2022.6.22 Lecture 11

10:25 – 11:55

Lecture on

Magnetic Properties of Materials

磁性 (Magnetism)

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Review

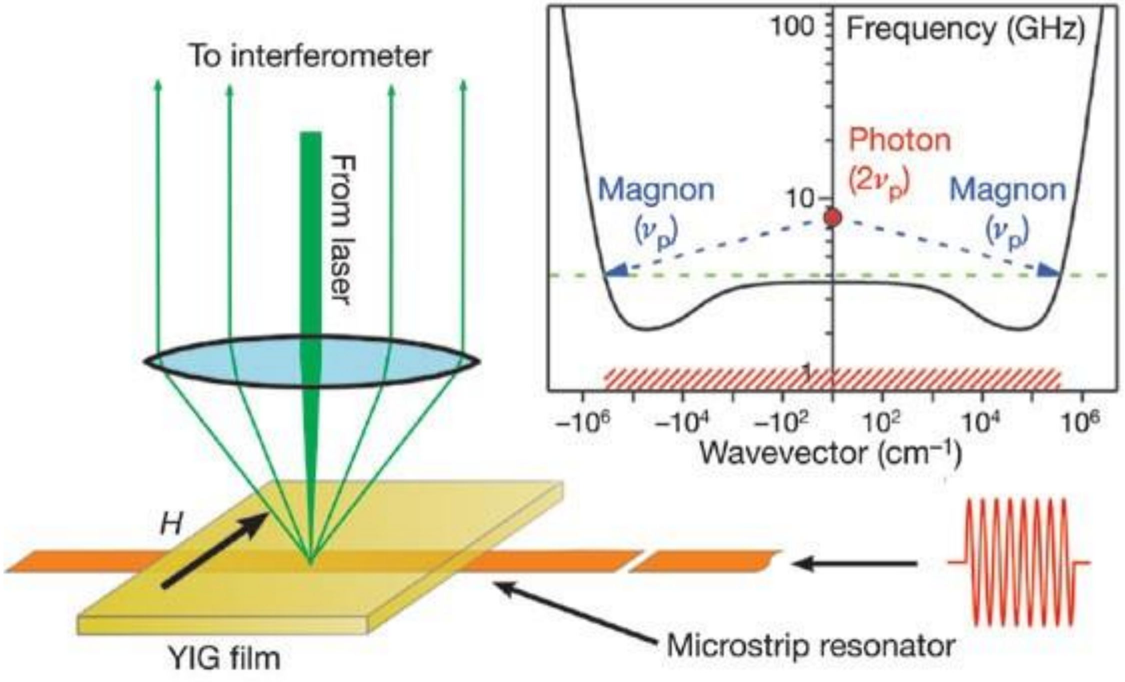
- Spin wave (classical) in a ferromagnet
- Quantization of spin wave (magnon)
- Magnons in an anti-ferromagnet
- Magnon approximation for weak excitations in ferro- and anti-ferromagnets
- Nambu-Goldstone mode in phase transition
- Experiments on magnons

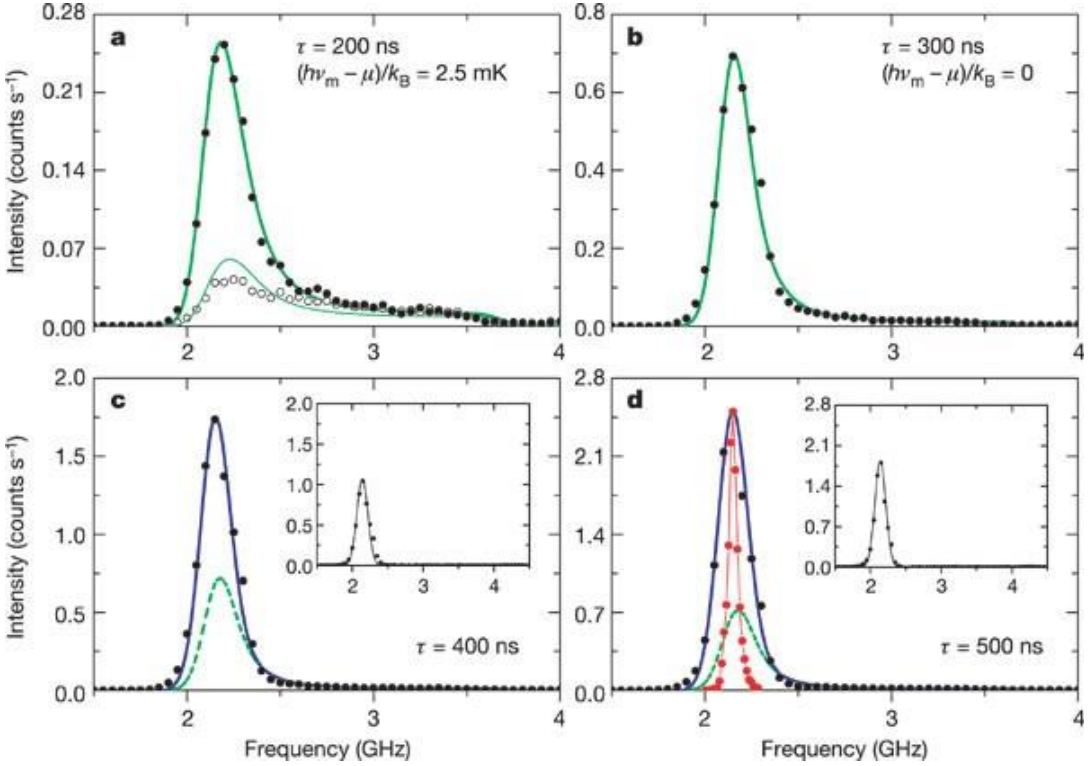
Outline

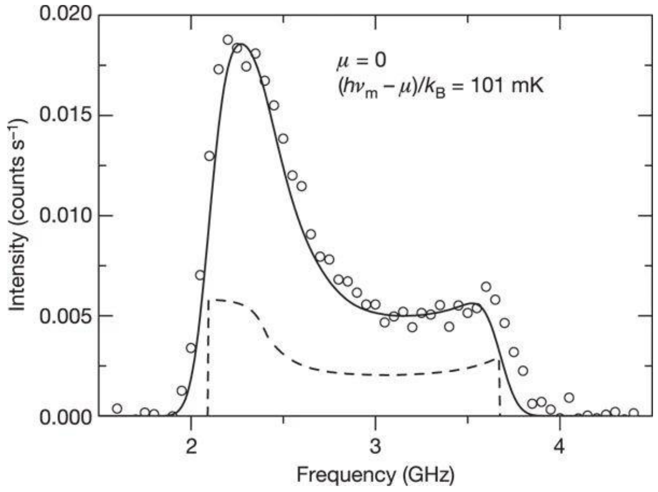
- Ferromagnetic and Antiferromagnetic resonance
- Spin wave resonance
- Experiments on magnons
- Scaling relations
- Renormalization group
- Derivation of scaling ansatz

