Lecture on

Semiconductors / 半導体

(Physics of semiconductors)

2021.6.02 Lecture 08

10:25 – 11:55

Institute for Solid State Physics, University of Tokyo

Shingo Katsumoto

Review of the last week

Chapter 6 Homo and hetero junctions

pn homo junctions

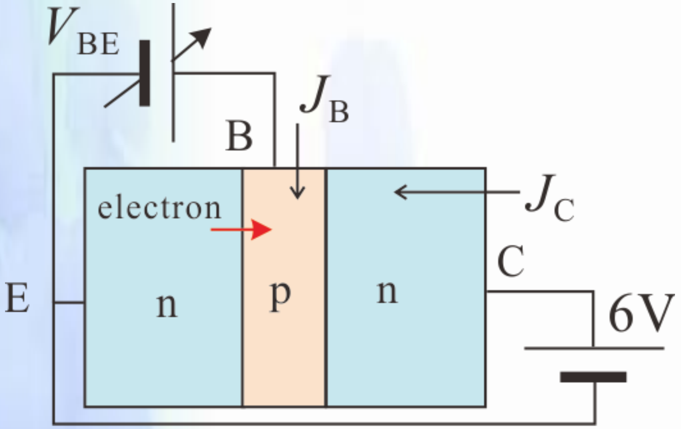
Solar cells

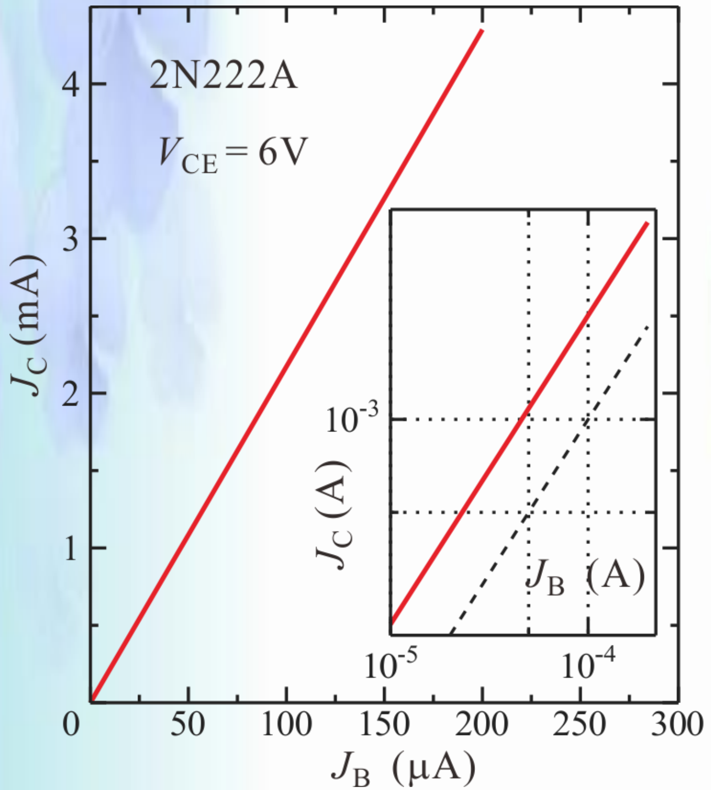
Bipolar transistors

John Bardeen, William Shockley,
Walter Brattain 1948 Bell Labs.

Gerald Pearson, Daryl Chapin
and Calvin Fuller at Bell labs. 1954

Current amplification: Linearization with quantity selection

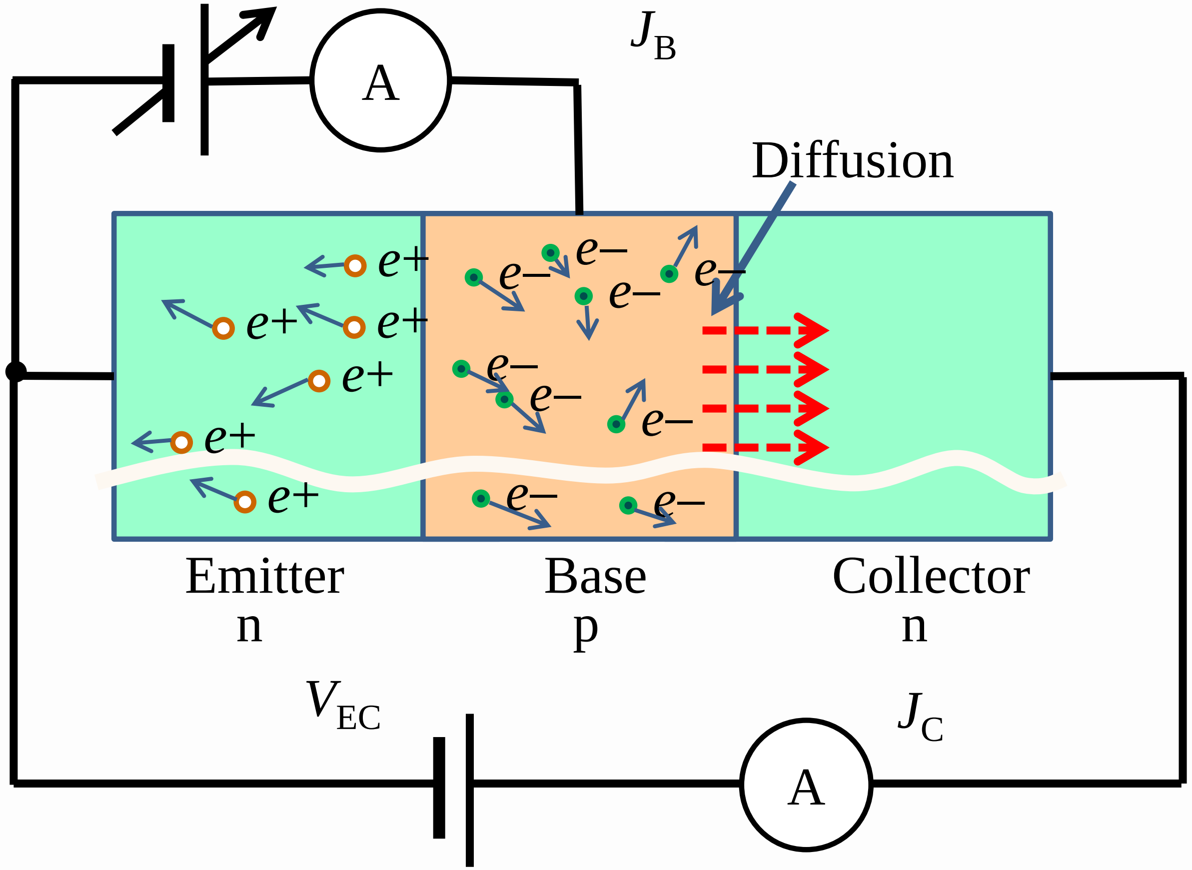




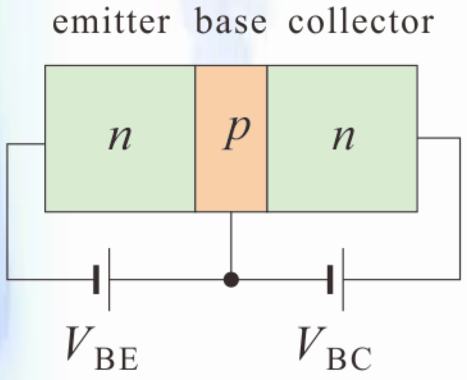
$$J_C = h_{FE}\, J_B$$

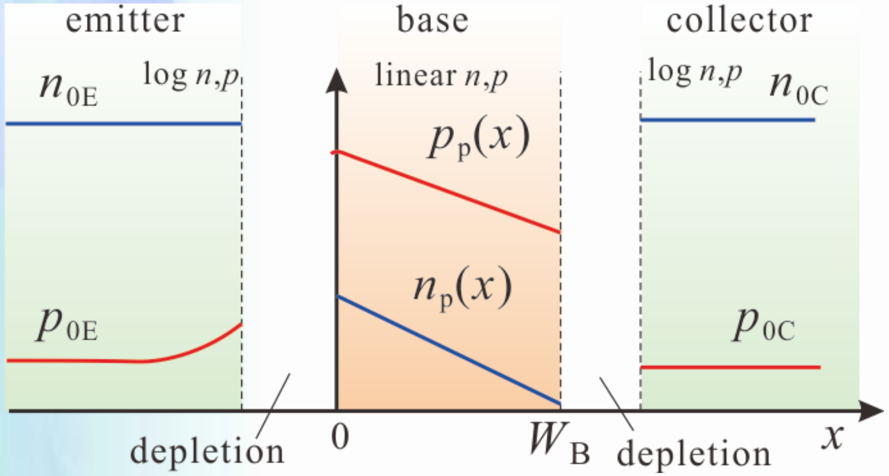
Emitter-common current gain

How a bipolar transistor amplifies signal?



Expression of h_{FE}





Sweeping out of minority carriers at the depletion edge $\quad n_p(W_{\mathrm{B}}) = n_{p0} \exp \dfrac{-eV_{\mathrm{BC}}}{k_{\mathrm{B}}T} \approx 0$

Diffusion current in the base: constant $\quad \dfrac{dn_p}{dx}$: constant $\quad n_p(x)$: linear in x

Device cross section A $\quad j_{\mathrm{D}e} = -D_e \dfrac{dn_p}{dx} \approx eD_e \dfrac{n_p(0)}{W_{\mathrm{B}}} = \dfrac{J_{\mathrm{C}}}{A}$

The law of mass action $\quad n_{p0} \approx \dfrac{n_i^2}{N_{\mathrm{A}}}$

$$J_{\mathrm{C}} \approx \frac{eAD_e n_{p0}}{W_{\mathrm{B}}} \exp \frac{eV_{\mathrm{BE}}}{k_{\mathrm{B}}T} \approx \frac{eAD_e n_i^2}{W_{\mathrm{B}} N_{\mathrm{A}}} \exp \frac{eV_{\mathrm{BE}}}{k_{\mathrm{B}}T} \equiv J_{\mathrm{S}} \exp \frac{eV_{\mathrm{BE}}}{k_{\mathrm{B}}T}$$

$$J_{\mathrm{B}h} = \frac{eAD_h}{L_h} p_{n\mathrm{E}}(0) = \frac{eAD_h}{L_h} p_{n\mathrm{E}0} \exp \frac{eV_{\mathrm{BE}}}{k_{\mathrm{B}}T} = \frac{eAD_h}{L_h} \frac{n_i^2}{N_{\mathrm{D}}} \exp \frac{eV_{\mathrm{BE}}}{k_{\mathrm{B}}T}$$

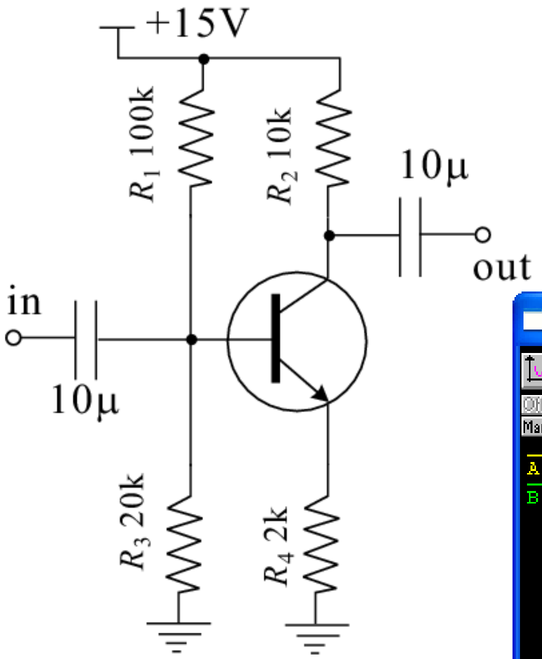
Recombination current: $\quad J_{\mathrm{Br}} = \dfrac{Q_e}{\tau_b} = \dfrac{en_p(0)AW_{\mathrm{B}}}{2\tau_b} \exp \dfrac{eV_{\mathrm{BE}}}{k_{\mathrm{B}}T}$

