Lecture on

Semiconductors / 半導体

(Physics of semiconductors)

2021.5.26 Lecture 07

10:25 – 11:55

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Review of last week

Concept of exciton-polariton (continued)

Chapter 5 Semi-classical treatment of transport

Transport coefficient

Classical transport: Boltzmann equation

Currents: particle flows

Drude formula, Diffusion current, Hall effect

Various scatterings

Heat transport, Thermoelectric effect

Boltzmann equation and thermoelectric constants

For the thermoelectric effect, we need to consider (only) ∇T

The distribution function in lhs is replaced with unperturbed one.

$$\boldsymbol{v}\cdot\nabla f+\frac{\boldsymbol{F}}{m^*}\nabla_v f=-\frac{f-f_0}{\tau(E)}\approx\left[\boldsymbol{v}\cdot\nabla+\frac{\boldsymbol{F}}{m^*}\nabla_{\boldsymbol{v}}\right]f_0$$

With $a\equiv-\frac{E-E_{\mathrm{F}}}{k_{\mathrm{B}}T}$

$$\frac{\partial f_0}{\partial T}=\frac{\partial f_0}{\partial E}\frac{\partial E}{\partial a}\frac{\partial a}{\partial T}=\frac{\partial f_0}{\partial E}(-k_{\mathrm{B}}T)\left(\frac{E-E_{\mathrm{F}}}{k_{\mathrm{B}}T^2}\right)=\frac{\partial f_0}{\partial E}\frac{E_{\mathrm{F}}-E}{T}$$

From the above we can rewrite

$$\nabla f_0=\nabla T\frac{E_{\mathrm{F}}-E}{T}\frac{\partial f_0}{\partial E},\qquad \nabla_v f_0=\nabla_v E\frac{\partial f_0}{\partial E}=m\boldsymbol{v}\frac{\partial f_0}{\partial E}$$

Then the Boltzmann equation gives

$$f=f_0-\tau(E)\boldsymbol{v}\cdot\left[-e\mathbf{E}+\frac{E_{\mathrm{F}}-E}{T}\nabla T\right]\frac{\partial f_0}{\partial E}$$

Substituting the above and $\mathbf{E}=(\mathcal{E}_x,0,0)$ into the current expression

$$j_x=-e\langle nv_x\rangle=-e\int_0^\infty v_x f(E)\mathscr{D}(E)dE$$

$$=e\int_0^\infty v_x^2\tau\left[-e\mathcal{E}_x+\frac{E_{\mathrm{F}}-E}{T}\frac{\partial T}{\partial x}\right]\frac{\partial f_0}{\partial E}\mathscr{D}(E)dE$$

Boltzmann equation and thermoelectric constants (2)

$$j_x = e\int_0^\infty v_x^2\tau\left[-e\mathcal{E}_x + \frac{E_\mathrm{F}-E}{T}\frac{\partial T}{\partial x}\right]\frac{\partial f_0}{\partial E}\mathscr{D}(E)dE = 0$$

$j_x = 0$ means the balancing of the drift current and the diffusion current

Then the Seebeck coefficient is calculated as

$$S = \frac{\mathcal{E}_x}{\partial T/\partial x} = \int_0^\infty v_x^2\tau\frac{E_\mathrm{F}-E}{eT}\frac{\partial f_0}{\partial E}\mathscr{D}(E)dE \Bigg/ \int_0^\infty v_x^2\tau\frac{\partial f_0}{\partial E}\mathscr{D}(E)dE$$

$$= \frac{1}{eT}\left[E_\mathrm{F} - \int_0^\infty \tau E^2\frac{\partial f_0}{\partial E}\mathscr{D}(E)dE \Bigg/ \int_0^\infty \tau E\frac{\partial f_0}{\partial E}\mathscr{D}(E)dE\right]$$

$$= \langle\tau E\rangle_E/\langle\tau\rangle_E$$

Maxwell approximation

$$\frac{\partial f_0}{\partial E} = -\frac{f_0}{k_\mathrm{B}T}$$

Energy dependence of relaxation time

$$\tau \propto E^s$$

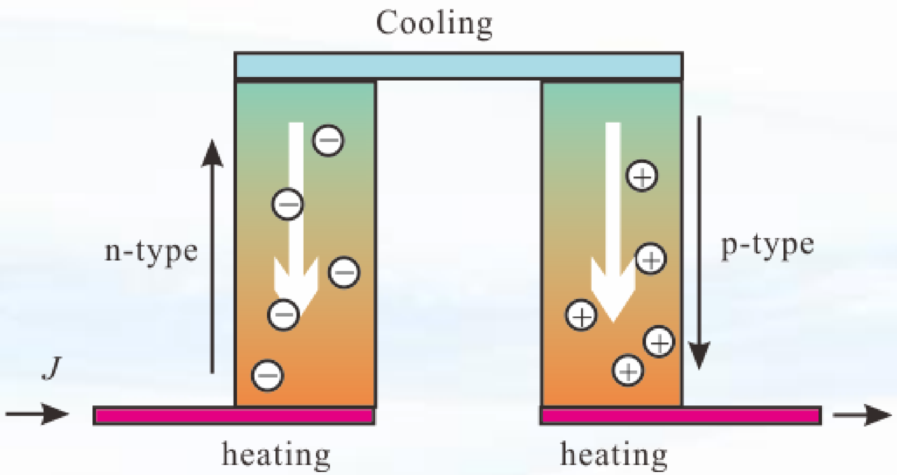
$$S = -\frac{1}{eT}\left[\frac{\langle\tau E\rangle_E}{\langle\tau\rangle_E} - E_\mathrm{F}\right] = -\frac{1}{eT}\left[\left(\frac{5}{2}+s\right)k_\mathrm{B}T - E_\mathrm{F}\right]$$

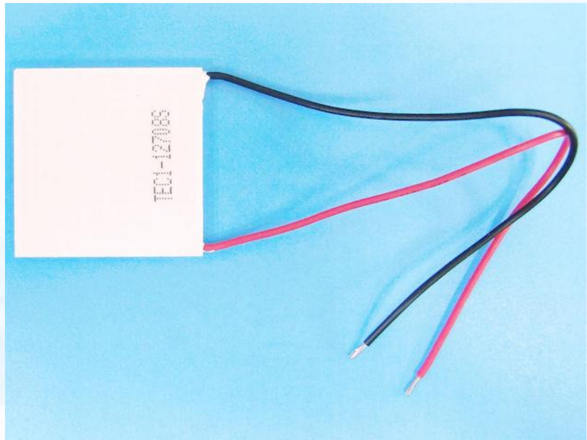
Seebeck measurement provides information on E_F and scattering mechanisms

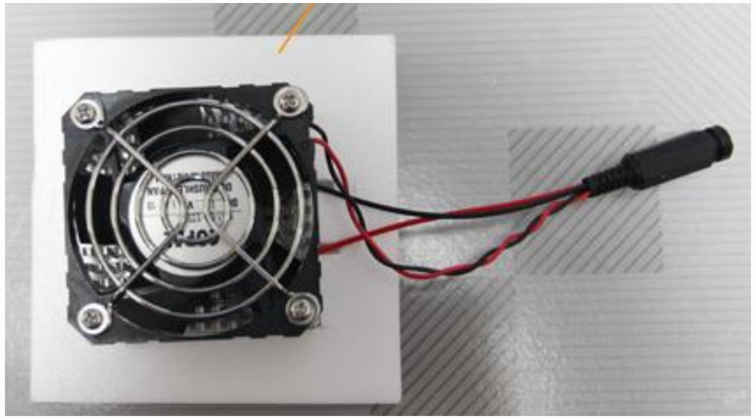
Peltier device

$$S = \frac{1}{qT}\left[\left(\frac{5}{2}+s\right)k_{\mathrm{B}}T - E_{\mathrm{F}}\right] \qquad \Pi = ST$$

Sign of the coefficient changes with carrier charge







Physics in spatially structured semiconductors

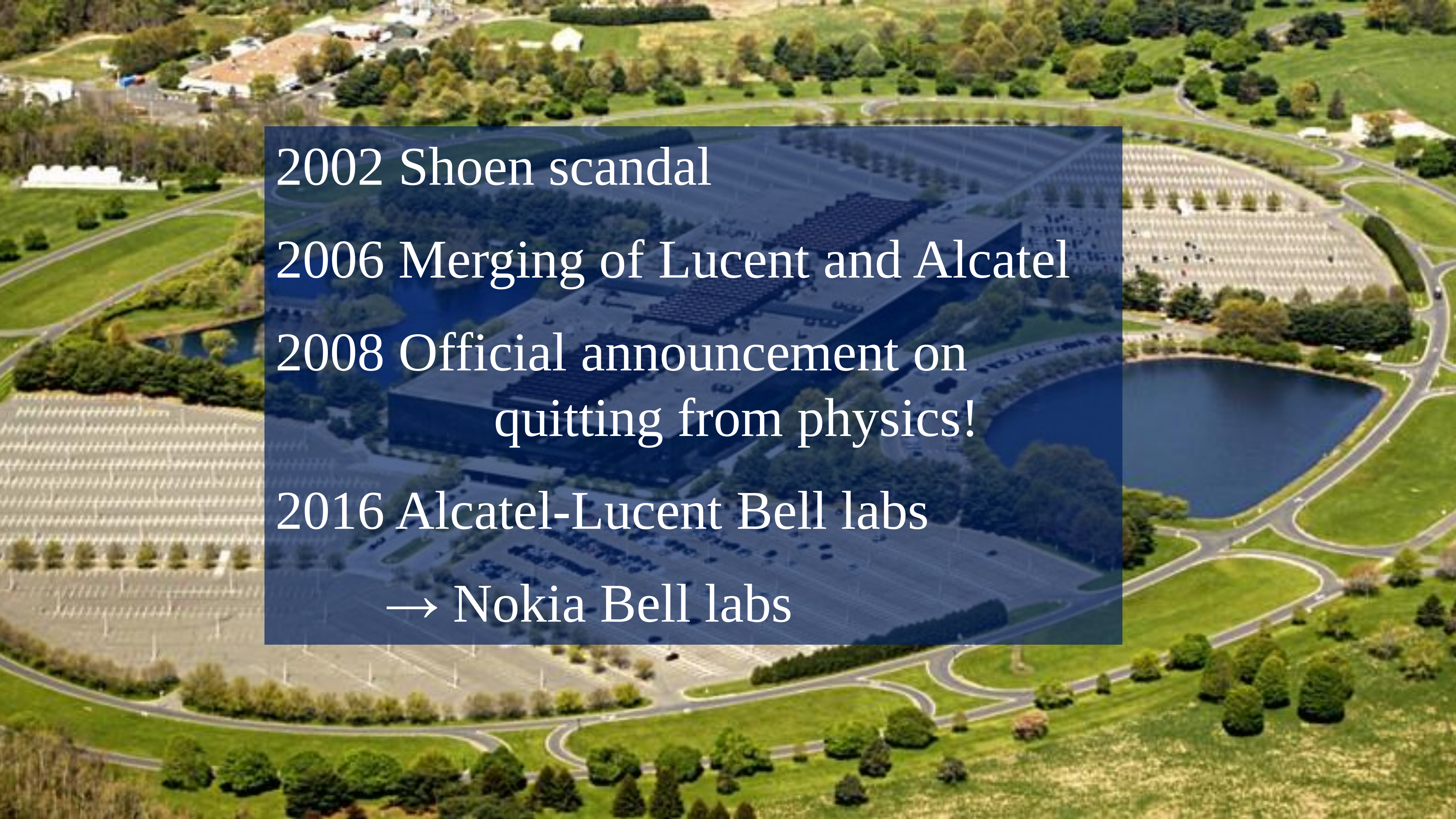
Our apparatus:

