2022.06.15 Lecture 10

Lecture on

10:25 – 11:55

Magnetic Properties of Materials

磁性 (Magnetism)

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Review

- Anti-ferromagnetic Heisenberg model : parallel field susceptibility
- Spin flop and metamagnetic transition

- Ferrimagnetism
- Molecular-field approximation

- Helimagnetism

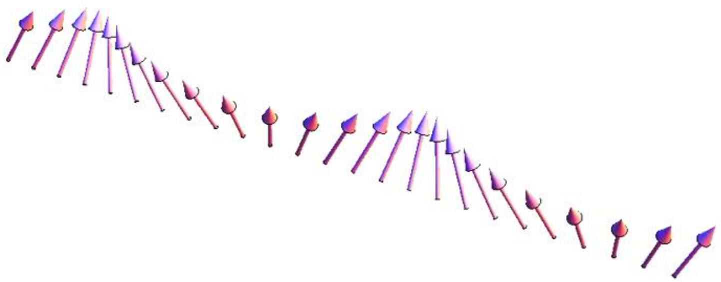
- Spin wave

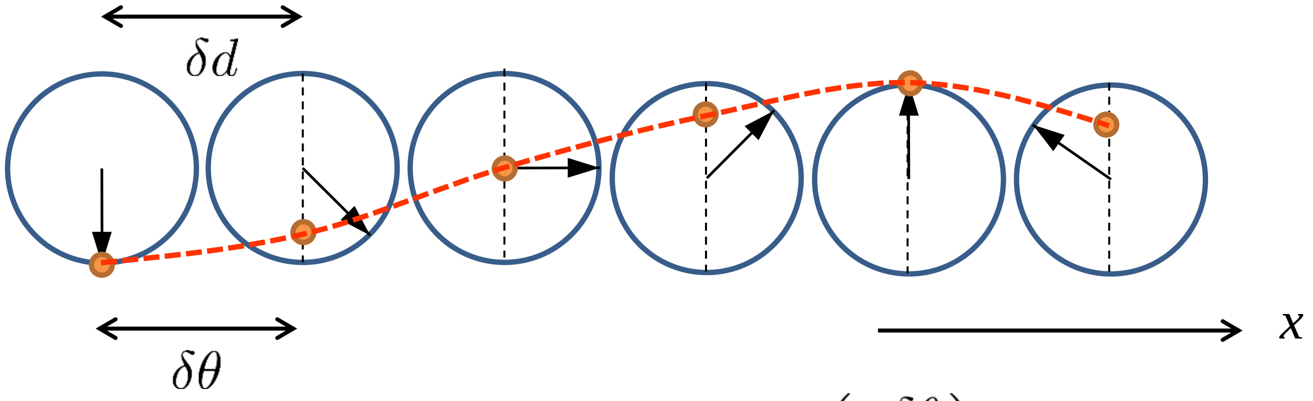
Outline

- Spin wave (classical) in a ferromagnet
- Quantization of spin wave (magnon)
- Magnons in an anti-ferromagnet
- Magnon approximation for weak excitations in ferro- and anti-ferromagnets
- Nambu-Goldstone mode in phase transition
- Experiments on magnons

Spin wave from phase shift of spin precessions

https://www.youtube.com/watch?v=pWQ3r-2Xjeo





snapshot $$S_x = S_0 \cos(n\delta\theta) = S_0 \cos\left(x\frac{\delta\theta}{\delta d}\right) = S_0 \cos kx$$

$$S = S_x + iS_y = S_0 \exp(ikx)$$

cf. Bloch electrons → Magnons can be described in magnetic Brillouin zone.

Total spin $\boldsymbol{S} = \sum_i \boldsymbol{S}_i$ Heisenberg equation: $i\hbar\frac{\partial \boldsymbol{S}}{\partial t} = [\boldsymbol{S}, \mathscr{H}]$

Such a motion of macroscopic magnetic moment can be confirmed by Ferromagnetic resonance (FMR)

Phase shifts of precessions with sites:

$$S_{ix} = A\cos(\omega_0 t + \theta_i), \quad S_{iy} = A\sin(\omega_0 t + \theta_i)$$

Then a snapshot should be expressed in a Fourier form.

However for ω_0 we need to consider the spin-spin interaction.

Equations of motion in the momentum space

Fourier transform, inverse Fourier transform:

$$S_{\boldsymbol{q}x} = \frac{1}{\sqrt{N}} \sum_j S_{jz} \exp(-i\boldsymbol{q}\cdot\boldsymbol{r}_j), \quad S_{jx} = \frac{1}{\sqrt{N}} \sum_{\boldsymbol{q}} S_{\boldsymbol{q}x} \exp(i\boldsymbol{q}\cdot\boldsymbol{r}_j)$$

Heisenberg Hamiltonian, equation of motion:

$$\mathscr{H} = -2J \sum_{\langle i,j\rangle} \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_j \ , \qquad i\hbar \frac{\partial \boldsymbol{S}}{\partial t} = [\boldsymbol{S}, \mathscr{H}]$$

Substituting the above Fourier transforms into equation of motion, we obtain a set of equations of motion in the momentum space as:

$$\begin{cases} i\hbar \dfrac{\partial S_{\boldsymbol{q}x}}{\partial t} = \dfrac{4i}{\sqrt{N}} J \sum_{\langle i,j\rangle} S_{iy} S_{jz} \exp(-i\boldsymbol{q}\cdot\boldsymbol{r}_i)\{1 - \exp[i\boldsymbol{q}\cdot(\boldsymbol{r}_i - \boldsymbol{r}_j)]\} \\ i\hbar \dfrac{\partial S_{\boldsymbol{q}y}}{\partial t} = -\dfrac{4i}{\sqrt{N}} J \sum_{\langle i,j\rangle} S_{ix} S_{jz} \exp(-i\boldsymbol{q}\cdot\boldsymbol{r}_i)\{1 - \exp[i\boldsymbol{q}\cdot(\boldsymbol{r}_i - \boldsymbol{r}_j)]\}. \end{cases}$$

Fourier transform of interaction *J*:

$$J_{\boldsymbol{q}} = \sum_j J \exp[i\boldsymbol{q}\cdot(\boldsymbol{r}_i - \boldsymbol{r}_j)]$$

(*i* can be taken somewhere)

Nearest neighbor approximation:

Small angle approximation, i.e., replace S_{jz} with S:

$$\begin{cases} \hbar \dfrac{\partial S_{\boldsymbol{q}x}}{\partial t} = 2[J_0 - J_{\boldsymbol{q}}] S S_{\boldsymbol{q}y}, \\ \hbar \dfrac{\partial S_{\boldsymbol{q}y}}{\partial t} = -2[J_0 - J_{\boldsymbol{q}}] S S_{\boldsymbol{q}x}. \end{cases}$$

Spin wave (ferromagnetic)

These are the equation we obtained in the last lecture but *B*:

$$\hbar \frac{d\boldsymbol{S}_{\boldsymbol{q}}}{dt} = -[2(J_{\boldsymbol{0}} - J_{\boldsymbol{q}})S + \mu B]\boldsymbol{e}_z \times \boldsymbol{S}_{\boldsymbol{q}}$$

These equation represents precession around z-axis (in Fourier space)

$$\left\{\begin{aligned} \hbar \frac{dS_{\boldsymbol{q}x}}{dt} &= \quad [2(J_{\boldsymbol{0}} - J_{\boldsymbol{q}})S + \mu B]S_{\boldsymbol{q}y}, \\ \hbar \frac{dS_{\boldsymbol{q}y}}{dt} &= -[2(J_{\boldsymbol{0}} - J_{\boldsymbol{q}})S + \mu B]S_{\boldsymbol{q}x}, \\ \hbar \frac{dS_{\boldsymbol{q}z}}{dt} &= 0 \end{aligned}\right.$$

Hence we write $S_{\boldsymbol{q}x} + iS_{\boldsymbol{q}y} \propto \exp[-i\epsilon_{\boldsymbol{q}} t/\hbar]$

to obtain the excitation energy $\epsilon_{\boldsymbol{q}} = 2(J_{\boldsymbol{0}} - J_{\boldsymbol{q}})S + \mu B$

Remember the message:

$$S_{ix} = A\cos(\omega_0 t + \theta_i), \quad S_{iy} = A\sin(\omega_0 t + \theta_i)$$

However for ω_0 we need to consider the spin-spin interaction.