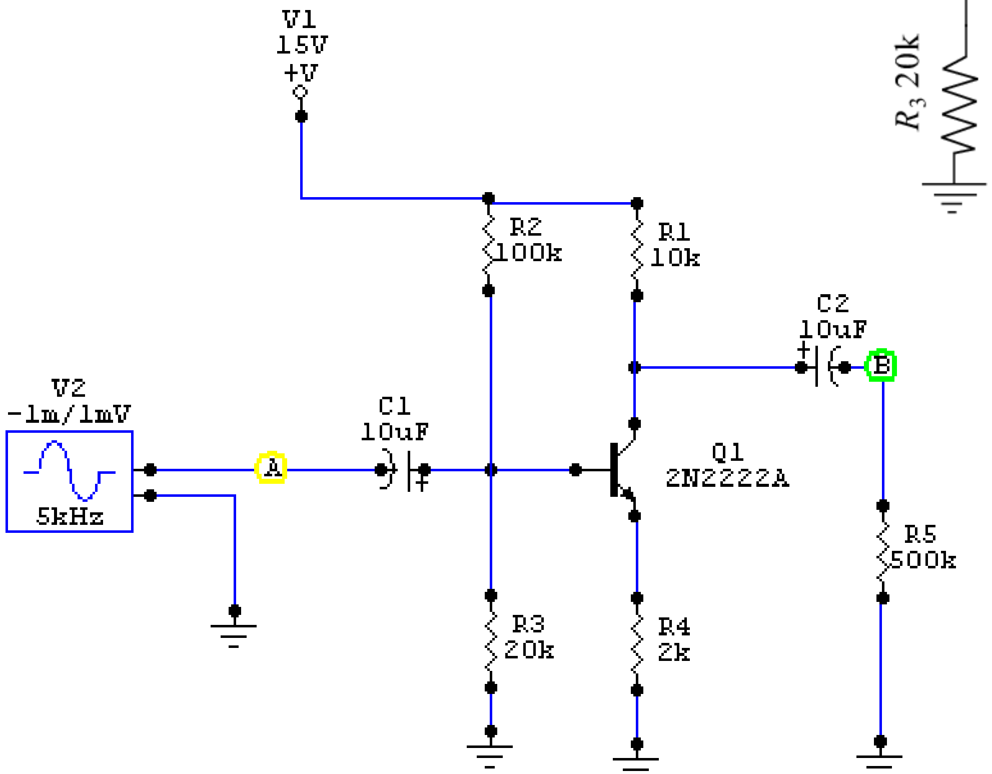
$$h_{\mathrm{FE}} = \left(\frac{D_h}{D_e} \frac{W_{\mathrm{B}}}{L_h} \frac{N_{\mathrm{A}}}{N_{\mathrm{B}}} + \frac{W_{\mathrm{B}}^2}{2\tau_b D_e} \right)^{-1}$$

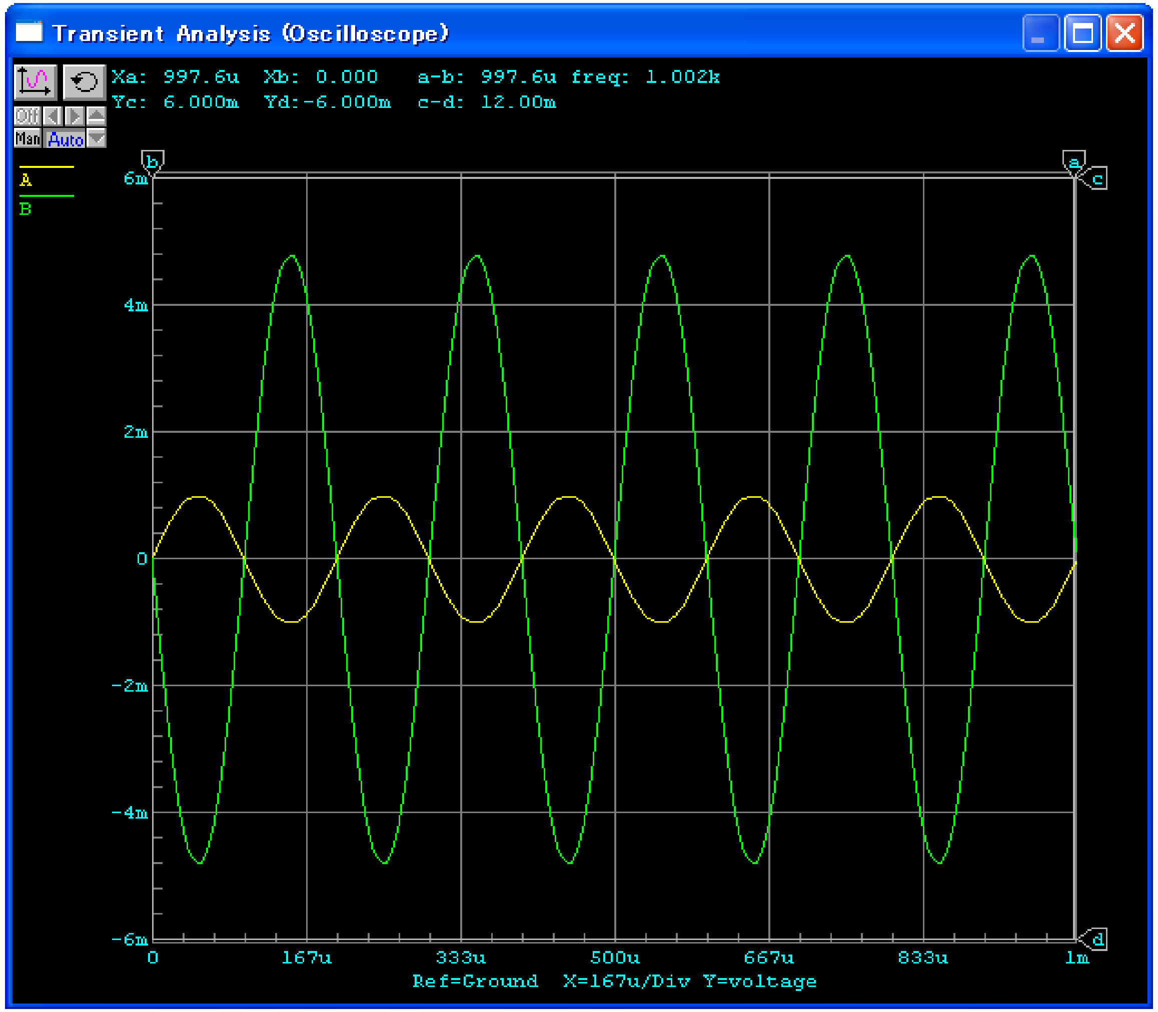
Example of an amplification circuit

$$\Delta V_{\mathrm{C}} = R_2 \Delta J_{\mathrm{C}} \approx R_2 \Delta J_{\mathrm{E}}$$

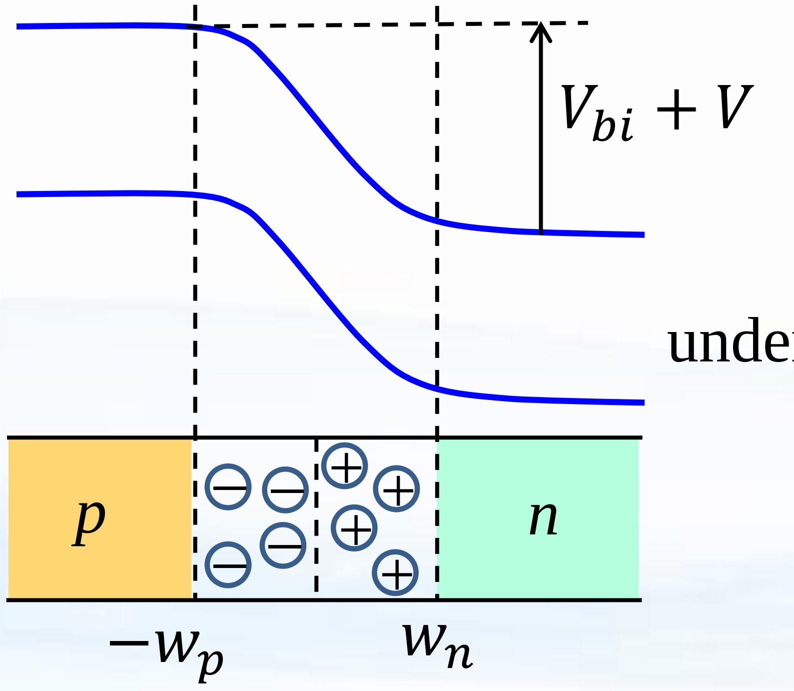
$$= R_2 \frac{\Delta V_{\mathrm{E}}}{R_4} = \frac{R_2}{R_4} \Delta V$$







Depletion layer width with reverse bias voltage



Poisson equation $$\frac{d^2\phi}{dx^2} = -aq(x) \quad (a \equiv (\epsilon\epsilon_0)^{-1})$$

charge: $$\begin{cases} q = -eN_A & (-w_p \le x \le 0), \\ q = eN_D & (0 \le x \le w_n) \end{cases}$$

under conditions

potential boundary: $$\begin{cases} \phi(-\infty) = 0 \\ \phi(-w_p) = 0, \quad \left.\frac{d\phi}{dx}\right|_{-w_p} = 0, \\ \phi(w_n) = V + V_{\rm bi}, \quad \left.\frac{d\phi}{dx}\right|_{w_n} = 0 \end{cases}$$

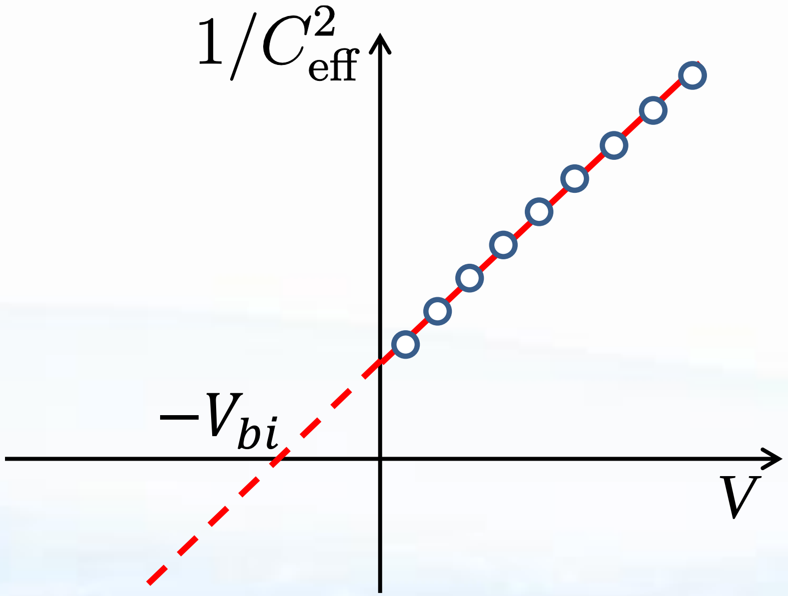
The integration gives $$\phi(x) = \begin{cases} \frac{aeN_A}{2}(x + w_p)^2 & (-w_p \le x \le 0), \\ V + V_{\rm bi} - \frac{aeN_D}{2}(x - w_n)^2 & (0 \le x \le w_n) \end{cases}$$

$$\lim_{x\to+0}\phi = \lim_{x\to-0}\phi,$$
$$\lim_{x\to+0}(d\phi/dx) = \lim_{x\to-0}(d\phi/dx)$$

$$w_p = \left[\frac{2\epsilon_0\epsilon(V + V_{\rm bi})}{eN_A}\cdot\frac{N_D}{N_D + N_A}\right]^{1/2}, \quad w_n = \left[\frac{2\epsilon_0\epsilon(V + V_{\rm bi})}{eN_D}\cdot\frac{N_A}{N_D + N_A}\right]^{1/2}$$

$$w_d = w_p + w_n = \left[\frac{2\epsilon_0\epsilon(V + V_{\rm bi})}{e}\cdot\frac{N_A + N_D}{N_A N_D}\right]^{1/2}.$$

