Lecture on

Semiconductors / 半導体

(Physics of semiconductors)

2021.6.16 Lecture 10

10:25 – 11:55

Institute for Solid State Physics, University of Tokyo

Shingo Katsumoto

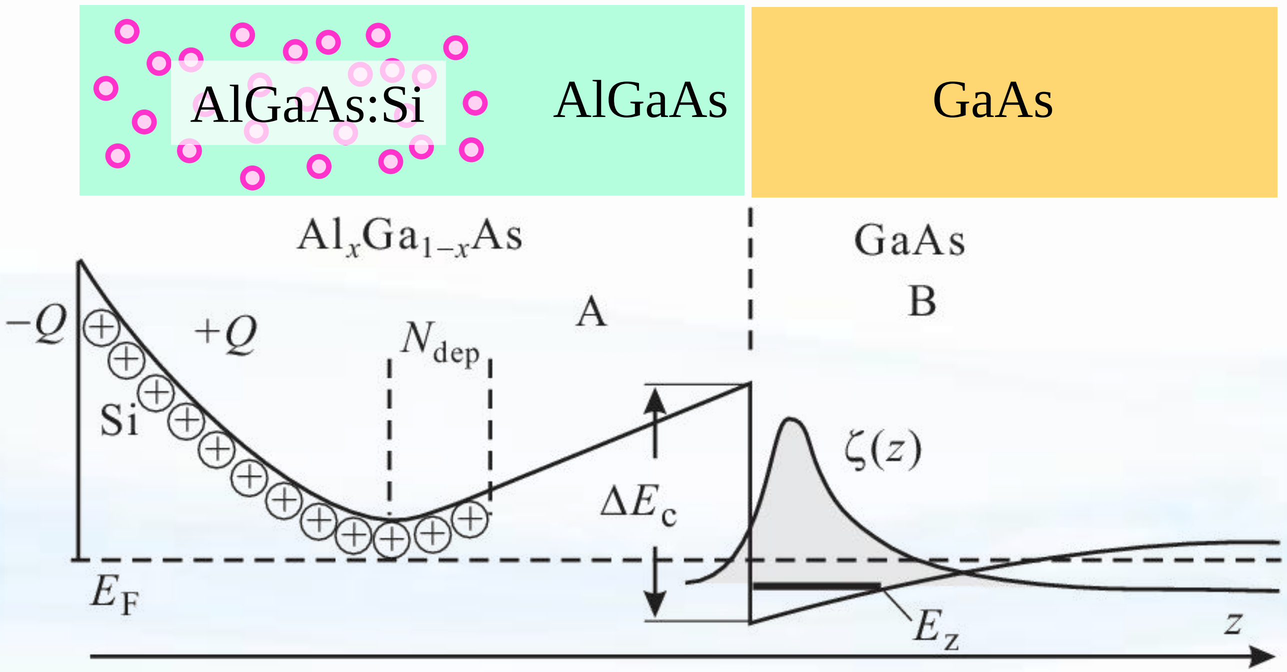
Review of last week

- Band discontinuity at heterojunction

Chapter 7 Quantum Structure (Quantum wells, wires, dots)

- Quantum wells
- Excitons in quantum wells
- Quantum barriers

Modulation doping and 2-dimensional electrons



Donor potential $V_D(z) = \frac{4\pi e^2}{\epsilon\epsilon_0} N_{\mathrm{dep}} z \quad z > 0$

$$\Psi(\boldsymbol{r}) = \psi(x, y)\zeta(z)$$

Electric field of sheet charge at z' $-\frac{4\pi e^2}{\epsilon\epsilon_0} n_{2d} |\zeta(z')|^2 |z - z'|$

$$V_{2d}(z) = -\frac{4\pi e^2}{\epsilon\epsilon_0} n_{2d}(E_z) \int_{-\xi}^{\infty} |\zeta(z')|^2 |z - z'| dz'$$

Heterointerface potential $V_h(z) = \Delta E_{\mathrm{c}}[1 - H(z)]$

Poisson-Schrödinger scheme

potential
$$V(z) = V_h(z) + \frac{4\pi e^2}{\epsilon\epsilon_0}\left[N_{\mathrm{dep}} z - n_{2d}(E_z) \int_{-\xi}^{\infty} |z - z'||\zeta(z')|^2 dz'\right]$$