- Band structure
- Effective mass approximation
- Carrier statistics
- Electron-photon couplings
- Thermodynamics
- Semi-classical transport (Boltzmann equation)

Chapter 6 Homo and hetero junctions

Bell laboratories ～1984

Bell laboratories 90’s
Lucent Technologies



2002 Shoen scandal

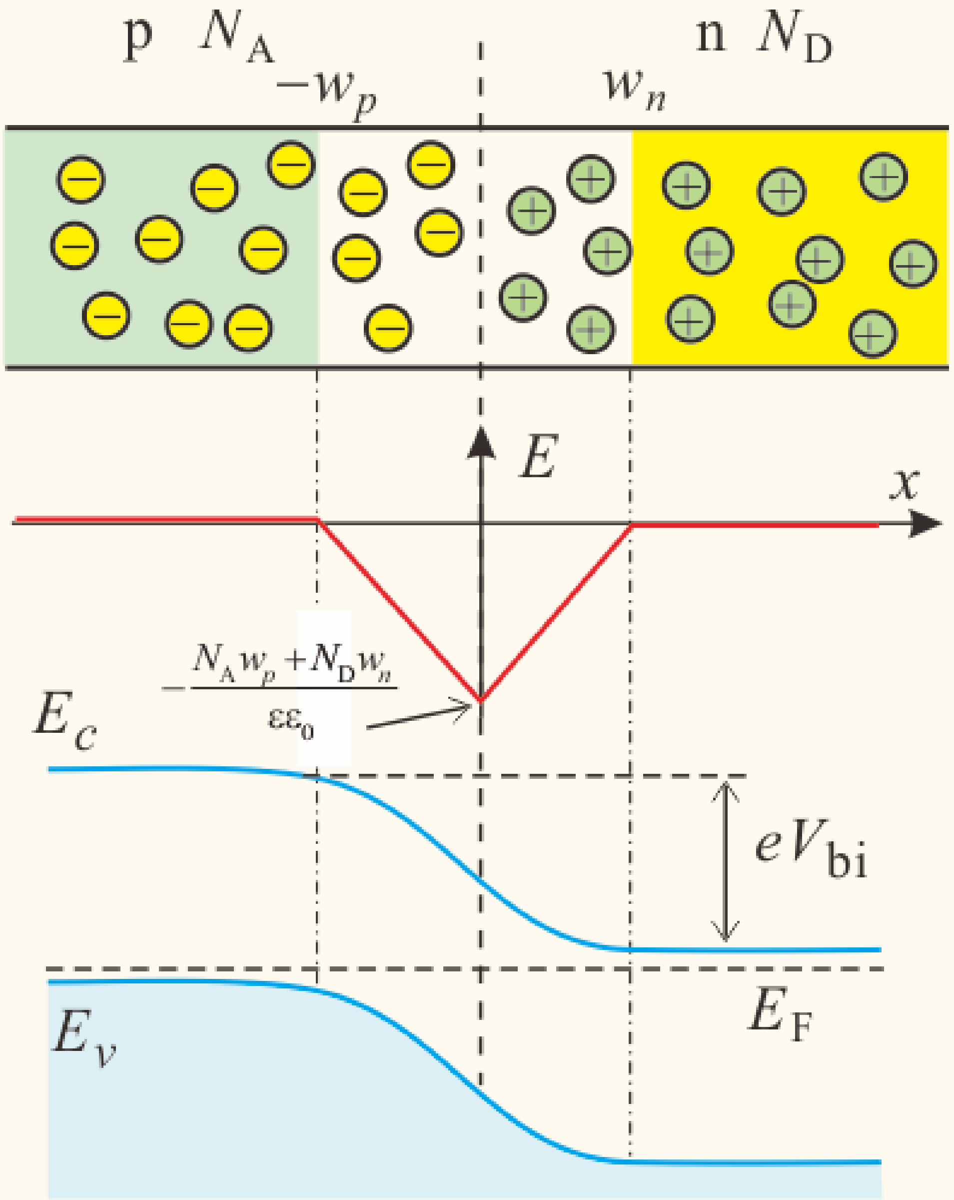
2006 Merging of Lucent and Alcatel

2008 Official announcement on quitting from physics!

2016 Alcatel-Lucent Bell labs

→ Nokia Bell labs

pn homo junctions



Fabrication of pn junctions:
- Epitaxial growth
- Counter doping
- Masked, focused doping

Equilibrium

Diffusion current:
Entropy increase

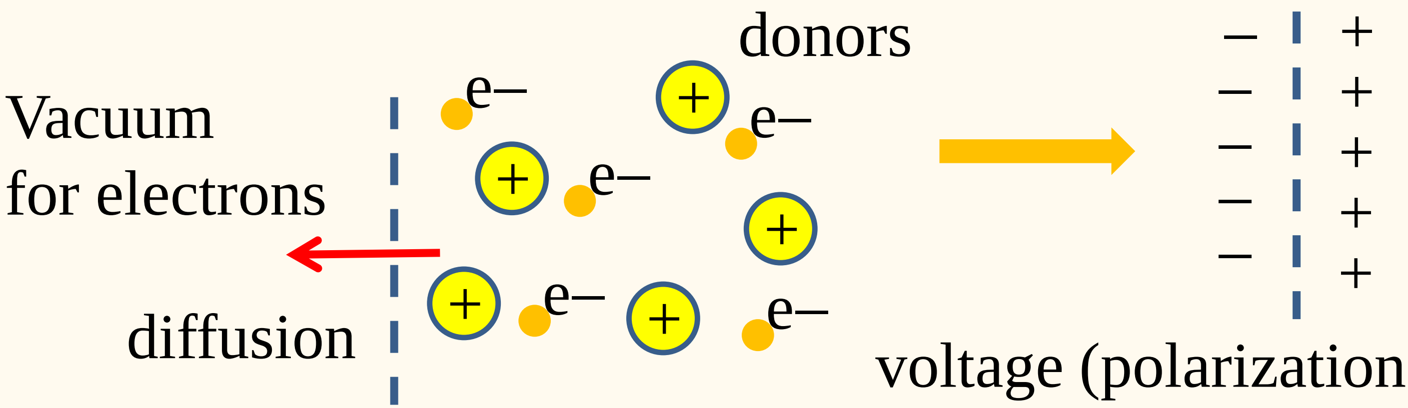
Balance:
Minimize Free energy

Drift current:
Internal energy decrease

- ➢ Carrier depletion layer (space charge layer)
- ➢ Built-in potential

pn junction thermodynamics

Consider electrons attached to a vacuum without work function



voltage (polarization) → energy cost

$$F = \underline{U} - T\underline{S}$$

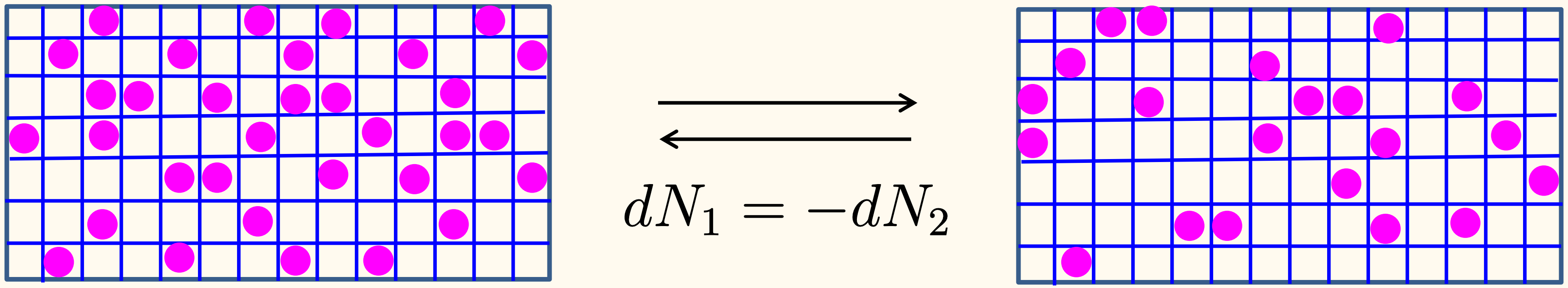
Minimization of F → Built-in (diffusion) voltage V_{bi}

Voltage (internal energy cost)

Diffusion (entropy)

number of sites: N

number of sites: N



$$dN_1 = -dN_2$$

particle number: N_1

particle number: N_2

Estimation of built-in potential

Notation of carrier concentration

	electrons	holes
n-layer	n_n	p_n
p-layer	n_p	p_p

$$n_n \sim N_\mathrm{D},\ p_p \sim N_\mathrm{A} \quad n_p = \frac{n_i^2}{p_p} \sim \frac{n_i^2}{N_\mathrm{A}}$$

Number of cases: $W = {}_N\mathrm{C}_{N_1}\,{}_N\mathrm{C}_{N_2}$

$N \gg N_1, N_2$ Stirling approximation: $\ln N! \approx N\ln N - N$

$$\ln W = \ln \frac{N!}{(N-N_1)!N_1!}\frac{N!}{(N-N_2)!N_2!} \qquad \frac{d\ln W}{dN_1} \approx \ln\frac{N_2}{N_1}\frac{N-N_1}{N-N_2} \approx \ln\frac{N_2}{N_1}$$:Mixing entropy

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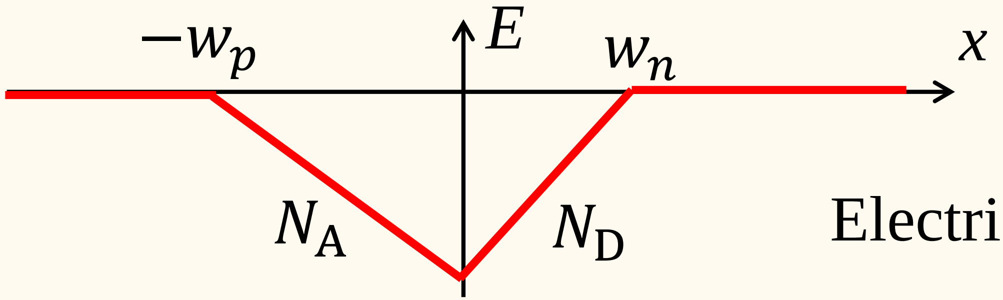
$$N_1 = n_n, \quad N_2 = n_p, \qquad F = U - TS = U - Tk_\mathrm{B}\ln W$$

$$\frac{dF}{dn_n} = 0 \ \rightarrow \ \frac{dU}{dn_n} = eV_\mathrm{bi} = k_\mathrm{B}T\frac{d\ln W}{dn_n} = k_\mathrm{B}T\ln\frac{n_n}{n_p}$$

$$np = n_i^2 = N_\mathrm{c}N_\mathrm{v}\exp\left(-\frac{E_\mathrm{g}}{k_\mathrm{B}T}\right) \ \rightarrow \qquad \approx k_\mathrm{B}T\ln\frac{N_\mathrm{D}N_\mathrm{A}}{n_i^2} = E_\mathrm{g} - k_\mathrm{B}T\ln\frac{N_\mathrm{c}N_\mathrm{v}}{N_\mathrm{D}N_\mathrm{A}}$$

