Exercise 0714 for "Semiconductors"

Problem setting: 30/06/2021 出題 2021 年 6 月 30 日
Solution submission deadline: 14/07/2021 解答提出期限 2021 年 7 月 14 日

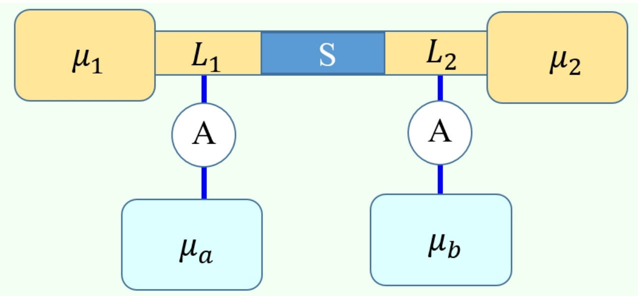
General notes / 一般的注意

The text part in the answer should be typed in English or Japanese. I hope you could avoid handwriting but if you have no way to do so, the handwriting should be as clear as possible. The scoring does not depend on the language. It doesn't matter if you are good at grammar, vocabulary, or sentences, but if I cannot catch the meaning, the scoring will get deducted regardless of English or Japanese. The answer sheet should be in small-sized (hopefully less than 1 MB) PDF format, which can be appropriately displayed by Adobe Reader. The file of the answer should be submitted through ITC-LSM.

解答のテキスト部分は極力手書きでないようにお願いします．英語，日本語のどちらでも良く，採点は言語に依存しません．文法や語法，文章の上手下手は問題にしませんが，意味が取れない場合は，英語日本語にかかわらず，減点します．解答は，ファイルサイズのできるだけ小さな (1 MB 以下が目安)，Adobe Reader できちんと表示できる PDF ファイルにまとめ，ITC-LSM を通して提出してください．

0714-1 Four terminal conductance

The general conductance formula for four terminal measurement is given in (8.41). However it is convenient to reduce the expression to the case of a quantum wire with two voltage terminals without feeding of current. That can be obtained with the simple following consideration.



Consider the four-terminal configuration shown in the left. μ_α ($\alpha = 1, 2, a, b$) are chemical potentials of four reservoirs. The transmission coefficient of the sample part (S) is $\mathcal{T}$. L_1 and L_2 are "perfect" leads without any backscattering inside. The current meters (A) indicate zero.

Assume flux 1 is applied from reservoir 1. $1 - \mathcal{T}$ is reflected and lead-1 thus contains flux $2 - \mathcal{T}$. Fermi's golden rule gives the number of electrons go into probe-a as

$$\frac{J_{\rm in}}{e} = \frac{2\pi}{\hbar}|t_{1a}|^2 D_a D_1(\mu_1 - \mu_a)(2 - \mathcal{T}).$$

Here D_x is the density of states in reservoir x, t_{xy} is the transition matrix element between reservoir x and y. The counter flow is

$$\frac{J_{\rm in}}{e} = \frac{2\pi}{\hbar}|t_{1a}|^2 D_a D_1(\mu_a - \mu_2)\mathcal{T}.$$

Equating these (because the net current is zero) gives

$$\mu_a = \frac{1}{2}[\mu_1(2 - \mathcal{T}) + \mu_2\mathcal{T}].$$

1. Express μ_b with μ_1, μ_2 and $\mathcal{T}$ as in the above for μ_a.
2. Obtain the expression for four-terminal conducatnce.
3. What happens for $\mathcal{T} = 1$?

0714-2 Cyclotron motion in graphene

Consider electrons in graphene under magnetic field. Let us treat the motion semi-classically.
Cyclotron frequency expression

$$\omega_{\rm c} = \frac{eB}{m^*}$$

cannot be applied because $m^* = 0$. Instead we use the relation $E = pc$.

1. Express the cyclotron radius with momentum and e and B.
2. A circular motion of electron gives kinetic phase and AB phase to the electron. Replace the momentum in the answer to 1. with $\hbar k$ and express the total phase acquired by the electron within a circular motion with the flux through the cyclotron circle and the flux quantum.
3. The acquired phase should be integer times 2π. Obtain the expression for cyclotron radius and energy.