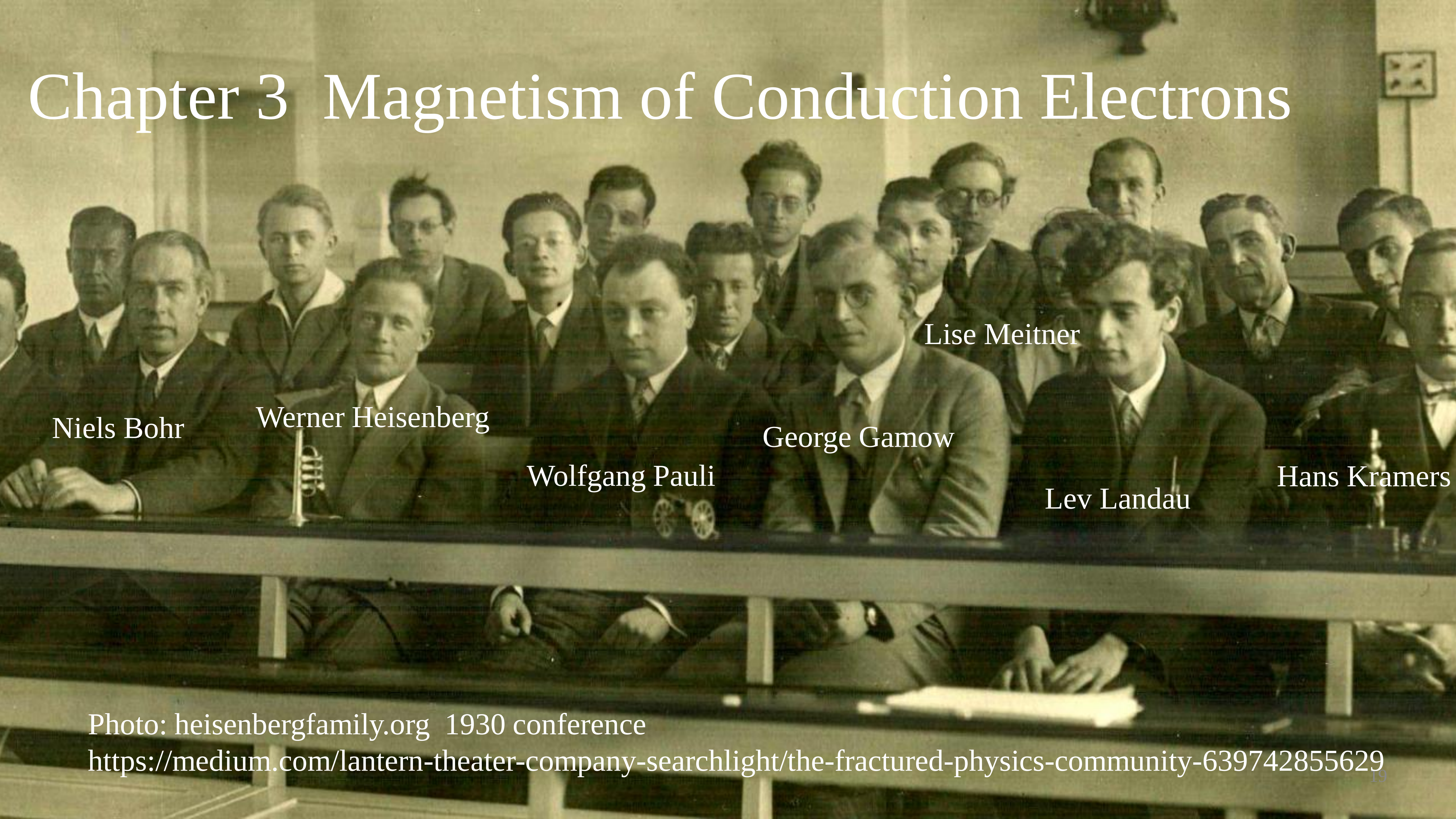


Photo: heisenbergfamily.org 1930 conference
https://medium.com/lantern-theater-company-searchlight/the-fractured-physics-community-639742855629