Effective capacitance and reverse bias voltage



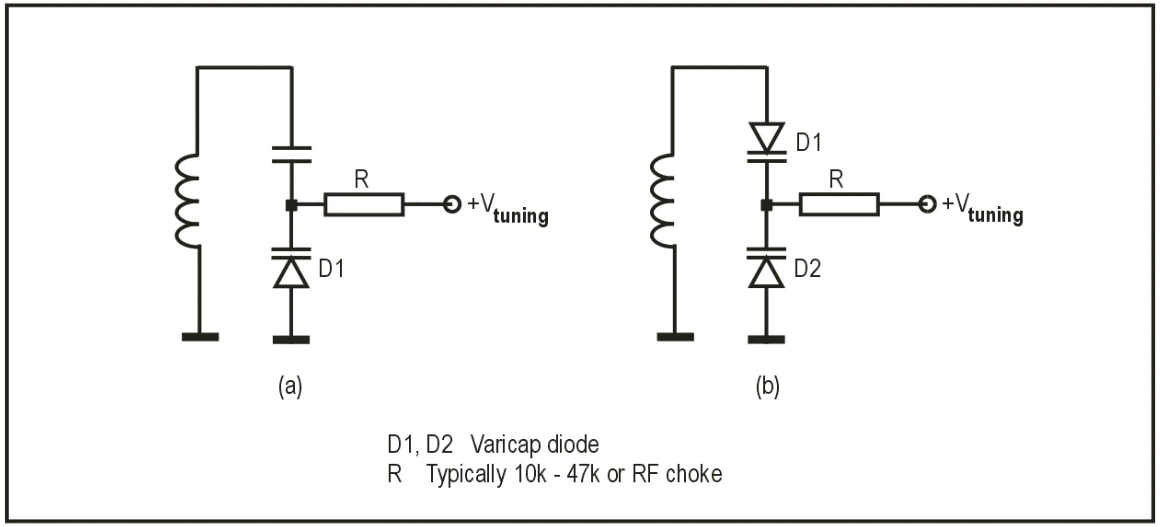
$$\frac{1}{C_{\mathrm{eff}}^2} = \frac{2}{\epsilon\epsilon_0 e N_{\mathrm{D}}}(V + V_{\mathrm{bi}})$$

This gives a way for the doping profiling.

Varicap diode

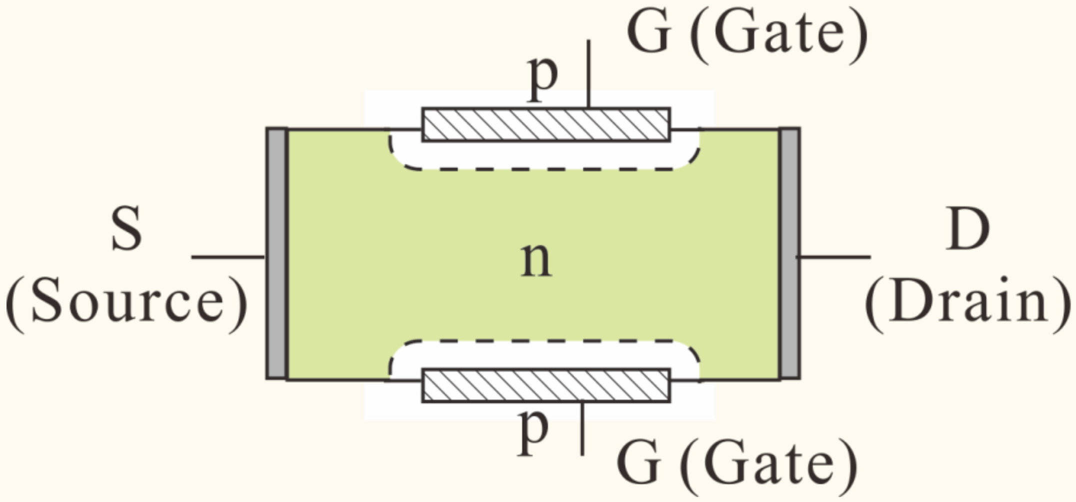
KB505

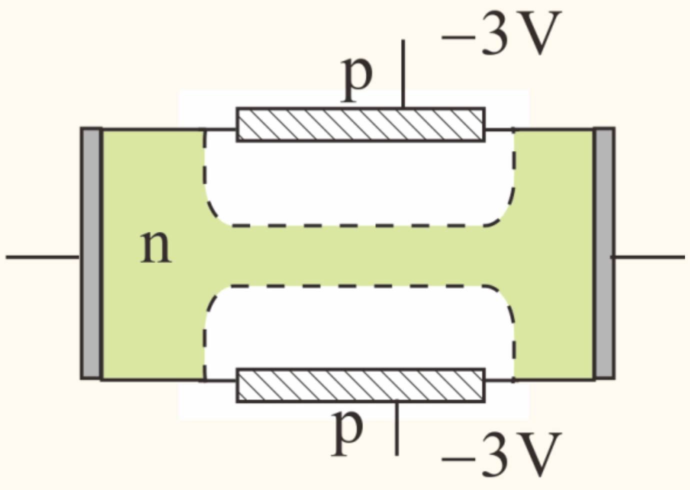
circuit example



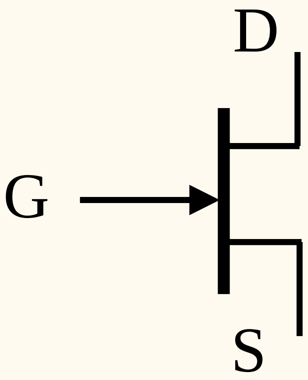
Frequency modulation
Phase lock loop

pn junction field effect transistor (JFET)

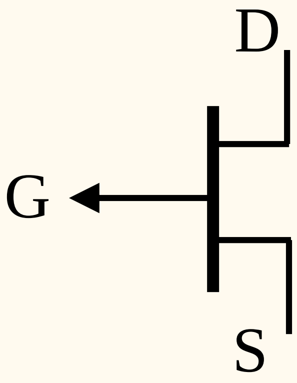




Circuit symbols

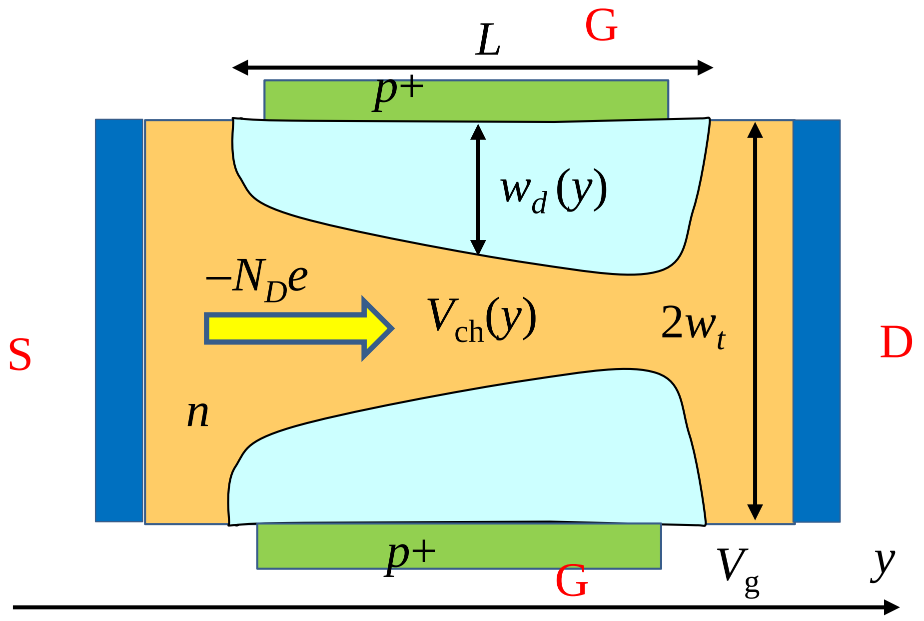


n-channel



p-channel

pn junction FET



$$V(y) = V_g + V_{bi} - V_{ch}(y)$$

$$w_d(y) = \sqrt{\frac{2\epsilon\epsilon_0 V(y)}{eN_D}}$$

$$J_{ch} = eN_D\mu_n \frac{dV_{ch}}{dy} \cdot 2[w_t - w_d(y)]W$$

conductivity — electric field — channel width

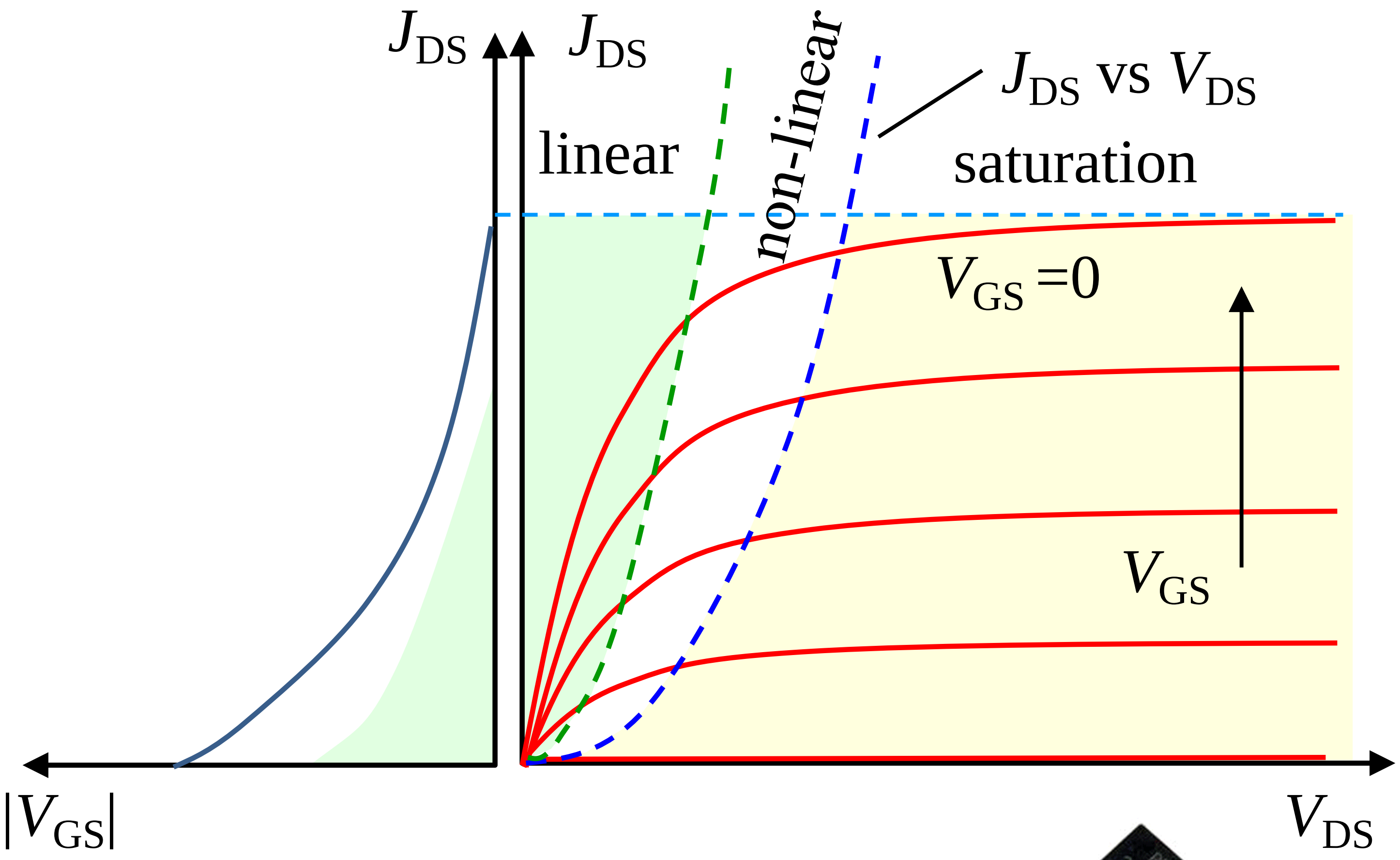
$$J_{ch}L = \int_0^L J_{ch}dy = 2eN_D\mu_n W \int_0^L (w_t - w_d)\frac{dV}{dy}dy = 2w_t eN_D\mu_n W \int_{V_0}^{V_L}\left(1 - \frac{w_d}{w_t}\right)dV$$

pinch off (internal) voltage: $w_d(V_c) = w_t \quad V_c = \frac{eN_D w_t^2}{2\epsilon\epsilon_0}$