Holstein-Primakoff transformation

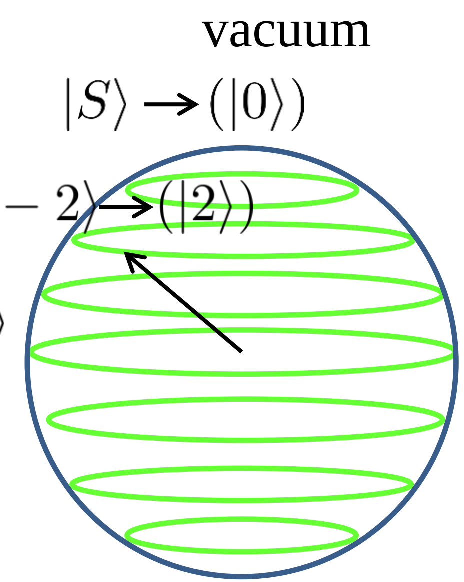
Let us consider the quantization of the spin wave.

Spin operator: $\boldsymbol{S}$ $\quad |m\rangle$: eigenfunction of S_z with eigenvalue of m.

We define up/down operator as: $S_{\pm} = S_x \pm S_y$

Then from the properties of spin operator:

$$S_+ |m\rangle = \sqrt{S(S+1) - m(m+1)}\, |m+1\rangle$$
$$S_- |m\rangle = \sqrt{S(S+1) - m(m-1)}\, |m-1\rangle$$



This is as if we are treating number states $|n\rangle$.

Let us introduce boson creation/annihilation operators: $a^\dagger,\ a$

$$\begin{cases} a_j |n_j\rangle = \sqrt{n_j}\, |n_j - 1\rangle \\ a_j^\dagger |n_j\rangle = \sqrt{n_j + 1}\, |n_j + 1\rangle \end{cases}$$

"Vacuum" of the boson: $|S\rangle \quad (S_z = S)$

n-boson state: $|S - n\rangle$

Then as is for ordinary boson operators, we obtain: $a|S\rangle = 0, \quad |S-n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n |S\rangle$

With number operator $\hat{n} = a^\dagger a$ we can write

$$S_z = S - \hat{n},$$
$$S_+ = \sqrt{2S - \hat{n}}\, a,$$
$$S_- = a^\dagger \sqrt{2S - \hat{n}}$$

Holstein-Primakoff transformation

Non-interacting spin wave approximation

In Holstein-Primakoff transformation we have nonlinear terms. → Interaction between the bosons.

Expand the square roots in Holstein-Primakoff transformation

$$\left.\begin{aligned} \hat{S}_{j+} &= \sqrt{2S}\left(1 - \frac{a_j^\dagger a_j}{4S} + \cdots\right) a_j, \\ \hat{S}_{j-} &= \sqrt{2S} a_j^\dagger \left(1 - \frac{a_j^\dagger a_j}{4S} + \cdots\right) \end{aligned}\right\}$$

Then the Hamiltonian is expanded as

$$\begin{aligned} \mathscr{H} &= -2\sum_{\langle i,j\rangle} J_{ij}\hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_j = -2\sum_{\langle i,j\rangle} J_{ij}\{\hat{S}_{iz}\hat{S}_{jz} + (\hat{S}_{i+}\hat{S}_{j-} + \hat{S}_{i-}\hat{S}_{j+})/2\} \\ &= -2\sum_{\langle i,j\rangle} J_{ij}\left[S^2 - S(\hat{n}_i + \hat{n}_j) + S(a_i^\dagger a_j + a_j^\dagger a_i) + \hat{n}_i\hat{n}_j - \frac{1}{4}a_i^\dagger a_j^\dagger a_j a_j - \frac{1}{4}a_j^\dagger a_j^\dagger a_j a_i + \cdots\right]. \end{aligned}$$

Take up to quadratic terms

$$\mathscr{H} = -2\sum_{\langle i,j\rangle} J_{ij}[S^2 - S(\hat{n}_i + \hat{n}_j) + S(a_i^\dagger a_j + a_j^\dagger a_i)]$$

Ferromagnetic spin wave: ferromagnetic magnon

Fourier transform of creation/annihilation operators:

$$a_{\boldsymbol{q}} = \frac{1}{\sqrt{N}} \sum_j a_j \exp(i\boldsymbol{q} \cdot \boldsymbol{r}),$$

$$a_{\boldsymbol{q}}^\dagger = \frac{1}{\sqrt{N}} \sum_j a_j \exp(-i\boldsymbol{q} \cdot \boldsymbol{r})$$

Substitute these to the approximated Hamiltonian →

Magnon Hamiltonian

$$\mathscr{H} = -2\sum_{\langle i,j \rangle} J_{ij} S^2 + 2\sum_{\boldsymbol{q}} [J_{\boldsymbol{0}} - J_{\boldsymbol{q}}] S a_{\boldsymbol{q}}^\dagger a_{\boldsymbol{q}}$$

$$= E_0 + \sum_{\boldsymbol{q}} \hbar\omega_{\boldsymbol{q}} a_{\boldsymbol{q}}^\dagger a_{\boldsymbol{q}}$$

Total magnetization: $M = \mu \left\langle \sum_i S_{iz} \right\rangle = \mu S N - \mu \sum_i \langle a_i^\dagger a_i \rangle = \mu S N - \mu \sum_{\boldsymbol{q}} n(\epsilon_{\boldsymbol{q}})$

with Bose distribution function: $n(\epsilon) = \left(\exp \frac{\epsilon}{k_{\mathrm{B}} T} - 1 \right)^{-1}$

Magnon dispersion

$$\hbar\epsilon_{\boldsymbol{q}} = 2S(J_{\boldsymbol{0}} - J_{\boldsymbol{q}}) = 2SJ\{2 - [\exp(iqa) + \exp(-iqa)]\} \simeq 2SJ \left[2 - 2\left(1 - \frac{(qa)^2}{2}\right) \right] = 2SJ(qa)^2$$

Then we obtain $M = \mu N \left[S - \zeta\left(\frac{3}{2}\right) \left(\frac{k_{\mathrm{B}} T}{8\pi J S} \right)^{3/2} \right]$ $\quad \zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^2}$ $\quad \zeta\left(\frac{3}{2}\right) \approx 2.612$

Why we have introduced the concept: Magnon?

Low temperature Magnetic moment:

$$M = \mu N \left[S - \zeta\left(\frac{3}{2}\right)\left(\frac{k_B T}{8\pi J S}\right)^{3/2} \right] \qquad \zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^2} \qquad \zeta\left(\frac{3}{2}\right) \approx 2.612$$

The internal energy is:

$$U = E_0 + \sum_{\boldsymbol{q}} n(\epsilon_{\boldsymbol{q}}) = E_0 + 12\pi J S N \zeta\left(\frac{5}{2}\right)\left(\frac{k_B T}{8\pi J S}\right)^{5/2} .$$

Then low temperature specific heat is obtained by

$$C = \frac{\partial U}{\partial T} = \frac{15}{4} N k_B \zeta\left(\frac{5}{2}\right)\left(\frac{k_B T}{8\pi J S}\right)^{3/2}$$

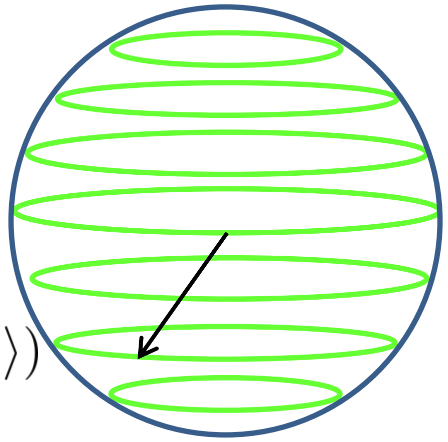
As above, by considering magnons we can calculate low energy excitations and obtain important quantities.

Magnons: Low temperature model of ferro (anti-ferro) magnets.