Estimation of depletion layer width



Charge neutrality: $w_n N_\mathrm{D} = w_p N_\mathrm{A}$

Electric field:
$$\begin{aligned} -\epsilon\epsilon_0 E(x) &= e[N_\mathrm{A}(2x + w_p) + N_\mathrm{D} w_n] \quad (x < 0) \\ &= e[N_\mathrm{A} w_p + N_\mathrm{D}(w_n - 2x)] \quad (x \geq 0) \end{aligned}$$

Built-in potential is $V_\mathrm{bi} = \int_{-w_p}^{w_n} (-E(x))dx = \frac{e}{\epsilon\epsilon_0}(N_\mathrm{D} + N_\mathrm{A}) w_n w_p = \frac{e}{\epsilon\epsilon_0}(N_\mathrm{D} + N_\mathrm{A}) \frac{N_\mathrm{D}}{N_\mathrm{A}} w_n^2$

$$V_\mathrm{bi} = \frac{1}{e}\left(E_\mathrm{g} - k_\mathrm{B} T \ln \frac{N_\mathrm{c} N_\mathrm{v}}{N_\mathrm{D} N_\mathrm{A}}\right)$$

$$\therefore \ w_n = \frac{1}{e}\sqrt{\frac{\epsilon\epsilon_0 N_\mathrm{A}}{(N_\mathrm{D} + N_\mathrm{A}) N_\mathrm{D}}\left(E_\mathrm{g} - k_\mathrm{B} T \ln \frac{N_\mathrm{c} N_\mathrm{v}}{N_\mathrm{A} N_\mathrm{D}}\right)}$$

$$w_p = \frac{1}{e}\sqrt{\frac{\epsilon\epsilon_0 N_\mathrm{D}}{(N_\mathrm{D} + N_\mathrm{A}) N_\mathrm{A}}\left(E_\mathrm{g} - k_\mathrm{B} T \ln \frac{N_\mathrm{c} N_\mathrm{v}}{N_\mathrm{A} N_\mathrm{D}}\right)}$$

Current-voltage characteristics

Electrons Equilibrium

$$n_n = N_c \exp\left(\frac{E_\mathrm{F} - E_c}{k_\mathrm{B}T}\right),$$

$$n_p = N_c \exp\left(\frac{E_\mathrm{F} - E_c - eV_\mathrm{bi}}{k_\mathrm{B}T}\right) = n_n \exp\left(-\frac{eV_\mathrm{bi}}{k_\mathrm{B}T}\right)$$

Current balance $J_{pn} = ev_n n_p \qquad J_{np} = ev_n n_n \exp\left(-\frac{eV_\mathrm{bi}}{k_\mathrm{B}T}\right)$

External voltage *V*

Forward bias (against V_{bi}) : lowers barrier for diffusion current n_n

$V_\mathrm{bi} \to V_\mathrm{bi} - V$

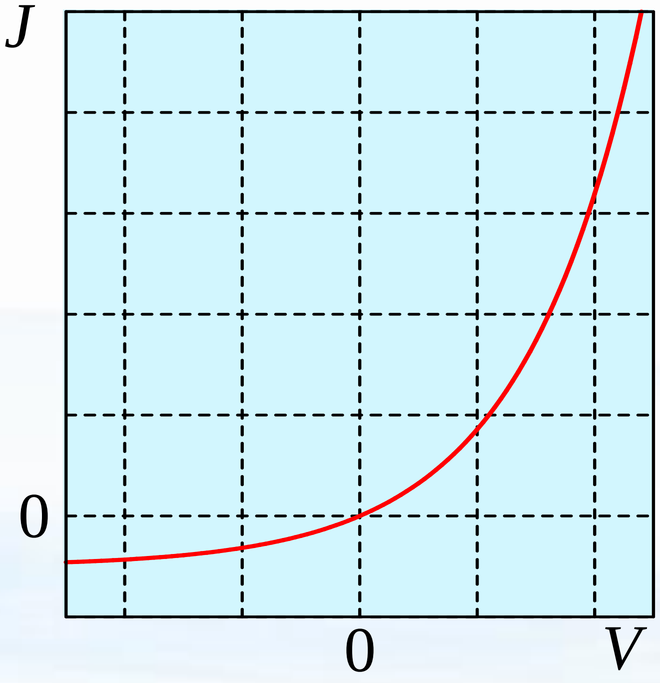
$$J_{np} = ev_n n_n \exp\left(-\frac{e(V_\mathrm{bi} - V)}{k_\mathrm{B}T}\right) = ev_n n_p \exp\left(\frac{eV}{k_\mathrm{B}T}\right)$$

$$J_\mathrm{e} = J_{np} - J_{pn} = ev_n n_p \exp\frac{eV}{k_\mathrm{B}T} - ev_n n_p = ev_n n_p\left[\exp\frac{eV}{k_\mathrm{B}T} - 1\right]$$

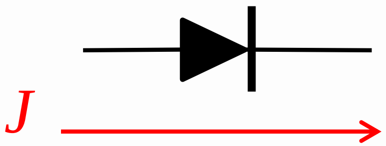
Electron, hole summation

$$J = e(v_n n_p + v_p p_n)\left[\exp\frac{eV}{k_\mathrm{B}T} - 1\right]$$

Injection of minority carriers



pn junction circuit symbol



$$J = \underbrace{e(v_n n_p + v_p p_n)}_{\text{minority carrier current}} \underbrace{\left[\exp \frac{eV}{k_\mathrm{B}T} - 1\right]}_{\text{barrier overflow}}$$

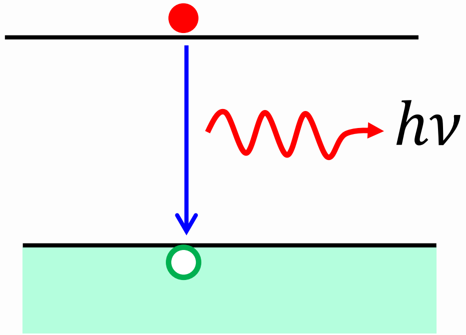
Forward bias region: minority carrier injection

Diffusion equation: $D_e \frac{d^2 n_p}{dx^2} = \frac{n_p - n_{p0}}{\tau_e}$

Minority carrier diffusion length: $L_e = \sqrt{D_e \tau_e},\ L_h = \sqrt{D_h \tau_h}$

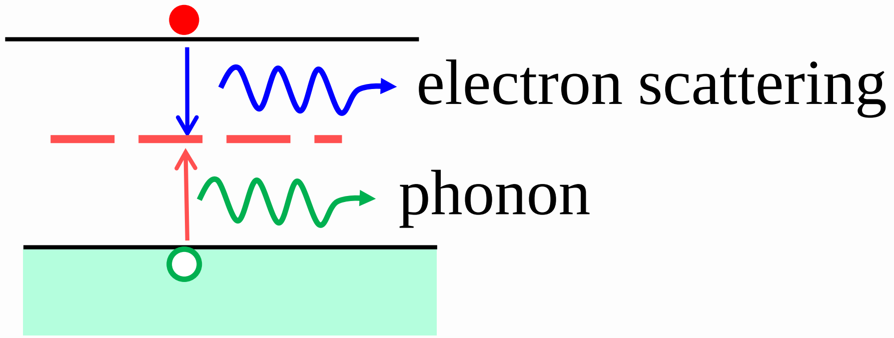
Fate of injected minority carriers

Radiative recombination

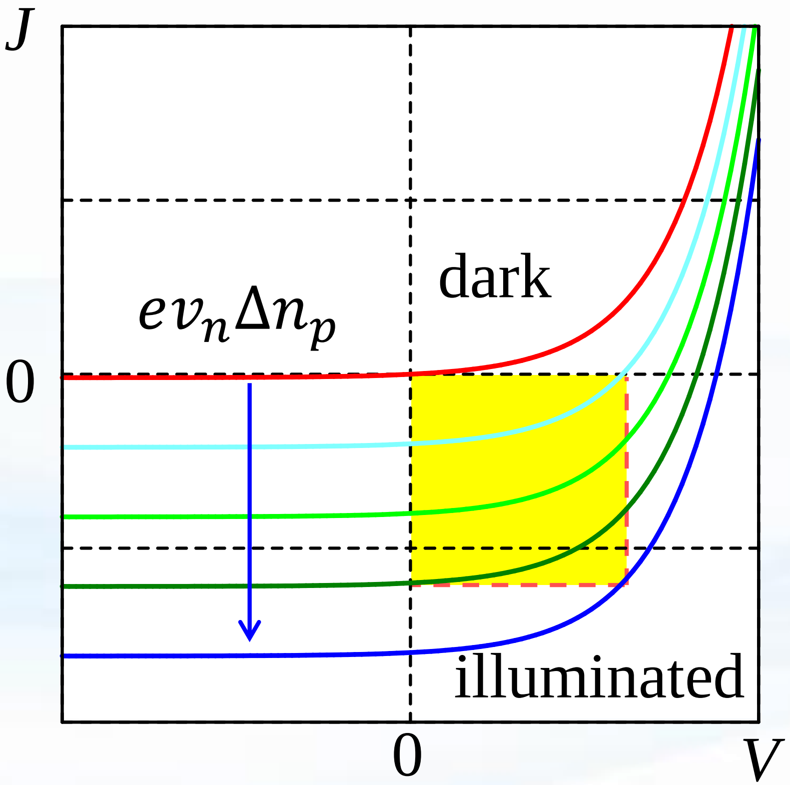


light emitting diode

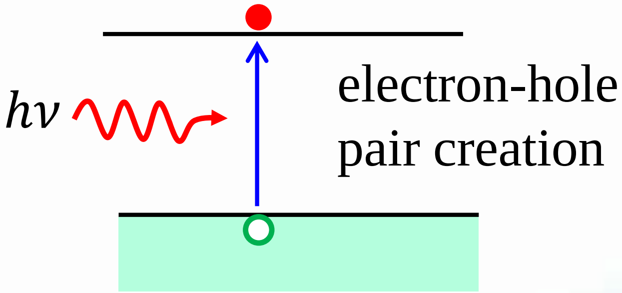
Non-radiative recombination



Solar cells (external injection of minority carriers)



Minority carriers which diffuse to the junction region are swept out to the other side.



$$J_{e0} = ev_n n_p \left[\exp\frac{eV}{k_{gB}T} - 1\right]$$

$$\left.\begin{array}{ll} \text{Voltage for } J = 0 & V_{\text{oc}} \\ \text{Current for } V = 0 & J_{\text{sc}} \end{array}\right\}$$

$$\text{Filling factor (FF)} = \frac{P_{\max}}{J_{\text{sc}}V_{\text{oc}}}$$

$$\begin{aligned} J_e &= ev_n n_p \exp\frac{eV}{k_{\rm B}T} - ev_n(n_p + \Delta n_p) \\ &= J_{n0} - \underline{ev_n \Delta n_p} \end{aligned}$$

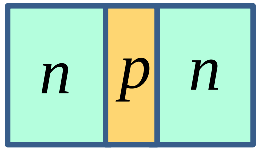
external injection

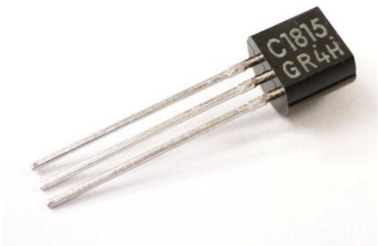
Gerald Pearson, Daryl Chapin
and Calvin Fuller at Bell labs. 1954

pn junction (bipolar) transistors

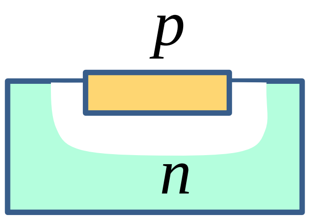
John Bardeen, William Shockley, Walter Brattain 1948 Bell Labs.