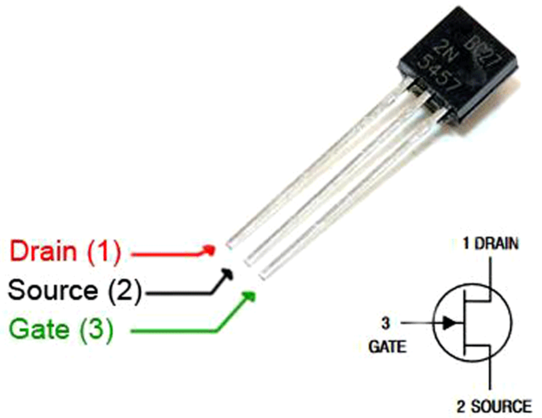
$$J_{ch} = \frac{2N_D e\mu_n W w_t}{L}\left[V_L - V_0 + \frac{2}{3\sqrt{V_c}}(V(V_0)^{3/2} - V(V_L)^{3/2})\right]$$

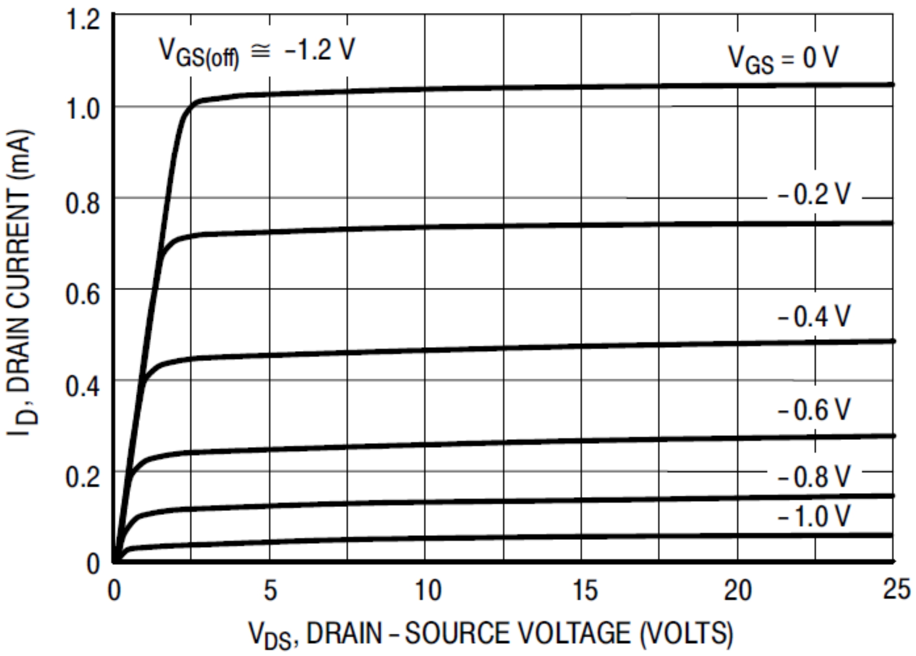
Only valid for $w_d < w_t/2$.

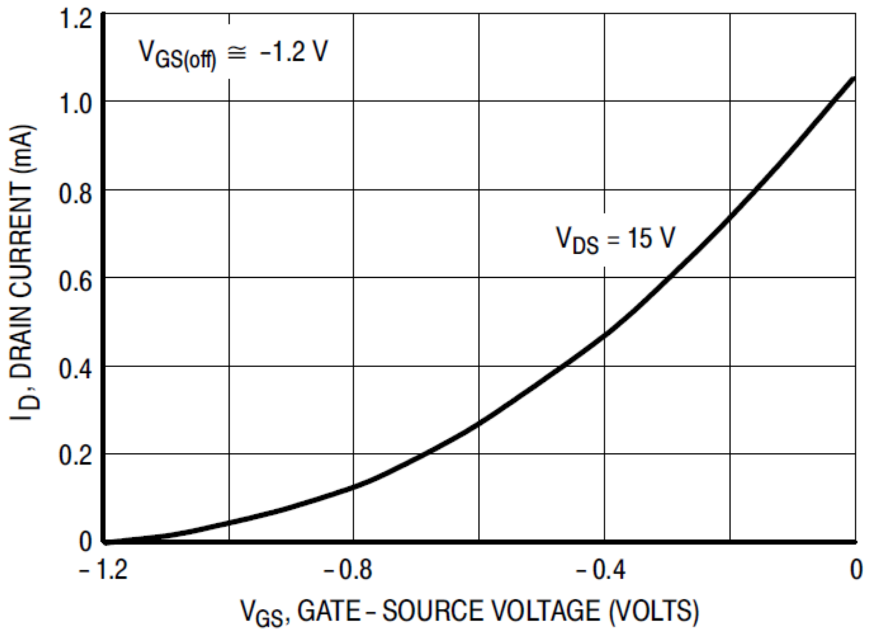
I-V characteristics of JFET



Example: 2N5457
n-channel
depletion-type



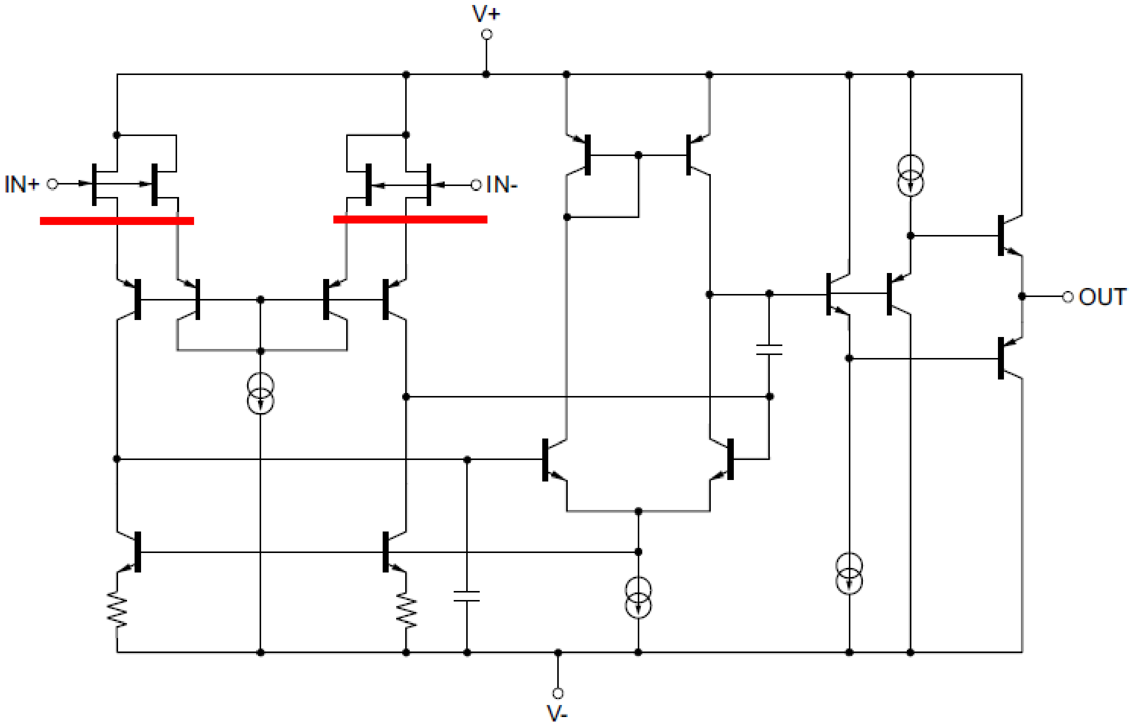




Application of JFET

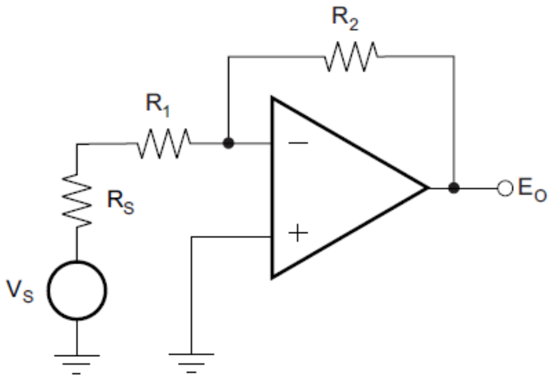
Low linearity → linearization with feedback with high gain
High input impedance, low bias current (operation at the reverse bias region)
: fit the input stage of operational amplifier

Example: OPA827



Copyright © 2016, Texas Instruments Incorporated

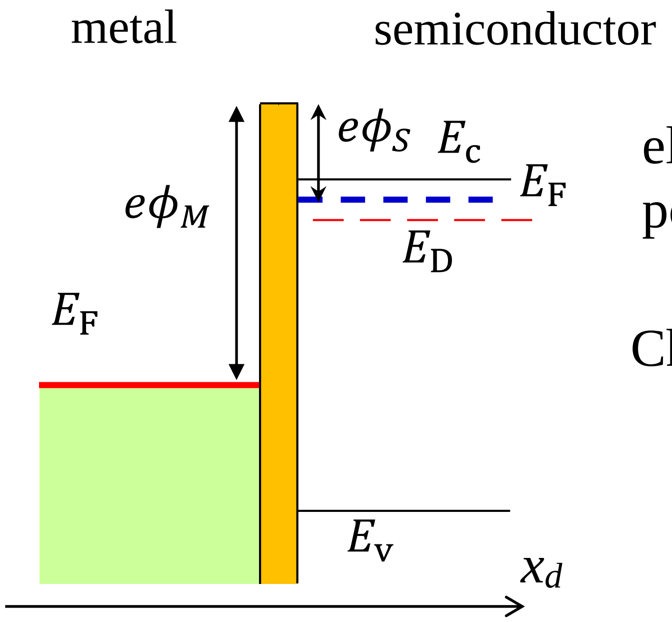
- Input voltage noise: 4 $nV/\sqrt{Hz}$ at 1 kHz
- Input bias current 10 pA max
- Input impedance 10^{13} Ω



Inverting amplifier

Schottky barrier (metal-semiconductor junction)

Walter Schottky
1886-1976



Q: space charge

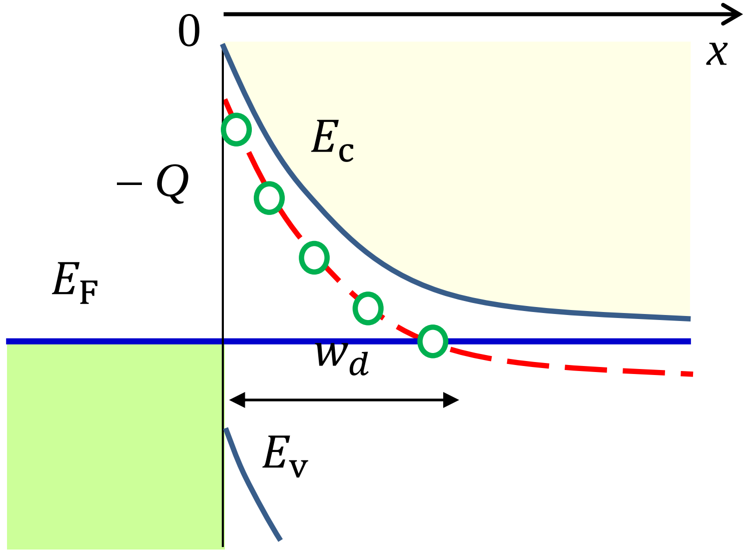
electrostatic potential

$$\phi(x_d) = \int_0^{x_d} (eN_D x - Q)/\epsilon\epsilon_0 dx = \frac{1}{\epsilon\epsilon_0}\left(\frac{eN_D}{2}x_d^2 - Qx_d\right)$$