Chapter 3 Magnetism of Conduction Electrons

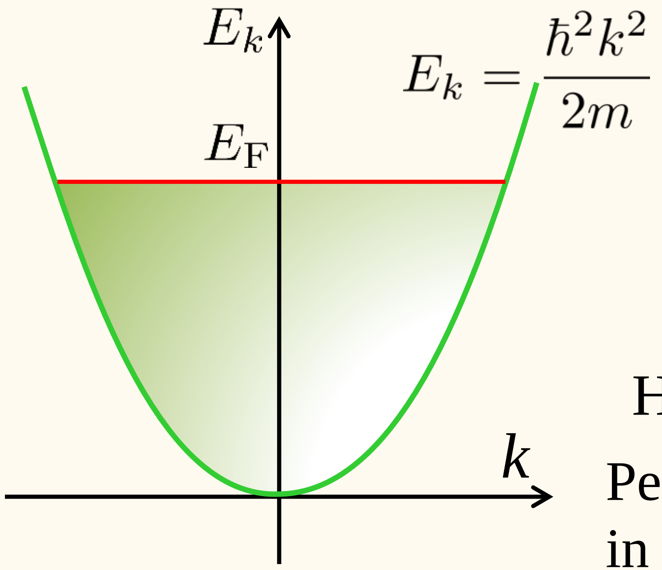
3.1 Pauli paramagnetism

3.2 Landau diamagnetism

3.3 An example of orbital diamagnetism

https://sci-toys.com/scitoys/scitoys/magnets/pyrolytic

Spin paramagnetism in free electrons



Hamiltonian: kinetic energy + spin

$$\mathscr{H} = \sum_{\boldsymbol{k}\sigma} E_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} + \frac{1}{2} g\mu_B B \sum_{\boldsymbol{k}\sigma} \sigma c^\dagger_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma}$$

spin variable: $\sigma \ : \ (\uparrow, \downarrow) \to (1, -1)$

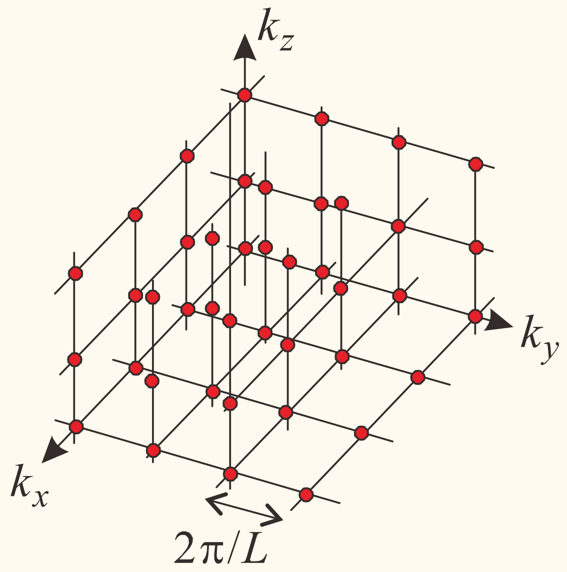
How to count k in metals?