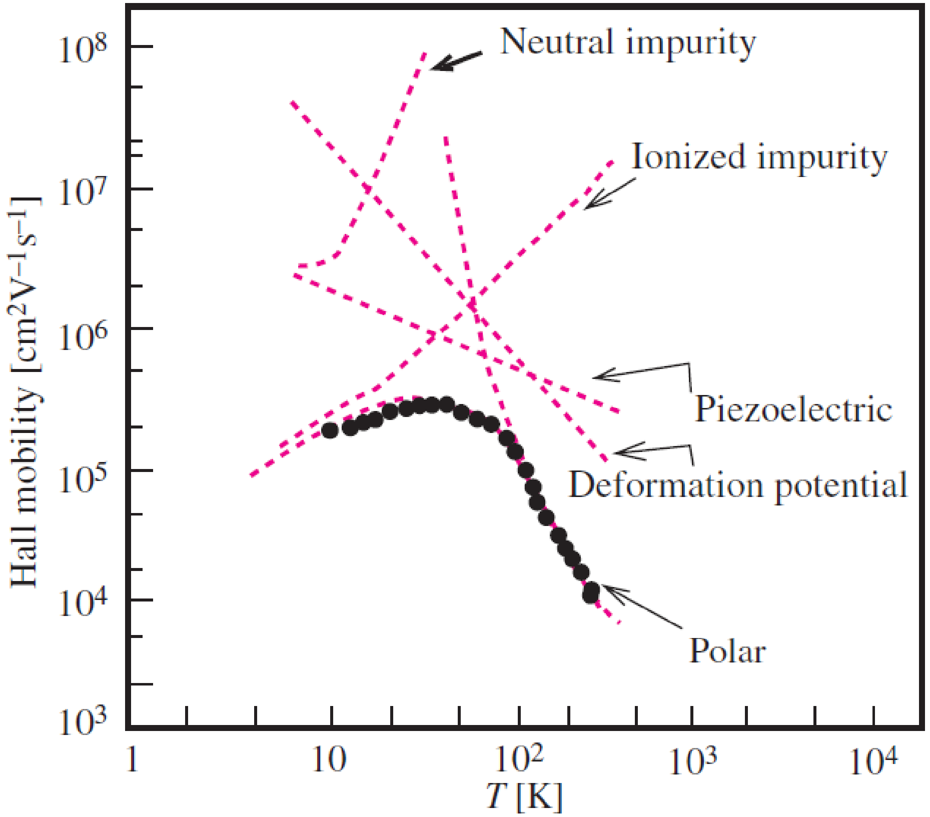
Schrödinger equation
$$\left[-\frac{\hbar^2}{2m^*(z)} \frac{\partial^2}{\partial z^2} + V(z)\right] \zeta(z) = E_z \zeta(z)$$

Boundary condition
$$\zeta(0)^{(\mathrm{A})} = \zeta(0)^{(\mathrm{B})}, \quad \frac{1}{m_{\mathrm{A}}^*} \frac{d\zeta^{(\mathrm{A})}}{dz}\bigg| = \frac{1}{m_{\mathrm{B}}^*} \frac{d\zeta^{(\mathrm{B})}}{dz}\bigg|$$

Electron mobility in MODFET

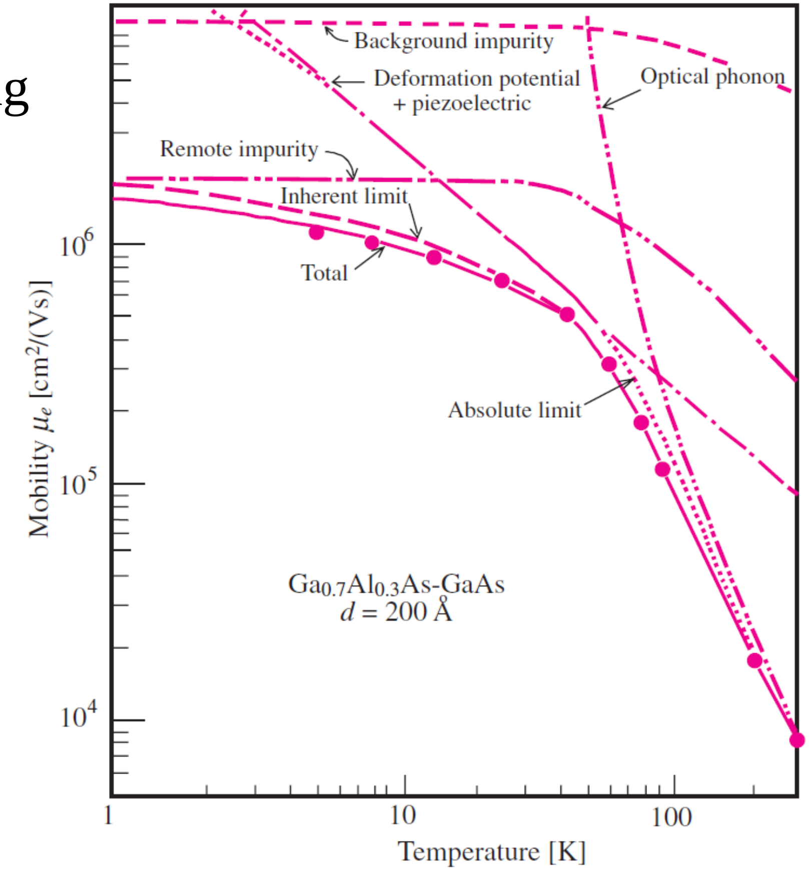
Matthiessen's rule (series connection of scattering)