Charge balance: $w_d = \dfrac{Q}{eN_D}$ $\phi_M - \phi_S - \phi(w_d) = 0$

$$Q = \sqrt{2\epsilon\epsilon_0 N_D e(\phi_M - \phi_S)} \quad \therefore w_d = \sqrt{\frac{2\epsilon\epsilon_0(\phi_M - \phi_S)}{eN_D}} \equiv \sqrt{\frac{2\epsilon\epsilon_0 V_s}{eN_D}}$$

Voltage V --> barrier height $e(V_s - V)$

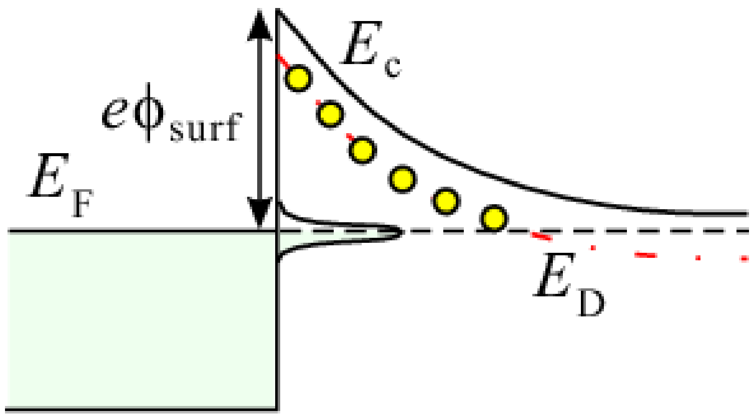


$$J = AT^2\left[\exp\left(\frac{e(V - V_s)}{k_B T}\right) - \exp\left(\frac{-eV_s}{k_B T}\right)\right]$$
$$= eAT^2 \exp\left(\frac{-eV_s}{k_B T}\right)\left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]$$

barrier overcoming current

No minority carrier injection

MES-FET



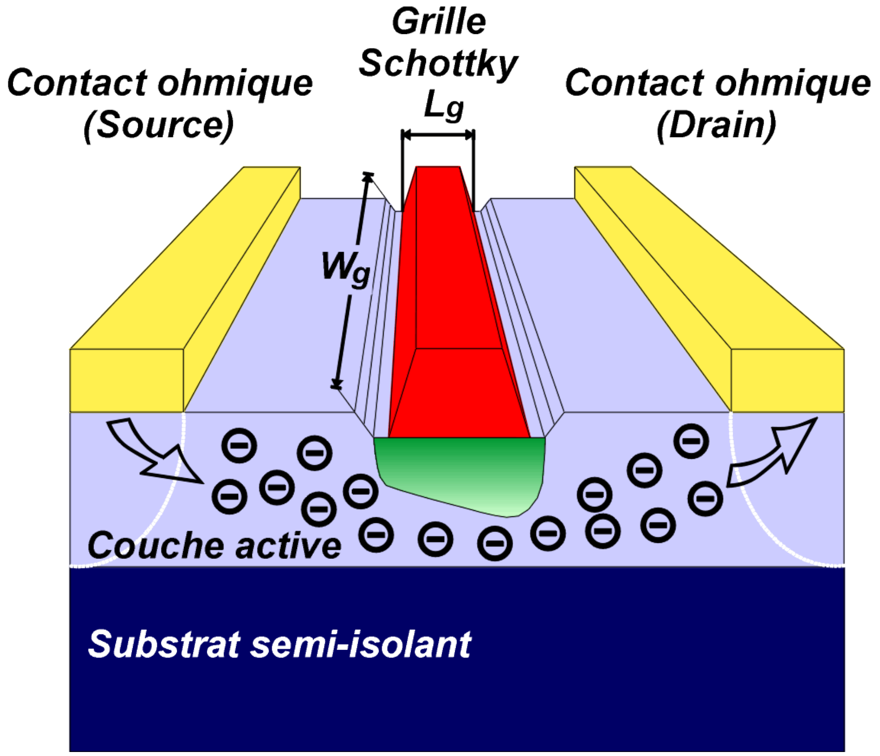
In most cases, the surface barrier is dominated by the pinning of Fermi level with the high density surface states.

MES (metal-semiconductor) FET: depletion layer device.

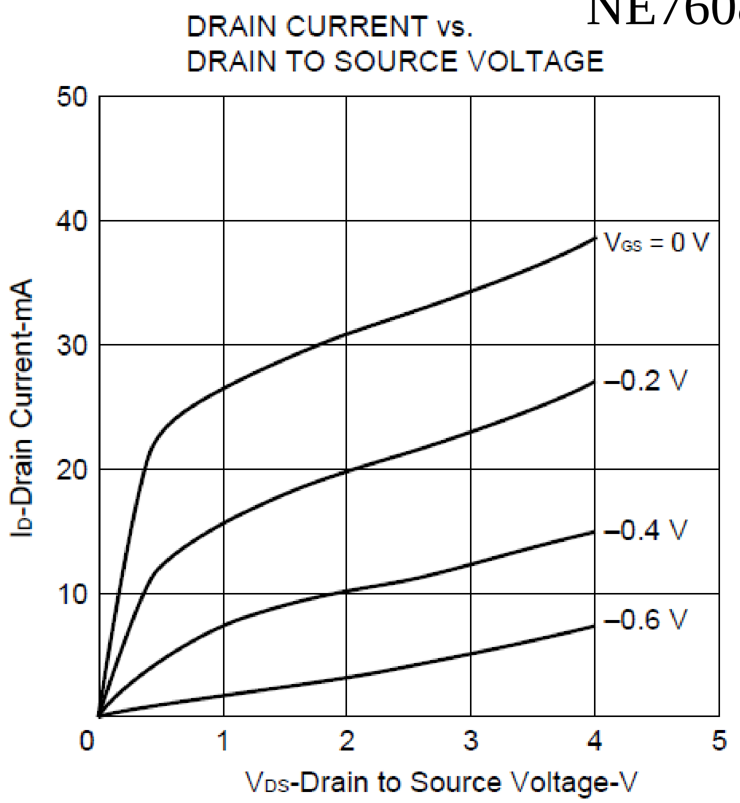
Complementary circuit is difficult.

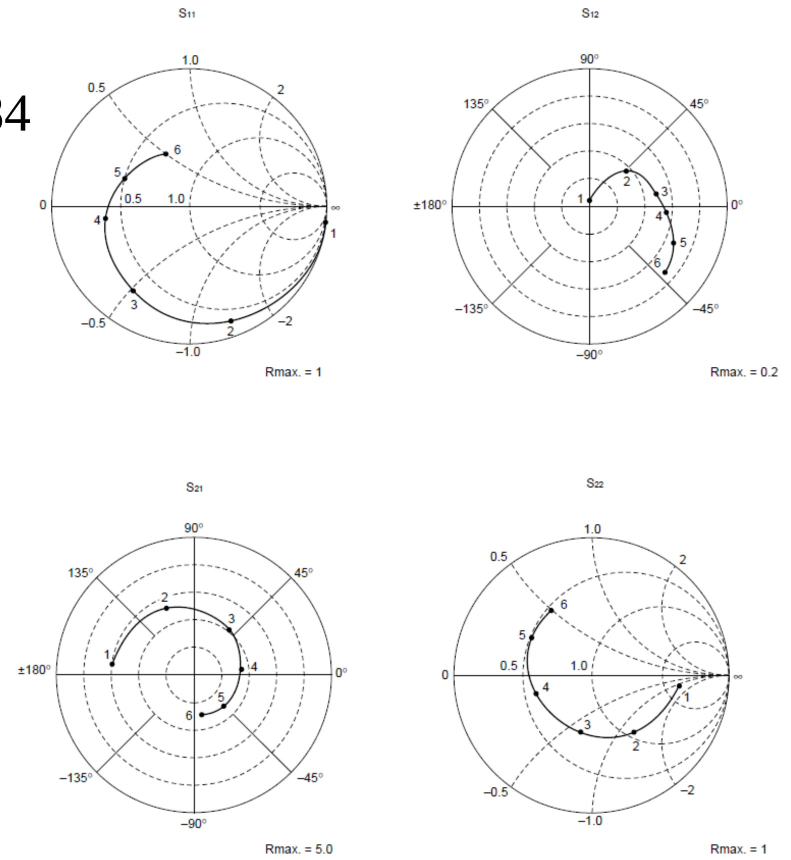
Saturation effect is a bit weak.

Data are given in S-parameters.

