Final report problems for "Semiconductors"

Problem setting: 14/07/2021 出題 2021 年 7 月 14 日
Solution submission deadline: 16/08/2021 解答提出期限 2021 年 8 月 16 日

General notes / 一般的注意

Choose three of eight problems and answer them. The difficulties of the problems are not equal. Some of them are easy, some not. But not tough anyway. You can choose the problems according to your interests and motivation.

The text part in the answer should be typed in English or Japanese. I hope you could avoid handwriting but if you have no way but to do so, the handwriting should be as clear as possible. The scoring does not depend on the language. It doesn't matter if you are good at grammar, vocabulary, or sentences, but if I cannot catch the meaning, the scoring will get deducted regardless of English or Japanese. The answer sheet should be in small-sized PDF format, which can be appropriately displayed by Adobe Reader. The file of the answer should be submitted through ITC-LSM.

8 問のうち 3 問を選んで答えてください．問題はとても簡単なものからそうでもないものまでありますが，いずれにしても大して難しくありません．あなたの興味と意欲に応じて選んでください．

解答のテキスト部分は極力手書きでないようにお願いします．英語，日本語のどちらでも良く，採点は言語に依存しません．文法や語法，文章の上手下手は問題にしませんが，意味が取れない場合は，英語日本語にかかわらず，減点します．解答は，ファイルサイズのできるだけ小さな Adobe Reader できちんと表示できる PDF ファイルにまとめ，ITC-LSM を通して提出してください．

FR1 Bipolar transistor

Let us consider a pn-junction of Si at the temperature 300 K. In the p-layer the acceptor (boron, B) concentration is 10^{21} m^{-3} and in the n-layer the donor (phosphorous, P) concentration is 10^{20} m^{-3}. The doping profile is abrupt.

(1) Obtain the built-in potential.
(2) Calculate the depletion layer widths for p- and n-layers at reverse bias voltage -5 V.
(3) Calculate the differential capacitance at reverse bias voltage -5 V for the area 1 mm$\times$1 mm.

Let us put another p-layer and make a pnp transistor (gedankenexperiment). Assume the followings: the hole diffusion length in the base is 10 μm; the electron diffusion length in the emitter is also 10 μm; the diffusion coefficients of minority carriers in the emitter and the base are the same ($D_e = D_h$).

(4) Calculate $h_{\rm FE}$ for base widths 0.5 μm and 0.1 μm. (Ignore depletion layer widths, other non-ideal factors. Calculate under the simplest approximation.)

To solve this problem use the discussions in 6A.1 (typos are corrected in July version).

FR2 SdH oscillation

Figure FR2.1 shows the Shubnikov-de Haas oscillation and the quantum Hall effect in two-dimensional electrons.

(1) Calculate the electron concentration from the low (<0.5 T) field data.

(2) Something happened around 0.65 T. What is it?

