

Kashiwa Science Camp 2019

# Feel Quantum Mechanics!

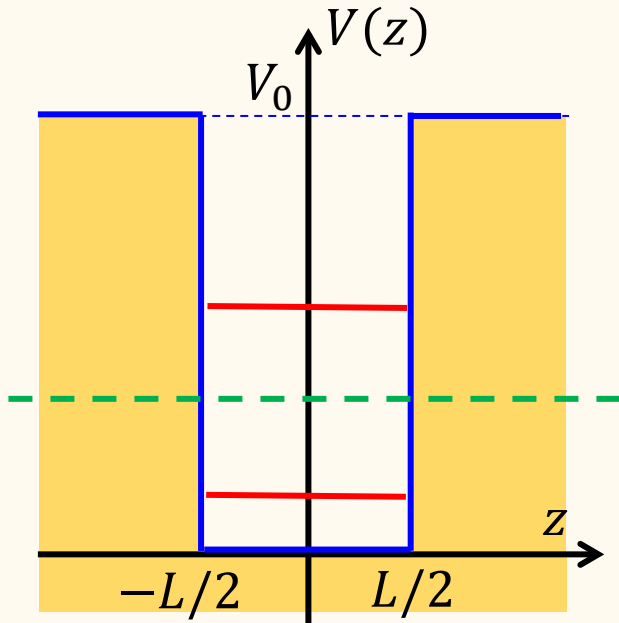
Shingo Katsumoto, Taketomo Nakamura  
Yoshiaki Hashimoto