$$\frac{1}{\tau_{\mathrm{total}}} = \sum_{\beta} \frac{1}{\tau_{\beta}} = \frac{1}{\tau_{\mathrm{defects}}} + \frac{1}{\tau_{\mathrm{cattier}}} + \frac{1}{\tau_{\mathrm{lattice}}} + \cdots \qquad \frac{1}{\mu_{\mathrm{total}}} = \sum_{\beta} \frac{1}{\mu_{\beta}} = \frac{1}{\mu_{\mathrm{defects}}} + \frac{1}{\mu_{\mathrm{cattier}}} + \frac{1}{\mu_{\mathrm{lattice}}} + \cdots$$



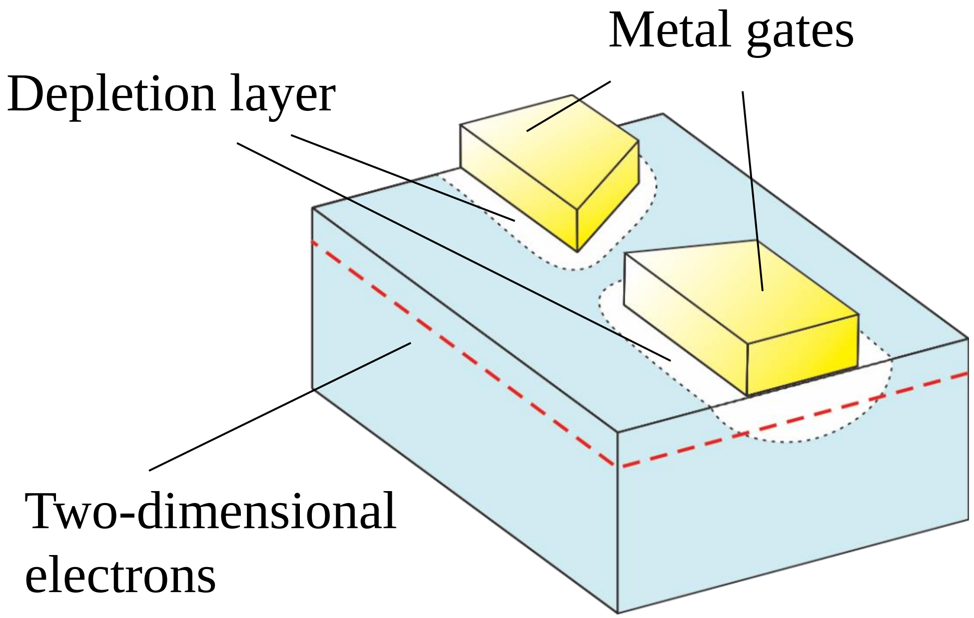
Fletcher *et al.*, J. Phys. C **5**, 212 (1972)

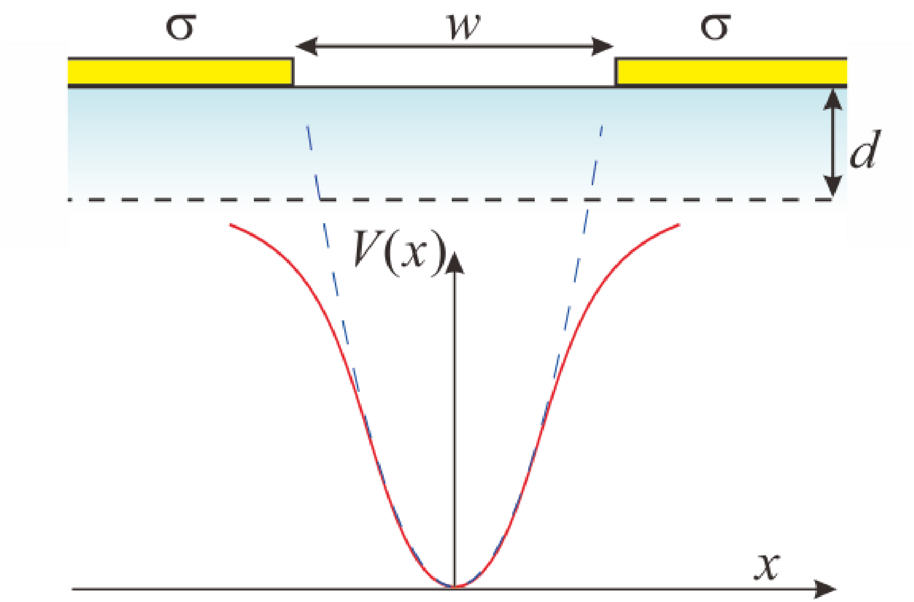
Reduction of impurity scattering by modulation doping structure