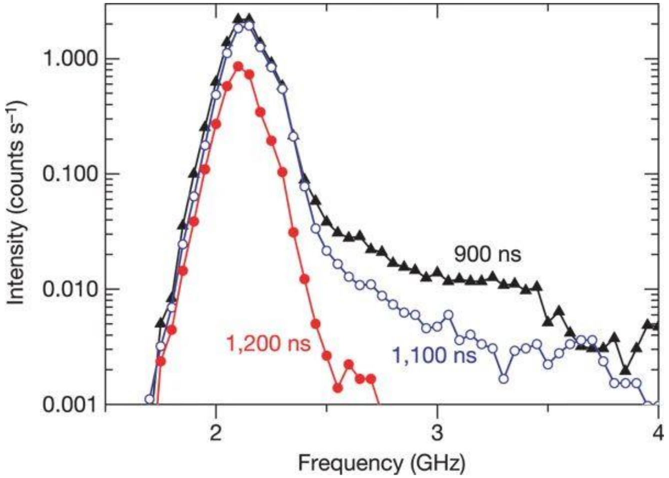
BEC of quasi-equilibrium magnons at room temperature

Demokritov et al., Nature 443, 431 (2006).









Ferromagnetic resonance

Free energy:

$$\mathscr{F} = \sum_{\langle i,j \rangle} \lambda_{ij} \boldsymbol{M}_i \cdot \boldsymbol{M}_j - \sum_{i,j} \boldsymbol{M}_i \mathbf{K}_{i,j} \boldsymbol{M}_j - \sum_i \boldsymbol{M}_i \cdot \left(\boldsymbol{H} - \mathbf{N} \sum_j \boldsymbol{M}_j \right),$$

In case of ferromagnetically ordered state:

$$= -\lambda M^2 + \underline{\boldsymbol{M} \cdot \mathbf{K} \boldsymbol{M}} - \boldsymbol{M} \cdot (\boldsymbol{H} - \underline{\mathbf{N} \boldsymbol{M}})$$

Anisotropic field | Zeeman | Demagnetizing field

Effective field other than ***H*** :

$$\boldsymbol{H}_{\text{eff}} = \lambda \boldsymbol{M} - (\boldsymbol{K} + \boldsymbol{N}) \boldsymbol{M}$$

Kinetic equation of macroscopic moment:

$$\frac{1}{\gamma} \frac{d\boldsymbol{M}}{dt} = \boldsymbol{M} \times (\boldsymbol{H} - \mathbf{K} \boldsymbol{M} - \mathbf{N} \boldsymbol{M})$$

γ: gyromagnetic ratio

When the anisotropy is uniaxial, the shape of the sample, the magnetic field are on line:

$$\omega = \gamma \sqrt{(H + (K_x - K_z + N_x - N_z)M)(H + (K_y - K_z + N_y - N_z)M)}$$

Antiferromagnetic resonance

Effective field for two magnetic sublattices (1,2) :

$$\boldsymbol{H}_{\text{eff}1} = -\lambda \boldsymbol{M}_2 + \mathbf{K}_{11}\boldsymbol{M}_1 + \mathbf{K}_{12}\boldsymbol{M}_2 + \mathbf{N}(\boldsymbol{M}_1 + \boldsymbol{M}_2)$$
$$\boldsymbol{H}_{\text{eff}2} = -\lambda \boldsymbol{M}_1 + \mathbf{K}_{21}\boldsymbol{M}_1 + \mathbf{K}_{22}\boldsymbol{M}_2 + \mathbf{N}(\boldsymbol{M}_1 + \boldsymbol{M}_2)$$

In the case of antiferromagnet: $\boldsymbol{M}_1 = -\boldsymbol{M}_2$ No demagnetizing effect!

Anisotropy tensor: $\mathbf{K}_{11} = \mathbf{K}_{22}, \quad \mathbf{K}_{12} = \mathbf{K}_{21}$

Assumption: Anisotropy energy $\mathscr{F}_{\text{A}}$ is uniaxial. $\mathscr{F}_{\text{A}} = -\frac{K_1}{2}(\cos^2\theta_1 + \cos^2\theta_2)$ $\theta_1,\ \theta_2$: angles to $\boldsymbol{M}_1,\ \boldsymbol{M}_2$

Anisotropy tensor: $K_{zz} = -\frac{K_1}{|\boldsymbol{M}_1|}, \quad (\text{others}) = 0$

Resonance frequencies:

$$\frac{\omega_{\pm}}{\gamma} = \sqrt{2\lambda K_1 + (K_1/|M_1|)^2} \pm H, \qquad H \leq H_{\text{c}},$$
$$\frac{\omega_{+}}{\gamma} = \sqrt{B^2 - 2\lambda K_1} \qquad H > H_{\text{c}}$$