2019年 柏サイエンスキャンプ  
量子力学を体感する  
第3日

東京大学  
理学部・理学系研究科  
物理学専攻  
物性研究所

勝本信吾, 中村壮智, 橋本義昭

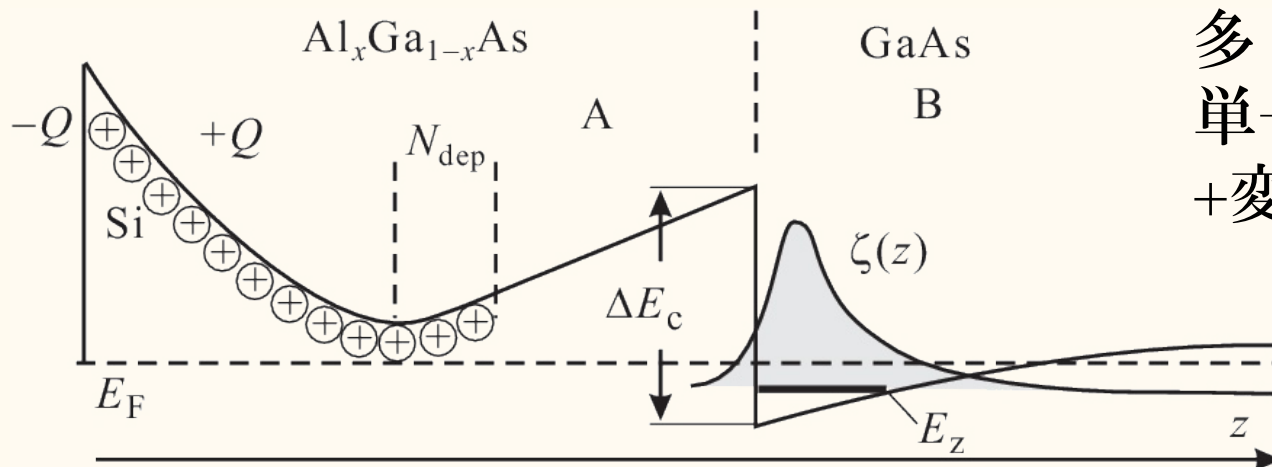
# 2次元電子系



量子井戸：Fermiエネルギーが基底準位とその上の準位の間にある →  $z$ 方向の運動の自由度はない。



$xy$ 面内には自由に動ける  
2次元電子系



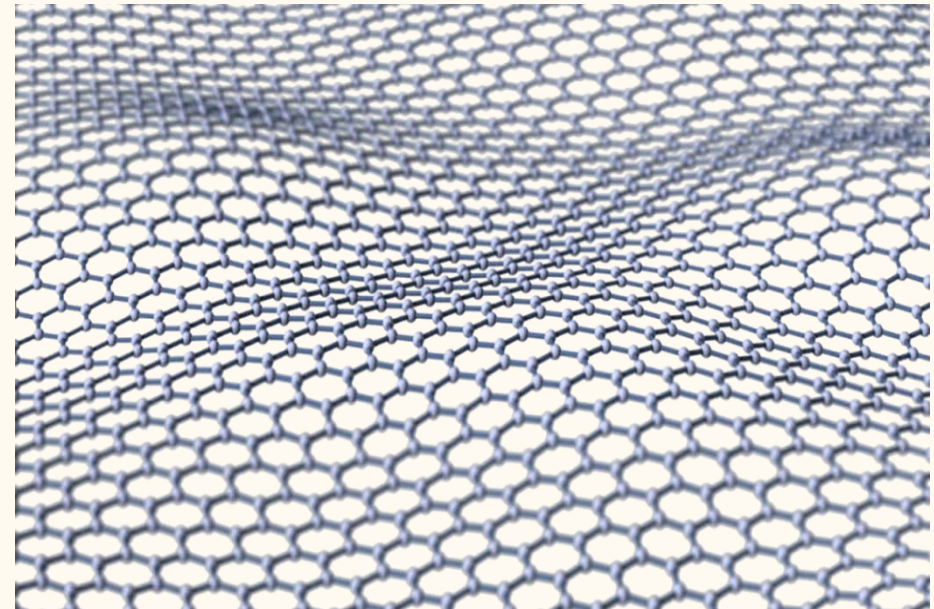
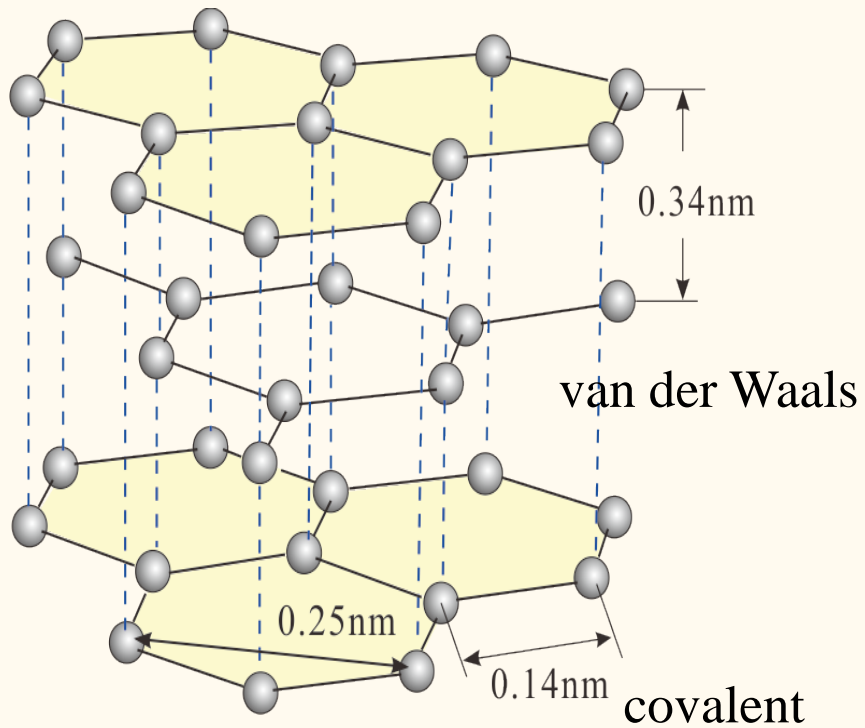
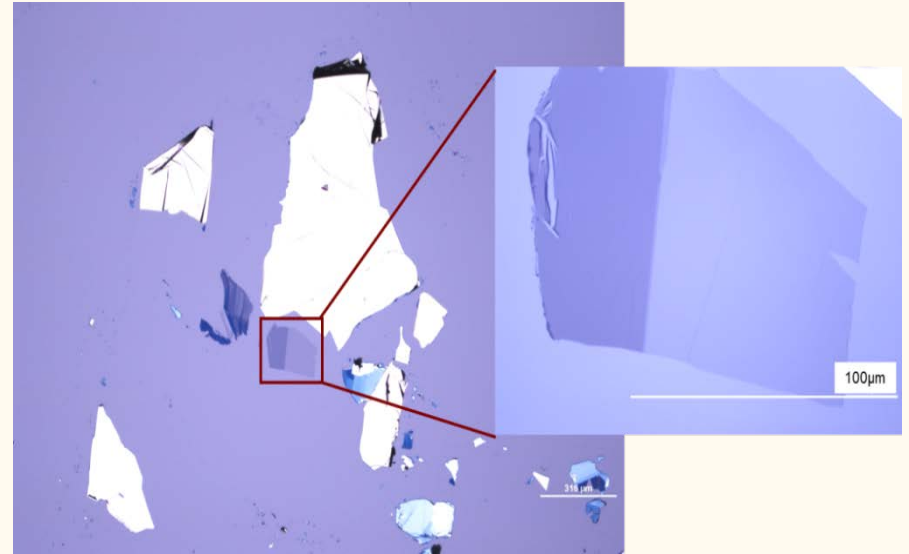
多くの場合  
単一ヘテロ接合  
+変調ドーピング