Periodic boundary condition in L-cube

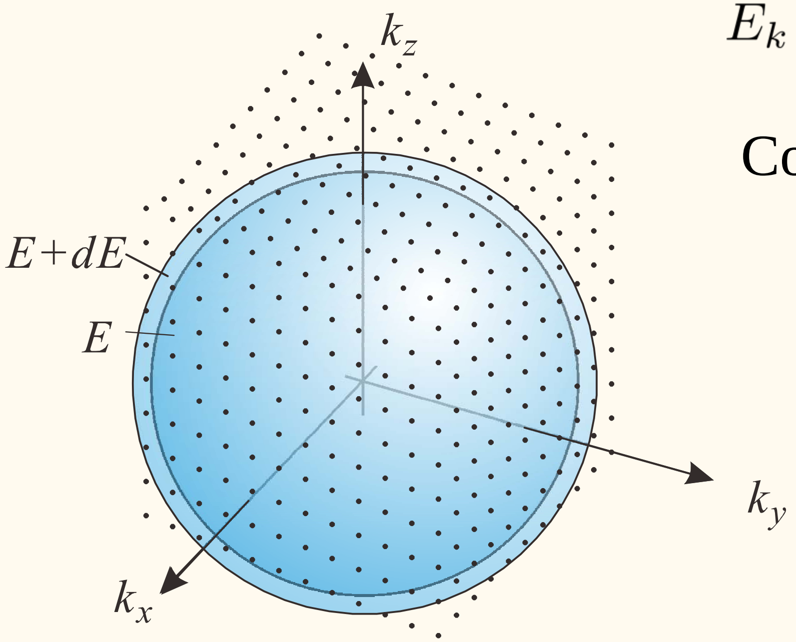
$$\boldsymbol{k} = \frac{2\pi}{L}(n_x, n_y, n_z) \quad (n_x, n_y, n_z : \ \text{integers})$$

$$E_k = \frac{\hbar^2 k^2}{2m}$$

Constant E sphere radius: $k_E = \frac{\sqrt{2mE}}{\hbar}$



(a)



(b)

$$\rho(E) = \frac{1}{L^3}\left(\frac{L}{2\pi}\right)^3 4\pi k_E^2 \frac{dk_E}{dE} = \frac{1}{\pi^2 \hbar^3}\sqrt{\frac{mE}{2}}$$

Pauli paramagnetism

Expectation value of magnetic moment: $$-\frac{g\mu_{\mathrm{B}}}{2}\sum_{\boldsymbol{k}\sigma}\sigma\left\langle c^{\dagger}_{\boldsymbol{k}\sigma}c_{\boldsymbol{k}\sigma}\right\rangle = \frac{g\mu_{\mathrm{B}}}{2}\sum_{\boldsymbol{k}}\left[f\left(E_{\boldsymbol{k}}-\frac{g\mu_{\mathrm{B}}B}{2}\right)-f\left(E_{\boldsymbol{k}}+\frac{g\mu_{\mathrm{B}}B}{2}\right)\right]$$

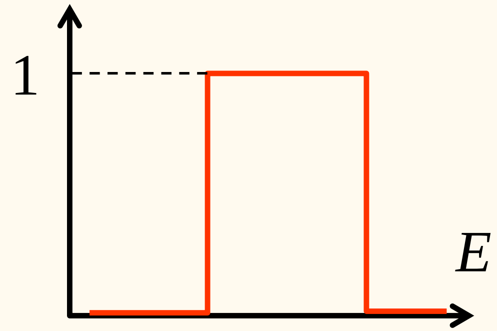
Fermi distribution function: $$f(E)=\frac{1}{\exp[(E-\mu)/k_{\mathrm{B}}T]+1}$$

Chemical potential μ is determined from $$N_{\mathrm{e}}=\int_0^{\infty}dE\rho(E)\left[f\left(E_{\boldsymbol{k}}-\frac{g\mu_{\mathrm{B}}B}{2}\right)+f\left(E_{\boldsymbol{k}}+\frac{g\mu_{\mathrm{B}}B}{2}\right)\right]$$

$$\mu\to E_{\mathrm{F}}$$

Magnetization: $$M=\frac{g\mu_{\mathrm{B}}}{2}\int_0^{\infty}dE\rho(E)\left[f\left(E_{\boldsymbol{k}}-\frac{g\mu_{\mathrm{B}}B}{2}\right)-f\left(E_{\boldsymbol{k}}+\frac{g\mu_{\mathrm{B}}B}{2}\right)\right]$$

$$\frac{\left[f\left(E_{\boldsymbol{k}}-\frac{g\mu_{\mathrm{B}}B}{2}\right)-f\left(E_{\boldsymbol{k}}+\frac{g\mu_{\mathrm{B}}B}{2}\right)\right]}{2\left(\frac{g\mu_{\mathrm{B}}B}{2}\right)}$$



$$E_{\mathrm{F}}-\frac{g\mu_{\mathrm{B}}B}{2} \qquad E_{\mathrm{F}}+\frac{g\mu_{\mathrm{B}}B}{2}$$

$$T\to 0$$

$$\frac{\partial M}{\partial B}\to$$

Pauli paramagnetic susceptibility

$$\chi_{\mathrm{Pauli}}=\left(\frac{g\mu_{\mathrm{B}}}{2}\right)^2[2\rho(E_{\mathrm{F}})]$$