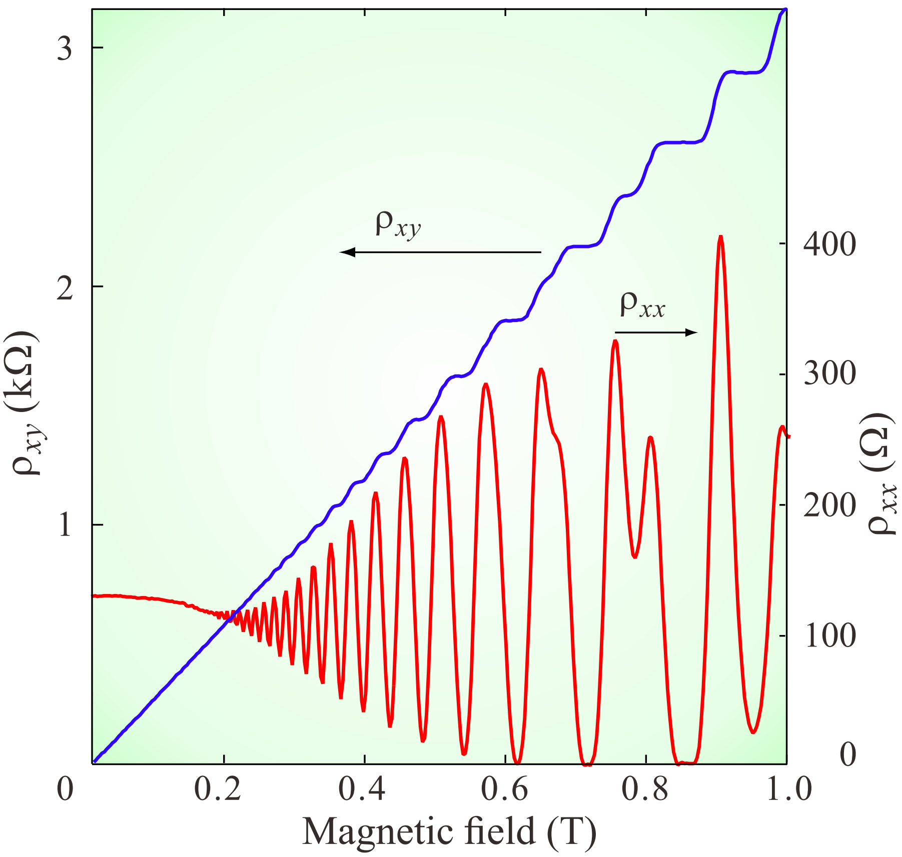

Fig. FR2.1 SdH oscillation in a 2DEG at an AlGaAs/GaAs interface (red line). Blue line is the Hall resistivity. T =30 mK.

FR3 Double barrier diode

Consider a double barrier resonant diode with GaAs as the well material and $Al_{0.4}Ga_{0.6}As$ as the barrier material. Lets adopt E_g=1.424 eV for GaAs and E_g=$1.424 + 1.265x + 0.265x^2$ (eV) for $Al_xGa_{1-x}As$ and $\Delta E_c : \Delta E_v$ =6:4. The electron effective mass in GaAs is $0.067m_0$ and ignore the change in AlxGa$_{1-x}$As. Consider n-type electrodes (note that in the lecture we considered p-type).

(1) Obtain the transfer matrix of 5nm thickness GaAs- $Al_{0.4}Ga_{0.6}As$.

(2) Calculate the transmission probability of resonant diode with two 5nm barriers and a 5nm well region as a function of incident energy (from 0 to the top of the barrier with an appropriate interval) and plot in a figure.

FR4 Triangular potential

Let us consider the rectangular potential illustrate in Fig.FR4.1.

(1) First consider the most coarse approximation. Choosing a kinetic energy E determines the effective potential with E/a. Now let us approximate the potential with a rectangular potential of width E/a, bottom V(0), infinite barrier height. Let m^* be the effective mass and obtain the eigen energies from lower level with index $n = 1, 2, \cdots$

(2) Compare the above result with more accurate one on Airy functions.

(3) Also try comparison with Wenzel-Kramers-Brillouin (WKB) approximation for wavefunction penetration into the barrier.

For the solution of triangular potential problem, see Appendix FRA.

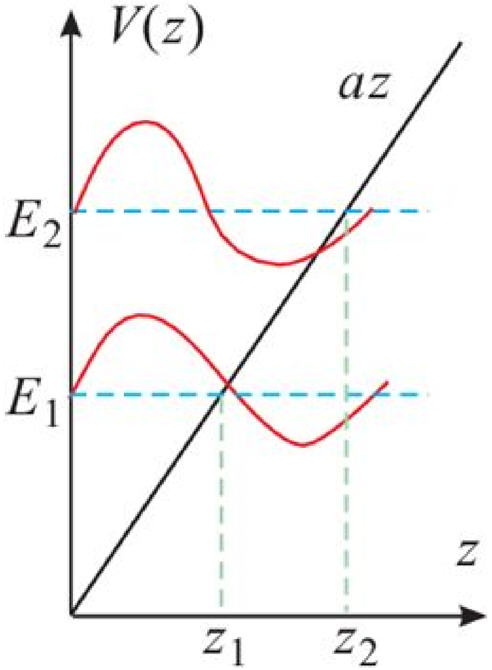

Fig. FR4.1 Schematic diagram of a triangular potential.

FR5 Edge mode transport

In Fig.FR5.1 the green region indicates 2DEG, 1 to 6 are the electric contacts, the yellow regions are metallic gates. The structure has a quantum point contact in the middle. In the integer quantum Hall state with filling factor ν, the sample has ν edge modes at sample-vacuum eeges. With applying gate voltage, we can tune the number of modes which transmit through the QPC, to χ. Other modes are completely reflected by the QPC. The current is through 1 and 4.

(1) Obtain the longitudinal resistance R_{L}, which is measured from the voltage between 2 and 3 V_{23} or 6 and 5 V_{65}.

(2) Obtain the Hall resistance R_{H}, measured from V_{26} or V_{35}.