# Graphene: A two-dimensional material

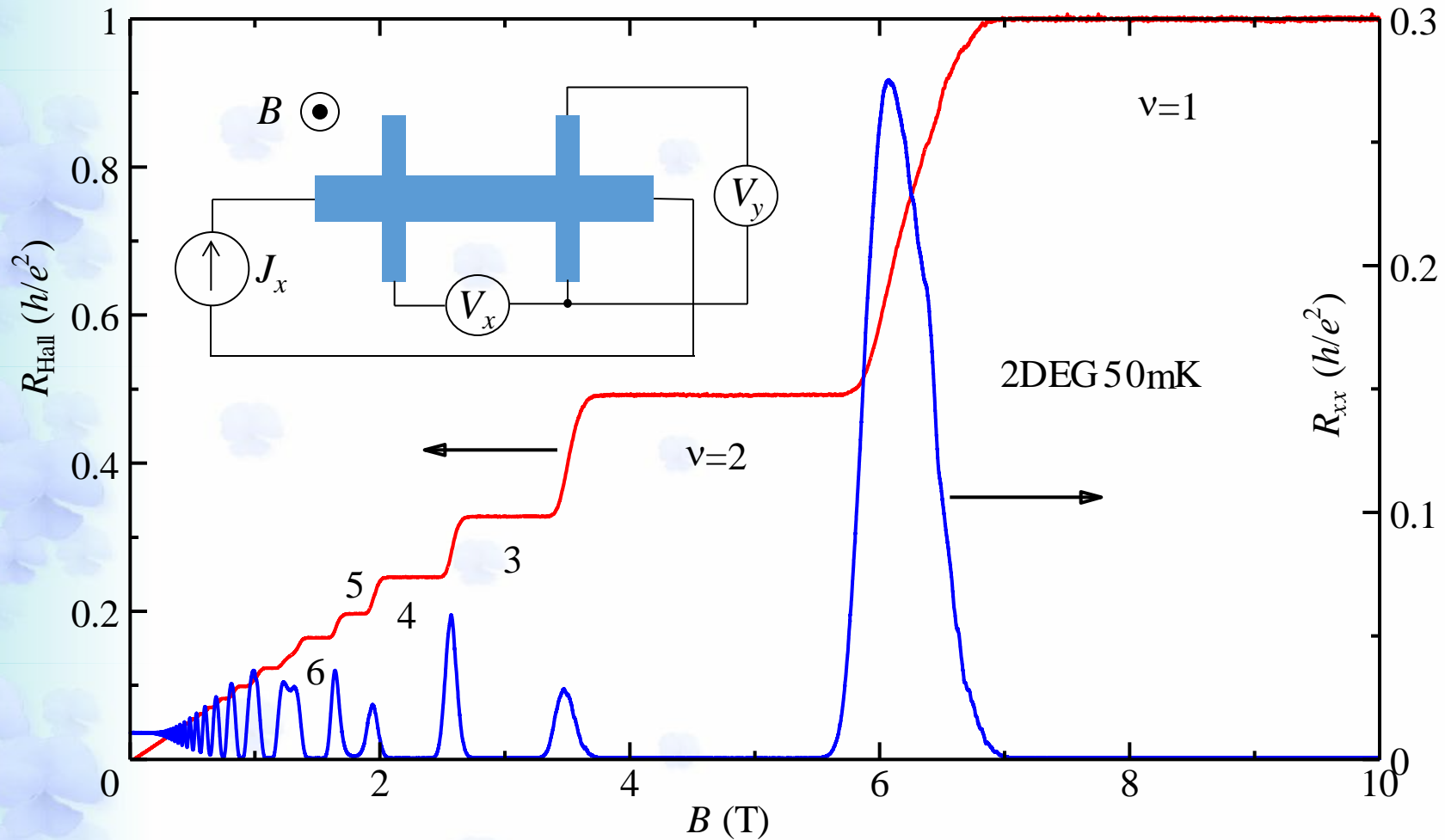
Graphite



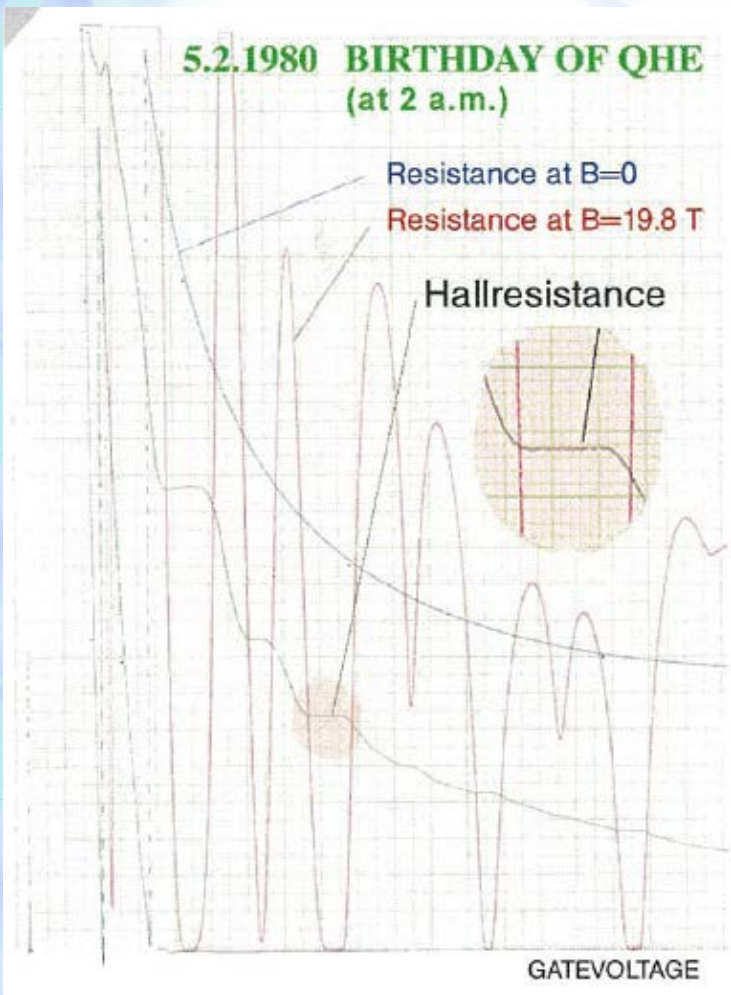
Graphene



# Integer Quantum Hall Effect



# Birthday of quantum Hall effect



**Notes 4/5.2.1980**

rotating sample holder

pin connections

$$E_u = R_H \cdot D \cdot j = \frac{1}{n \cdot e} \cdot B \cdot \frac{I}{b}$$

$$U_H = \frac{B}{n \cdot e} \cdot I$$

$$U_H = \frac{2\pi \hbar}{e \cdot e} \cdot \frac{I}{2\pi k} = \frac{h}{e^2} \cdot I$$

$$\frac{d}{dV} \frac{1}{\epsilon_0} = \frac{1}{e^2} = \rho_{xy} = \frac{1}{2} \cdot \frac{1}{\epsilon_0} \cdot \sqrt{\frac{\hbar}{e^2}} \Rightarrow 25813 \Omega$$

