Physics of Semiconductors (Appendix)

Shingo Katsumoto
Dept. Physics and Inst. for Solid State Physics, University of Tokyo

July 18, 2016

Appendix K: Confinement with a triangular potential

To solve a problem in final report, the following knowledge on quantum confinement with a triangular potential is required.

Let us approximate a confinement potential for two-dimensional electrons with a downward triangular potential. This is a simple but a good approximation for confinement at MOS interfaces. The problem can be viewed as a slope with a limit wall, which is a well known problem in elementary quantum mechanics. We take it as a one-dimensional potential problem along x-axis and write down the Schrödinger equation as

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x)\right)\psi=E\psi,\quad V(x)=\begin{cases}ax & (x>0,\quad a>0)\\ \infty & (x\le 0)\end{cases}. \tag{8.1}$$

The equation can be rewritten with a variable transformation

$$s=\left(\frac{2ma}{\hbar^2}\right)^{1/3}\left(x-\frac{E}{a}\right). \tag{8.2}$$

as

$$\frac{d^2\psi}{ds^2}=s\psi. \tag{8.3}$$

This differential equation is mathematically called Airy or Stokes-type differential equation. The solutions are called **Airy functions**, which are classified with the asymptotic behavior for $s\to\infty$, into Ai ($\psi\to 0$) and Bi ($\psi\to\infty$). They are plotted in Fig.8.1(b). As the basis of quantum mechanical wavefunctions we should adopt Ai for vanishment at the infinite point.

Asymptotic forms of Ai for $s\to\pm\infty$ are given as

$$\mathrm{Ai}(s)\sim\frac{1}{2\sqrt{\pi}s^{1/4}}\exp\left(-\frac{2}{3}s^{3/2}\right)\quad(s\to\infty) \tag{8.4}$$

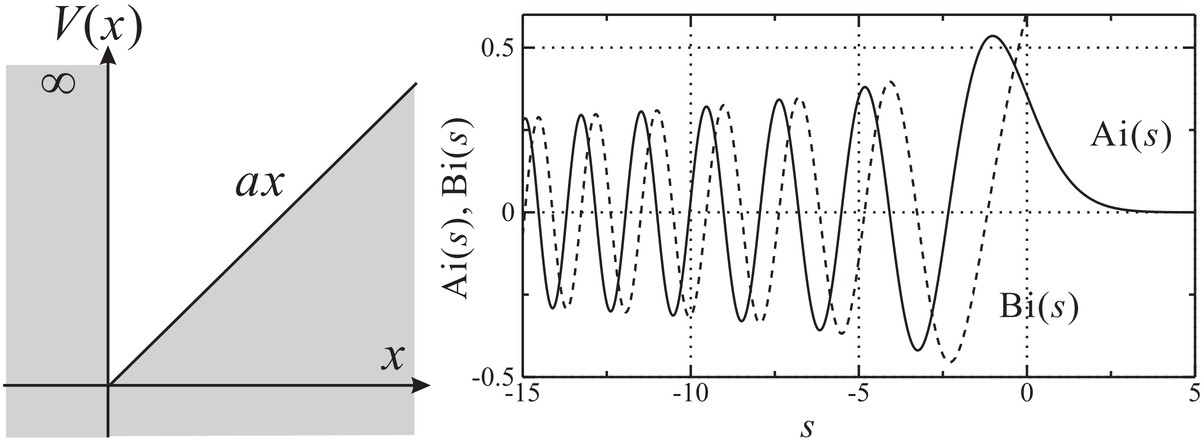
$$\sim\frac{1}{\sqrt{\pi}|s|^{1/4}}\cos\left(\frac{2}{3}|s|^{3/2}-\frac{\pi}{4}\right)\quad(s\to-\infty). \tag{8.5}$$

In $x<0$, $V=\infty$ and the boundary condition at $x=0$ is $\psi(+0)=0$. From the constraint, a zero of $\psi(x)$ should be placed at $x=0$. We write zeros of Ai from the smallest with ascending order $s_1, s_2, \cdots s_n, \cdots$. Then from eq.(8.2), the energy eigenvalue E_n correspondint to n is obtained as

$$E_n=-\left(\frac{\hbar^2a^2}{2m}\right)^{1/3}s_n, \tag{8.6}$$

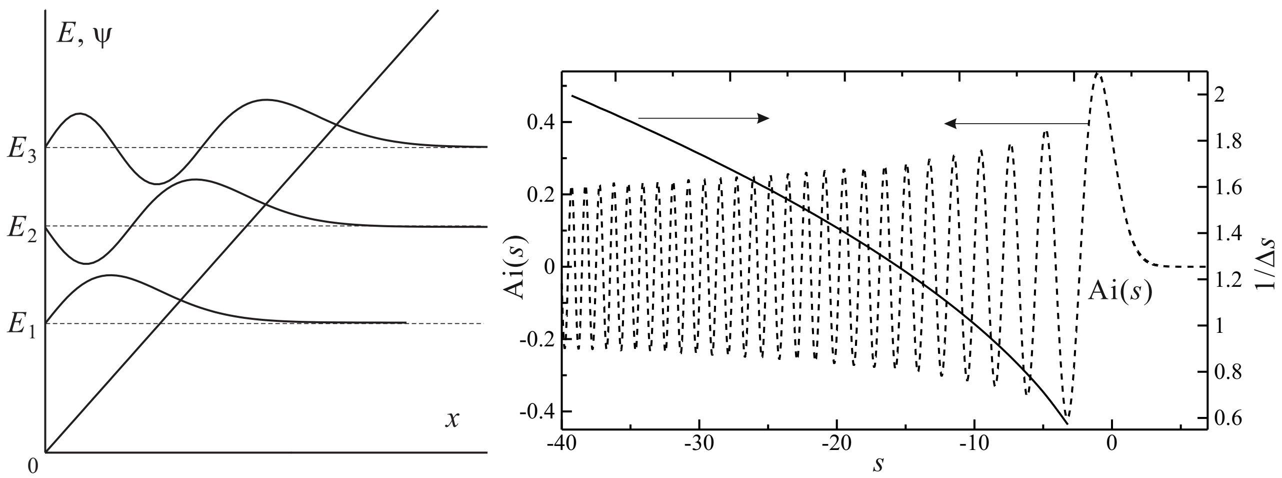
From the asymptotic form eq.(8.5), approximate values for s_n for large n are given as

$$s_n\sim-\left(\frac{3\pi(4n-1)}{8}\right)^{2/3}. \tag{8.7}$$



(a) (b)

Figure 8.1: (a) Schematic illustration of a triangular potential. (b) Airy functions.



(a) (b)

Figure 8.2: (a) Energy eigenvalues and eigenstates in a triangular confinement potential. Drawn from the lowest energy for $n = 1, 2, 3$. (b) Inverse of distance between neighboring zero Δs of Airy function Ai(s) as a function of s. Ai(s) is also plotted as a broken curve.