Walukiewicz *et al.* Phys. Rev. B **30**, 4571 (1984).

Formation of quantum wires: split gate



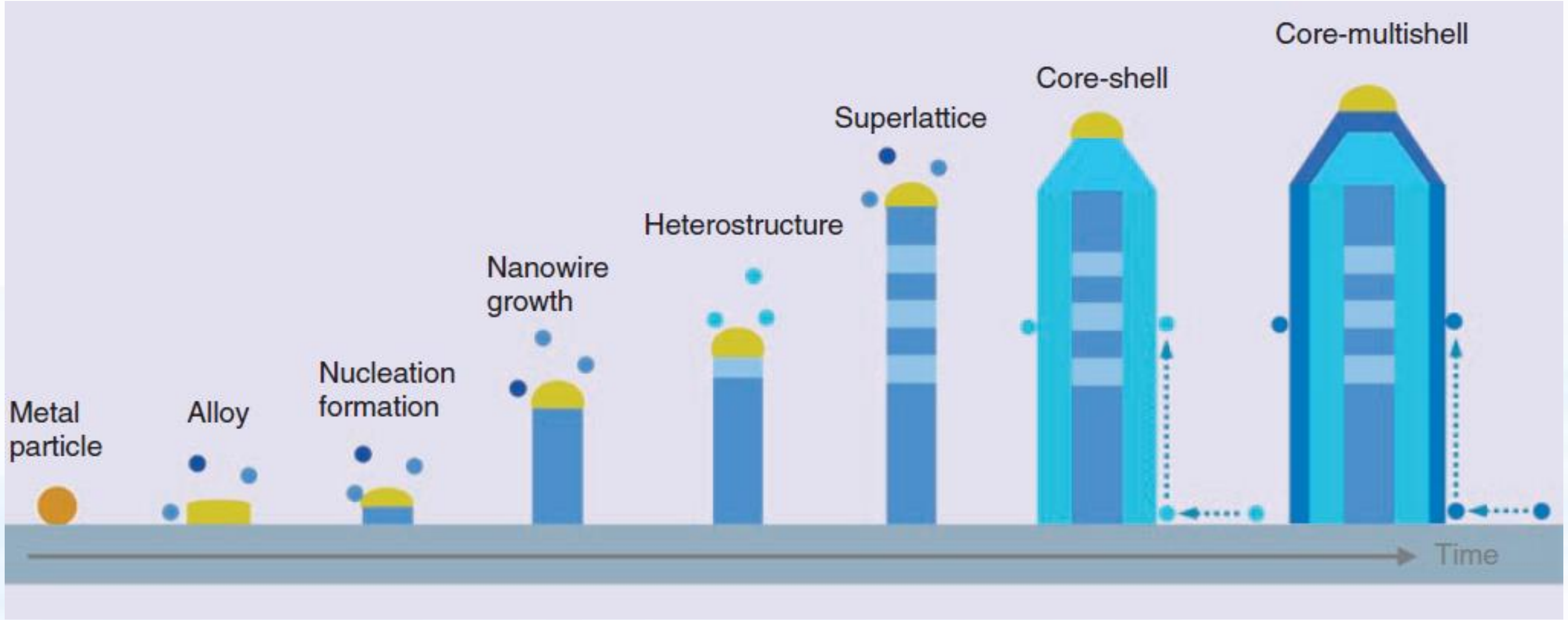


Two-dimensional electrons are pinched with depletion layers from Schottky gates to a one-dimensional system.