Spin wave modeling of anti-ferromagnets

Anti-ferromagnet → Decompose into A, B sublattices

A sublattice: we can consider magnon model



$|-S+2\rangle \rightarrow (|2\rangle)$

$|-S\rangle \rightarrow (|0\rangle)$

vacuum

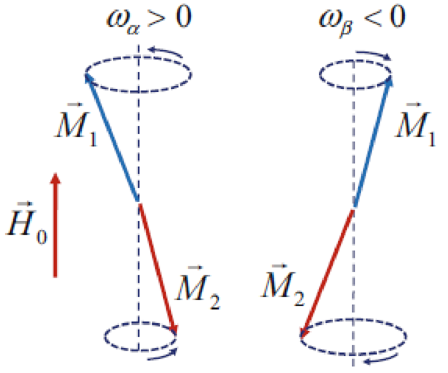
B sublattice: Magnetization is reversed

Take the vacuum as $|0\rangle_{\mathrm{B}} = |-S\rangle$

Boson creation/annihilation operators $b_j^\dagger, \quad b_j$

Then the Holstein-Primakoff transform is

$$\left.\begin{aligned} S_{jz} &= -S + b_j^\dagger b_j, \\ S_{j+} &= b_j^\dagger \sqrt{2S - b_j^\dagger b_j}, \\ S_{j-} &= \sqrt{2S - b_j^\dagger b_j}\, b_j \end{aligned}\right\}$$



Quadratic Hamiltonian : $\mathscr{H} = -\alpha_z |J| N S^2 + 2|J|S \sum_{\langle i,j \rangle} (a_i^\dagger a_i + b_j^\dagger b_j + a_i b_j + a_i^\dagger b_j^\dagger) \qquad i \in \mathrm{A}, \quad j \in \mathrm{B}$

Spin wave modeling of anti-ferromagnets (2)

Fourier transformation of creation/annihilation operators

$$\left.\begin{aligned} a_i &= \sqrt{\frac{2}{N}}\sum_{\boldsymbol{q}} a_{\boldsymbol{q}} \exp(-i\boldsymbol{q}\cdot\boldsymbol{r}_i), \\ b_j &= \sqrt{\frac{2}{N}}\sum_{\boldsymbol{q}} b_{\boldsymbol{q}} \exp(-i\boldsymbol{q}\cdot\boldsymbol{r}_j) \end{aligned}\right\}$$

Momentum representation of the Hamiltonian

$$\mathscr{H} = -\alpha_z|J|NS^2 + 2\alpha_z|J|S\sum_{\boldsymbol{q}}[a_{\boldsymbol{q}}^\dagger a_{\boldsymbol{q}} + b_{\boldsymbol{q}}^\dagger b_{\boldsymbol{q}} + \gamma(\boldsymbol{q})(a_{\boldsymbol{q}}^\dagger b_{\boldsymbol{q}}^\dagger + a_{\boldsymbol{q}} b_{\boldsymbol{q}})]$$

with nearest neighbor summation

$$\gamma(\boldsymbol{q}) = \alpha_z^{-1}\sum_{\boldsymbol{\rho}} \exp(-i\boldsymbol{q}\cdot\boldsymbol{\rho})$$

But the above Hamiltonian is still not diagonalized. (Néel ordered state is not true ground state.)

Bogoluibov transformation

$(a_{\boldsymbol{q}}, b_{\boldsymbol{q}}) \to (\alpha_{\boldsymbol{q}}, \beta_{\boldsymbol{q}})$

$$\left.\begin{aligned} a_{\boldsymbol{q}} &= \cosh\theta_{\boldsymbol{q}}\alpha_{\boldsymbol{q}} - \sinh\theta_{\boldsymbol{q}}\beta_{\boldsymbol{q}}^\dagger, \\ b_{\boldsymbol{q}} &= \cosh\theta_{\boldsymbol{q}}\beta_{\boldsymbol{q}} - \sinh\theta_{\boldsymbol{q}}\alpha_{\boldsymbol{q}}^\dagger. \end{aligned}\right\}$$

Bosonic commutation relations

$$[\alpha_{\boldsymbol{q}}, \alpha_{\boldsymbol{q}}^\dagger] = 1, \quad [\beta_{\boldsymbol{q}}, \beta_{\boldsymbol{q}}^\dagger] = 1, \quad [\alpha_{\boldsymbol{q}}, \beta_{\boldsymbol{q}}] = [\alpha_{\boldsymbol{q}}^\dagger, \beta_{\boldsymbol{q}}^\dagger] = 0$$

Spin wave modeling of anti-ferromagnets (3)

$$\mathscr{H} = -\alpha_z|J|NS^2 + 2\alpha_z|J|S\sum_{\boldsymbol{q}}[(\cosh 2\theta_{\boldsymbol{q}} - \gamma(\boldsymbol{q})\sinh\theta_{\boldsymbol{q}})(\alpha^\dagger_{\boldsymbol{q}}\alpha_{\boldsymbol{q}} + \beta^\dagger_{\boldsymbol{q}}\beta_{\boldsymbol{q}} + 1) - 1 \\ - (\sinh 2\theta_{\boldsymbol{q}} - \gamma(\boldsymbol{q})\cosh 2\theta_{\boldsymbol{q}})(\alpha_{\boldsymbol{q}}\beta_{\boldsymbol{q}} + \alpha^\dagger_{\boldsymbol{q}}\beta^\dagger_{\boldsymbol{q}})]$$

Condition for diagonalization: $\sinh 2\theta_{\boldsymbol{q}}/\cosh 2\theta_{\boldsymbol{q}} = \tanh 2\theta_{\boldsymbol{q}} = \gamma(\boldsymbol{q})$

Diagonalized Hamiltonian: $\mathscr{H} = -\alpha_z|J|NS^2 + 2\alpha_z|J|S\sum_{\boldsymbol{q}}[(\sqrt{1-\gamma(\boldsymbol{q})^2} - 1) + \sqrt{1-\gamma(\boldsymbol{q})^2}(\alpha^\dagger_{\boldsymbol{q}}\alpha_{\boldsymbol{q}} + \beta^\dagger_{\boldsymbol{q}}\beta_{\boldsymbol{q}})]$

Ground state energy: $-\alpha_z|J|NS^2 + 2\alpha_z|J|S\sum_{\boldsymbol{q}}[(\sqrt{1-\gamma(\boldsymbol{q})^2} - 1)$

Néel ordered state energy

Energy lowering due to hybridization of $S, S-1, \cdots, -S$ states

⇩

$\langle S_{jz}\rangle$ is shorter than S

How? $\langle S_{jz}\rangle = S - \frac{2}{N}\sum_{\boldsymbol{q}}\sinh\theta_{\boldsymbol{q}} = S - \frac{1}{N}\sum_{\boldsymbol{q}}\left(\frac{1}{\sqrt{1-\gamma(\boldsymbol{q})^2}} - 1\right)$

Spin wave modeling of anti-ferromagnets (4)

$$\langle S_{jz}\rangle = S - \Delta \qquad E_0 = N|J|\alpha_z S(S+\epsilon)$$

Lattice	Square	Simple Cubic	Body Centered Cubic
Δ	0.917	0.078	0.0593
ϵ	$0.158{+}0.0062S^{-1}$	$0.097{+}0.0024S^{-1}$	$0.073{+}0.0013S^{-1}$

Magnon dispersion $\epsilon_{\boldsymbol{q}} = 2\alpha_z|J|S\sqrt{1-\gamma(\boldsymbol{q})^2}$ Simple cubic case $\gamma(\boldsymbol{q}) = \cos\frac{q_x}{2}\cos\frac{q_y}{2}\cos\frac{q_z}{2}$

Asymptotic form $q \to 0$ $\epsilon_{\boldsymbol{q}} = 2\sqrt{2\alpha_z}|J|Saq$

Internal energy $U = E_0 + \frac{\pi^2}{15}N\left(\frac{k_B T}{2\sqrt{2\alpha_z}|J|S}\right)^3 k_B T$