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Acm}$$

$$\epsilon_0 = 0.8854 \cdot 10^{-12} \frac{As}{Vm}$$

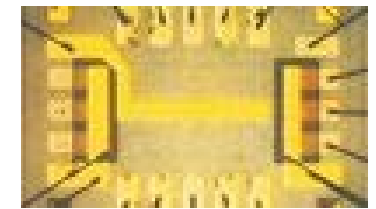
$$\sqrt{\frac{\epsilon_0}{\mu_0}} = 2.65 \cdot 10^{-3} \sqrt{2}^{-1}$$

$$\sqrt{\frac{\hbar}{e^2}} = 376.7 \Omega$$

$$25813 \Omega : N$$

1M $\Omega$ parallel	25813	$\rightarrow$ 25763.46
	129065	12742.04
	645325	6411.27
	226.13	326.25
	2151.68	2146.47

quantized resistances with and without the input resistance of the x-y recorder

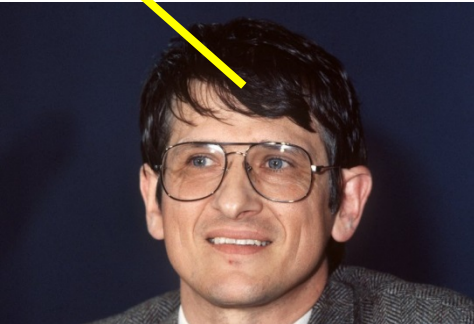
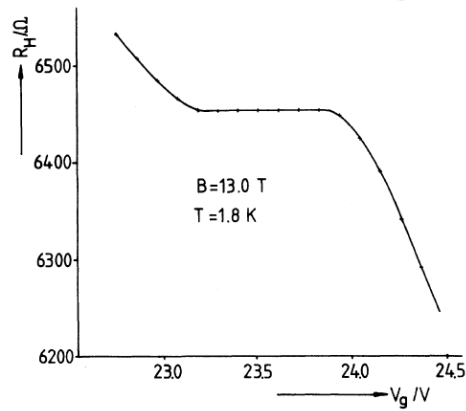
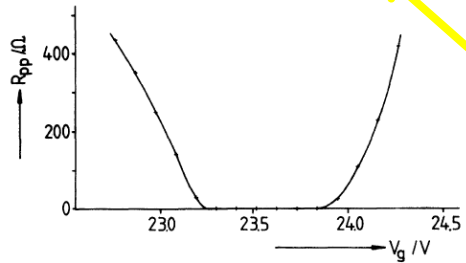


Klaus von Klitzing

# Discovery of quantum Hall effect

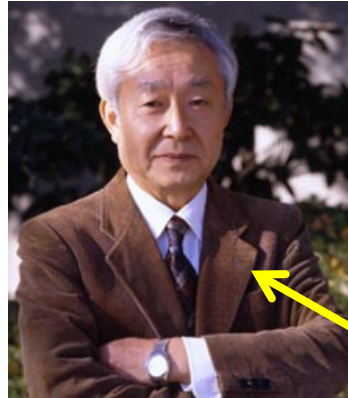


Klaus von Klitzing

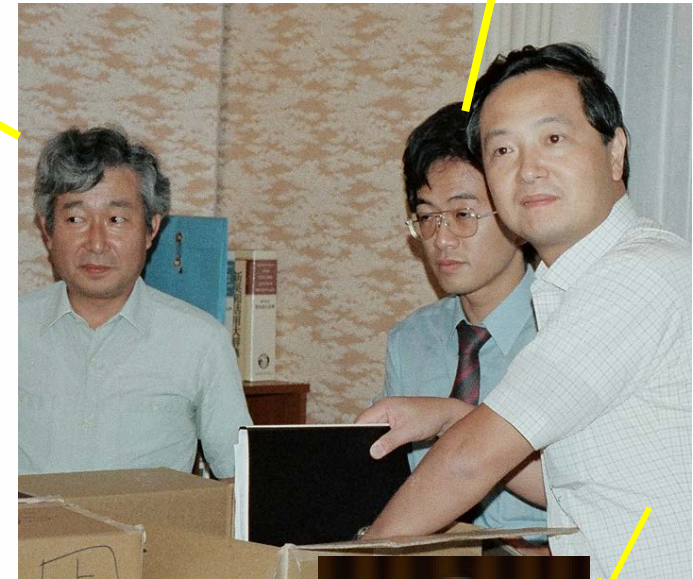


Shinji Kawaji

Phys. Rev. Lett. **45**, 494 (1980)