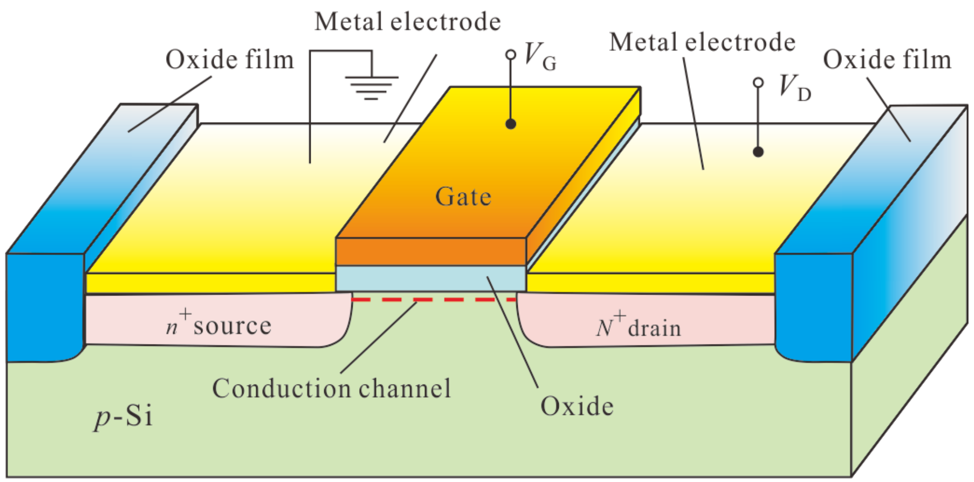


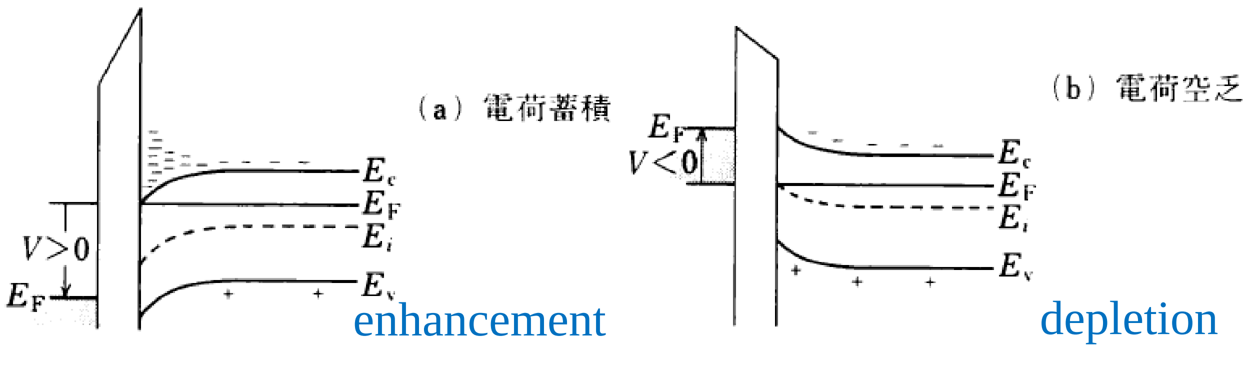
NE76084

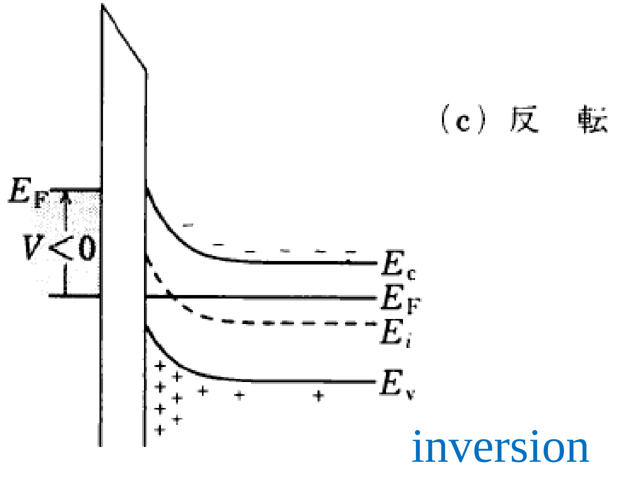




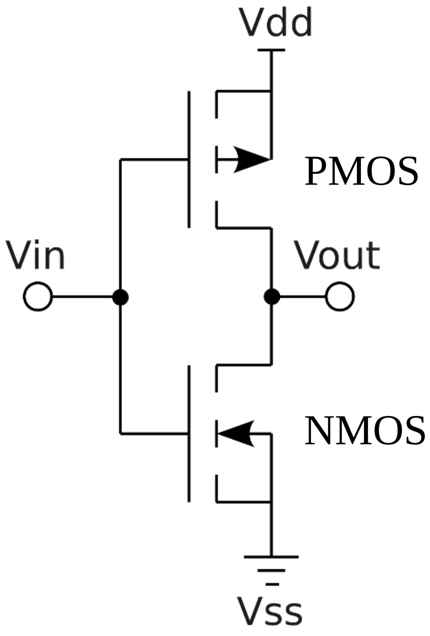
MOS FET



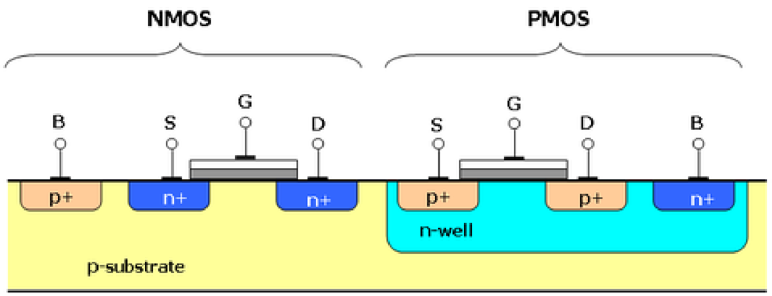




Complementary metal-oxide-semiconductor (CMOS) circuit



Simplified CMOS inverter circuit



Low leakage current

Single gate input both on/off switch

FinFET

Improvement of MOS FET

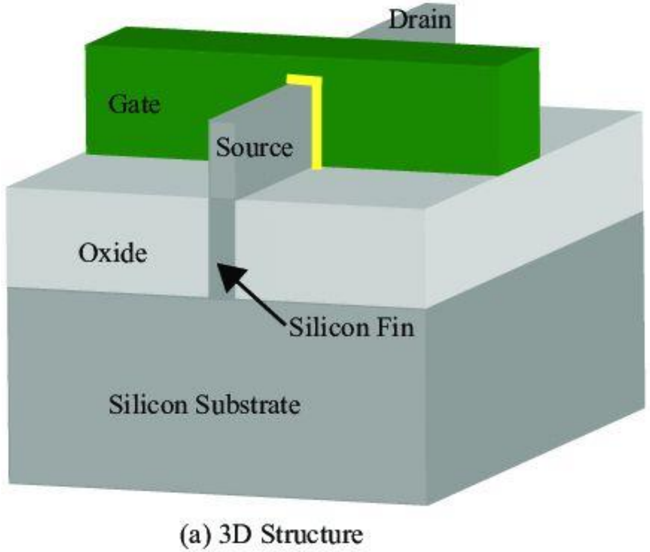
High -κ materials for dielectrics other than

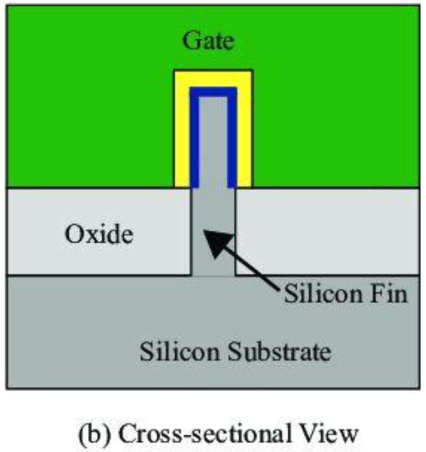
	κ
SiO_2	3.9
HfO_4Si	11
Si_3N_4	7
Al_2O_3	9
ZrO_2	25
HfO_2	25

Low voltage action requirement:

Multi-gate structure to wrap up the conduction channel

FinFET structure



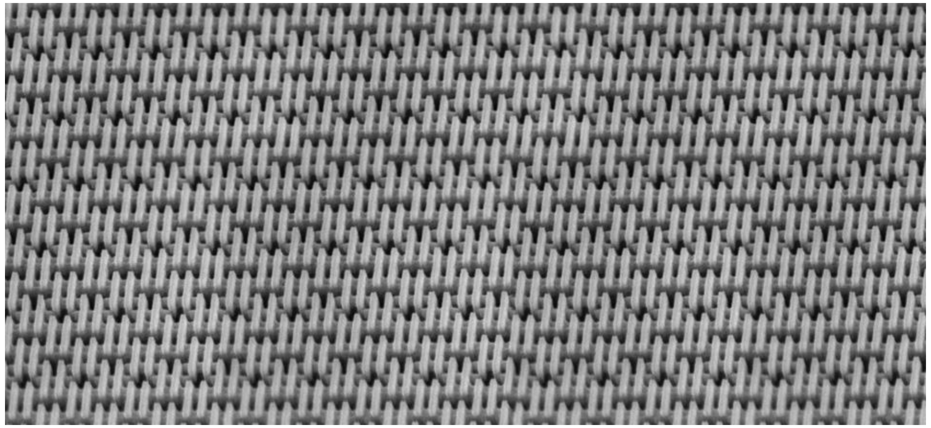


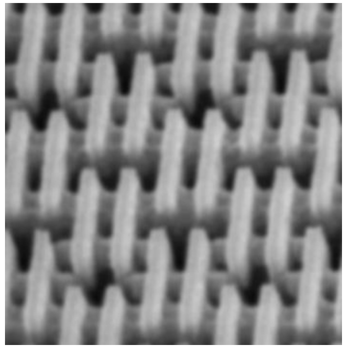
Pinch-off the channel with wrapping gate

Less than 1 ps

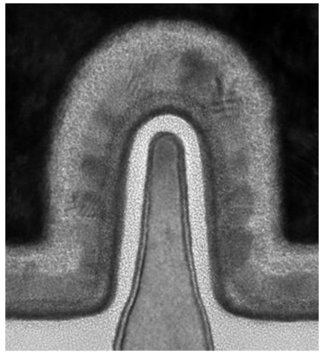
Less than 1 V

Inversion conductive mode





TSMC 7 nm transistors



TSMC 30 nm gate (2012)

Heterojunction and envelope function

Effective mass approximation

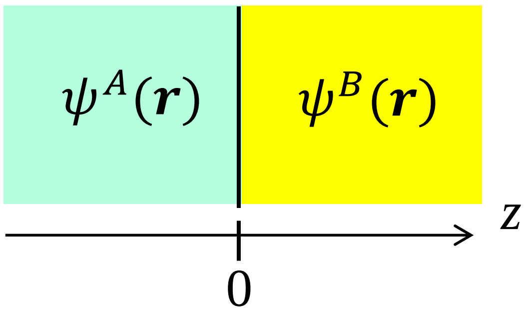
$$\left[-\frac{\hbar^2\nabla^2}{2m^*}+U(\boldsymbol{r})\right]f(\boldsymbol{r})=Ef(\boldsymbol{r})$$

$f(\boldsymbol{r})$: envelope function

This holds for spatially slow perturbation $U(\boldsymbol{r})$.

Then how about heterointerface?

$$\psi^{(\mathrm{A})}(\boldsymbol{r})=\sum_l f_l^{(\mathrm{A})}(\boldsymbol{r})u_{l\boldsymbol{k}}^{(\mathrm{A})}(\boldsymbol{r}),\quad \psi^{(\mathrm{B})}(\boldsymbol{r})=\sum_l f_l^{(\mathrm{B})}(\boldsymbol{r})u_{l\boldsymbol{k}}^{(\mathrm{B})}(\boldsymbol{r})$$



1. For simplicity we assume $u_{l\boldsymbol{k}}^{(\mathrm{A})}(\boldsymbol{r})=u_{l\boldsymbol{k}}^{(\mathrm{B})}(\boldsymbol{r}),\quad \partial\epsilon_l^{(\mathrm{A})}/\partial\boldsymbol{k}=\partial\epsilon_l^{(\mathrm{B})}/\partial\boldsymbol{k}$

Then continuity condition at z =0 becomes $f_l^{(\mathrm{A})}(\boldsymbol{r}_{xy},0)=f_l^{(\mathrm{B})}(\boldsymbol{r}_{xy},0)$

In xy-plane, the Bloch theorem tells $f_l^{(\mathrm{A,B})}=\frac{1}{\sqrt{S}}\exp(i\boldsymbol{k}_{xy}\cdot\boldsymbol{x})\underline{\chi_l^{(\mathrm{A,B})}(z)}$

envelope function for z

For z-freedom, we apply $k\cdot p$ perturbation.

$$\mathscr{D}^{(0)}\left(z,-i\hbar\frac{\partial}{\partial z}\right)\boldsymbol{\chi}=\epsilon\boldsymbol{\chi}$$

Heterojunction and envelope function (2)

The elements of $\mathscr{D}^{(0)}$ are

$$\mathscr{D}^{(0)}_{lm}\left(z, \frac{\partial}{\partial z}\right) = \left[\epsilon_l(z) + \frac{\hbar^2 k_{xy}^2}{2m_0} - \frac{\hbar^2}{2m_0}\frac{\partial^2}{\partial z^2}\right]\delta_{lm} + \frac{\hbar \boldsymbol{k}_{xy}}{m_0}\cdot\langle l|\boldsymbol{p}_{xy}|m\rangle - \frac{i\hbar}{m_0}\langle l|p_z|m\rangle\frac{\partial}{\partial z}$$

with $\epsilon_l(z) = \epsilon_l^{(\mathrm{A})} \quad (z<0), \quad \epsilon_l^{(\mathrm{B})} \quad (z \geq 0)$

$$V_l(z) \equiv \begin{cases} 0 & z<0 \quad (z \in \mathrm{A}) \\ \epsilon_l^{(\mathrm{B})} - \epsilon_l^{(\mathrm{A})} & z \geq 0 \quad (z \in \mathrm{B}). \end{cases}$$

$$\sum_{m=1}^{N}\left\{\left[\epsilon_{m0}^{(\mathrm{A})} + V_m(z) + \frac{\hbar^2 k_{xy}^2}{2m_0} - \frac{\hbar^2}{2m_0}\frac{\partial^2}{\partial z^2}\right]\delta_{lm} - \frac{i\hbar}{m_0}\langle l|\hat{p}_z|m\rangle\frac{\partial}{\partial z} + \frac{\hbar \boldsymbol{k}_{xy}}{m_0}\cdot\langle l|\hat{\boldsymbol{p}}_{xy}|m\rangle\right\}\chi_m = \epsilon\chi_l$$

Continuity condition:

$$\mathscr{A}^{(\mathrm{A})}\chi^{(\mathrm{A})}(z_0=0) = \mathscr{A}^{(\mathrm{B})}\chi^{(\mathrm{B})}(0)$$