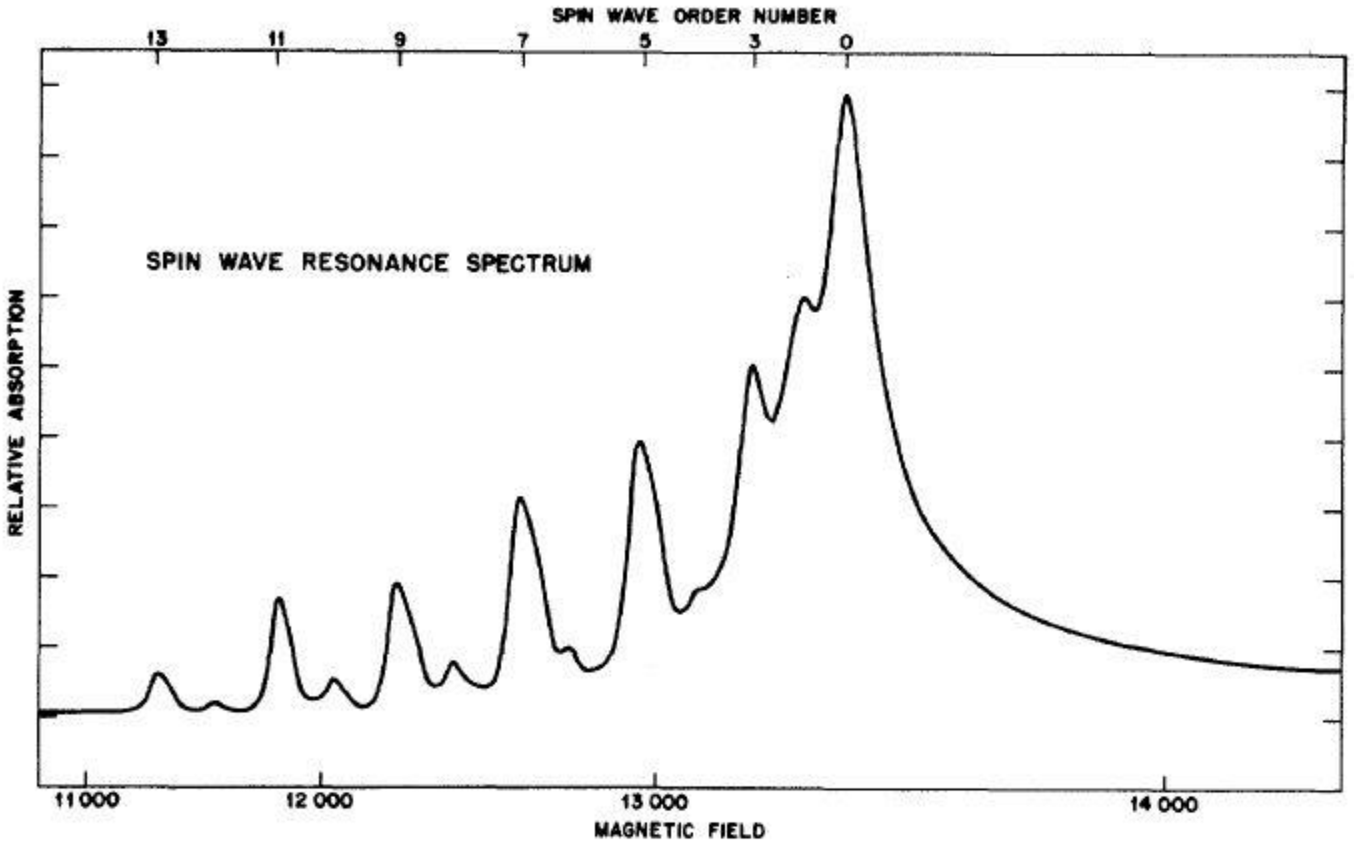
Critical field of spin-flop transition: $H_{\text{c}} = \sqrt{2\lambda K_1}$

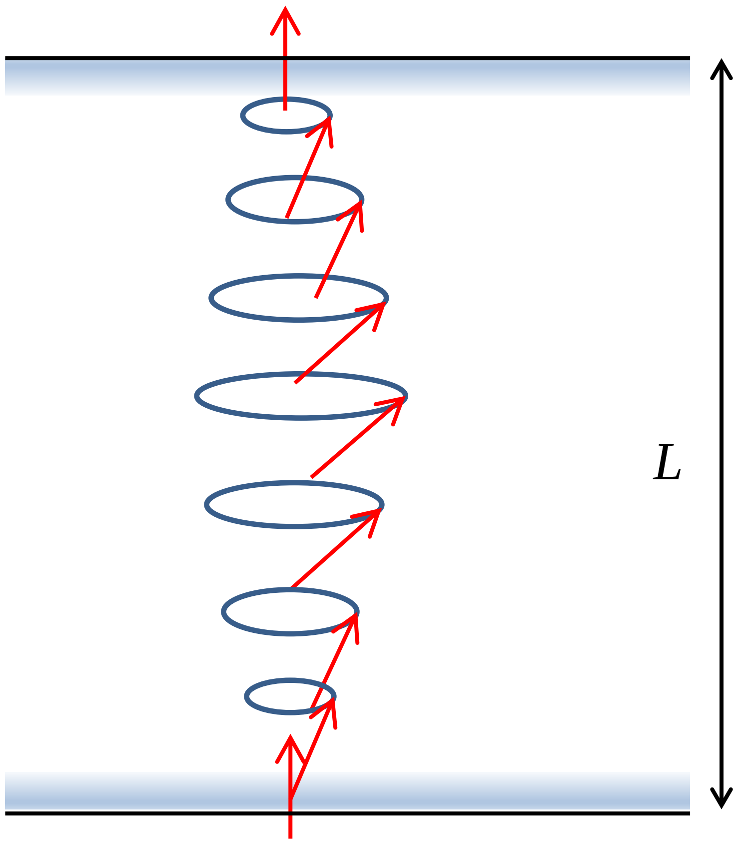
When the anisotropy is small: $\frac{\omega_{\pm}}{\gamma} = \sqrt{2\lambda K_1} \pm H, \quad H \leq H_{\text{c}}$

Spin wave (Magnon) resonance

Ferromagnetic spin wave dispersion relation: $$\omega_k = \gamma H + \frac{2SJ}{\hbar}(ka)^2$$

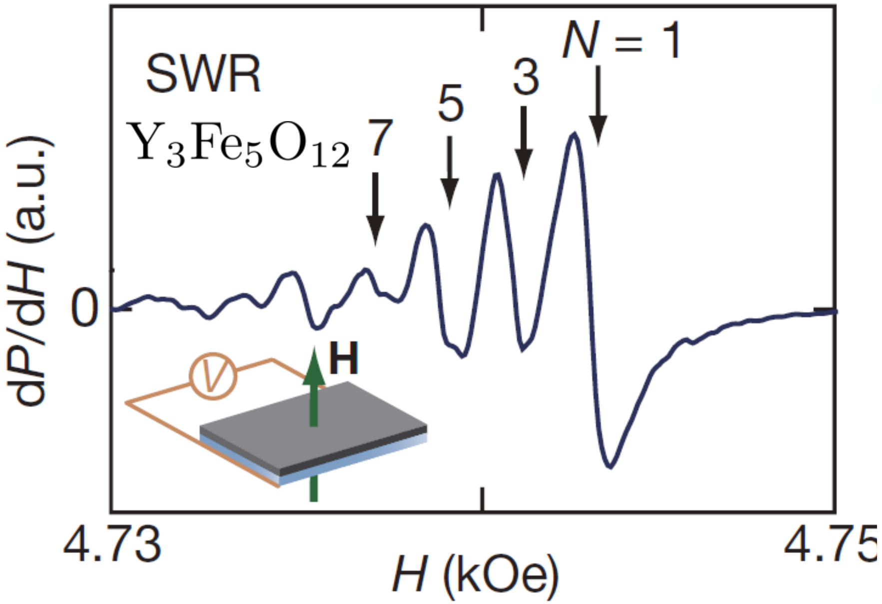
Standing wave condition: $$k = \frac{n\pi}{L}, \quad n = 1, 2, \cdots$$





Seavey, Tannenwald, PRL **1**, 168 (1958).