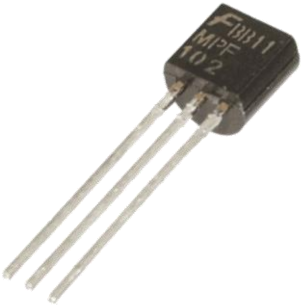
Bipolar junction transistor



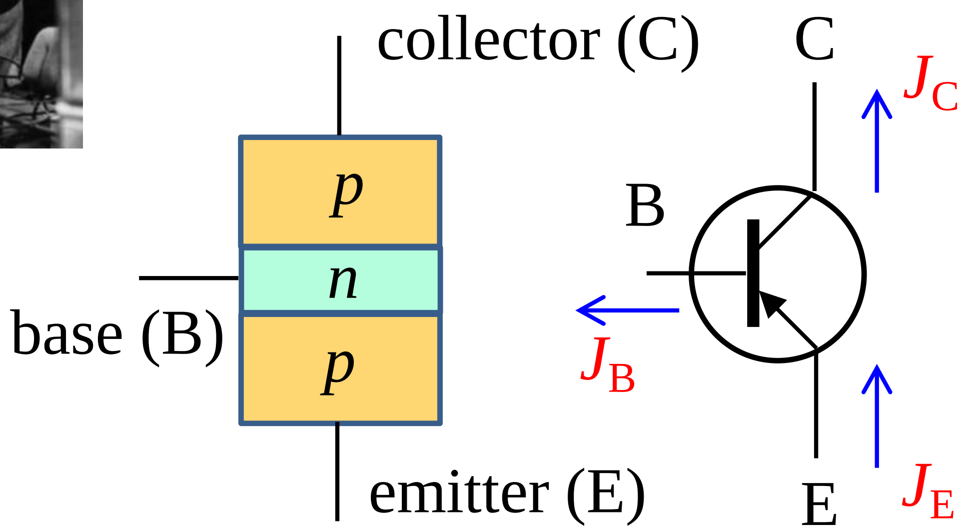


Field effect transistor

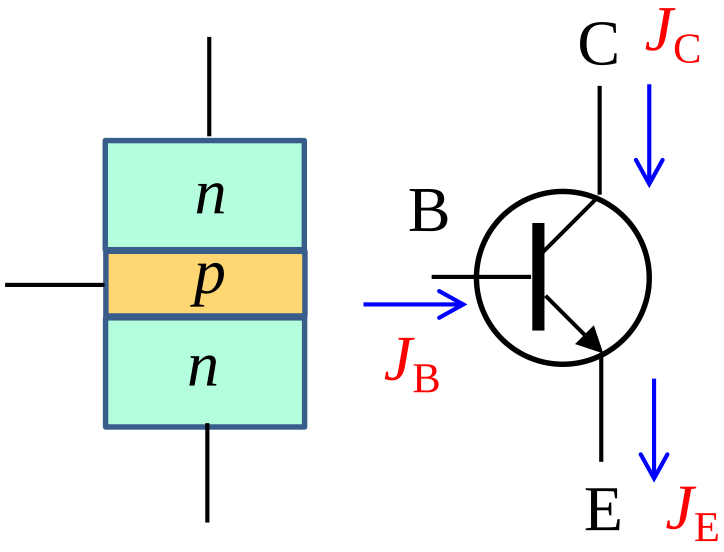




Bipolar transistor structures and symbols



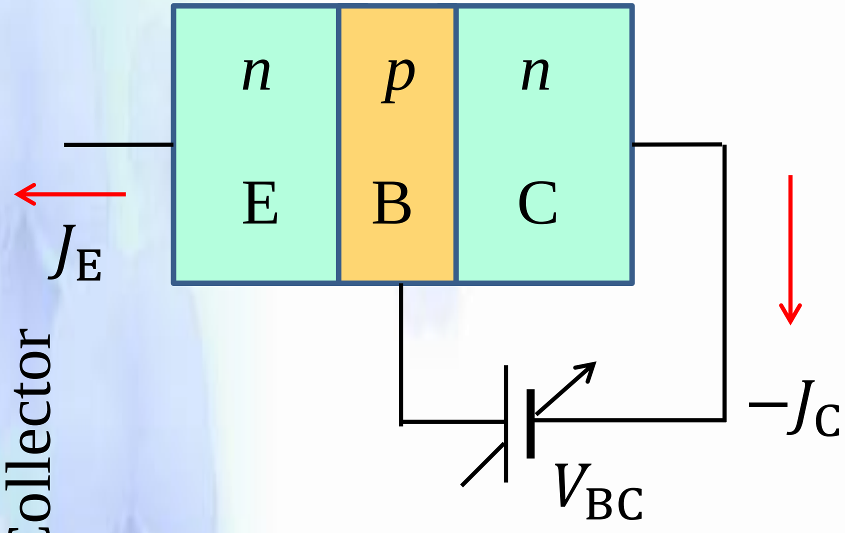
PNP type

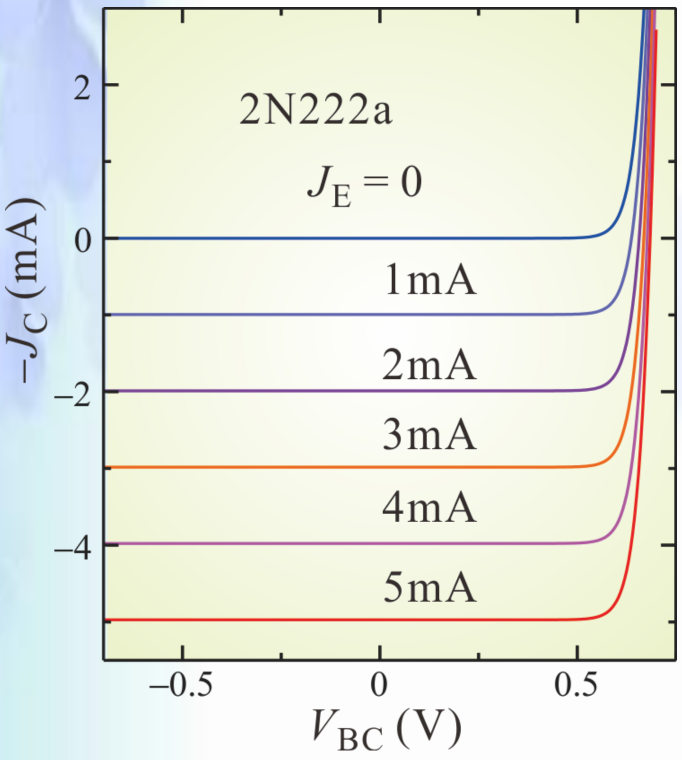


NPN type

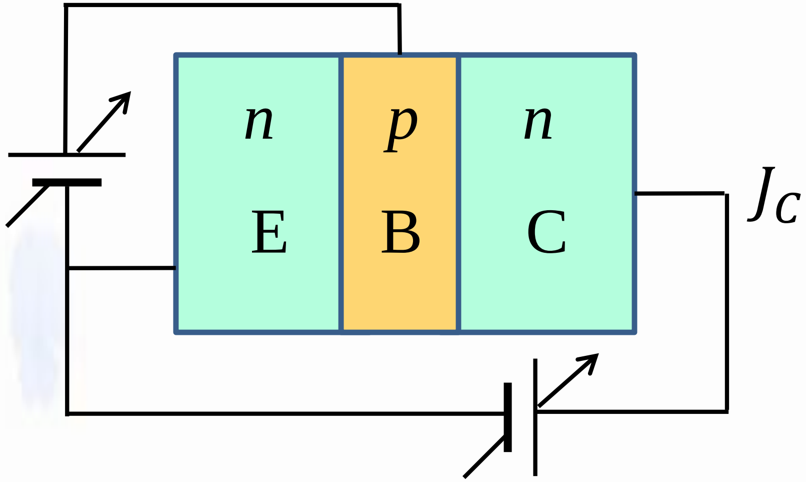
Base-Collector, Collector-Emitter characteristics