Results

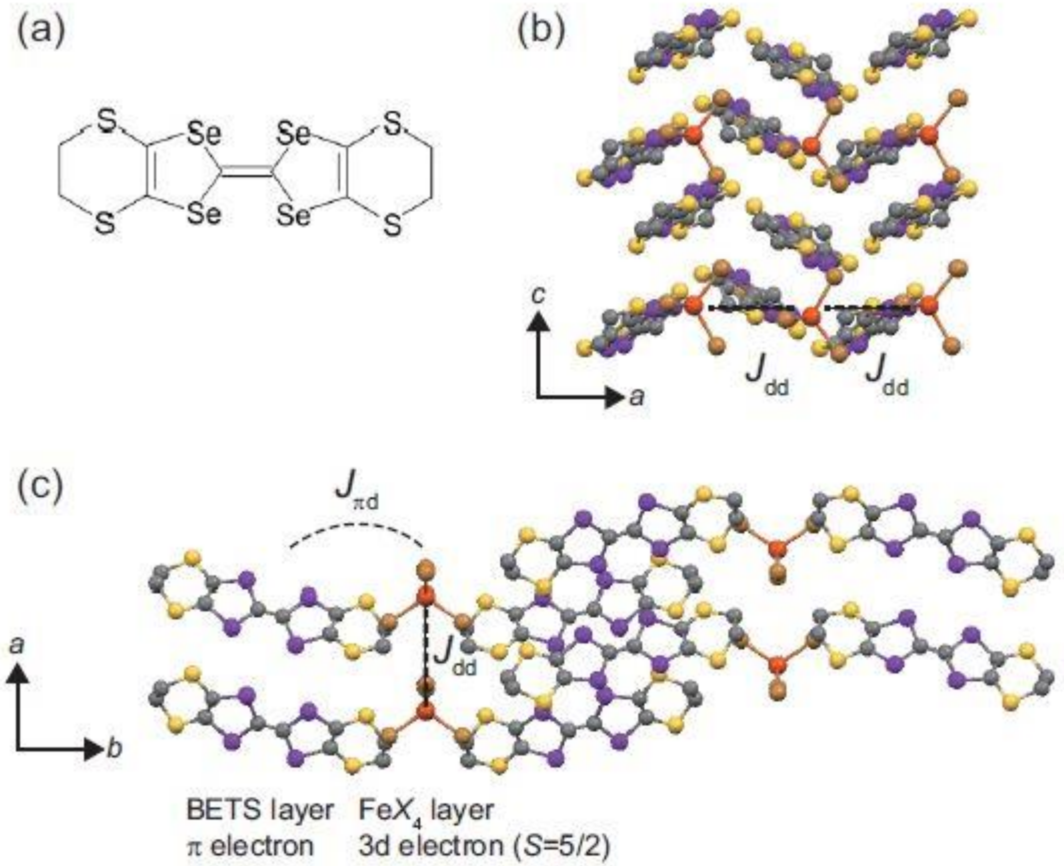
Lattice	1D Chain	2D Square Lattice	3D Simple Cubic
$-\frac{E_0}{\alpha_z|J|NS^2}$	$1{+}0.363S^{-1}$	$1{+}0.158S^{-1}$	$1{+}0.097S^{-1}$
$\frac{C}{Nk_B}$	$\frac{2\pi}{3}\left(\frac{k_B T}{2\alpha_z|J|S}\right)$	$\frac{14.42}{\pi}\left(\frac{k_B T}{2\alpha_z|J|S}\right)^2$	$4\sqrt{3}\frac{\pi^2}{5}\left(\frac{k_B T}{2\alpha_z|J|S}\right)^3$
ΔS	Diverge	0.197	0.078

Specific heat of an organic anti-ferromagnet

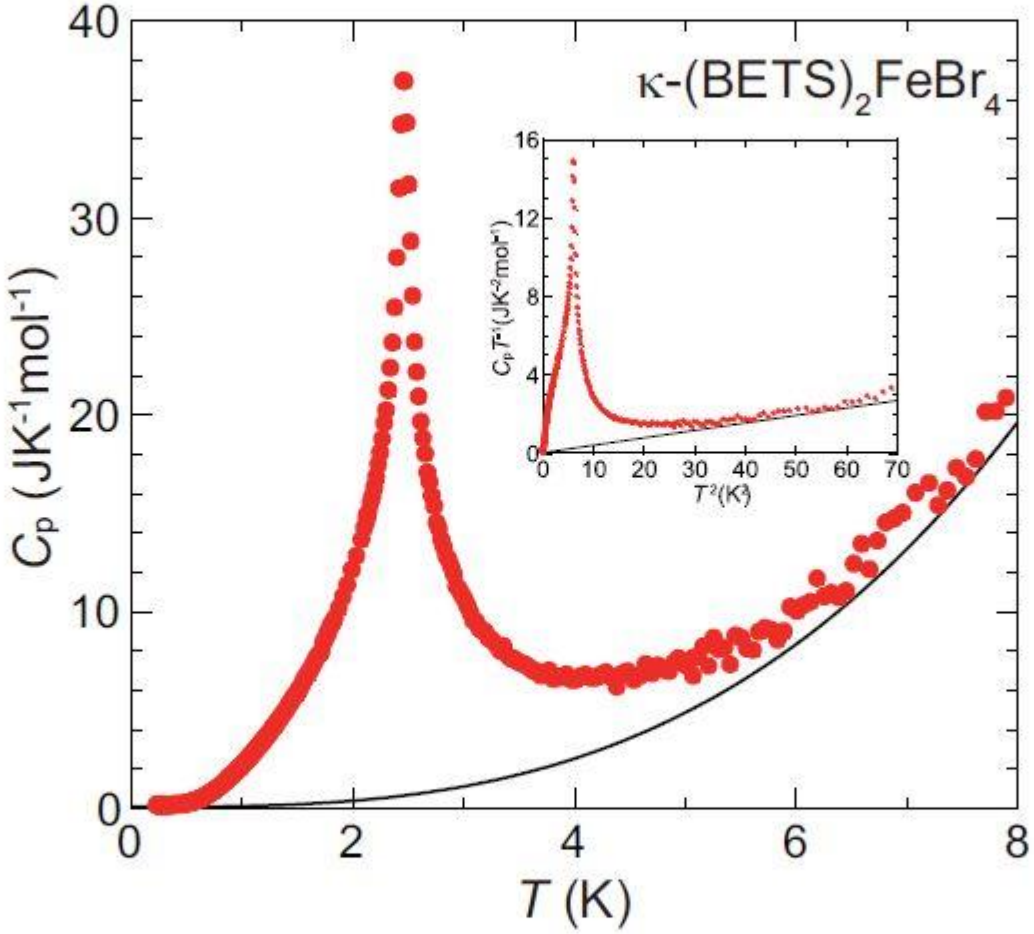
κ-$(\mathrm{BETS})_2\mathrm{FeBr}_4$

Anti-ferromagnetic T_N : 2.5 K

Superconducting T_c : 1.1 K

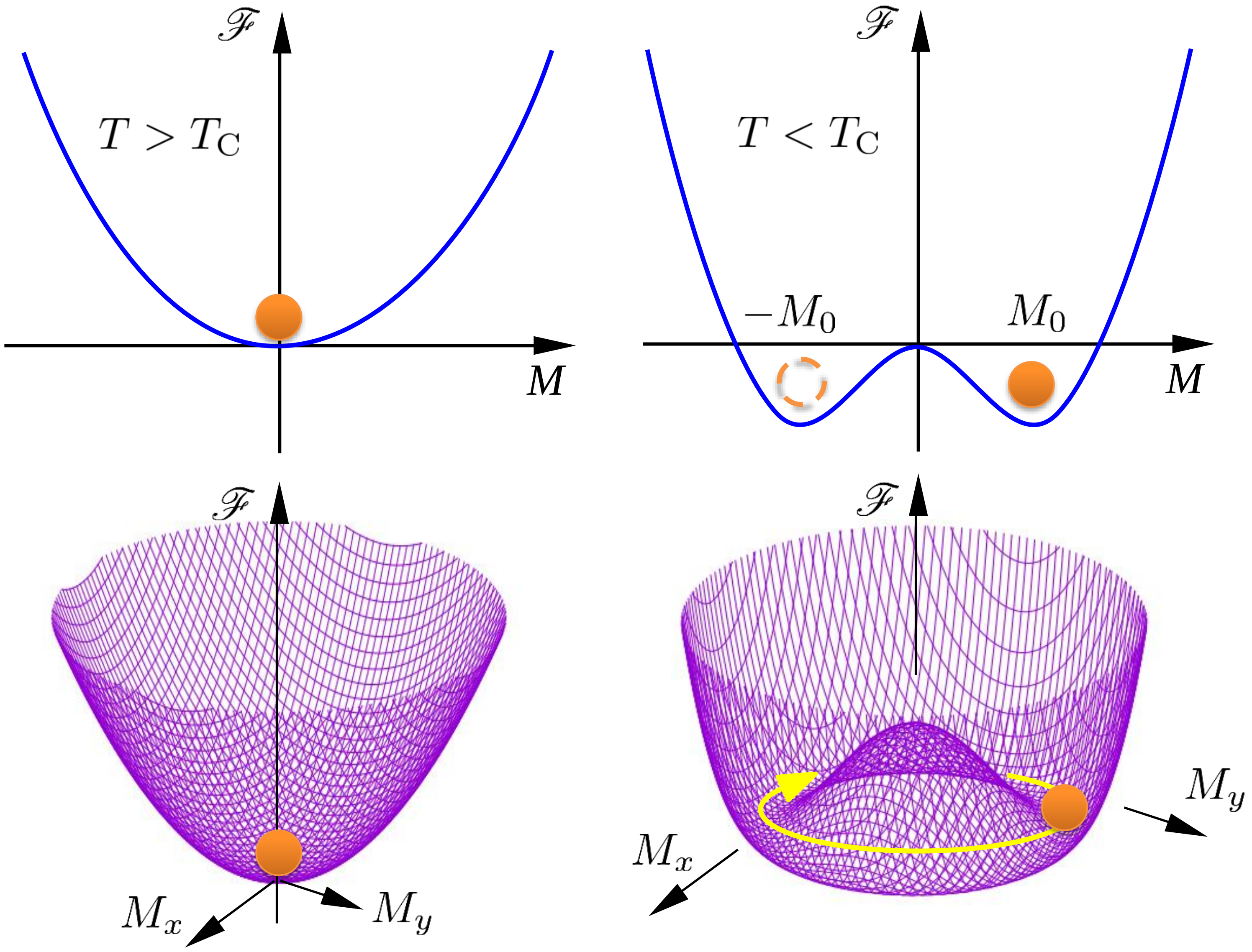


Normal metal specific heat: $C_\mathrm{m} = AT + BT^3$



Fukuoka et al., PRB **93**, 245136 (2016)