Magnon (spin wave) resonance in thin films



Kajiwara et al., Nature **464**, 262 (2010)

Spin wave resonance in a thin film

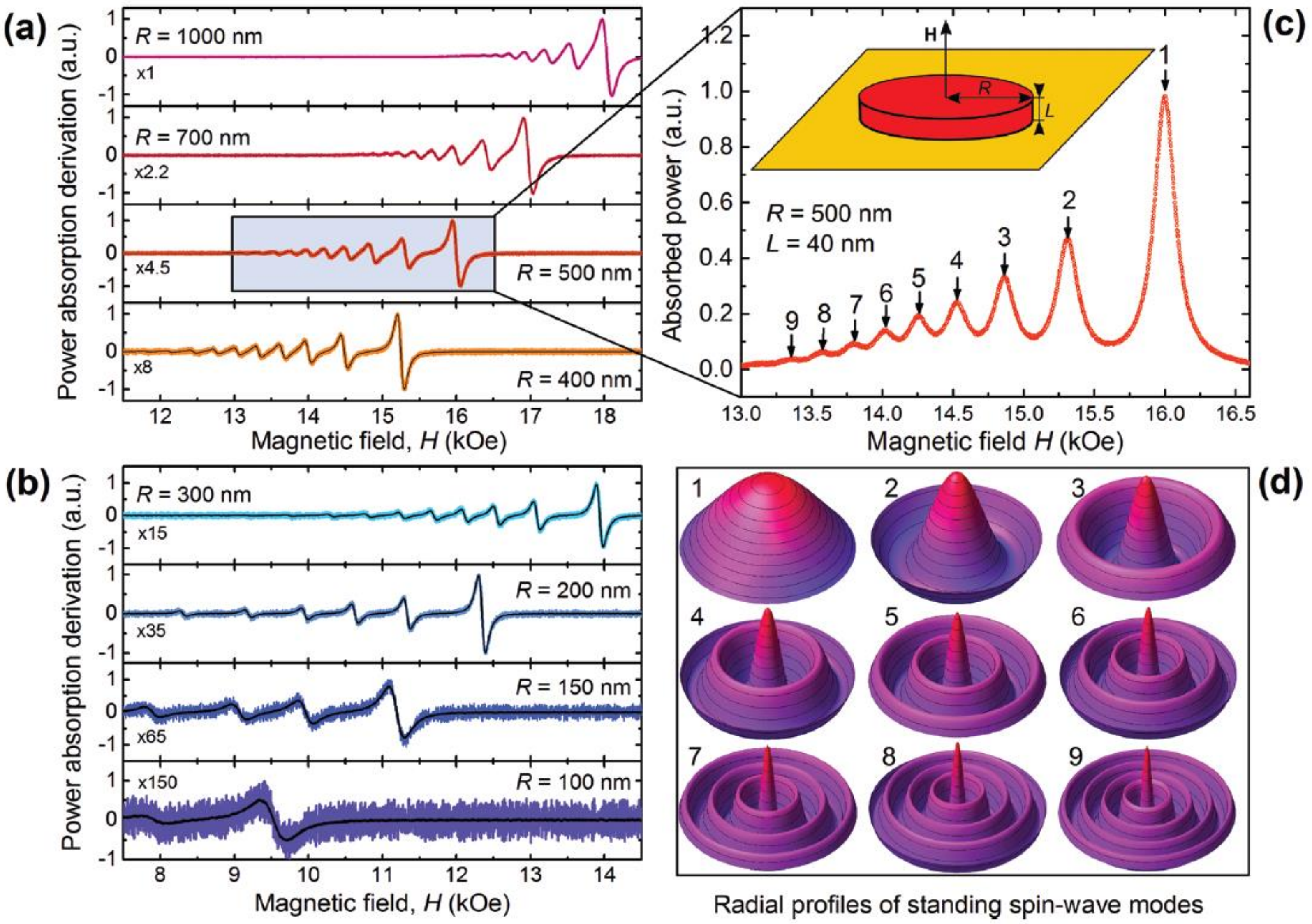
$$\frac{m}{\gamma} = \frac{B}{\mu_0} - 4\pi M_S + \frac{2Ak^2}{M_S}$$

$$k = \frac{p\pi}{L} \quad p : \text{odd number},$$

$$L : \text{ film thickness} \qquad A = \frac{\alpha_z S^2 J}{a}$$

Kittel Phys. Rev. **110**, 1295 (1958)