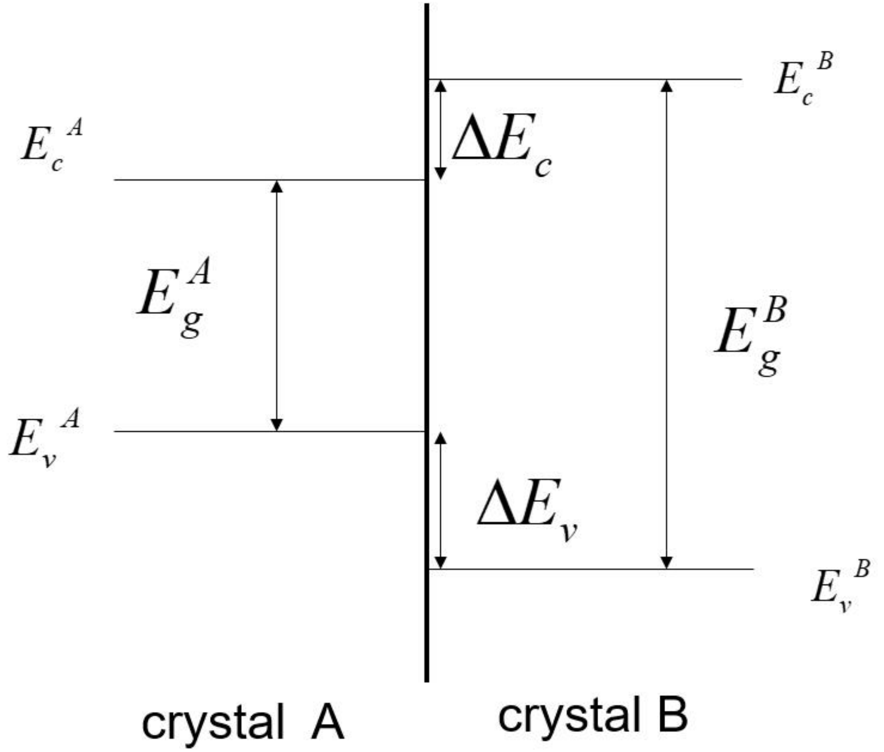
$$\mathscr{A}_{lm} = -\frac{\hbar^2}{2m_0}\left[\delta_{lm}\frac{\partial}{\partial z} + \frac{2i}{\hbar}\langle l|p_z|m\rangle\right]$$

continuity in derivative ($\frac{\partial}{\partial z}$) — discontinuity ($\langle l|p_z|m\rangle$)

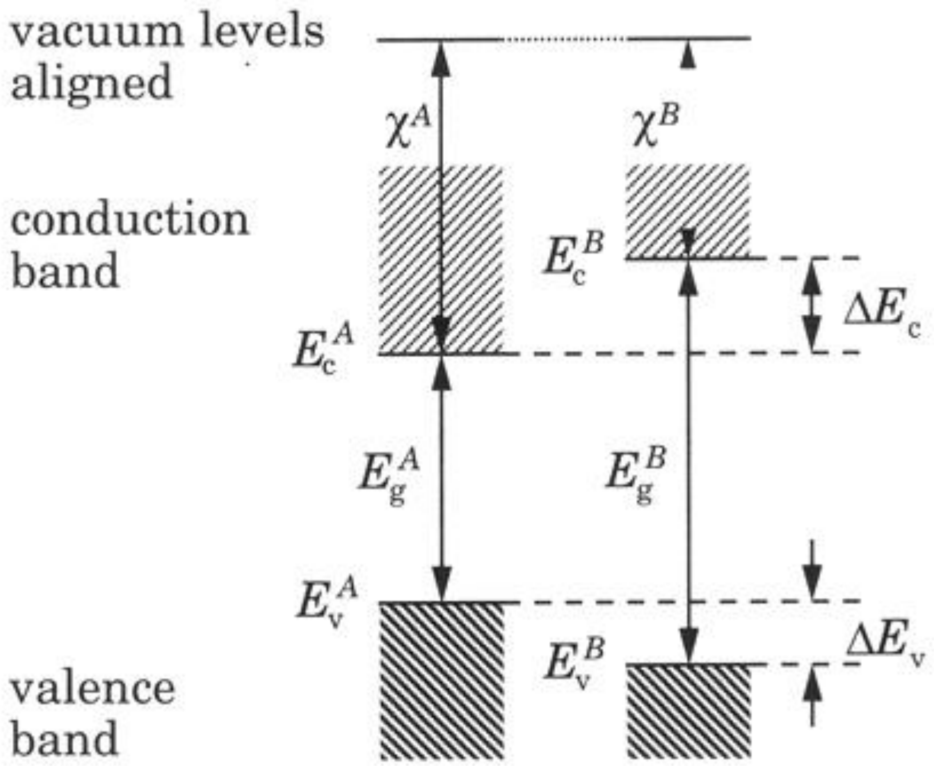
When the band mixing effect is ignorable, we can also apply the effective mass approximation for the heterojunctions.

Band discontinuity

Band discontinuity parameters

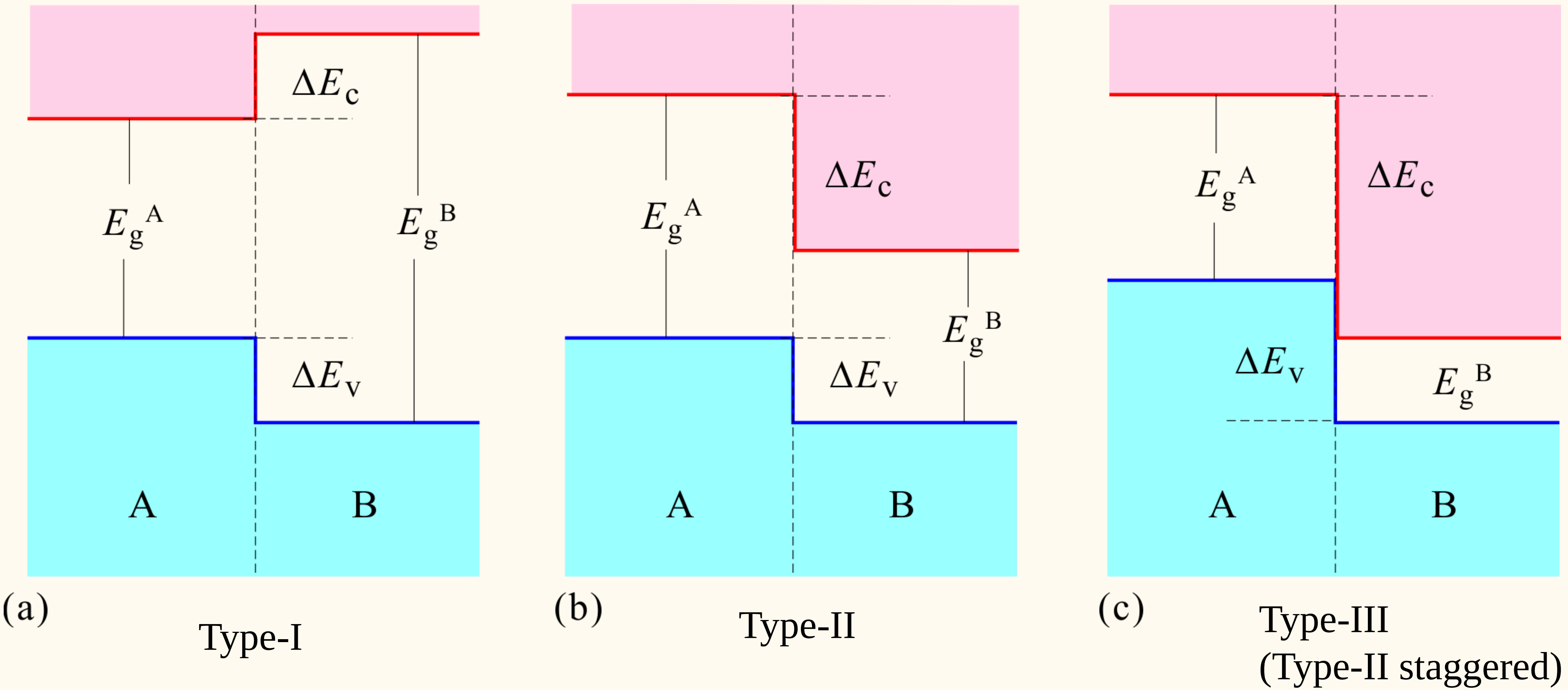


Anderson's rule: affinity from the vacuum level determines the alignment



R. L. Anderson, IBM J. Res. Dev. **4**, 283 (1960).

Heterojunction types

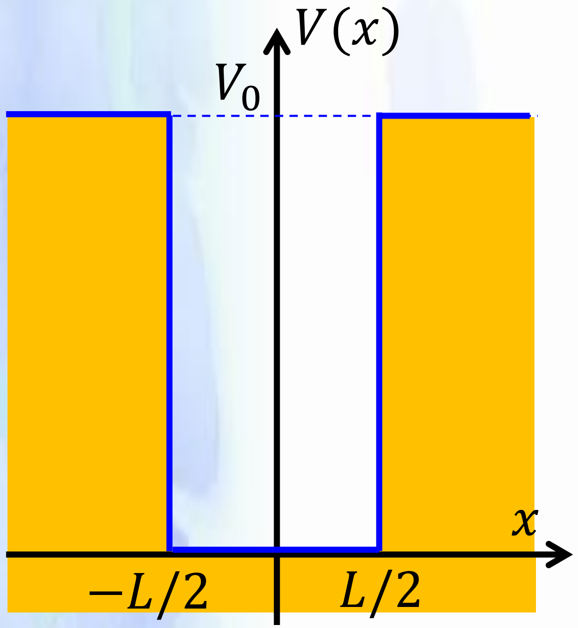


Chapter 7 Quantum Structure (Quantum wells, wires, dots)

Zhores I. Alferov | Herbert Kroemer | Jack S. Kilby

The Nobel Prize in Physics 2000 was awarded *"for basic work on information and communication technology"* with one half jointly to Zhores I. Alferov and Herbert Kroemer *"for developing semiconductor heterostructures used in high-speed- and opto-electronics"* and the other half to Jack S. Kilby *"for his part in the invention of the integrated circuit"*.

Quantum well (elementary quantum mechanics)



Outside the well: $\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V_0\right]\psi = E\psi, \quad x \leq -\frac{L}{2}, \ \frac{L}{2} \leq x, \quad \kappa \equiv \frac{\sqrt{2m|E-V_0|}}{\hbar}$

$$\psi(x) = \begin{cases} C_1 \exp(i\kappa x) + C_2 \exp(-i\kappa x) & E \geq V_0, \\ D_1 \exp(\kappa x) + D_2 \exp(-\kappa x) & E < V_0. \end{cases}$$

States localized inside the well: $E < V_0$ $\quad \frac{L}{2} < x \rightarrow D_1^+ = 0, \quad x < -\frac{L}{2} \rightarrow D_2^- = 0$

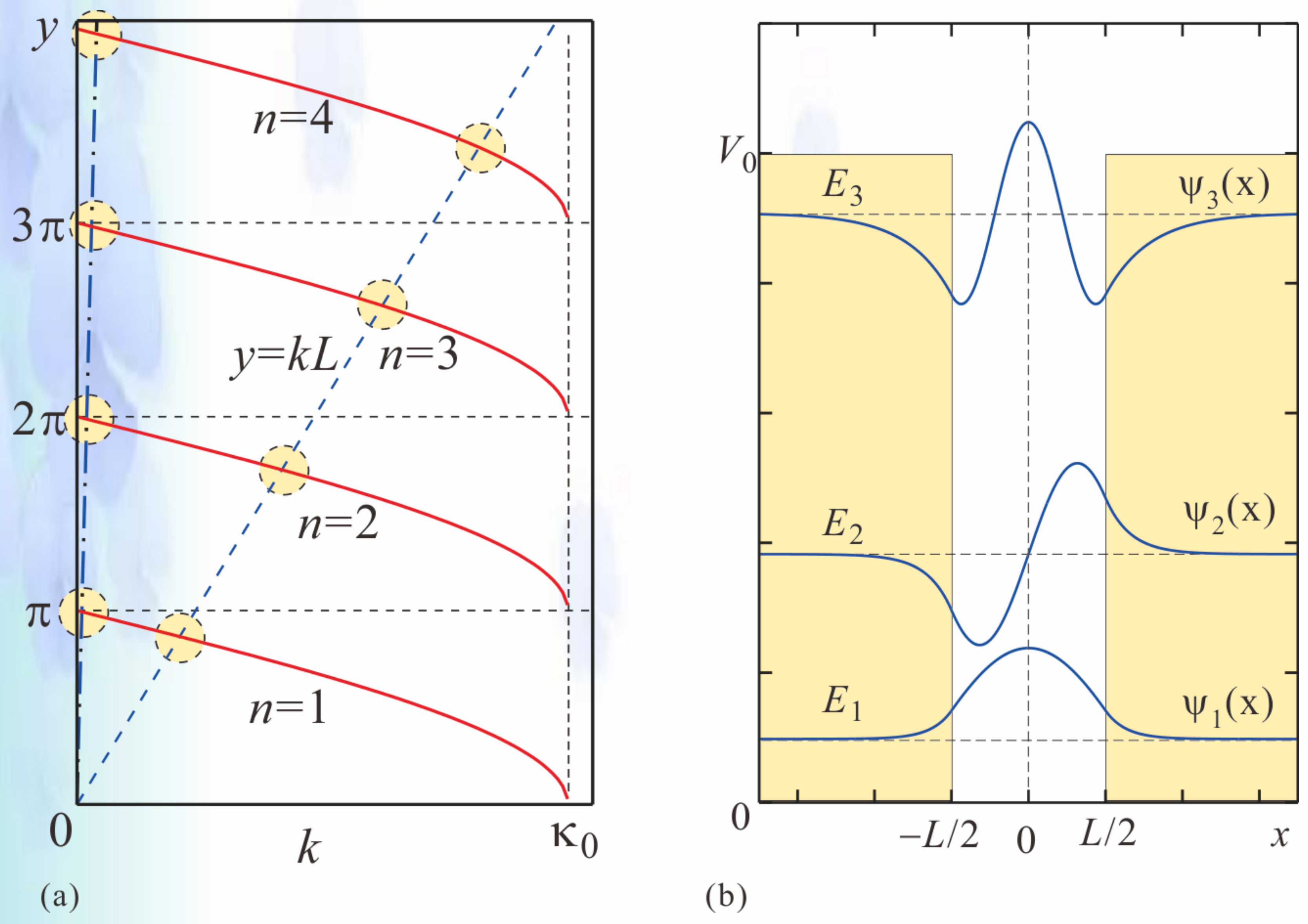
Inside the well: $\psi(x) = C_1 \exp(ikx) + C_2 \exp(-ikx), \ k \equiv \frac{\sqrt{2mE}}{\hbar}, \ x \in \left[-\frac{L}{2}, \frac{L}{2}\right]$