Spontaneous Symmetry Breaking and Nambu-Goldstone mode



Spontaneous symmetry breaking

Nambu-Goldstone theorem

When a spontaneous symmetry breaking takes place, a mode with zero energy at long wavelength limit appears.

$$E = mc^2$$

↓ ↓

0 0 massless

Nambu-Goldston mode

(Nambu-Goldston boson)

Magnons in the case of ferromagnets (type-B) and anti-ferromagnets (type-A).

Generalization of Nambu-Goldstone theorem (column)

Nambu-Goldstone mode, Higgs mode → Birth of particle mass; Standard theory of elementary particles

The theories based on the principle prevail all over the physics.

However, there still have been many open questions!

An example: According to the primitive statement, the number of NG mode should be the same as that of broken symmetries.

	Broken symmetry	Number of NG modes	Number of broken symmetry
N-G theorem		x	$y = x$
Crystal	Translational symmetry	3	3
3D Ferromagnet	Rotational symmetry	1	2
Spinor BEC	Rotational symmetry	2	3
Skymion crystal	Translational symmetry	1	2

Extended theorem (2012) : $x = y - \mathrm{rank}\,\langle [Q_a, Q_b] \rangle /2$

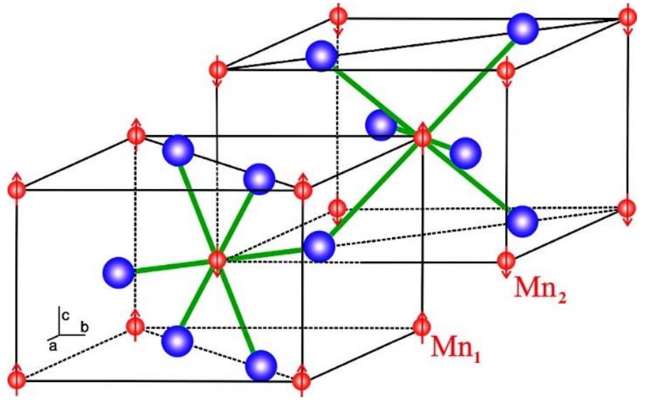
Watanabe, Murayama PRL **108**, 25162 (2012); Hidaka PRL **110**, 091601 (2013); Hidaka, Minami PTEP 2020, 033A01

Magnon dispersion relation measurement in MnF_2

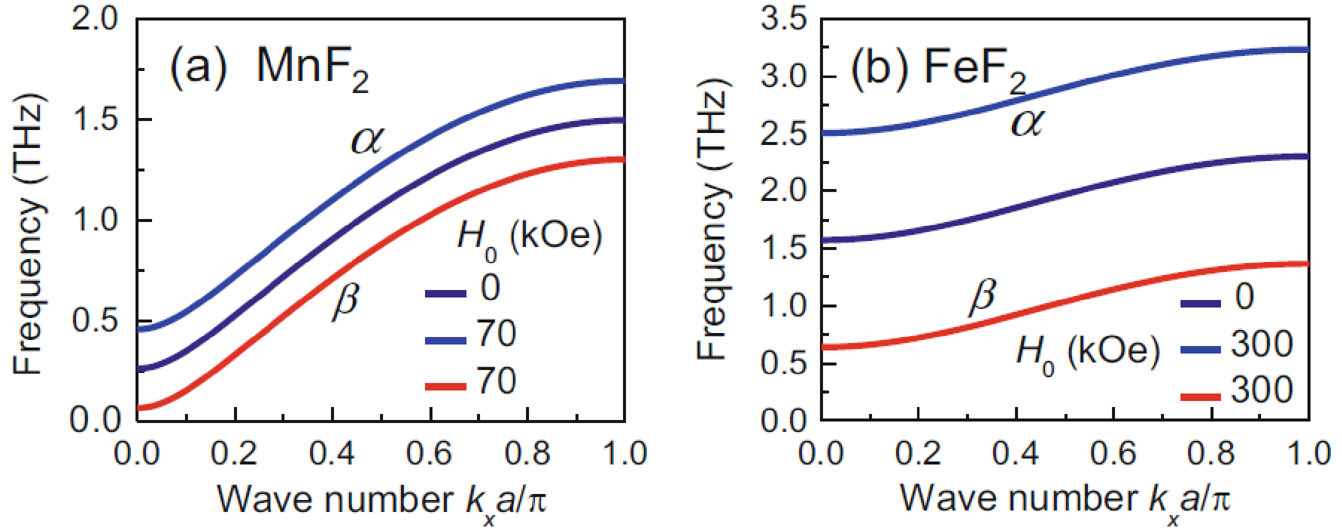
Neutron time-of-flight (TOF) measurement

$$\mathscr{H}_{\text{int}} = -\boldsymbol{\mu}_{\text{e}} \cdot \boldsymbol{B}_{\text{n}} \qquad \boldsymbol{B}_{\text{n}} = \text{rot}\left(\boldsymbol{\mu}_{\text{n}} \times \frac{\boldsymbol{r}}{r^3}\right)$$ neutron magnetic field

electron moment

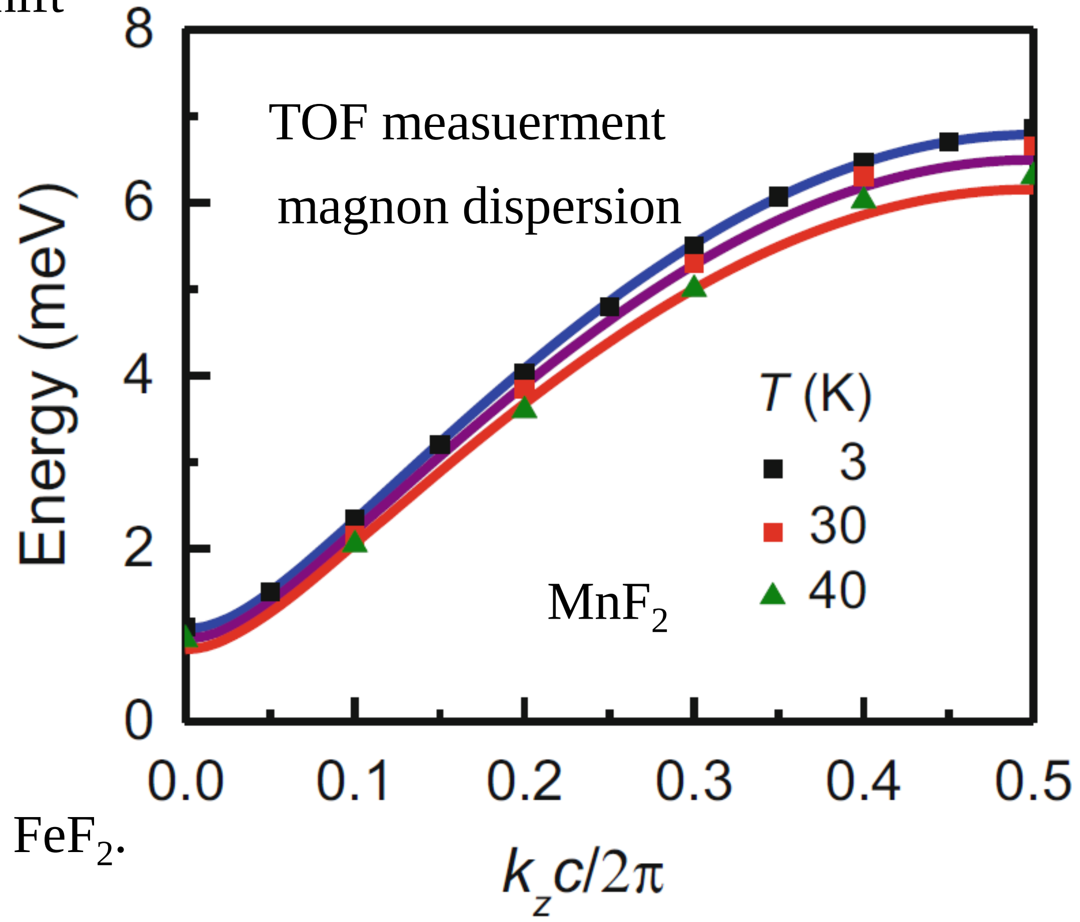


neutron inelastic scattering
$$\begin{cases} \hbar\boldsymbol{k} \longrightarrow \hbar(\boldsymbol{k}-\boldsymbol{q}) & \text{momentum shift} \\ \Delta E = \dfrac{\hbar^2}{2M}(-2\boldsymbol{k}\cdot\boldsymbol{q}+\boldsymbol{q}^2) & \text{energy loss} \end{cases}$$