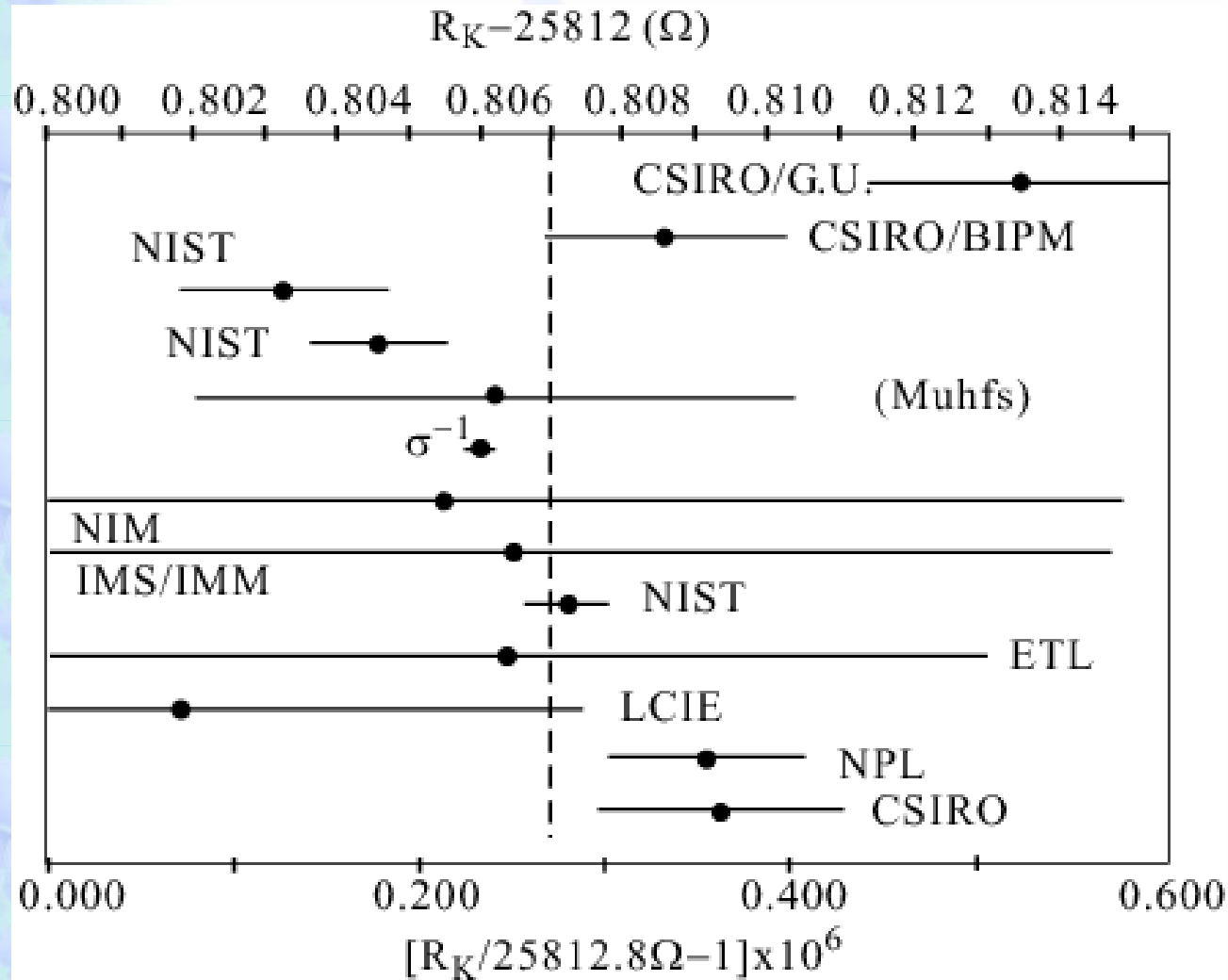
Yasuhiro Iye



Tsuneya Ando



# Standard of resistance



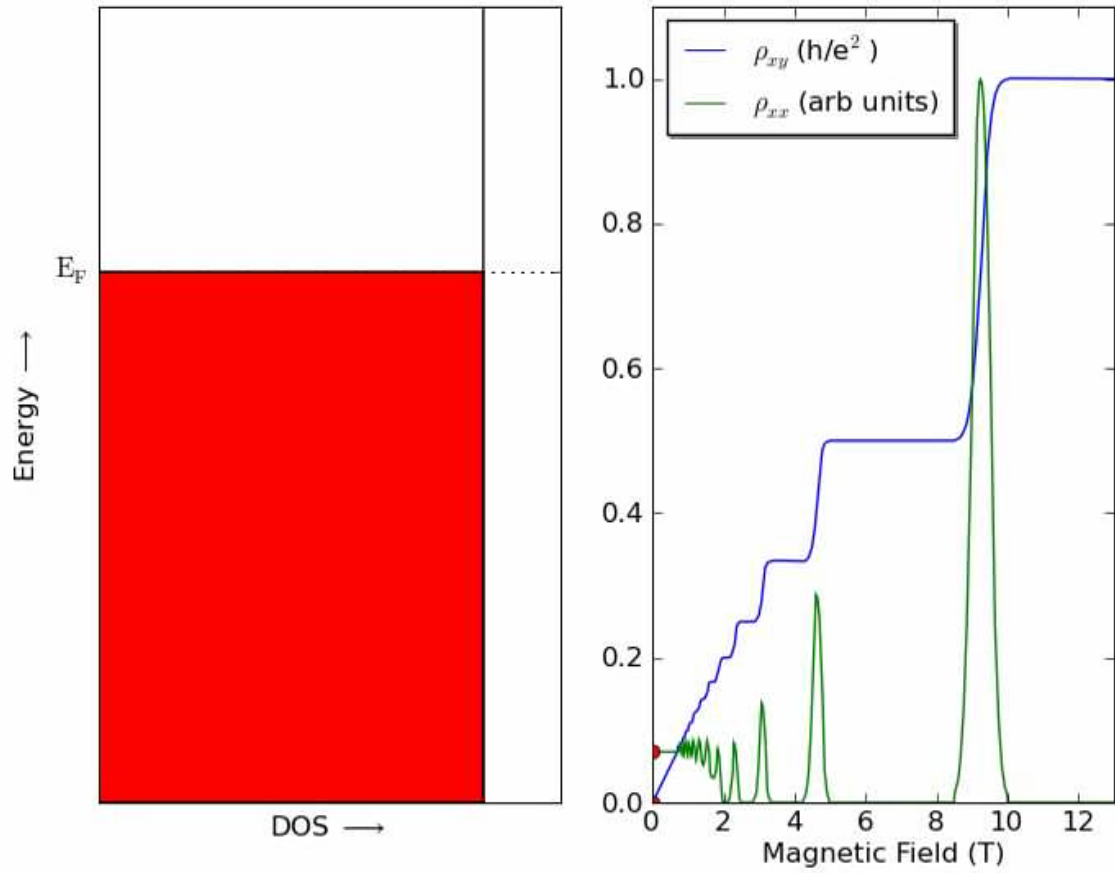
$$R_K = \frac{h}{e^2}$$

von Klitzing constant

25812.8074434(84) V/A

$\pm 3.2 \times 10^{-10}$

# IQHE and Landau quantization

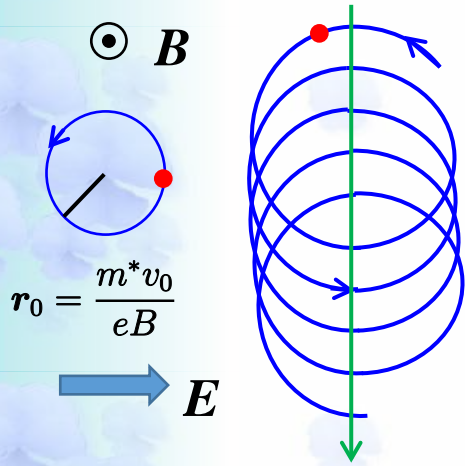


From Wikipedia



# Two dimensional electrons under magnetic field

Lorentz force  $m^* \ddot{\mathbf{r}} = -e\mathbf{v} \times \mathbf{B}$



$$\mathbf{r} = \mathbf{R} + r_0(\cos \omega_c t, \sin \omega_c t)$$

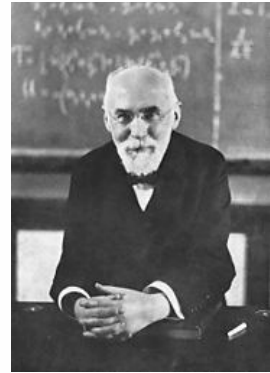
$$\omega_c \equiv \frac{eB}{m^*}, \quad r_0 \equiv \frac{v_0}{\omega_c}$$