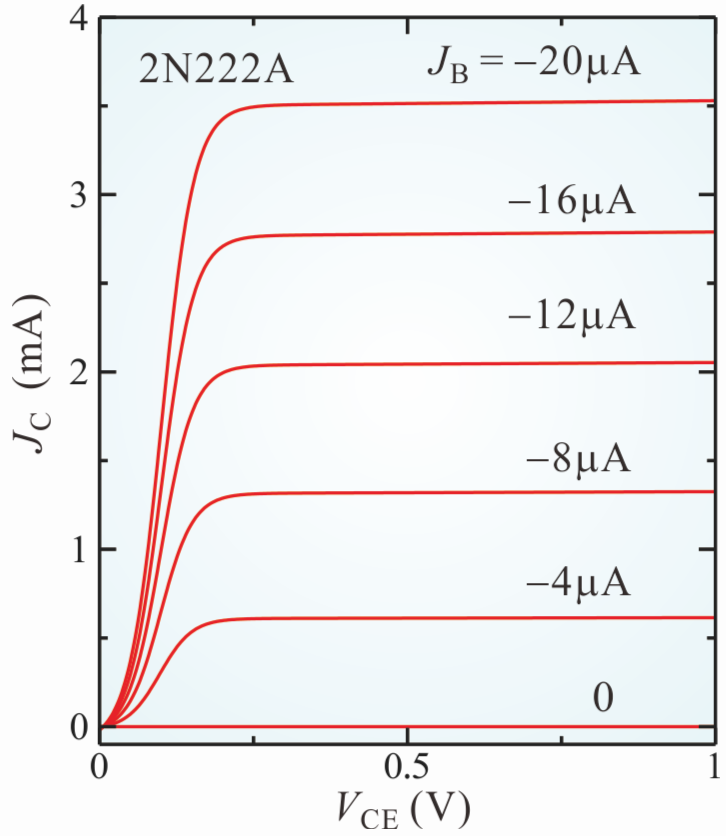
Base-Collector

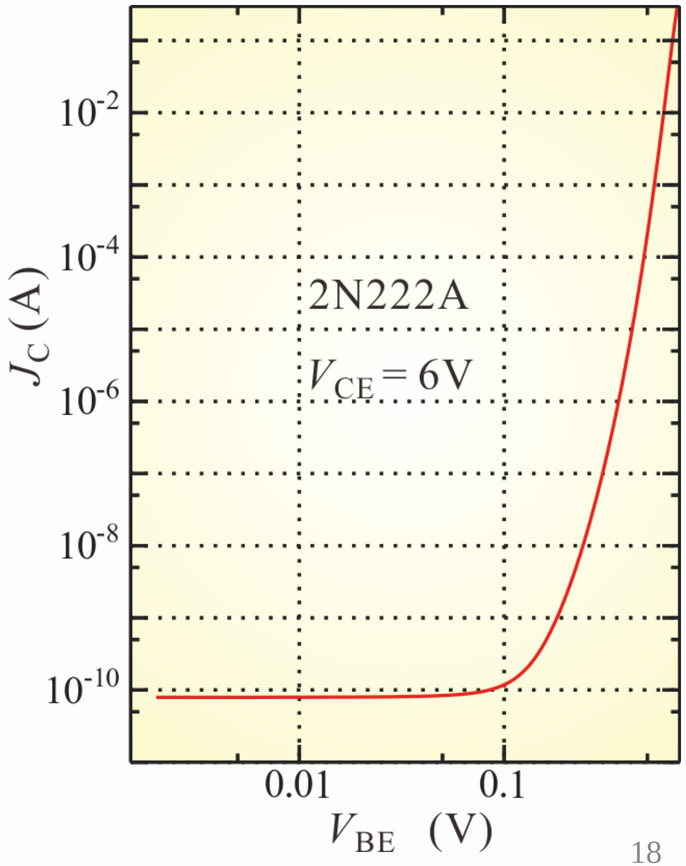




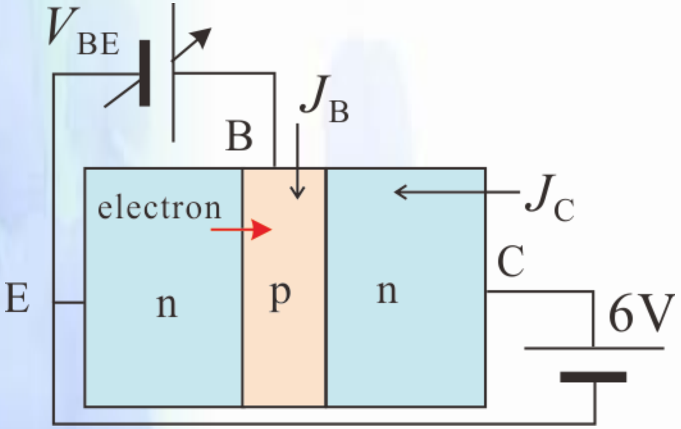
Collector-Emitter

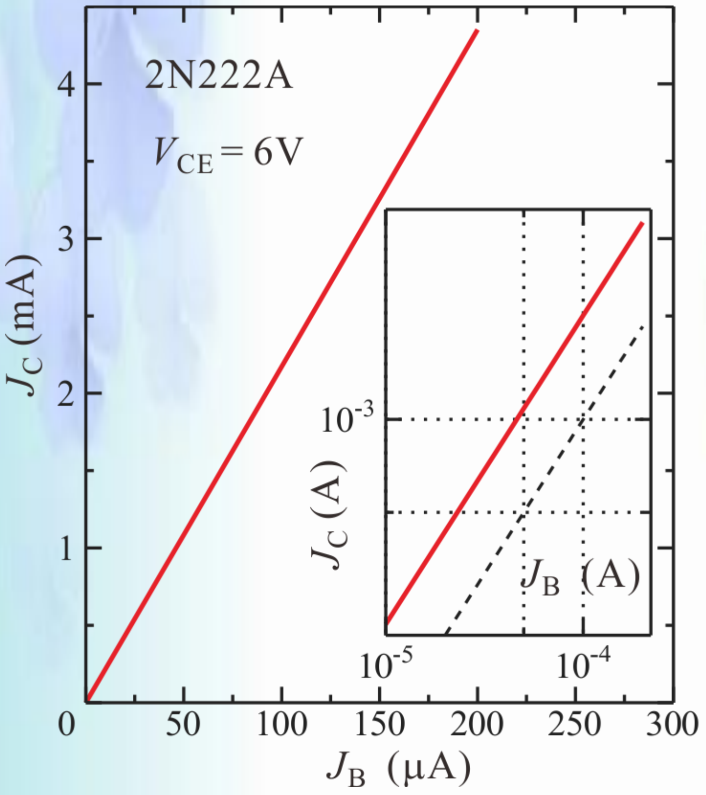






Current amplification: Linearization with quantity selection

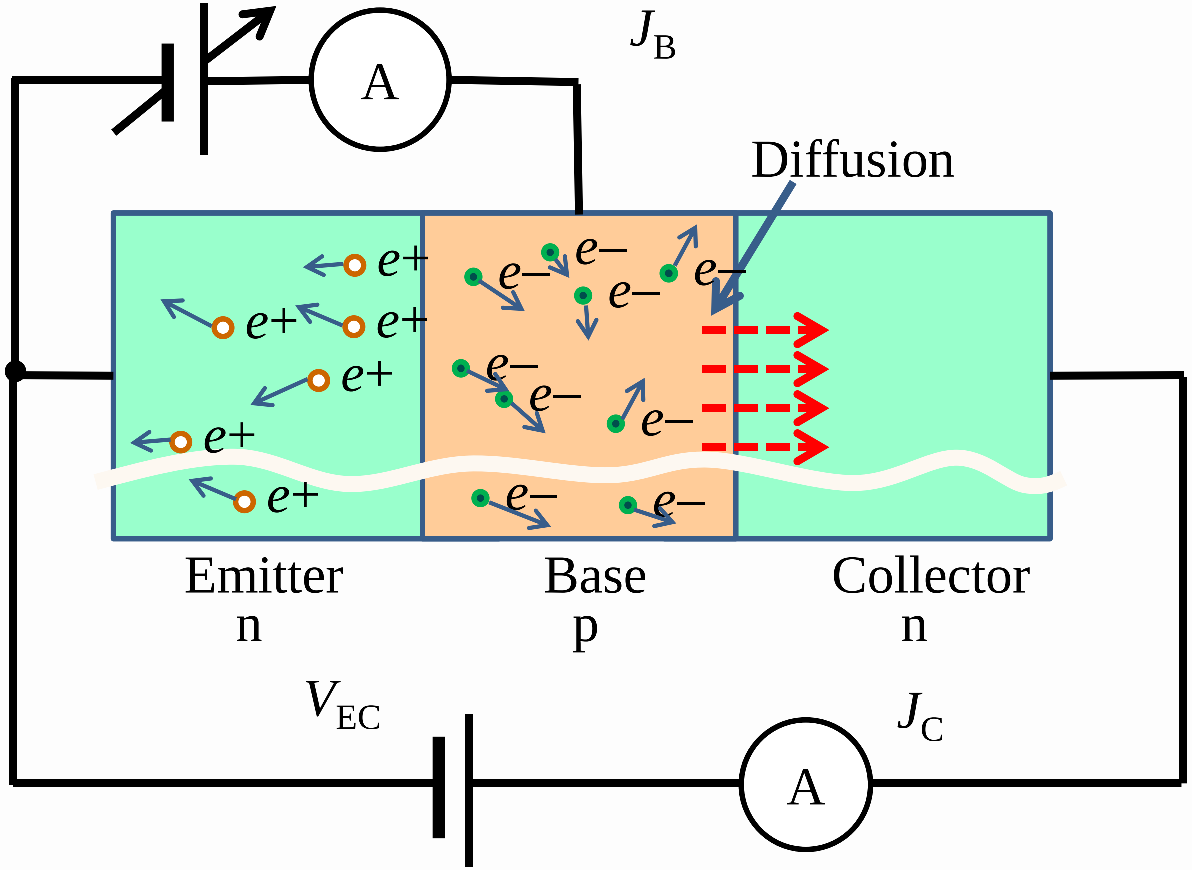




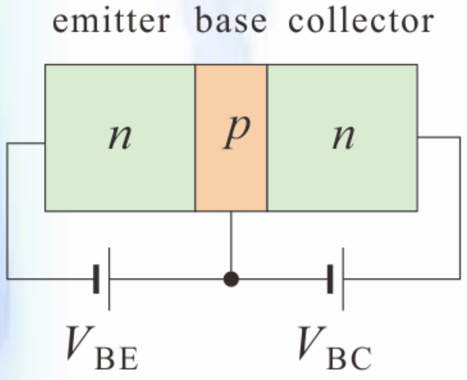
$$J_C = \underline{h_{FE}}\, J_B$$

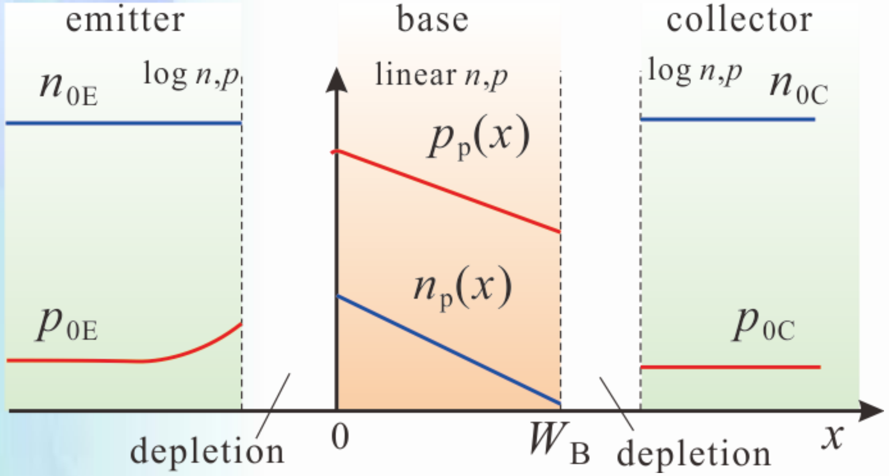
Emitter-common current gain

How a bipolar transistor amplifies signal?



Expression of h_{FE}





Sweeping out of minority carriers at the depletion edge $$n_p(W_{\mathrm{B}}) = n_{p0}\exp\frac{-eV_{\mathrm{BC}}}{k_{\mathrm{B}}T} \approx 0$$

Diffusion current in the base: constant $$\frac{dn_p}{dx} \ : \ \text{constant} \qquad n_p(x) \ : \text{linear in } x$$

Device cross section A $$j_{\mathrm{D}e} = -D_e\frac{dn_p}{dx} \approx eD_e\frac{n_p(0)}{W_{\mathrm{B}}} = \frac{J_{\mathrm{C}}}{A}$$

The law of mass action $$n_{p0} \approx \frac{n_i^2}{N_{\mathrm{A}}}$$

$$J_{\mathrm{C}} \approx \frac{eAD_e n_{p0}}{W_{\mathrm{B}}}\exp\frac{eV_{\mathrm{BE}}}{k_{\mathrm{B}}T} \approx \frac{eAD_e n_i^2}{W_{\mathrm{B}}N_{\mathrm{A}}}\exp\frac{eV_{\mathrm{BE}}}{k_{\mathrm{B}}T} \equiv J_{\mathrm{S}}\exp\frac{eV_{\mathrm{BE}}}{k_{\mathrm{B}}T}$$

$$J_{\mathrm{B}h} = \frac{eAD_h}{L_h}p_{n\mathrm{E}}(0) = \frac{eAD_h}{L_h}p_{n\mathrm{E}0}\exp\frac{eV_{\mathrm{BE}}}{k_{\mathrm{B}}T} = \frac{eAD_h}{L_h}\frac{n_i^2}{N_{\mathrm{D}}}\exp\frac{eV_{\mathrm{BE}}}{k_{\mathrm{B}}T}$$

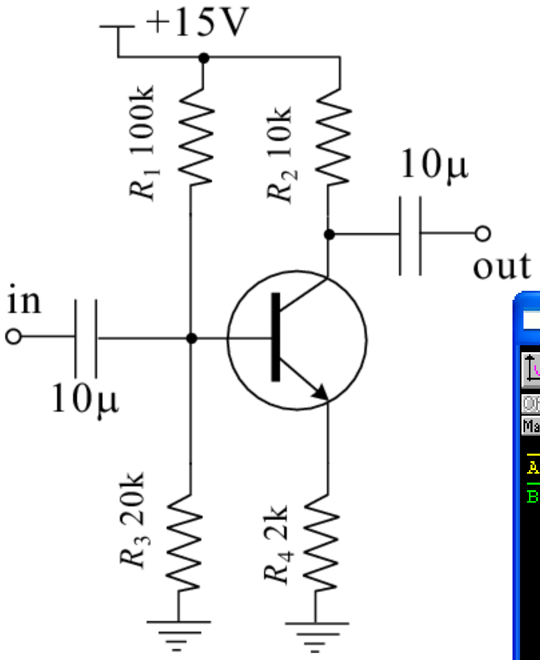
Recombination current: $$J_{\mathrm{Br}} = \frac{Q_e}{\tau_b} = \frac{en_p(0)AW_{\mathrm{B}}}{2\tau_b}\exp\frac{eV_{\mathrm{BE}}}{k_{\mathrm{B}}T}$$