Landau quantization

Hamiltonian free electron + magnetic field

$$\mathscr{H} = \frac{1}{2m}\sum_i (\boldsymbol{p}_i + e\boldsymbol{A})^2$$

Landau gauge: $\boldsymbol{A} = (0, Bx, 0)$

$$\boldsymbol{B} = \mathrm{rot}\,\boldsymbol{A} = (0, 0, B)$$

Schrödinger equation:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \left(\frac{\partial}{\partial y} - i\frac{eB}{\hbar}x\right)^2\right]\psi = E\psi$$

Homogeneous for *y* and *z*

→ Functional form assumption: $\psi = \exp[i(k_y y + k_z z)]u(x)$

Differential equation for *x*

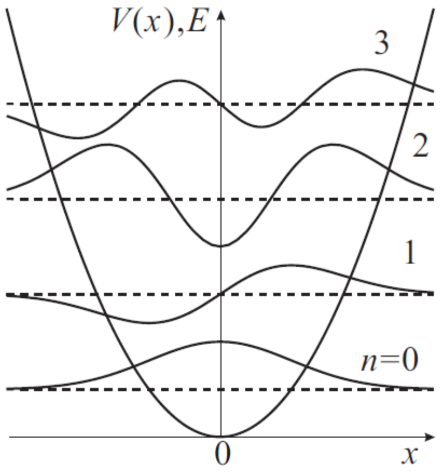
$$-\frac{\hbar^2}{2m}\left[\frac{d^2 u}{dx^2} + \left(k_y - \frac{eB}{\hbar}x\right)^2 u\right] = \left(E - \frac{\hbar^2 k_z^2}{2m}\right)u$$