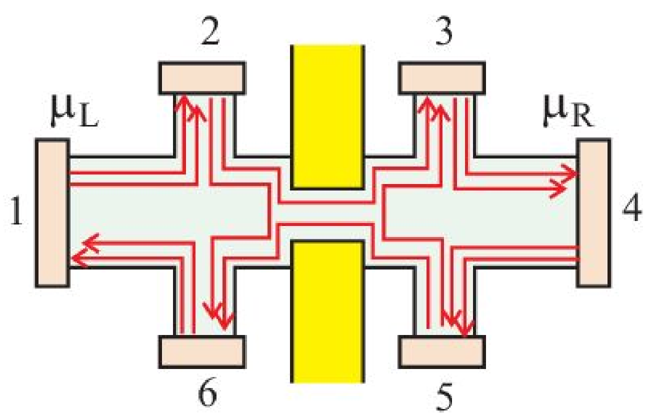

Fig. FR5.1 Illustration of the sample with two edge modes ($\nu = 2$). Yellow regions around the center are Schottky-type split gates.

FR6 Quantum Hall conductivity puzzle(?)

Consider a 2DEG under IQHE with ν =1. The edge modes can bring finite current without energy dissipation and the resistance is zero. The conductance of one-dimensional edge mode is then the inverse of the resistance and infinity. Let us write the quantum resistance h/e^2 as $R_{\rm q}$.

Two dimensional resistivity tensor under the condition is

$$\boldsymbol{\rho} = \begin{pmatrix} 0 & R_{\rm q} \\ -R_{\rm q} & 0 \end{pmatrix}.$$

Then the two dimensional conductivity tensor defined by the inverse of resistivity tensor

$$\boldsymbol{\sigma} = \boldsymbol{\rho}^{-1} = \frac{1}{R_{\rm q}^2} \begin{pmatrix} 0 & -R_{\rm q} \\ R_{\rm q} & 0 \end{pmatrix}.$$

That is $\sigma_{xx} = 0$! Does the calculation contain an error? If it does, what is the error? Or can you solve the puzzle?

FR7 Absorption coefficient of a nanowire

For a bulk (three-dimensional) semiconductor crystal, the frequency-dependence of the absorption coefficient $\alpha(\omega)$ is given by

$$\alpha(\omega) \propto (\hbar\omega - E_{\rm g})^{1/2} \quad (\hbar\omega \geq E_{\rm g})$$

as in Eq. (4.68). With a similar analysis to that used in deriving the above relation, find the frequency dependence of the absorption coefficient for an infinitesimally thin semiconducting “nanowire.” The answer should be in the form

$$\alpha(\omega) \propto (\hbar\omega - E_{\rm g})^{\gamma} \quad (\hbar\omega > E_{\rm g}).$$

And for the answer, it is enough just to obtain the exponent γ.

FR8 Basics in band theory

(1) Show that tight-binding approximation to the simple cubit lattice gives the dispersion as

$$E_n(\boldsymbol{k}) = E_n - \alpha_n - 2t \sum_{j=x,y,x} \cos k_j a. \tag{FR8.1}$$

Apply the same to the body-centered cubic and the face-centered cubit structures.

(2) Wavefunctions at the top of valence band (Γ-point) in sp^3-bonding diamond structure semiconductors can be written to the second order of $k \cdot p$ approximation as

$$\text{Heavy hole band: } \left| \frac{3}{2}, \pm\frac{1}{2} \right\rangle = \frac{1}{\sqrt{6}} \left\{ 2|z\rangle \begin{pmatrix} \alpha \\ \beta \end{pmatrix} - (|x\rangle \pm i|y\rangle) \begin{pmatrix} \beta \\ \alpha \end{pmatrix} \right\}, \tag{FR8.2}$$

$$\text{Light hole band: } \left| \frac{3}{2} \pm \frac{3}{2} \right\rangle = \frac{1}{\sqrt{2}} (|x\rangle \pm i|y\rangle) \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \tag{FR8.3}$$

$$\text{Spin split-off band: } \left| \frac{1}{2}, \pm\frac{1}{2} \right\rangle = \frac{1}{\sqrt{3}} \left\{ |z\rangle \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + (|x\rangle + i|y\rangle) \begin{pmatrix} \beta \\ \alpha \end{pmatrix} \right\}. \tag{FR8.4}$$

where α and β are spin part of the wavefunction, and $|x\rangle$, $|y\rangle$, $|z\rangle$ are just showing the symmetry along the axes. The upper and lower rows for α and β correspond to the double sign $\pm$ or $\mp$.

Show that these functions diagonalize the spin-orbit interaction

$$H_{\rm so} = \frac{C_{\rm so}}{r^3}(\boldsymbol{l}\cdot\boldsymbol{\sigma}), \tag{FR8.5}$$

where $\boldsymbol{l}$ is the orbital angular momentum and $\boldsymbol{\sigma}$ is the vector of Pauli spin matrices.