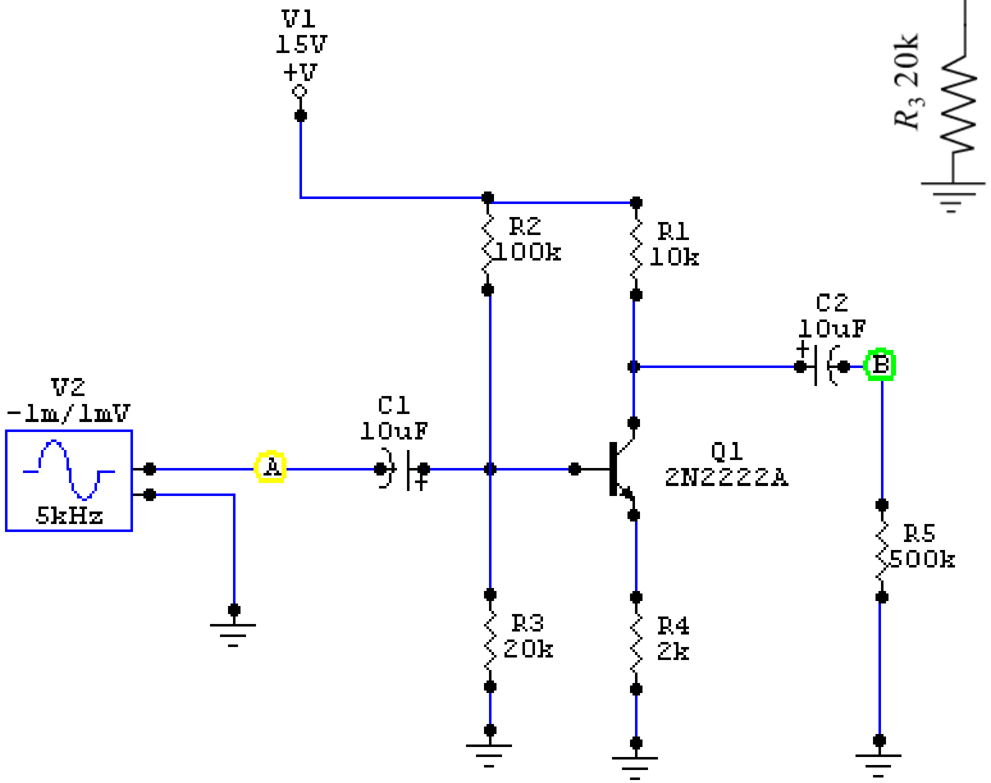
$$h_{\mathrm{FE}} = \left(\frac{D_h}{D_e}\frac{W_{\mathrm{B}}}{L_h}\frac{N_{\mathrm{A}}}{N_{\mathrm{B}}} + \frac{W_{\mathrm{B}}^2}{2\tau_b D_e}\right)^{-1}$$

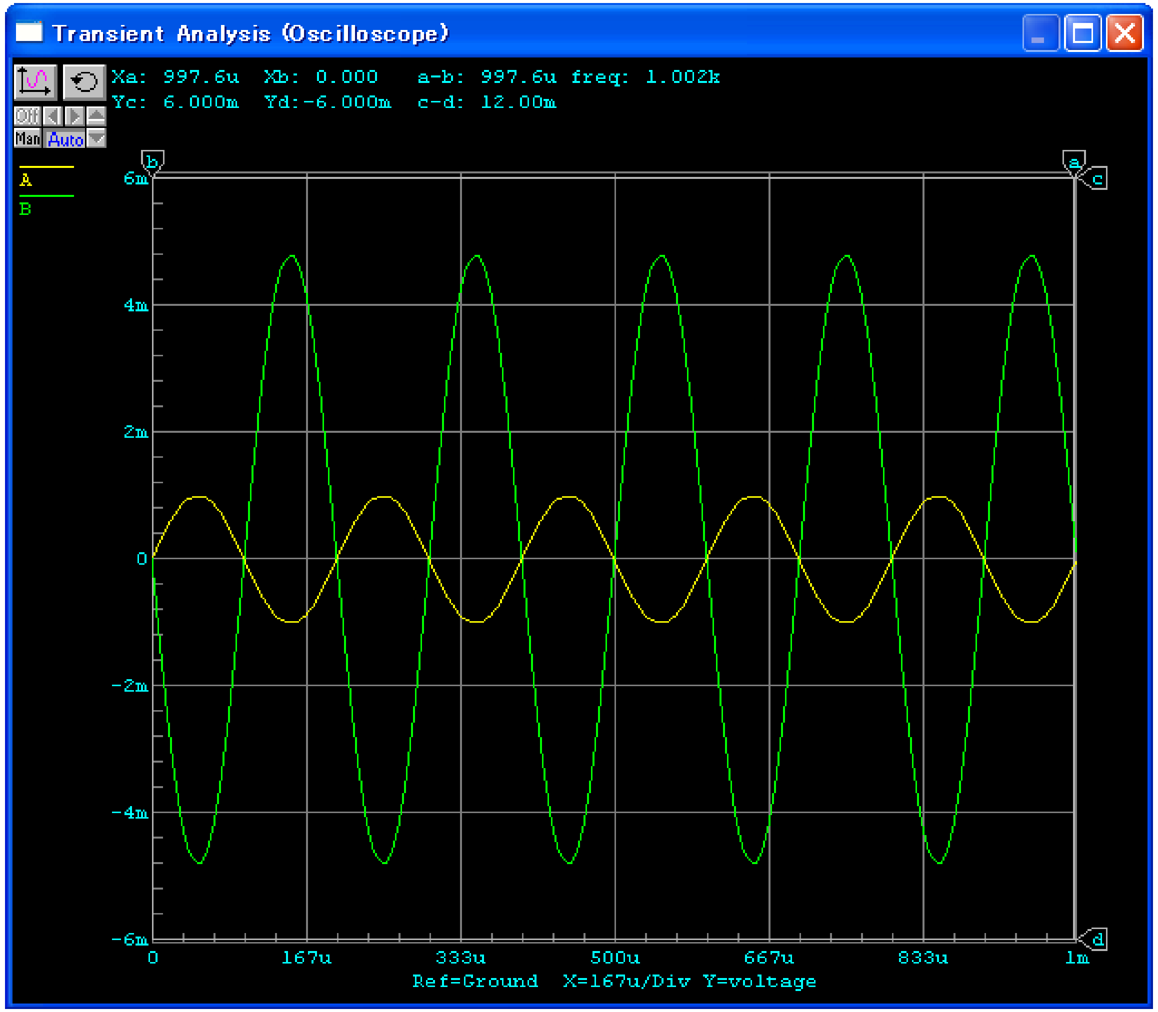
Example of an amplification circuit

$$\Delta V_{\mathrm{C}} = R_2 \Delta J_{\mathrm{C}} \approx R_2 \Delta J_{\mathrm{E}}$$

$$= R_2 \frac{\Delta V_{\mathrm{E}}}{R_4} = \frac{R_2}{R_4} \Delta V$$



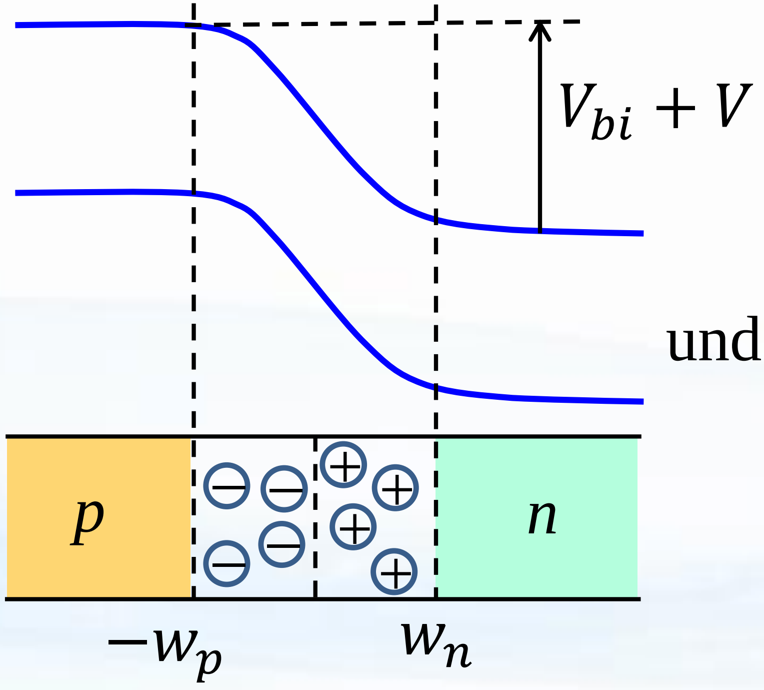



Transient Analysis (Oscilloscope)

Xa: 997.6u Xb: 0.000 a-b: 997.6u freq: 1.002k
Yc: 6.000m Yd:-6.000m c-d: 12.00m

Ref=Ground X=167u/Div Y=voltage

Depletion layer width with reverse bias voltage



Poisson equation $$\frac{d^2\phi}{dx^2} = -aq(x) \quad (a \equiv (\epsilon\epsilon_0)^{-1})$$

charge: $$\begin{cases} q = -eN_A & (-w_p \le x \le 0), \\ q = eN_D & (0 \le x \le w_n) \end{cases}$$

under conditions

potential boundary: $$\begin{cases} \phi(-\infty) = 0 & \\ \phi(-w_p) = 0, & \left.\frac{d\phi}{dx}\right|_{-w_p} = 0, \\ \phi(w_n) = V + V_{\rm bi}, & \left.\frac{d\phi}{dx}\right|_{w_n} = 0 \end{cases}$$