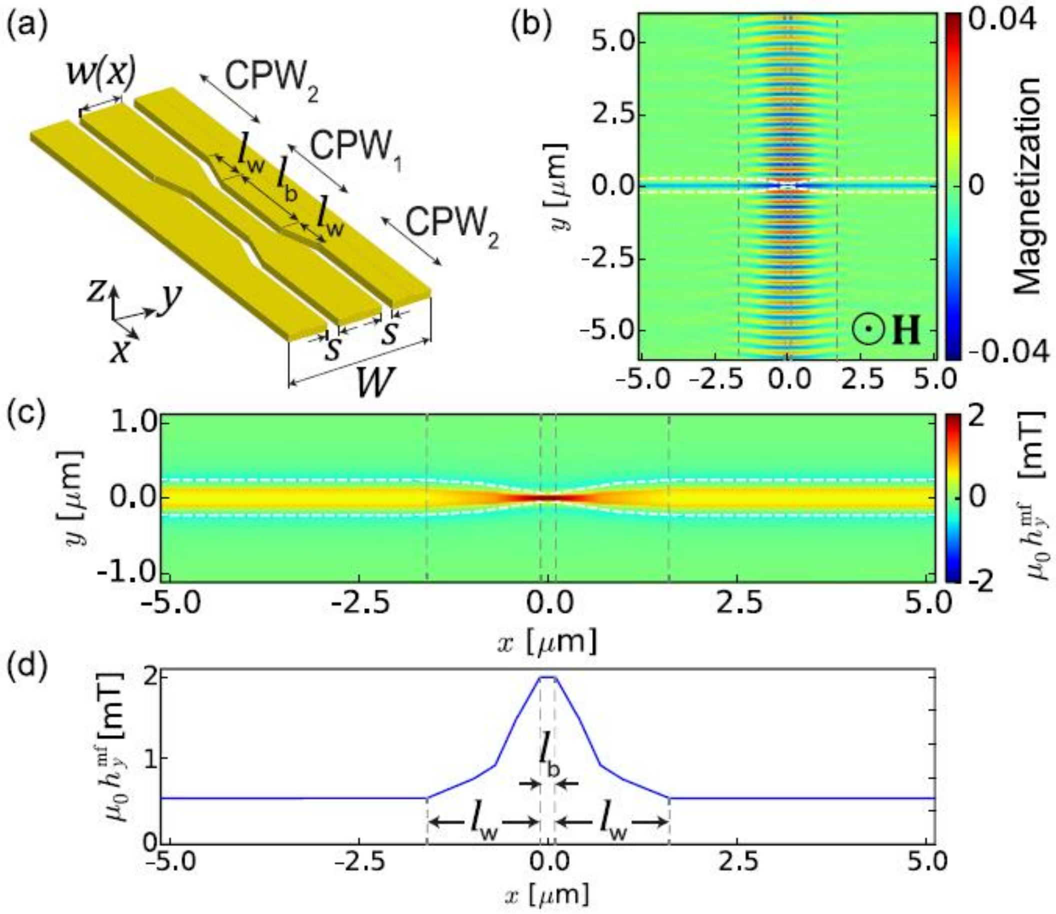
CoFe Nano disk



Dobrovolskiy et al., Nanoscale **12**, 21207 (2020).

Real space imaging of magnons

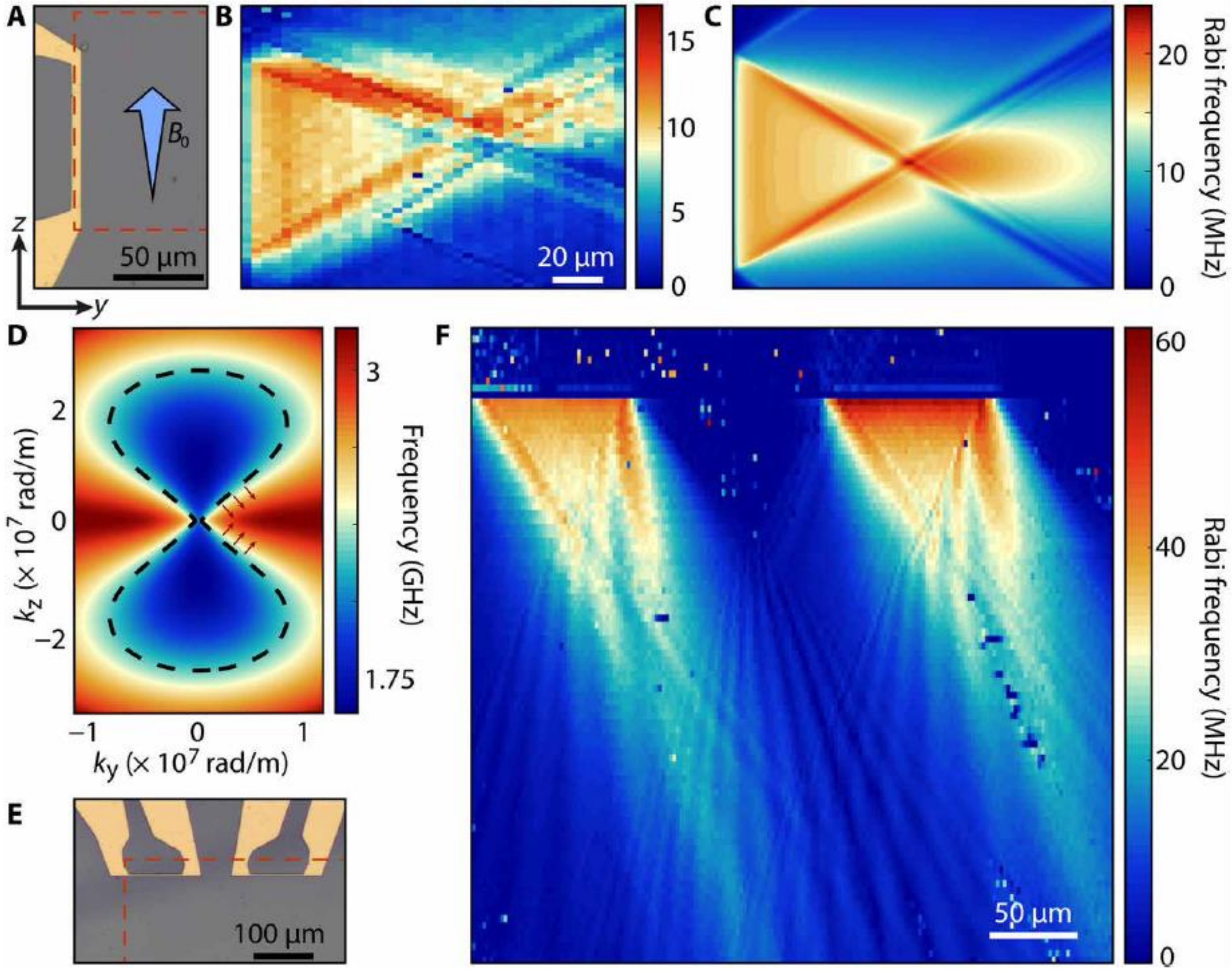
YIG



Gruszecki et al. Sci. Rep. **6**, 22367 (2016)

YIG

NV center ESR detection of magnon fields



Bertelli et al., Sci. Adv. **6**, eabd3556 (2020).

Section 5.10

Renormalization group and scaling theory of phase transition

Kenneth Wilson

1936 - 2013

1982 Nobel prize

Jacques Friedel

1921 - 2014

Jun Kondo

1930 - 2022

Correlation function

Magnetic moment (local) density : $m(\boldsymbol{r})$

Free energy density: $f(m(\boldsymbol{r}, \nabla m(\boldsymbol{r})) = f_0 + \frac{a}{2}m^2 + \frac{b}{4}m^4 + c|\nabla m|^2 - hm$

Free energy functional: $\mathscr{F}\{m(\boldsymbol{r})\} = \int_V d\boldsymbol{r}' f(m(\boldsymbol{r}'), \nabla' m(\boldsymbol{r}'))$

Partition function: $Z = \int \mathcal{D}m(\boldsymbol{r}) \exp\left[-\frac{\mathscr{F}\{m(\boldsymbol{r})\}}{k_{\mathrm{B}}T}\right]$ $\int \mathcal{D}m(\boldsymbol{r})$: functional integral

Probability density of realization of *m*(**r**) : $p\{m(\boldsymbol{r})\} = \frac{1}{Z}\exp\left[-\frac{\mathscr{F}\{m(\boldsymbol{r})\}}{k_{\mathrm{B}}T}\right]$