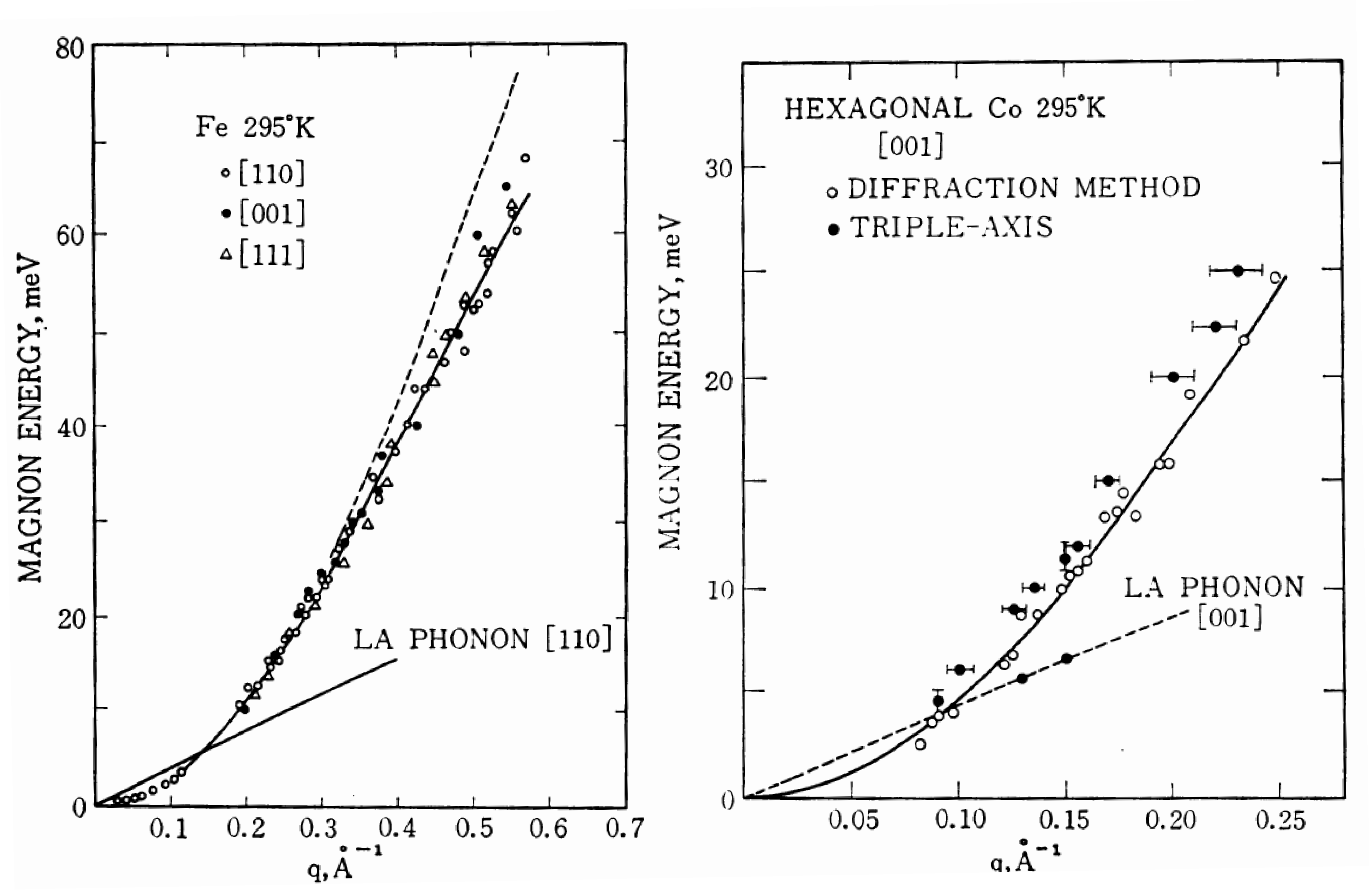
Calculated dispersions of anti-ferromagnetic magnons in MnF_2 and FeF_2.

(Taken from *Fundamentals of Magnonics*.)



Low et al., JAP **35**, 998 (1964).

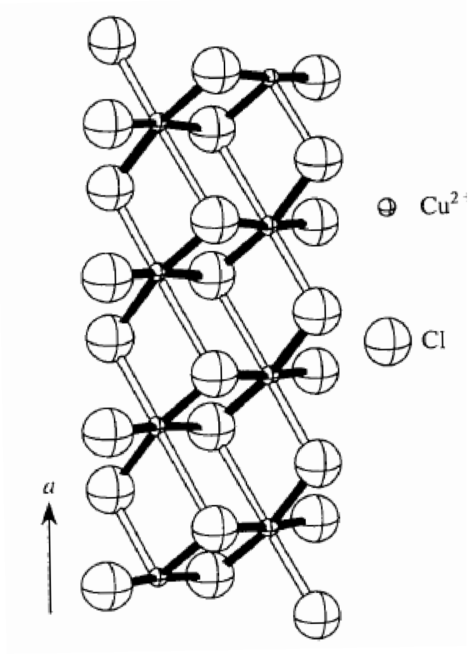
Magnon dispersions in metallic ferromagnets

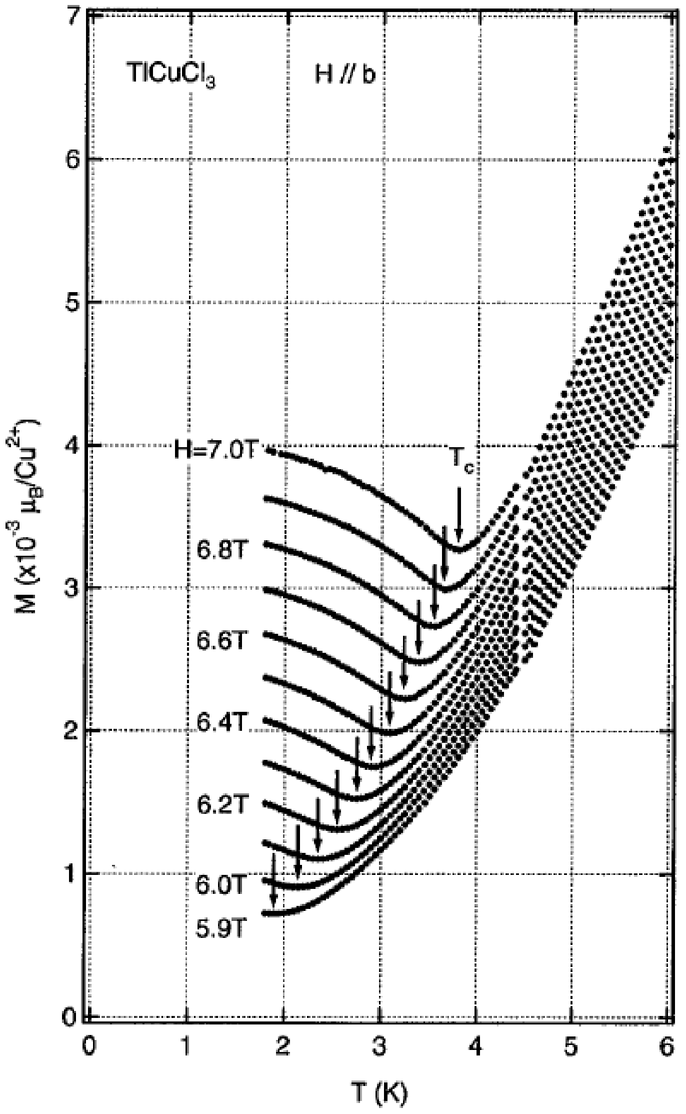


Bose-Einstein condensation of magnons

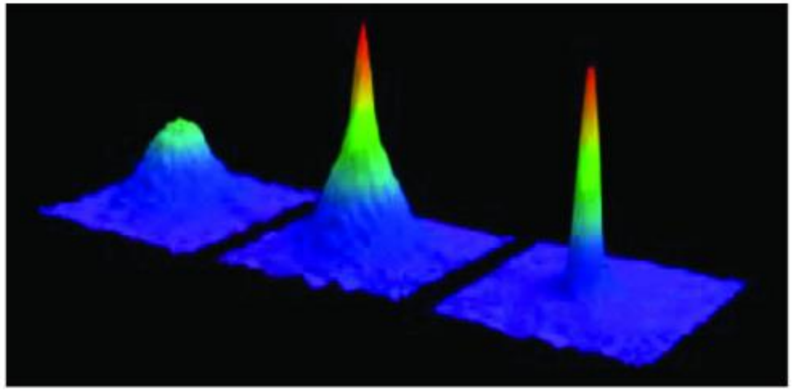
Magnons: not completely bosons (para statistics) however can be treated as “hard core” bosons