Cyclotron frequency, radius

$\mathbf{R}$  : Guiding center

$$m^* \ddot{\mathbf{r}} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

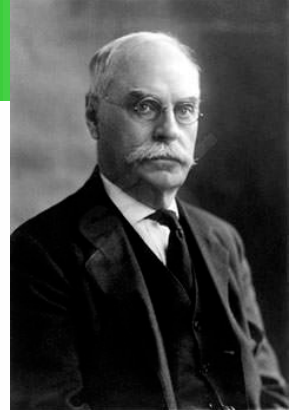
$\mathbf{R}$ : Moves vertically to  $\mathbf{E}$   
with constant velocity  $E/B$



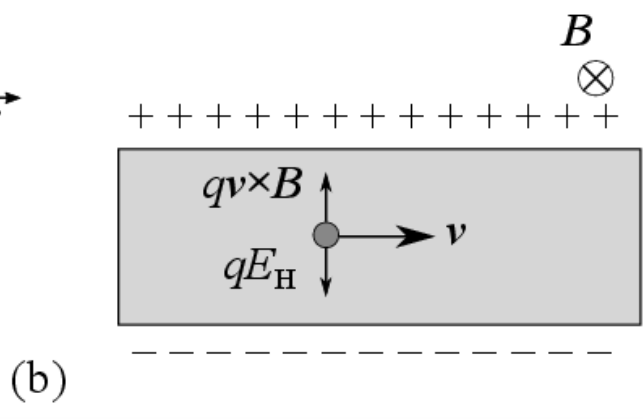
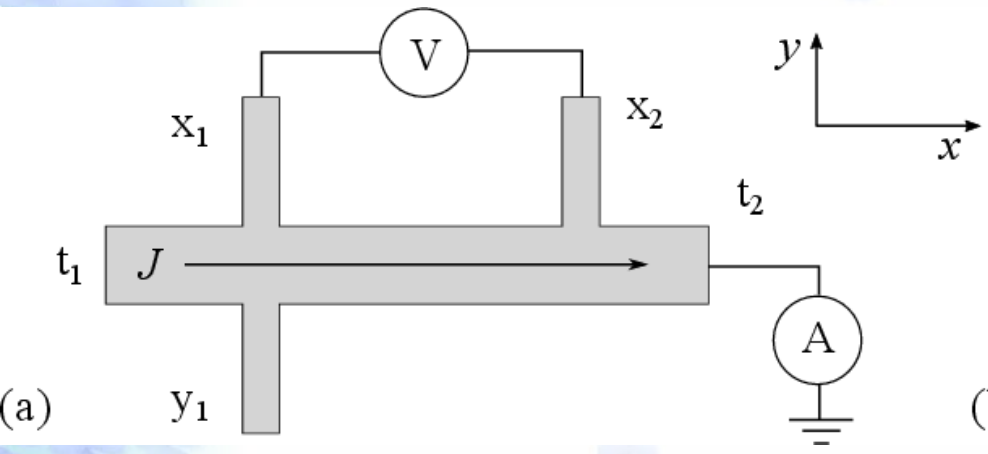
Hendrik Lorentz  
1853 - 1928



# Hall 効果



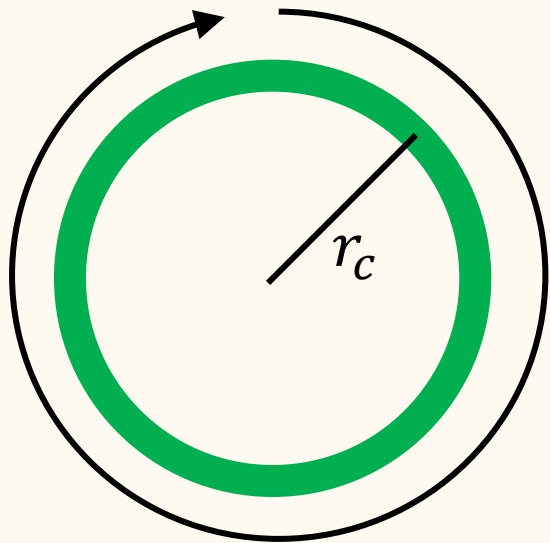
Edwin Hall  
1855-1938



$$E_H = vB = \frac{j}{q\rho} B, \quad R_H \equiv \frac{E_H}{jB} = \frac{1}{q\rho} \quad j: \text{電流密度}$$