Expectation value of physical quantity *A* $\langle A \rangle = \frac{1}{Z}\int \mathcal{D}m(\boldsymbol{r}) A \exp\left[-\frac{\mathscr{F}\{m(\boldsymbol{r})\}}{k_{\mathrm{B}}T}\right]$

Temperature dependence assumption: $a = \alpha(T - T_{\mathrm{C}}) \quad (\alpha > 0), \quad b = \mathrm{const.}\ (> 0)$

Correlation function (2)

Correlation function of order parameter fluctuation:

$$g(\boldsymbol{r}) = \langle (m(0) - \langle m(0) \rangle)(m(\boldsymbol{r}) - \langle m(\boldsymbol{r}) \rangle) \rangle = \langle m(0) m(\boldsymbol{r}) \rangle - \langle m(0) \rangle \langle m(\boldsymbol{r}) \rangle$$

The second term = 0 for $T > T_\mathrm{C}$

Fourier representation:

$$m(\boldsymbol{r}) = \frac{1}{\sqrt{V}} \sum_{\boldsymbol{k}} m_{\boldsymbol{k}} \exp(i\boldsymbol{k} \cdot \boldsymbol{r}) \qquad m_{-\boldsymbol{k}} = m_{\boldsymbol{k}}^*$$

Because

$$(m_k + m_{-k})(m_k e^{ikr} + m_{-k} e^{-ikr}) = 2|m_k|^2 e^{-ikr} + 2|m_{-k}|^2 e^{ikr}$$

And from translational invariance:

$$g(\boldsymbol{r}) = \frac{1}{V} \sum_{\boldsymbol{k}} \langle |m_{\boldsymbol{k}}|^2 \rangle \exp(-i\boldsymbol{k} \cdot \boldsymbol{r})$$

Free energy:

$$\mathscr{F} = V f_0 + \sum_{\boldsymbol{k}} |m_{\boldsymbol{k}}|^2 \left(\frac{a}{2} + c k^2 \right) + \frac{b}{4V} \sum_{\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4 = \boldsymbol{0}} m_{\boldsymbol{k}_1} m_{\boldsymbol{k}_2} m_{\boldsymbol{k}_3} m_{\boldsymbol{k}_4}$$

Ignore 4th order

Weight function:

$$p\{m(\boldsymbol{r})\} = \frac{1}{Z} \exp\left[-\frac{\mathscr{F}\{m(\boldsymbol{r})\}}{k_\mathrm{B} T} \right]$$

$$\frac{1}{Z} \exp\left[-\frac{2}{k_\mathrm{B} T} {\sum_{\boldsymbol{k}}}' \left(\frac{a}{2} + c k^2 \right) \left(m_{\boldsymbol{k}}^{(\mathrm{r})2} + m_{\boldsymbol{k}}^{(\mathrm{i})2} \right) \right]$$

Sum over independent $\boldsymbol{k}$ $\qquad \mathrm{Re}[m_{\boldsymbol{k}}] = m_{\boldsymbol{k}}^{(\mathrm{r})}, \quad \mathrm{Im}[m_{\boldsymbol{k}}] = m_{\boldsymbol{k}}^{(\mathrm{i})}$

Correlation function (3) Scaling relations

Finally $$g(\boldsymbol{r}) = \frac{1}{V}\sum_{\boldsymbol{k}}{}' \frac{k_\mathrm{B}T}{a+2ck^2} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} = k_\mathrm{B}T \int_0^\infty \frac{e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}}{2ck^2+a}\frac{d^3k}{(2\pi)^3} = \frac{k_\mathrm{B}T}{8\pi d}\frac{\exp(-r/\xi)}{r}, \quad \xi = \sqrt{\frac{2c}{a}}$$

Yukawa type function
Exponential decay + (1/r)

Correlation length

$T < T_\mathrm{C}$ $\quad \tilde{g}(\boldsymbol{r}) = \langle m(0)m(\boldsymbol{r})\rangle$ differs from $g(\boldsymbol{r})$ $\quad r \to \infty$

Appearance of long range order

Scaling relations temperature $t \equiv (T - T_\mathrm{C})/T_\mathrm{C}$ magnetic field h

Critical exponents:

$$\begin{aligned}
\text{Specific heat}: &\quad C \sim |t|^{-\alpha}, \\
\text{Magnetization (order parameter)}: &\quad m \sim |t|^{\beta} \quad (t<0) \\
&\quad m \sim h^{1/\delta} \quad (t=0), \\
\text{Susceptibility}: &\quad \chi \sim |t|^{-\gamma}, \\
\text{Correlation length}: &\quad \xi \sim |t|^{-\nu}
\end{aligned}$$

Scaling relations

$$g(\boldsymbol{r}) \sim \frac{\exp(-r/\xi)}{r^{d-2+\eta}}$$

d : dimensionality

GL theory gives $\alpha = 0, \;\; \beta = 1/2, \;\; \gamma = 1, \;\; \delta = 3, \;\; \nu = 1/2, \;\; \eta = 0$

Scaling relations:
$$\begin{cases} \gamma = (2-\eta)\nu, \\ \alpha + 2\beta + \gamma = 2, \\ \beta + \gamma = \beta\delta. \end{cases}$$

Scaling ansatz: Critical behavior is described by a single relevant parameter. $\dfrac{h}{|t|^{\Delta}}$ Δ: Gap exponent

Free energy expression: $$f_s \sim |t|^{2-\alpha} f_{\pm}\left(\frac{h}{|t|^{\Delta}}\right)$$

$$m(h=0) \sim -\frac{\partial f_s}{\partial h} \sim |t|^{2-\alpha-\Delta} f'_{\pm}(0) \sim |t|^{\beta} \quad (t<0)$$

$$\chi \sim -\frac{\partial^2 f}{\partial h^2} \sim |t|^{2-\alpha-2\Delta} f''_{\pm}(0) \sim |t|^{-\gamma}$$

$$\beta = 2 - \alpha - \Delta$$
$$-\gamma = 2 - \alpha - 2\Delta$$
$$\therefore \;\; \Delta = \beta + \gamma$$

Scaling ansatz and scaling relations

$$f_s \sim |t|^{2-\alpha} f_\pm \left(\frac{h}{|t|^\Delta} \right) \qquad t \to 0 \qquad \frac{h}{|t|^\Delta} \to \infty$$

Then we assume the asymptotic form: $f'_\pm(x) \sim x^{\lambda_\pm} \quad (x \to \infty)$

From the scaling relations: $m \sim |t|^\beta f'_\pm \left(\frac{h}{|t|^\Delta} \right) \sim \frac{h^{\lambda_\pm}}{|t|^{\Delta\lambda_\pm - \beta}}$