$TlCuCl_3$





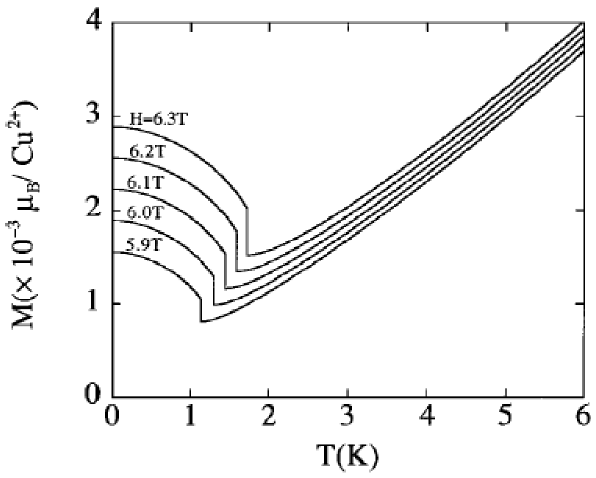
Sharp enhancement of magnetization

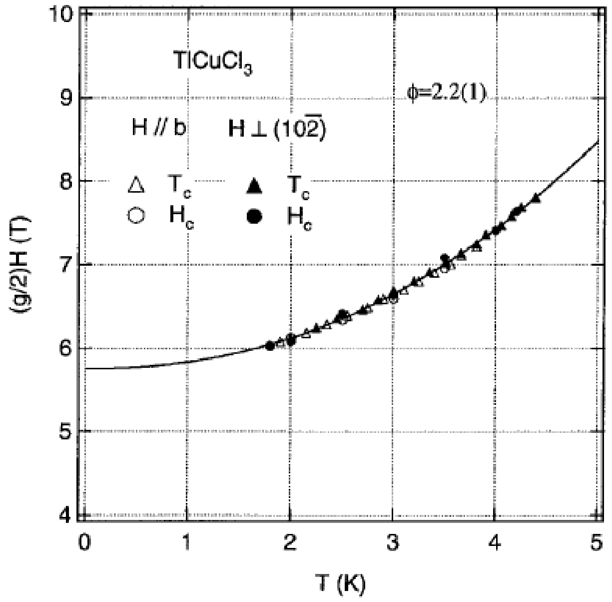


BEC in cold Rb atom ensemble

Sharp increase in particle density

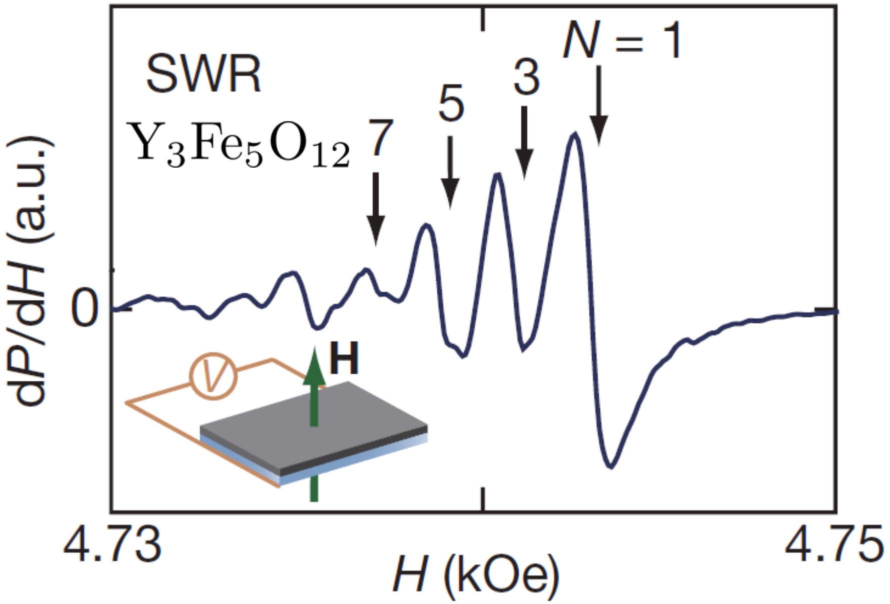
Theory





Nikuni et al., PRL **84**, 5868 (2000)

Magnon (spin wave) resonance in thin films



Kajiwara et al., Nature **464**, 262 (2010)

Spin wave resonance in a thin film

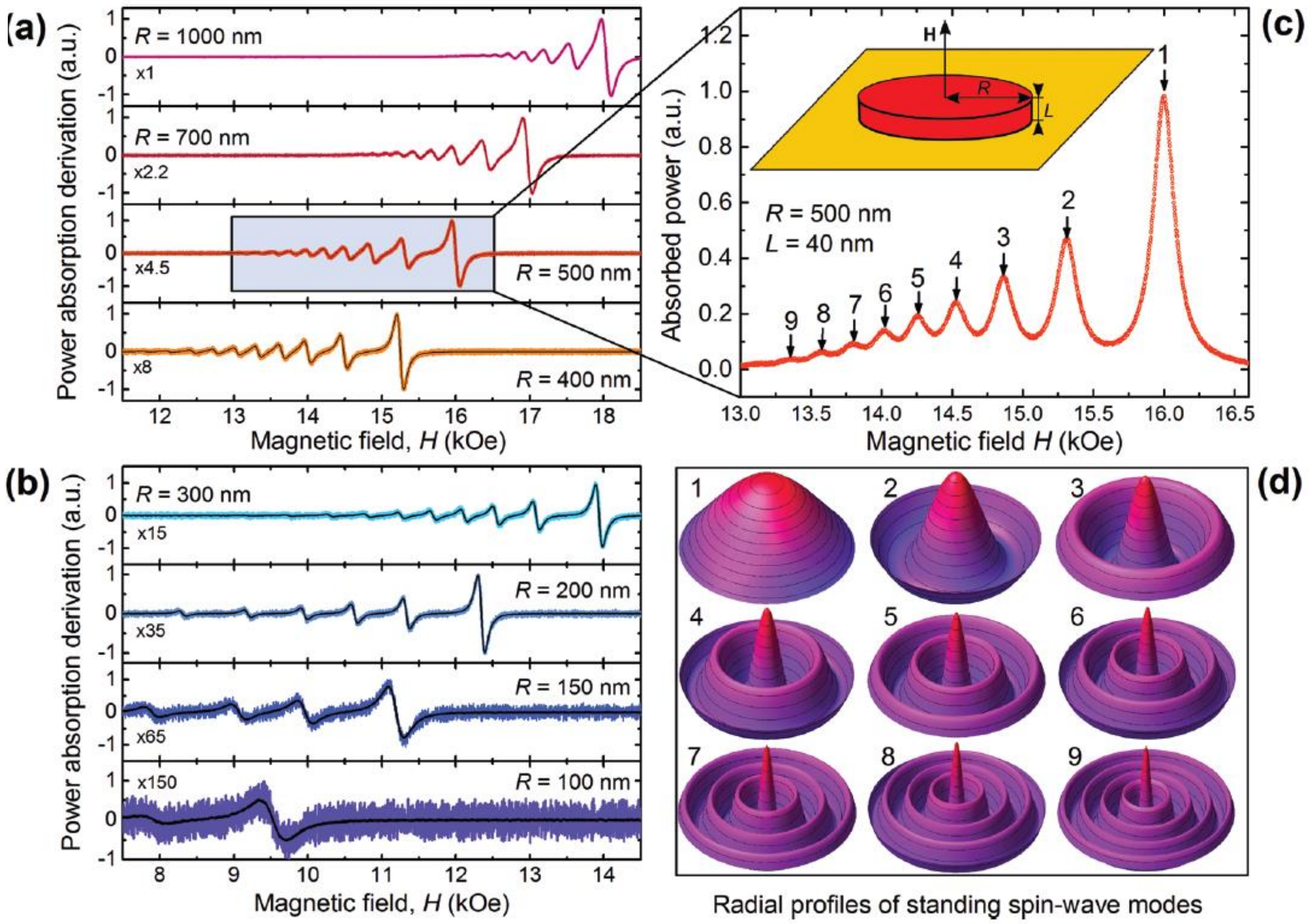
$$\frac{m}{\gamma} = \frac{B}{\mu_0} - 4\pi M_S + \frac{2Ak^2}{M_S}$$