3次元, 2次元問わず成立します

# Landau量子化 (の前期量子論版)



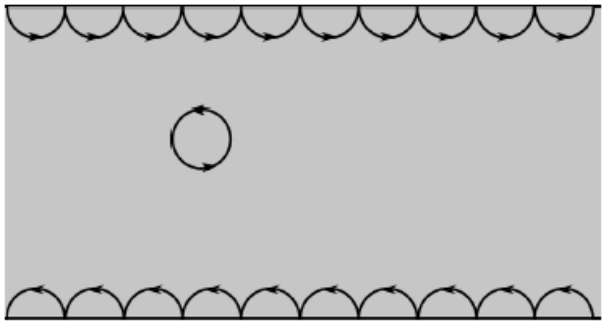
$$\begin{aligned}\Delta\theta &= \frac{1}{\hbar} \oint_C \mathbf{p} \cdot d\mathbf{s} \\ &= \frac{1}{\hbar} \oint_C (m\mathbf{v} + e\mathbf{A}) \cdot d\mathbf{s} \\ &= \frac{1}{\hbar} (2\pi r_c m v + e\pi r_c^2 B) \\ &= \frac{\pi r_c^2 |e| B}{\hbar} = 2\pi \frac{\phi}{\phi_0} \quad \left( \phi_0 \equiv \frac{h}{|e|} \right)\end{aligned}$$

$$\phi = n\phi_0 \quad (n = 1, 2, 3, \dots)$$

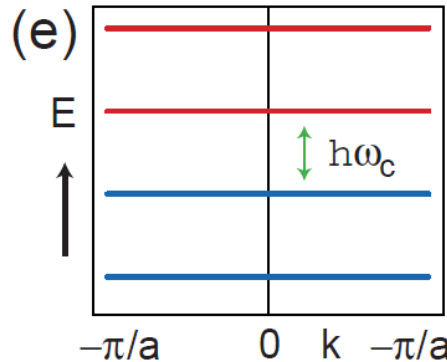
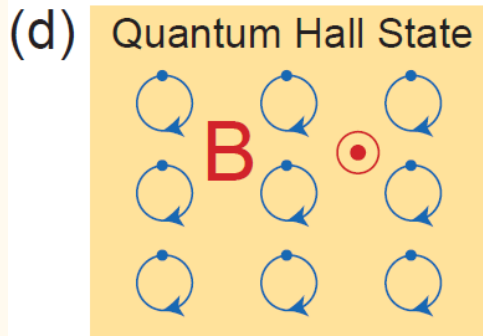
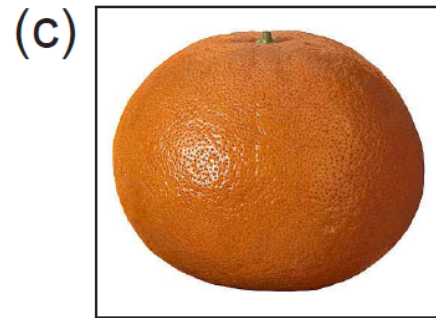
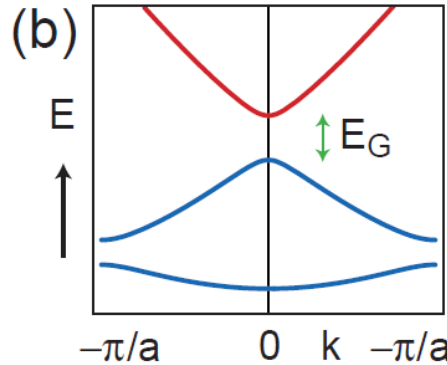
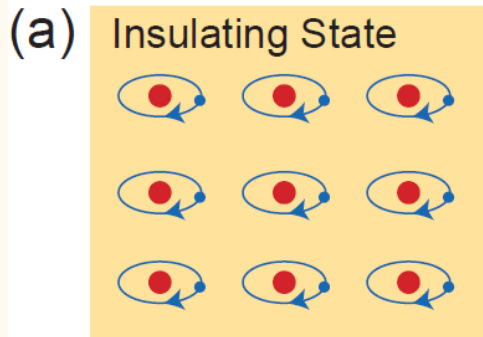
$$E = \frac{mv^2}{2} = \frac{m}{2} (\omega_c r_c)^2 = \frac{m\omega_c}{2\pi} \frac{|e|B}{m} (\pi r_c^2)$$

$$E_n = n\hbar\omega_c \quad \text{正確には } E_n = \hbar\omega_c \left( n + \frac{1}{2} \right) \quad (n = 0, 1, 2, \dots)$$

# 量子ホール端状態

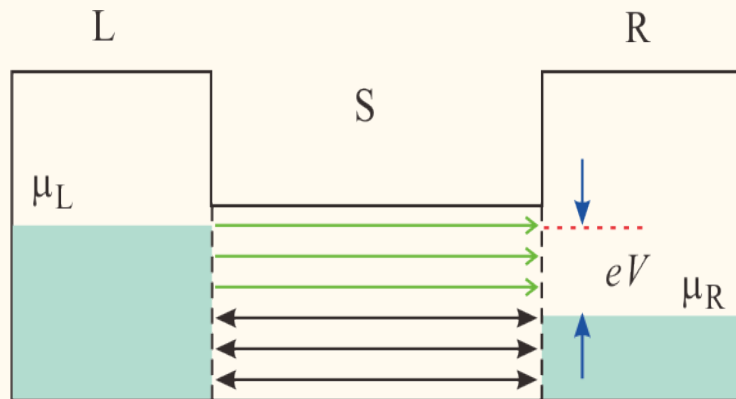


量子化スキッピング軌道  
カイラル(方向性) 1次元系  
Fermiエネルギー以下のLandau準位  
数だけ並列に走っている



物質中の  
トポロジー  
に関係した  
境界状態

# Conductance quantum



L, R : Particle reservoirs

Thermal equilibrium:  
well defined chemical potentials

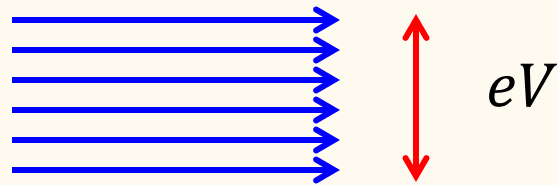
Instantaneous thermalization:  
particles lose quantum coherence

$$j(k) = \frac{e}{L} v_g = \frac{e}{\hbar L} \frac{dE(k)}{dk} \quad L: \text{wavefunction normalization length}$$