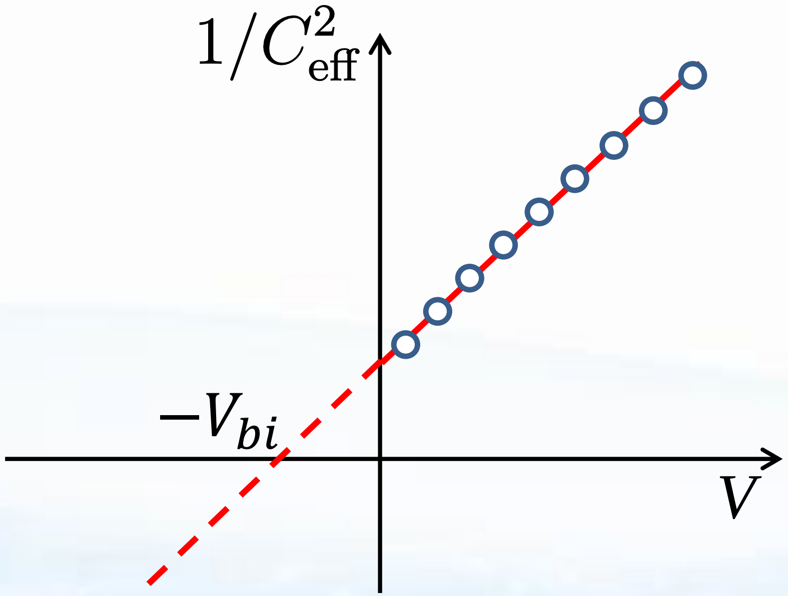
The integration gives $$\phi(x) = \begin{cases} \frac{aeN_A}{2}(x+w_p)^2 & (-w_p \le x \le 0), \\ V + V_{\rm bi} - \frac{aeN_D}{2}(x-w_n)^2 & (0 \le x \le w_n) \end{cases}$$

$$\lim_{x\to+0}\phi = \lim_{x\to-0}\phi,$$
$$\lim_{x\to+0}(d\phi/dx) =$$
$$\lim_{x\to-0}(d\phi/dx)$$

$$w_p = \left[\frac{2\epsilon_0\epsilon(V+V_{\rm bi})}{eN_A}\cdot\frac{N_D}{N_D+N_A}\right]^{1/2}, \quad w_n = \left[\frac{2\epsilon_0\epsilon(V+V_{\rm bi})}{eN_D}\cdot\frac{N_A}{N_D+N_A}\right]^{1/2}$$

$$w_d = w_p + w_n = \left[\frac{2\epsilon_0\epsilon(V+V_{\rm bi})}{e}\cdot\frac{N_A+N_D}{N_AN_D}\right]^{1/2}.$$

Effective capacitance and reverse bias voltage

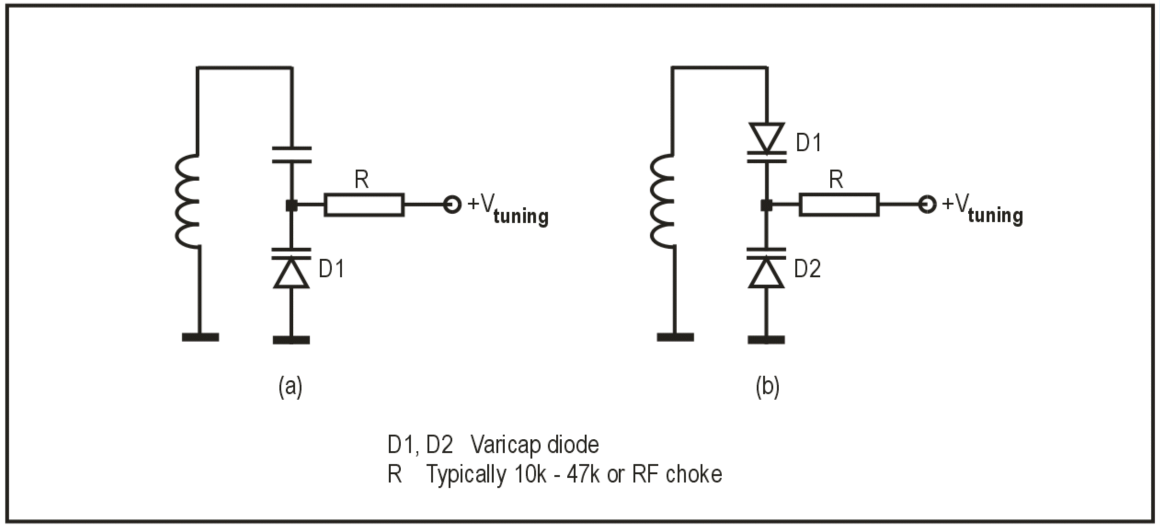


$$\frac{1}{C_{\mathrm{eff}}^2} = \frac{2}{\epsilon\epsilon_0 e N_{\mathrm{D}}}(V + V_{\mathrm{bi}})$$

This gives a way for the doping profiling.

Varicap diode

circuit example

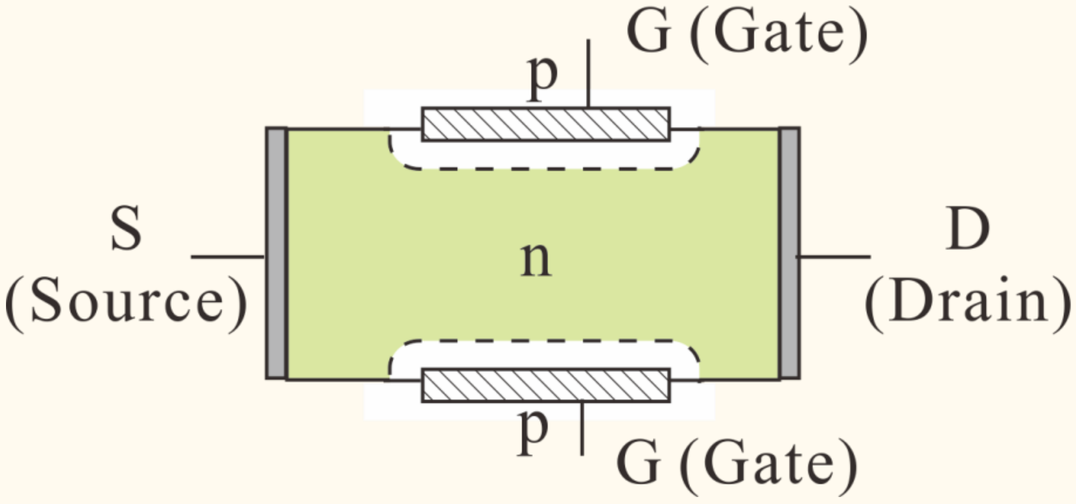


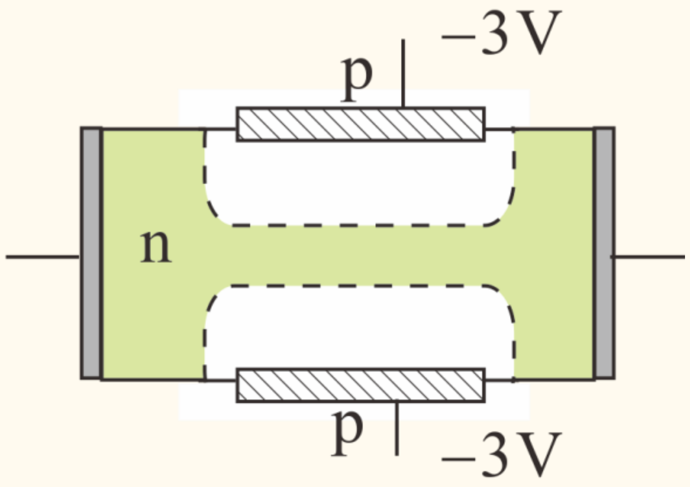
D1, D2 Varicap diode
R Typically 10k - 47k or RF choke

KB505

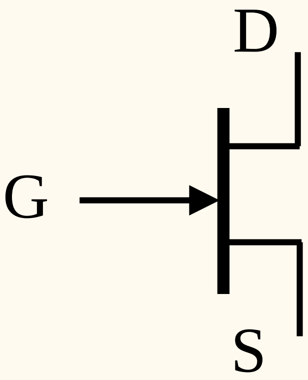
Frequency modulation
Phase lock loop

pn junction field effect transistor (JFET)

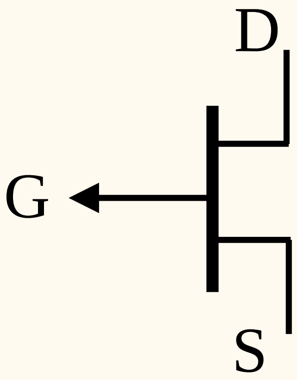




Circuit symbols

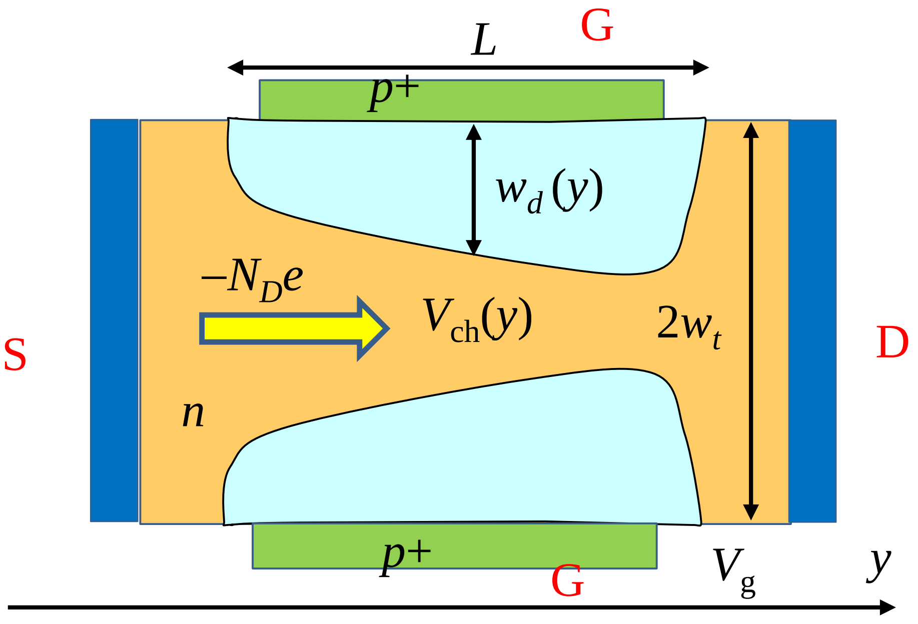


n-channel



p-channel

pn junction FET



$$V(y) = V_{\mathrm{g}} + V_{\mathrm{bi}} - V_{\mathrm{ch}}(y)$$

$$w_d(y) = \sqrt{\frac{2\epsilon\epsilon_0 V(y)}{eN_{\mathrm{D}}}}$$

$$J_{\mathrm{ch}} = \underbrace{eN_{\mathrm{D}}\mu_n}_{\text{conductivity}} \underbrace{\frac{dV_{\mathrm{ch}}}{dy}}_{\text{electric field}} \cdot \underbrace{2[w_t - w_d(y)]W}_{\text{channel width}}$$