Appendix FRA: Eigenstates in a triangular potential

Let us consider one dimensional triangular potential on x-axis. The time independent Schrödinger equation is written as

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x)\right)\psi = E\psi, \quad V(x)=\begin{cases} ax & (x>0, \quad a>0) \\ \infty & (x\le 0)\end{cases}. \tag{FRA.1}$$

With adopting the transformation of the variable

$$s=\left(\frac{2ma}{\hbar^2}\right)^{1/3}\left(x-\frac{E}{a}\right), \tag{FRA.2}$$

the Schrödinger equation is transformed into

$$\frac{d^2\psi}{ds^2}=s\psi. \tag{FRA.3}$$

The equation is now in the form of differential equation called Airy's (or Stokes') differential equation.

The solutions to (FRA.3) are called **Airy functions** and classified with the asymptotic behavior in $s\to\infty$ into Ai for $\psi\to 0$ and Bi for $\psi\to\infty$. Some representatives of them are plotted in Fig.**??**(b).

As basis for the bound state wavefuntions, we should adopt Ai, which are zero at infinity. The asymptotic form for $s\to\pm\infty$ is given as

$$\mathrm{Ai}(s) \sim \frac{1}{2\sqrt{\pi}s^{1/4}}\exp\left(-\frac{2}{3}s^{3/2}\right) \quad (s\to\infty) \tag{FRA.4}$$

$$\sim \frac{1}{\sqrt{\pi}|s|^{1/4}}\cos\left(\frac{2}{3}|s|^{3/2}-\frac{\pi}{4}\right) \quad (s\to-\infty). \tag{FRA.5}$$

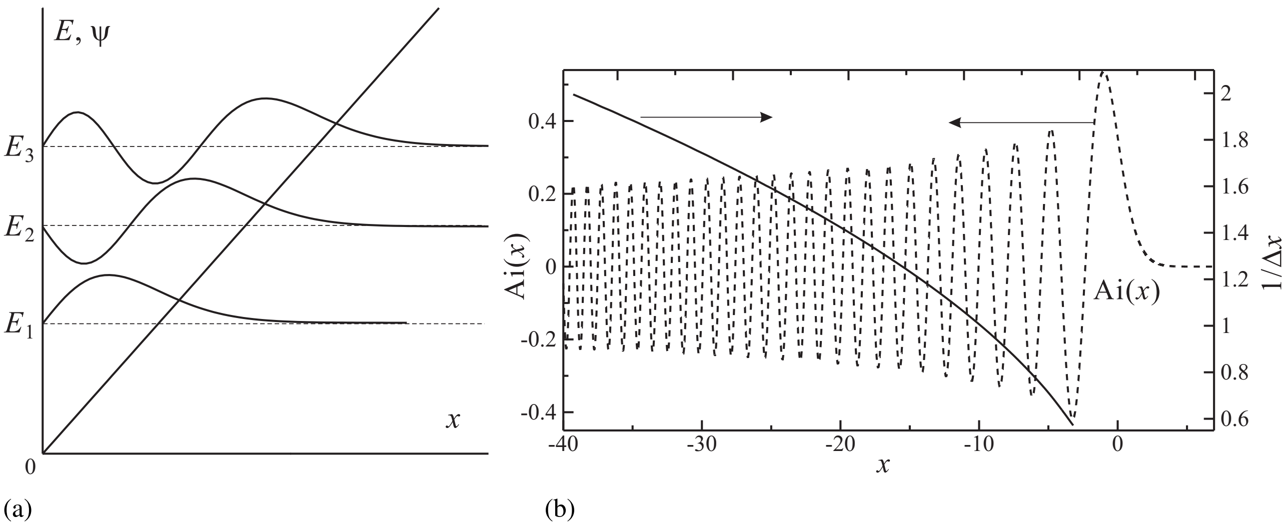

Fig. FRA.1 (a) illustrates eigenenergies and eigenfunctions in a triangular potential for $n=1,2,3$ from the ground state. (b) Let Δx be the intervals of zero points in Airy function. In this figure $1/\Delta x$ is plotted against the midpoints between the zero points. The broken line is Airy function.

In $x < 0$, $V = \infty$ and $\psi = 0$, the boundary condition at $x = 0$ is thus $\psi(+0) = 0$. Ai(s) has many zeros and the boundary condition requires that one of which must fit to $x = 0$. Let us write such zero points as $s_1, s_2, \cdots s_n, \cdots$ in the order of the absolute value of s, then the energy eigenvalue E_n is obtained from (FRA.2) as

$$E_n = -\left(\frac{\hbar^2 a^2}{2m}\right)^{1/3} s_n. \tag{FRA.6}$$

From the asymptotic form (FRA.5),

$$s_n \sim -\left(\frac{3\pi(4n-1)}{8}\right)^{2/3} \tag{FRA.7}$$

is the asymptotic solution of s_n for $n \to \infty$.