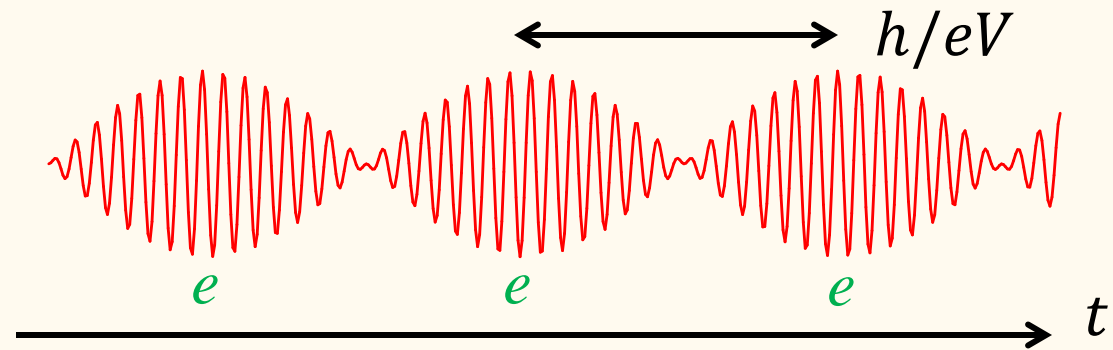
$$J = \int_{k_R}^{k_L} j(k) \frac{L}{2\pi} dk = \frac{e}{h} \int_{\mu_R}^{\mu_L} dE = \frac{e}{h} (\mu_L - \mu_R) = \frac{e^2}{h} V$$

$$G = \frac{J}{V} = \frac{e^2}{h} \equiv G_q \quad \text{Conductance quantum} \quad \left( \frac{2e^2}{h} \equiv G_q \text{ spin freedom} \right)$$

# 1次元系の電気伝導度



できるだけ速く  
独立状態を繰り返し作る



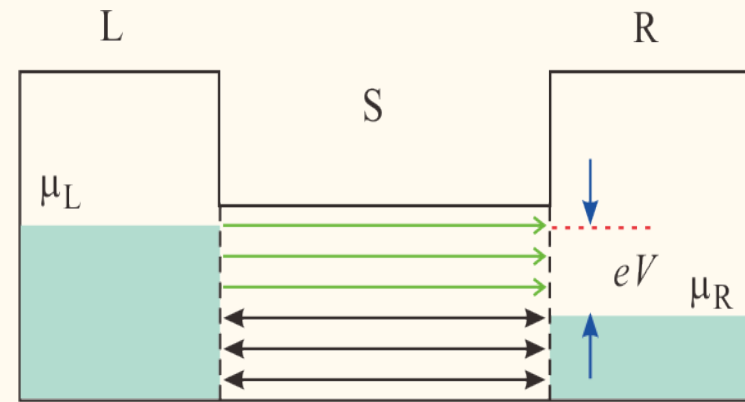
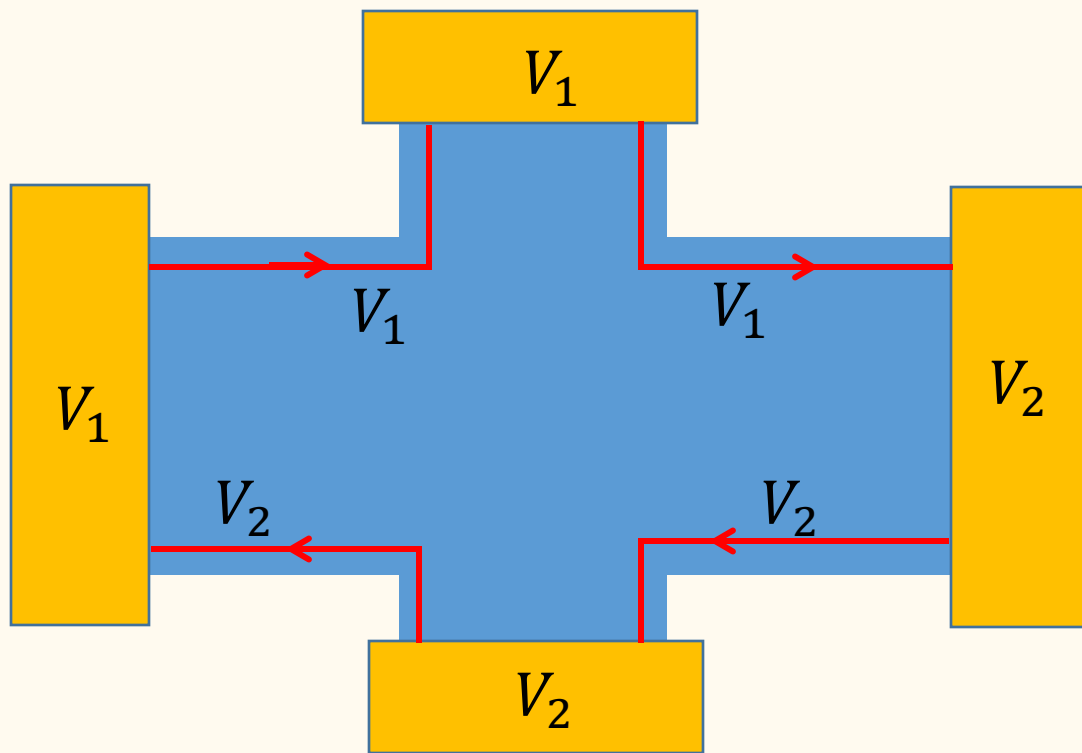
単位時間あたりに運ばれる電荷

$$J = \frac{eV}{h} \times e = \frac{e^2}{h} V$$

伝導度量子（量子化伝導度）

$$G_q \equiv \frac{e^2}{h}$$

# ホール伝導度の量子化



量子ホール端状態  
両端一組で  
一人前の1次元系