For m to be finite at $t \to 0$: $\lambda_\pm = \frac{\beta}{\Delta} = \frac{\beta}{\beta + \gamma}$ Compare with $m \sim h^{1/\delta}$ then $\delta = \frac{\beta + \gamma}{\beta}$

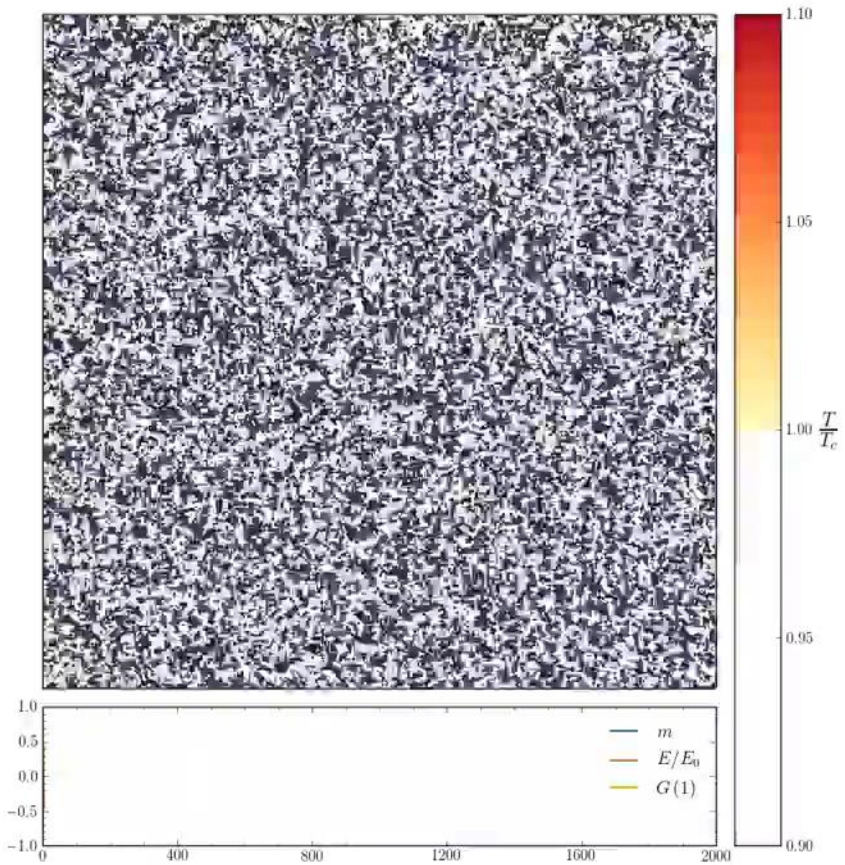
Hyperscaling relation: $2 - \alpha = d\nu$

Ising model

Directions of spins are limited to z

$$\mathscr{H} = -J \sum_{\langle i,j \rangle} S_i S_j - h \sum_i S_i$$

Solution: 1d Ising, 2d Onsager

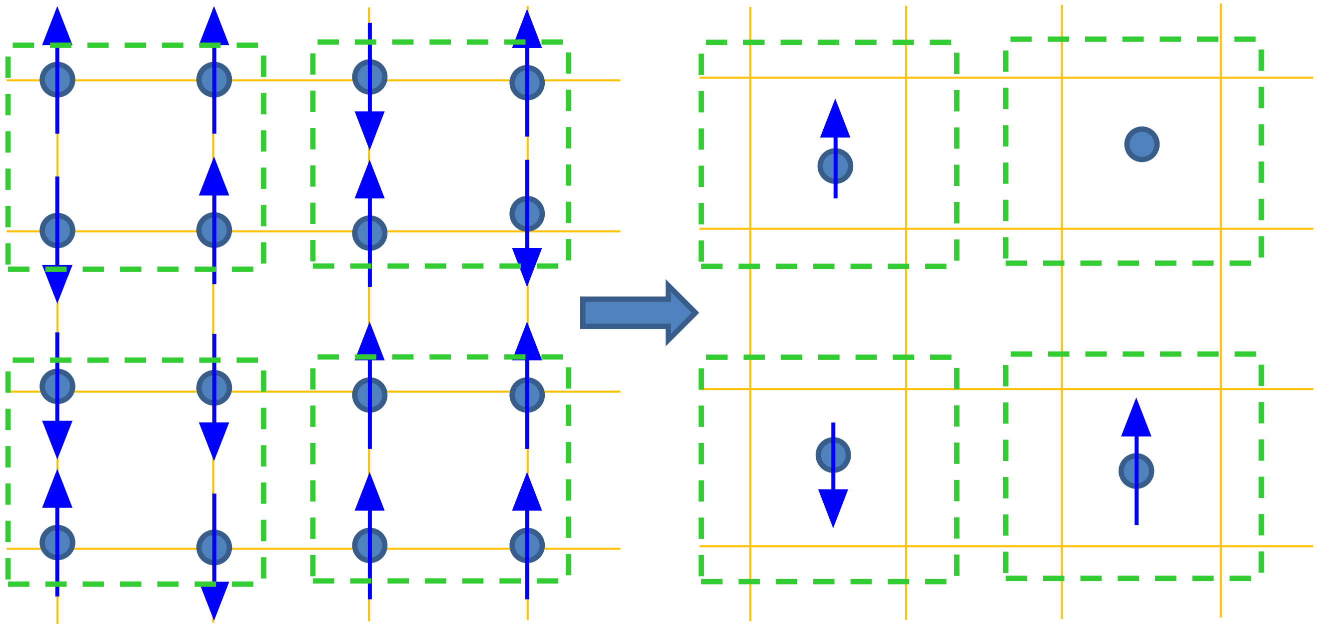


Model (Universality class)	α	β	γ	δ
2D Ising	0	1/8	7/4	15
3D Ising	0.115	0.324	1.239	4.82
3D XY	−0.01	0.34	1.32	4.9
3D Heisenberg	−0.11	0.36	1.39	4.9
Mean field approximation	0	1/2	1	3

https://www.youtube.com/watch?v=kjwKgpQ-l1s

Renormalization group

2D square lattice Ising model



Four spins are averaged and coarse-grained to a single spin.

$$s_q = \frac{1}{4}\sum_i s_{qi} \qquad \sqrt{4} = 2$$

Renormalization group transformation of scaling factor 2.