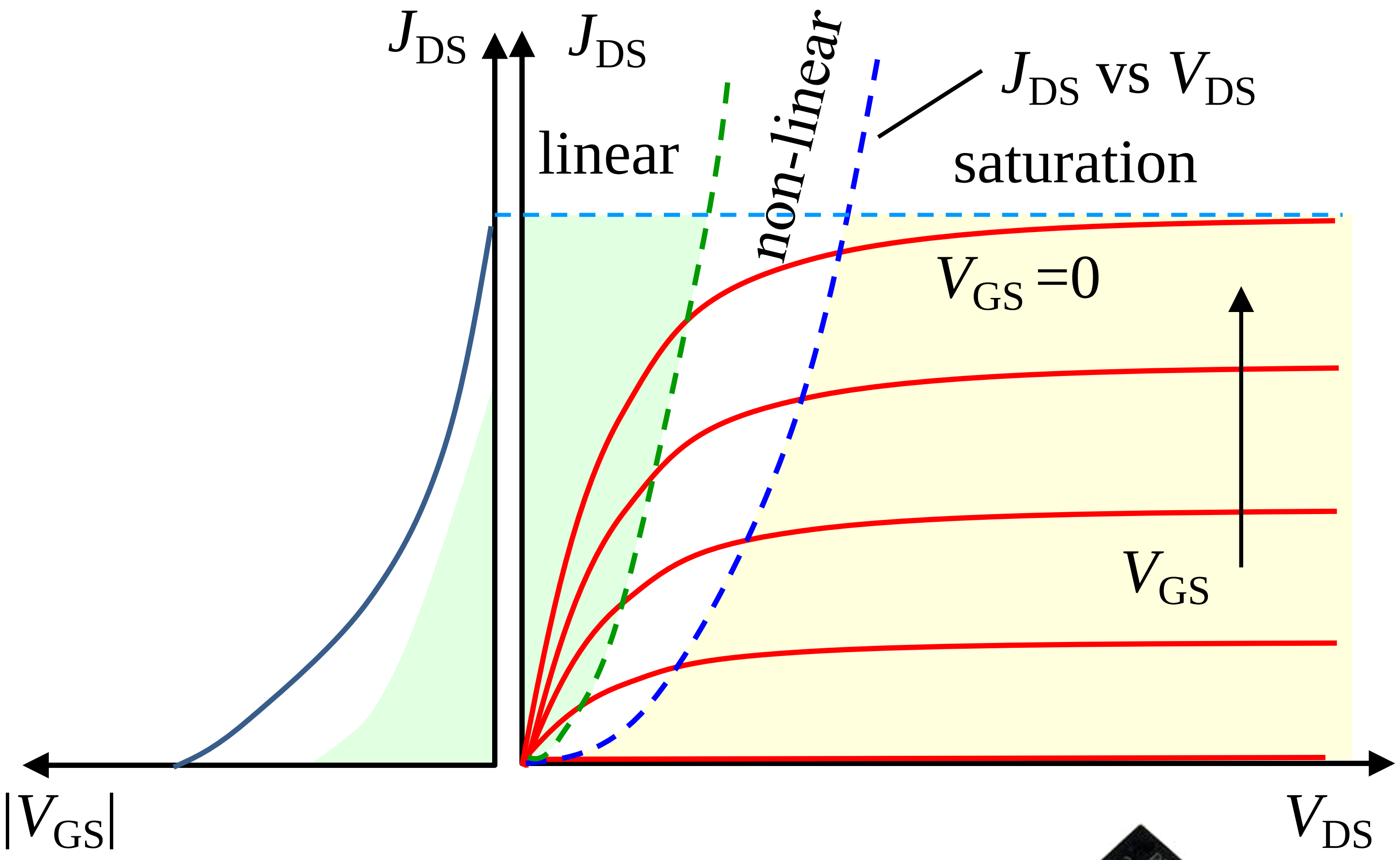
$$J_{\mathrm{ch}}L = \int_0^L J_{ch}dy = 2eN_D\mu_n W \int_0^L (w_t - w_d)\frac{dV}{dy}dy = 2w_t eN_D\mu_n W \int_{V_0}^{V_L}\left(1 - \frac{w_d}{w_t}\right)dV$$

pinch off (internal) voltage: $w_d(V_c) = w_t \quad V_c = \dfrac{eN_D w_t^2}{2\epsilon\epsilon_0}$

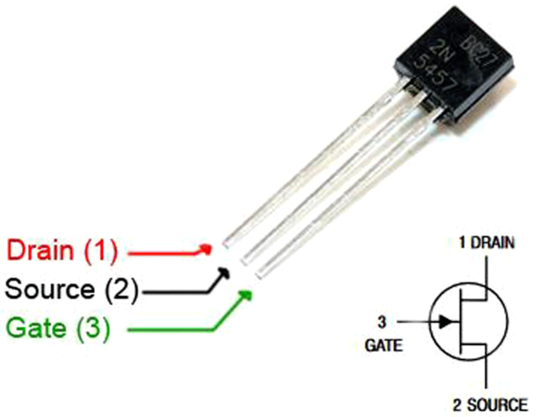
$$J_{\mathrm{ch}} = \frac{2N_D e\mu_n W w_t}{L}\left[V_L - V_0 + \frac{2}{3\sqrt{V_c}}\left(V(V_0)^{3/2} - V(V_L)^{3/2}\right)\right]$$

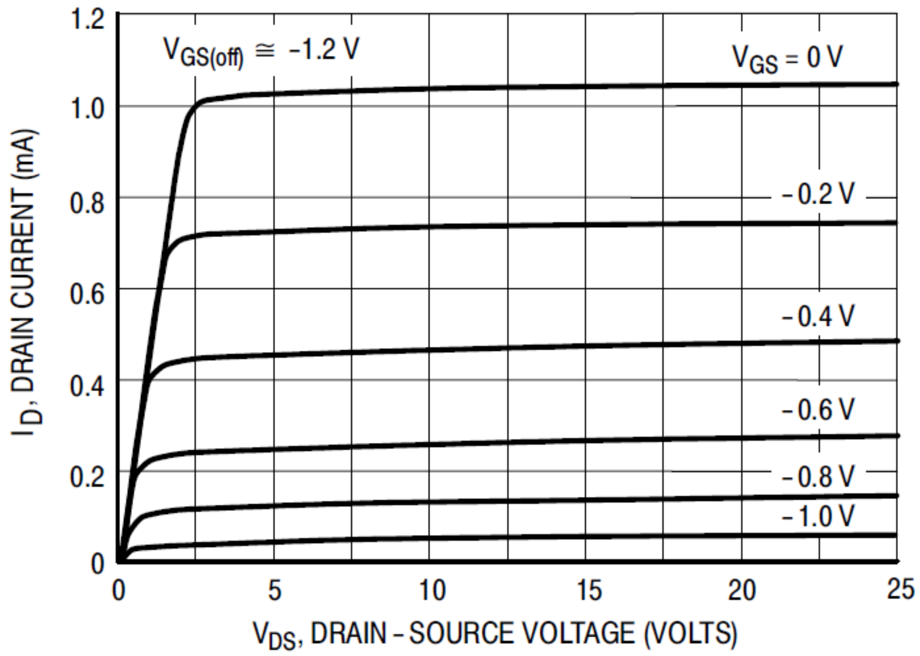
Only valid for $w_d < w_t/2$.

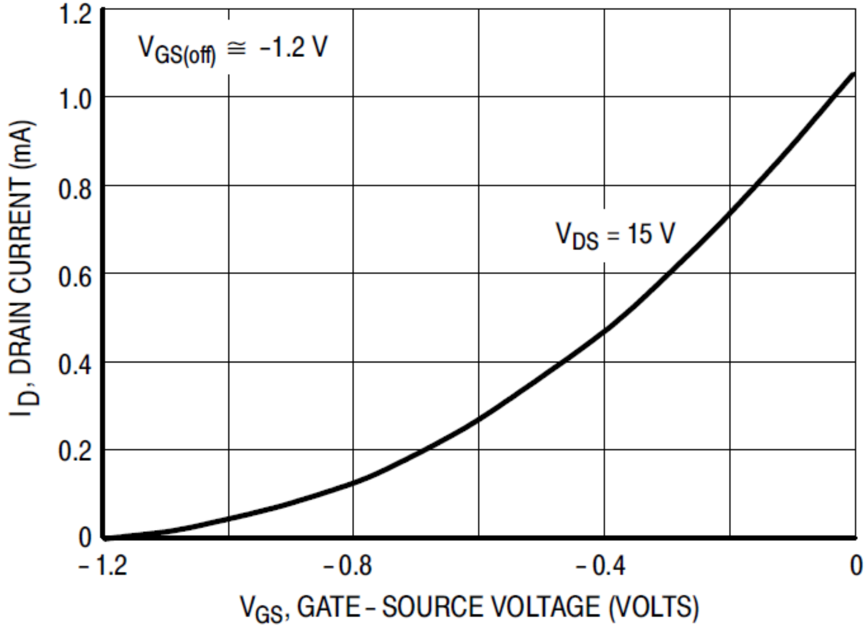
I-V characteristics of JFET



Example: 2N5457
n-channel
depletion-type







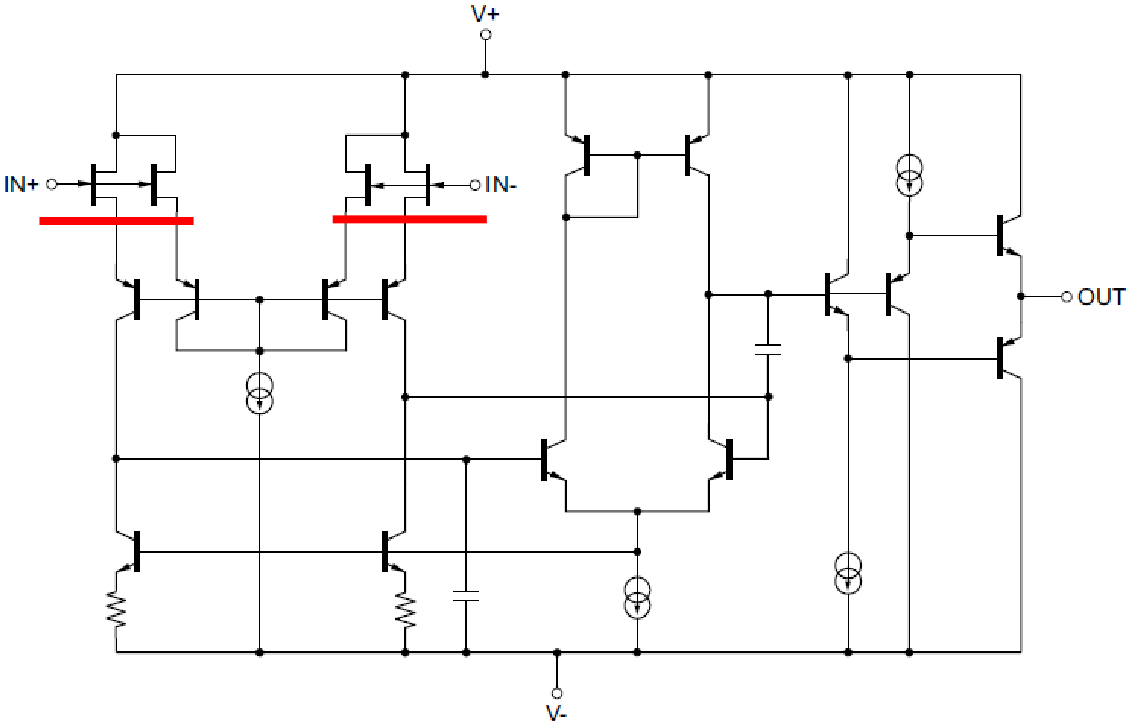
Application of JFET

Low linearity → linearization with feedback with high gain

High input impedance, low bias current (operation at the reverse bias region)

: fit the input stage of operational amplifier

Example: OPA827



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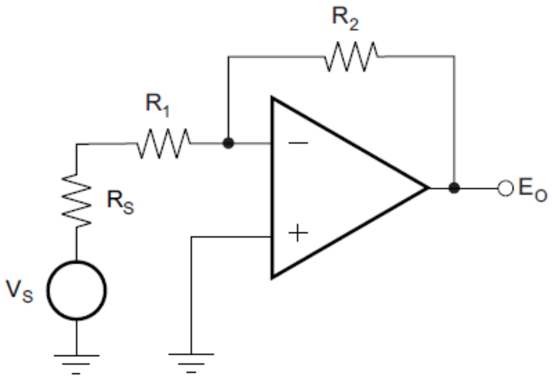
- Input voltage noise: 4 nV/$\sqrt{\text{Hz}}$ at 1 kHz
- Input bias current 10 pA max
- Input impedance 10^{13} Ω



Inverting amplifier