Envelope function connection

Continuity
$$\begin{cases} C_1 \exp(ikL/2) + C_2 \exp(-ikL/2) = D_2^+ \exp(-\kappa L/2), \\ C_1 \exp(-ikL/2) + C_2 \exp(ikL/2) = D_1^- \exp(-\kappa L/2), \end{cases}$$

Differentiability
$$\begin{cases} ikC_1 \exp(ikL/2) - ikC_2 \exp(-ikL/2) = -\kappa D_2^+ \exp(-\kappa L/2), \\ ikC_1 \exp(-ikL/2) - ikC_2 \exp(ikL/2) = \kappa D_1^- \exp(-\kappa L/2), \end{cases}$$

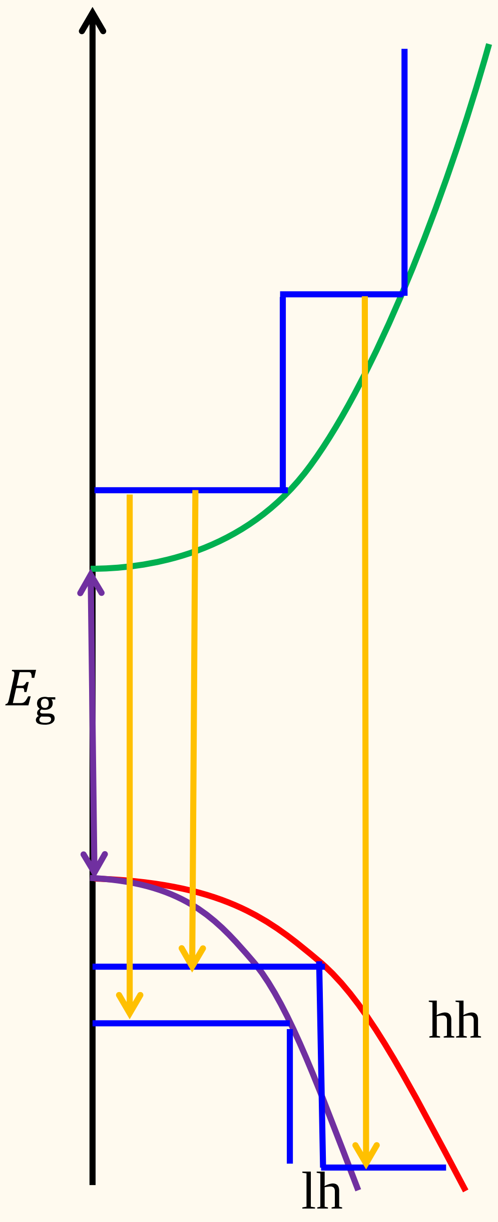
Quantum well



$$kL = -2\arctan\frac{k}{\sqrt{\kappa_0^2 - k^2}} + n\pi$$

$$\kappa_0^2 \equiv \frac{2mV_0}{\hbar^2}, \quad n = 1, 2, \cdots$$

Optical absorption of quantum wells



$$\left.\begin{aligned}\psi_e(\boldsymbol{r}) &= \phi_e(z)\exp(i\boldsymbol{k}_{xy}\cdot\boldsymbol{r}_{xy})u_c(\boldsymbol{r}),\\ \psi_h(\boldsymbol{r}) &= \phi_h(z)\exp(i\boldsymbol{k}_{xy}\cdot\boldsymbol{r}_{xy})u_v(\boldsymbol{r}).\end{aligned}\right\}$$

Envelope functions

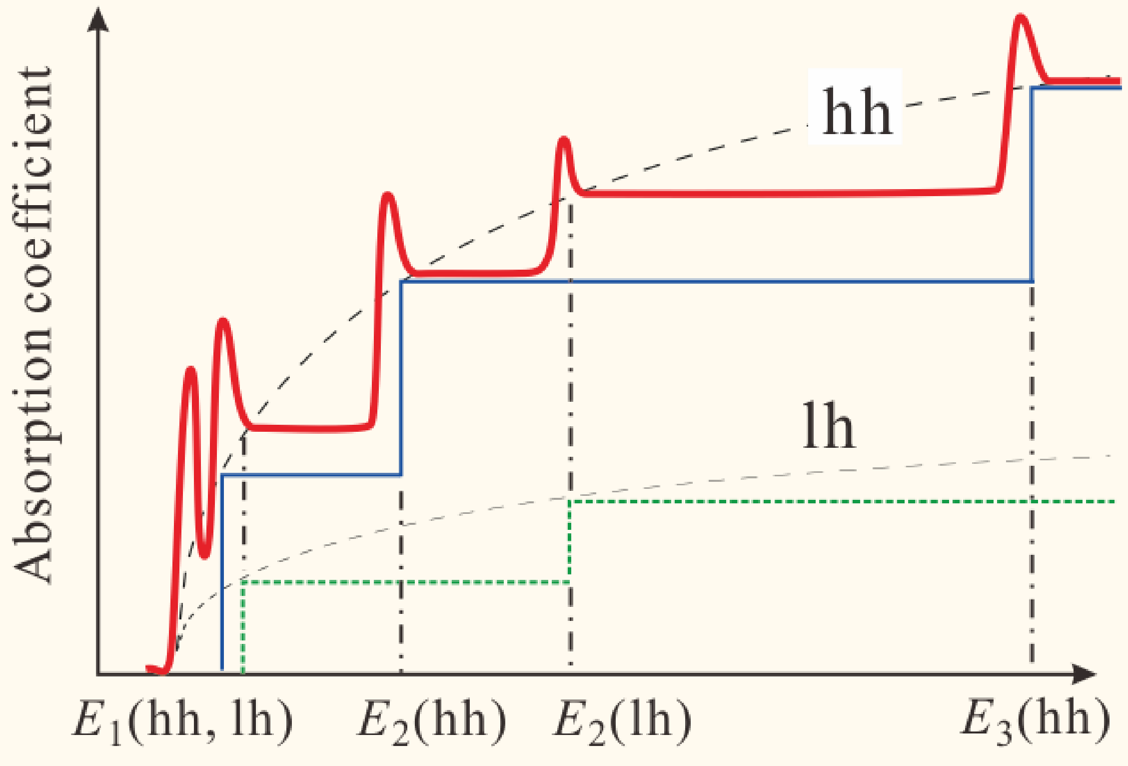
Lattice periodic functions

Direct transition rate: $P_{cv} \propto \langle u_c(\boldsymbol{r})|\boldsymbol{\nabla}|u_v(\boldsymbol{r})\rangle \int_{-\infty}^{\infty} dz\phi_e(z)^*\phi_h(z)$

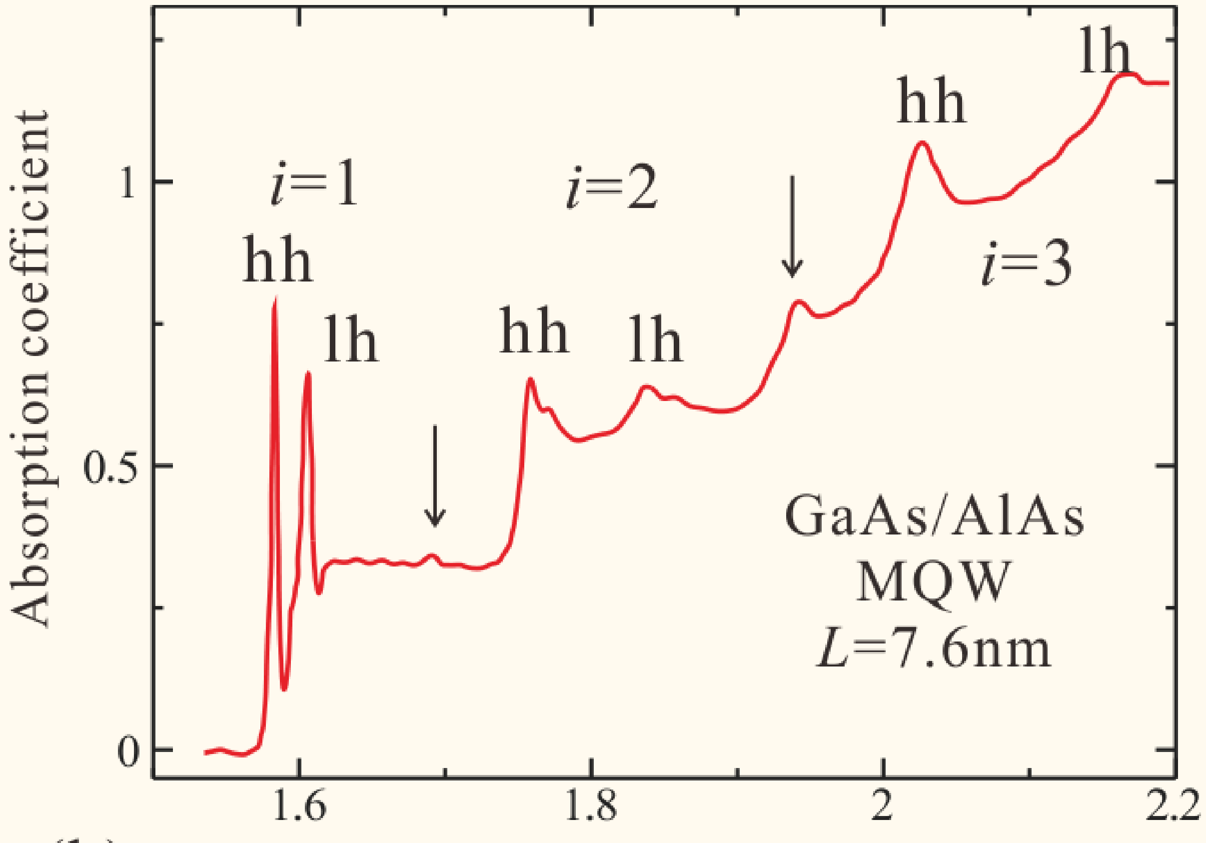
Transition energy: $E = E_{\mathrm{g}} + \Delta E_n^{(eh)} + \dfrac{\hbar^2}{2\mu}k_{xy}^2$

Two dimensional density of states: $\dfrac{dn}{dE} = \dfrac{m^*}{2\pi\hbar^2}H(E)$ $\quad$ ($H(x)$: Heaviside function)

Optical absorption of quantum wells



(a)



(b)