$k = \dfrac{p\pi}{L}$ $\;p$: odd number,

L : film thickness $\quad A = \dfrac{\alpha_z S^2 J}{a}$

Kittel Phys. Rev. **110**, 1295 (1958)

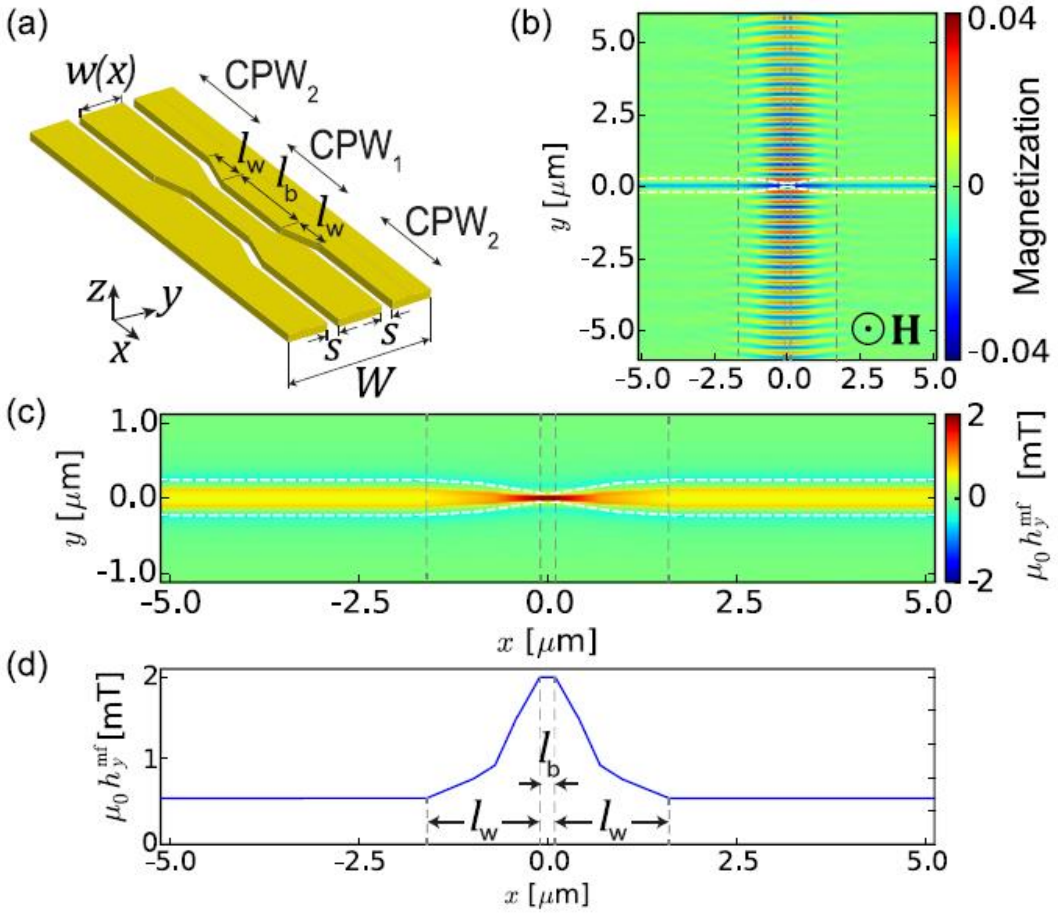
CoFe Nano disk



Dobrovolskiy et al., Nanoscale **12**, 21207 (2020).

Real space imaging of magnons

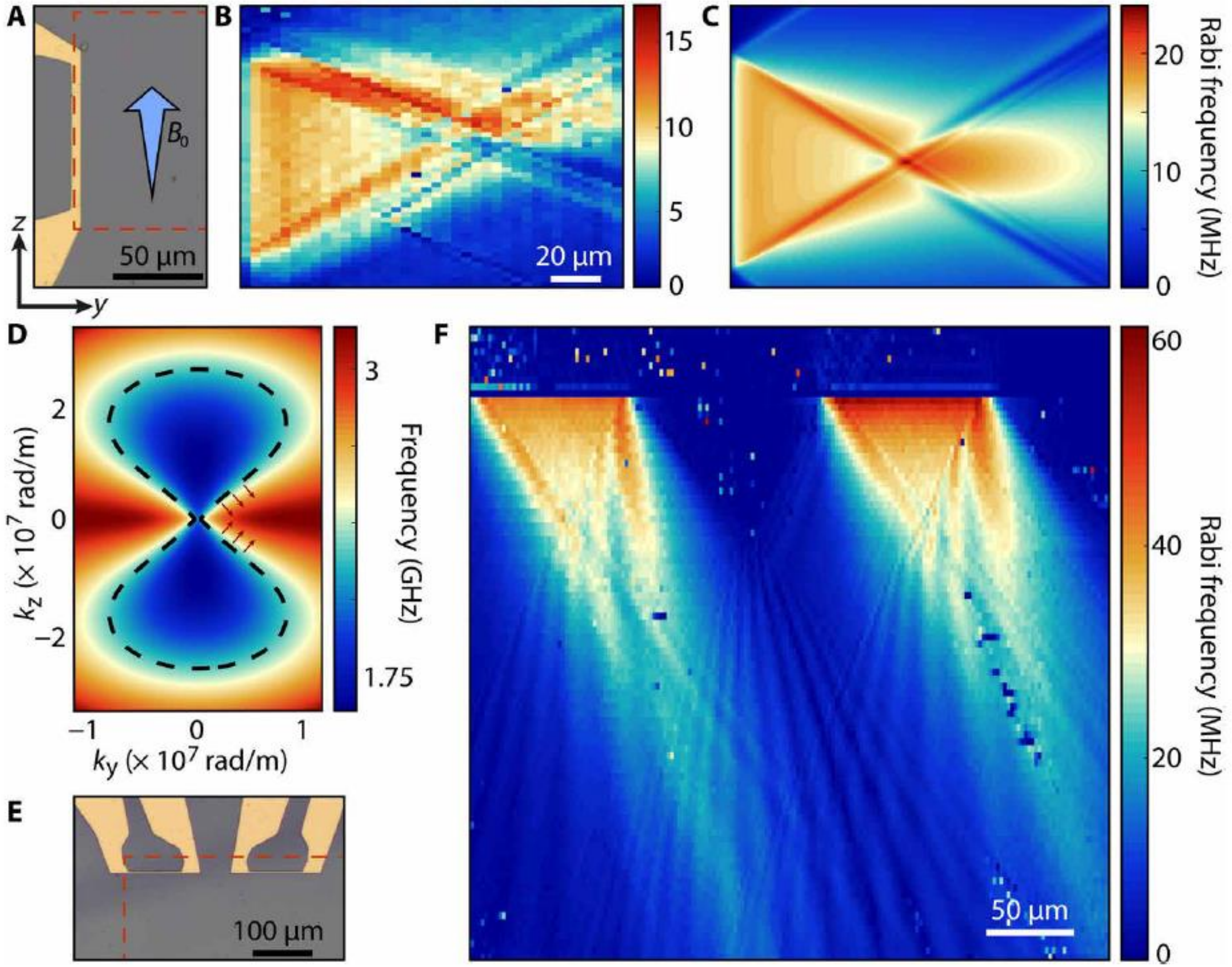
YIG



Gruszecki et al. Sci. Rep. **6**, 22367 (2016)

YIG

NV center ESR detection of magnon fields



Bertelli et al., Sci. Adv. **6**, eabd3556 (2020).

Summary

- Spin wave (classical) in a ferromagnet
- Quantization of spin wave (magnon)
- Magnons in an anti-ferromagnet
- Magnon approximation for weak excitations in ferro- and anti-ferromagnets
- Nambu-Goldstone mode in phase transition
- Experiments on magnons