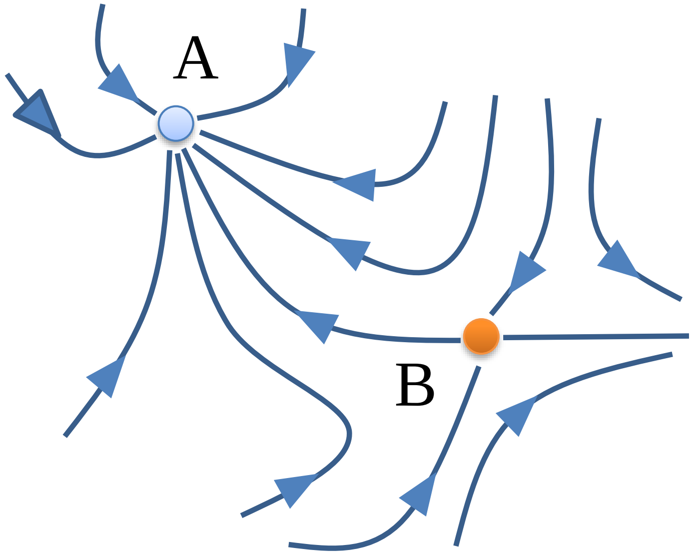
Renormalization group transformation of scaling factor *x* as $\mathcal{R}(x)$

$$\mathscr{H}' = \mathcal{R}(x)\mathscr{H}$$

$$\mathcal{R}(x')\mathcal{R}(x) = \mathcal{R}(x'x)$$

Ordinary no inverse element: Semigroup

Flow diagram



Flow diagram

$\mathcal{R}(x)$ x: continuous variable

Renormalization group transform → changes system parameters

Continuous movement of the system in the parameter space

⇓

Flow diagram

Complete order, complete disorder: the parameters do not change

⇒ Stable fixed points

Just on the critical point: Unstable fixed point

Derivation of scaling ansatz

t: Temperature, h : Magnetic field

Renormalization group transform with scaling factor x
$$\begin{cases} t' = g_1^{(x)}(t,h) \\ h' = g_2^{(x)}(t,h) \end{cases}$$

Expansion around the unstable fixed point $t = h = 0$
$$\begin{cases} t' \simeq \Lambda_{11}(x)t + \Lambda_{21}(x)h, \\ h' \simeq \Lambda_{21}(x)t + \Lambda_{22}(x)h \end{cases}$$

Symmetry difference between t and h. h is reversed by reversing the magnetization but t is not.

Then they cannot have linear relations: $\Lambda_{12}(x) = \Lambda_{21}(x) = 0$

$$(\Lambda_{11}(x))^n = \Lambda_{11}(x^n), \quad (\Lambda_{22}(x))^n = \Lambda_{22}(x^n)$$

This should hold for any x >1, natural number n

Then $\Lambda_{11}(x),\ \Lambda_{22}(x)$ should be power functions of x.

Derivation of scaling ansatz (2)

Hence we write $\Lambda_{11}(x) = x^{\lambda_1}, \quad \Lambda_{22}(x) = x^{\lambda_2}$

Let us consider the case of starting at (t , h).

n-times operation of RGT with SF x System temperature $t_0 = x^{n\lambda_1} t$ is far from the critical point (assume)

Correlation length $\dfrac{\xi(t)}{\xi(t_0)} = x^n = \left(\dfrac{t}{t_0}\right)^{-1/\lambda_1}$

Remember $\xi \sim |t|^{-\nu} \qquad \nu = \lambda_1^{-1}$

In a d-dimensional system, $f(t, h)$ becomes x^d times of the original.

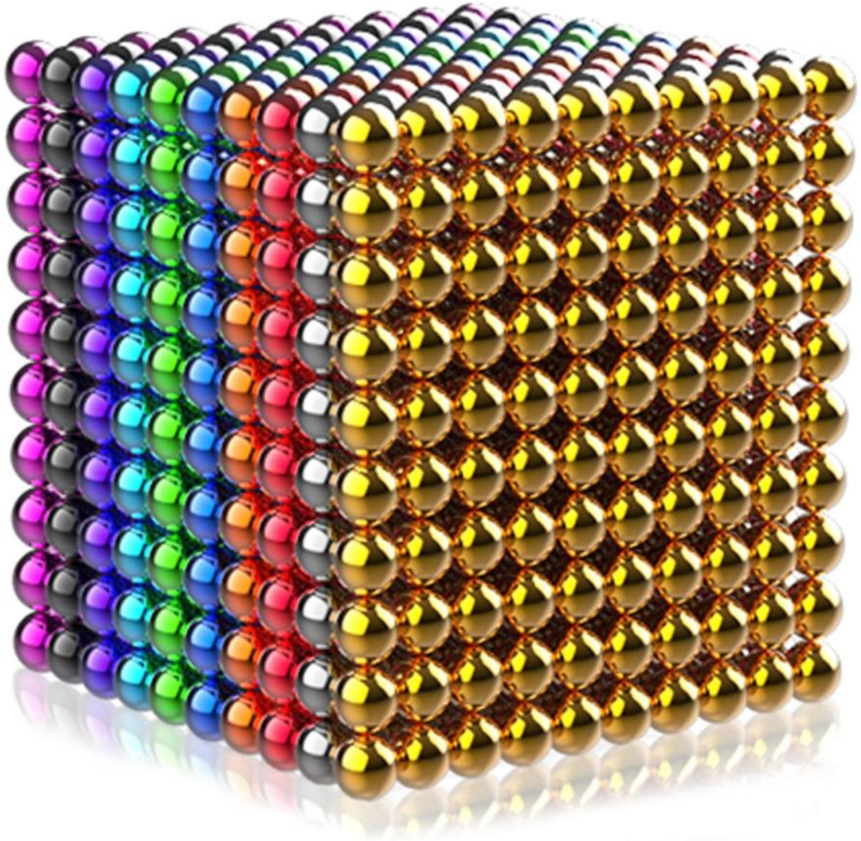
$$x^{nd} f(t, h) = f(x^{n\lambda_1} t, x^{n\lambda_2} h) = f(t_0, (t/t_0)^{-\lambda_2/\lambda_1} h)$$

Hence by some function $f_{\pm}(x)$ we can write

$$f(t, h) = t^{d/\lambda_1} f_{\pm}(t^{-\lambda_2/\lambda_1} h) = t^{d\nu} f_{\pm}\left(\frac{h}{t^{\Delta}}\right) \qquad \Delta = \frac{\lambda_2}{\lambda_1}$$

Chapter 6

Magnetism of Itinerant Electron Systems



Magnetic Puzzle

Summary

- Ferromagnetic and Antiferromagnetic resonance
- Spin wave resonance
- Experiments on magnons
- Scaling relations
- Renormalization group
- Derivation of scaling ansatz