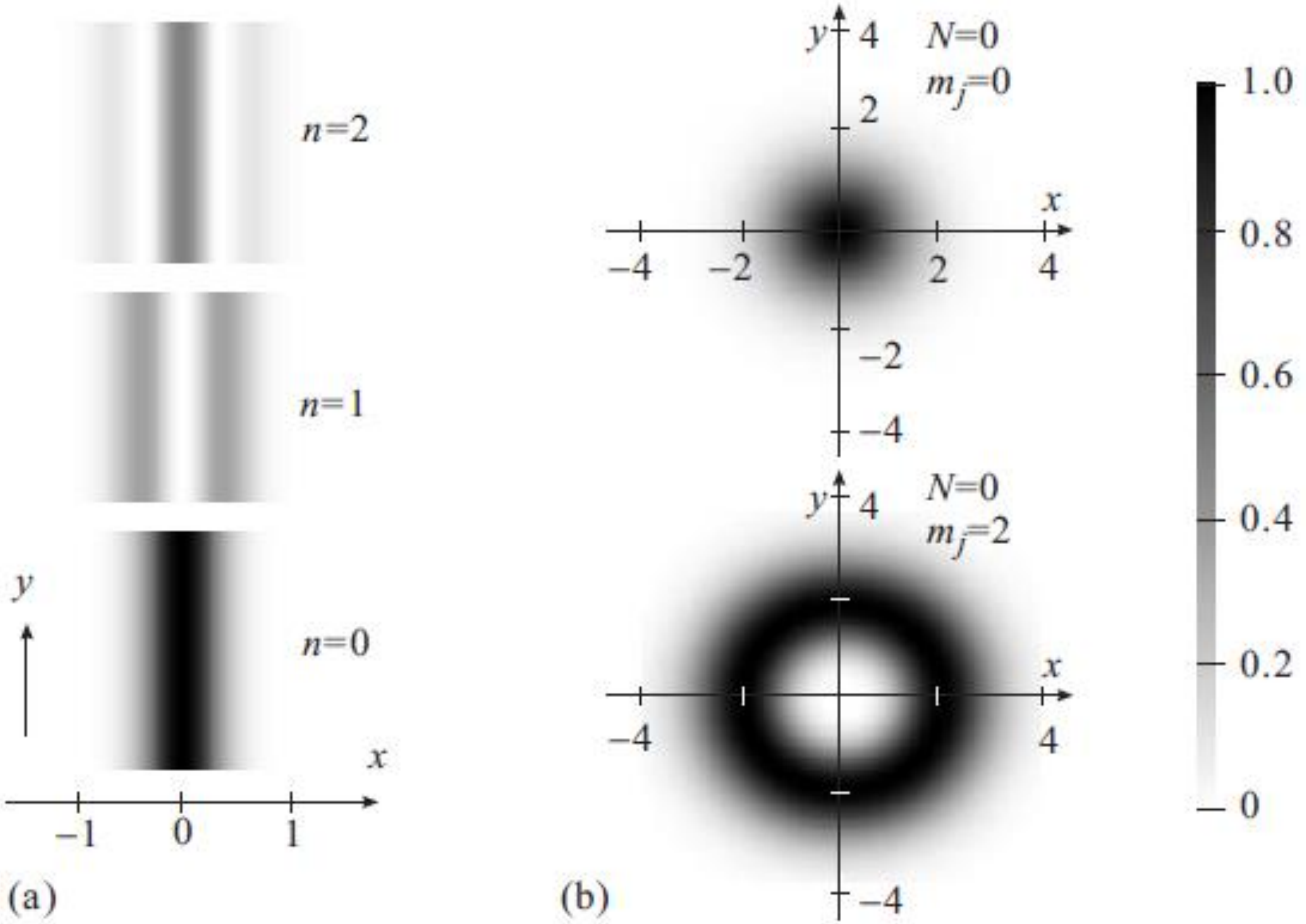
Harmonic oscillator at $x_c = \frac{\hbar k_y}{eB}$ $\quad \frac{m\omega_c^2}{2} = \frac{(eB)^2}{2m}$ $\quad \therefore\ \omega_c = \frac{eB}{m}$: Cyclotron frequency

Landau quantization

$$E(n, k_z) = \frac{\hbar^2 k_z^2}{2m} + \left(n + \frac{1}{2}\right)\hbar\omega_c = \frac{\hbar^2 k_z^2}{2m} + (2n+1)\mu_B B \quad (n = 0, 1, 2, \cdots)$$

Landau quantization: forms of wavefunctions





Diagonalize X

Diagonalize $X^2 + Y^2$ ← Symmetric gauge

$$\boldsymbol{A} = \boldsymbol{B} \times \boldsymbol{r}/2$$

Orbital diamagnetism

How to count density of states?

Periodic boundary condition in a cube with side length L

z-direction $\quad k_z = \frac{2\pi}{L} n_z \; (n_z = 0, \pm 1, \cdots) \quad E_z = \frac{\hbar^2 k_z^2}{2m}$ Number of k_z below E_z $\quad \frac{2L\sqrt{2mE_z}}{h}$

y-direction $\quad k_y = \frac{2\pi}{L} n_y \; (n_y = 0, \pm 1, \cdots)$

x-direction $\quad -\frac{L}{2} \le x_c \le \frac{L}{2} \qquad -\frac{L}{2} \le \frac{\hbar}{eB} k_y = \frac{\hbar}{eB} \frac{2\pi}{L} n_y \le \frac{L}{2} \qquad \therefore \; |n_y| \le \frac{eBL^2}{4\pi\hbar}$

Landau level degeneracy (in xy-plane) is $\frac{eBL^2}{h}$

the number of states below the total energy:

$$\Omega(E) = \frac{L^3}{h^2}\sqrt{8meB}\sum_{n=0}^{n_{\max}}\sqrt{E-(2n+1)\mu_B B} \qquad n_{\max} = \mathrm{int}\left(\frac{E-\mu_B B}{2}\right)$$

Free energy:

$$F = N\mu - 2k_B T\int \frac{d\Omega}{dE}\ln\{1+\exp[-(E-\mu)/k_B T]\}dE$$

Orbital diamagnetism (2)

Partial integration

$$\int \frac{d\Omega}{dE}\ln\{1+\exp[-(E-\mu)/k_{\mathrm{B}}T]\}dE = -\int \Omega(E)\left(-\frac{1}{k_{\mathrm{B}}T}\right)\frac{\exp[-(E-\mu)/k_{\mathrm{B}}T]}{1+\exp[-(E-\mu)/k_{\mathrm{B}}T]}dE$$

$$= \frac{1}{k_{\mathrm{B}}T}\int\left[\int \Omega(E)dE\right]\frac{d}{dE}\frac{1}{1+\exp[(E-\mu)/k_{\mathrm{B}}T]}dE$$

$$= \frac{1}{k_{\mathrm{B}}T}\frac{2\sqrt{8m}}{3}\frac{eBL^3}{h^2}\int\sum_{n=0}^{n_{\max}}[E-(2n+1)\mu_{\mathrm{B}}B]^{3/2}\frac{d}{dE}\frac{1}{1+\exp[(E-\mu)/k_{\mathrm{B}}T]}dE$$

$$F = N_{\mathrm{e}}\mu - A\int\phi(E)\frac{d}{dE}\frac{1}{1+\exp[(E-\mu)/k_{\mathrm{B}}T]}dE$$

$$\begin{cases} A = \dfrac{16L^3}{3\pi^2\hbar^3}m^{3/2}(\mu_{\mathrm{B}}B)^{5/2}, \\ \phi(E) = \displaystyle\sum_{n=0}^{n_{\max}}\left[\frac{E}{2\mu_{\mathrm{B}}B}-\left(n+\frac{1}{2}\right)\right]^{3/2} \end{cases}$$

$$T \to 0 \qquad F = N_{\mathrm{e}}E_{\mathrm{F}} - A\phi(E_{\mathrm{F}})$$

$$\mu_{\mathrm{B}} = \frac{e\hbar}{2m}$$

Orbital diamagnetism (3)

To calculate $\phi(E) = \sum_{n=0}^{n_{\max}} \left[\frac{E}{2\mu_{\mathrm{B}} B} - \left(n + \frac{1}{2} \right) \right]^{3/2}$

We use an asymptotic expansion $x \gg 1$ $\sum_{n=0}^{n_{\max}} \left[x - \left(n + \frac{1}{2} \right) \right]^{3/2} \approx \frac{2}{5} x^{5/2} - \frac{1}{16} x^{1/2} + \cdots$

which can be obtained by applying Euler-Maclaurin formula to $F(y) = (x-y)^{3/2}$

$$\sum_{n=0}^{n_0} F(n+1/2) \approx \int_0^{n_0+1} dy F(y) - \frac{1}{24} [F'(n_0+1) - F'(0)] \approx \frac{2}{5} x^{5/2} - \frac{1}{16} x^{1/2}$$

The free energy: $F = \text{const.} - \frac{L^3}{3} \rho(E_{\mathrm{F}}) (\mu_{\mathrm{B}} B)^2 + \cdots$

Landau orbital diamagnetism: $\chi_{\mathrm{Landau}} = -\frac{2}{3} \rho(E_{\mathrm{F}}) \mu_{\mathrm{B}}^2$

Total susceptibility of free electrons: $\chi = \chi_{\mathrm{Pauli}} + \chi_{\mathrm{Landau}} = \frac{4}{3} \rho(E_{\mathrm{F}}) \mu_{\mathrm{B}}^2$

Summary

- Magnetic resonance (continued)
- Spin Hamiltonian
- Example of analyzing experimental data on electron paramagnetic resonance
- Application of paramagnetism: magnetic refrigeration

Chapter 3 Magnetism of conduction electrons

- Pauli paramagnetism
- Landau diamagnetism