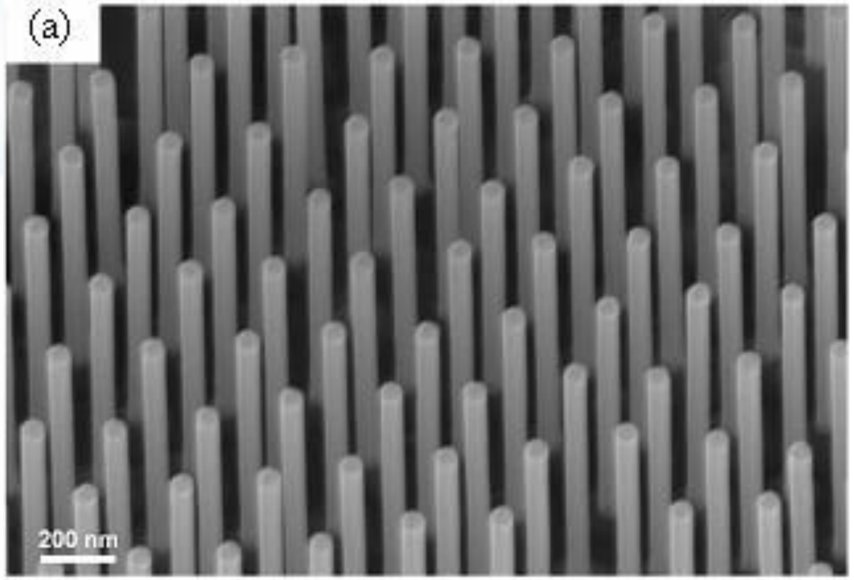
Electric field along z-axis can be approximated as $$\mathcal{E}_z(d) = \frac{-\sigma}{2\pi\epsilon\epsilon_0}\left[\pi + \arctan\frac{x - w/2}{d} - \arctan\frac{x + w/2}{d}\right]$$

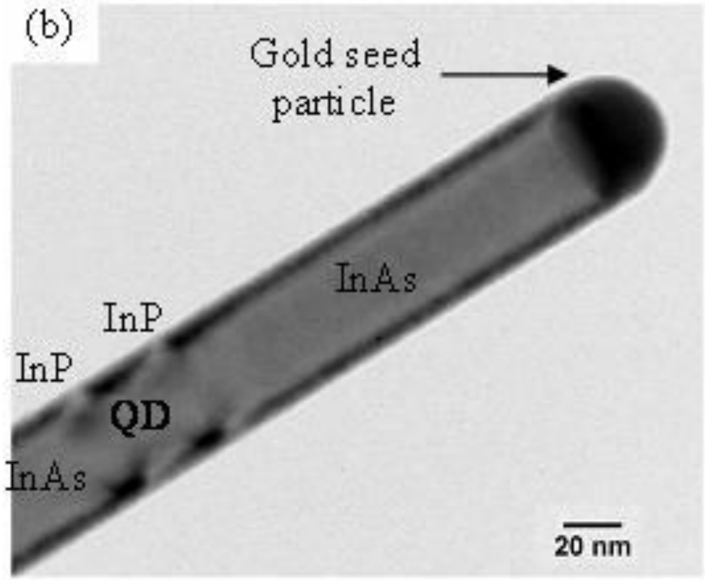
The bottom part of the confinement potential can be approximated by harmonic potential.

Self-assembled nano-wires



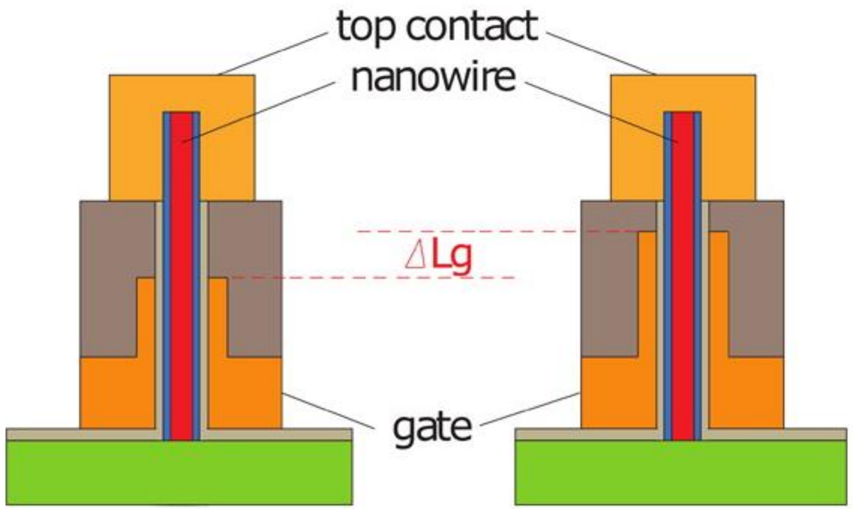
G. Zhang et al.
NTT technical
Review

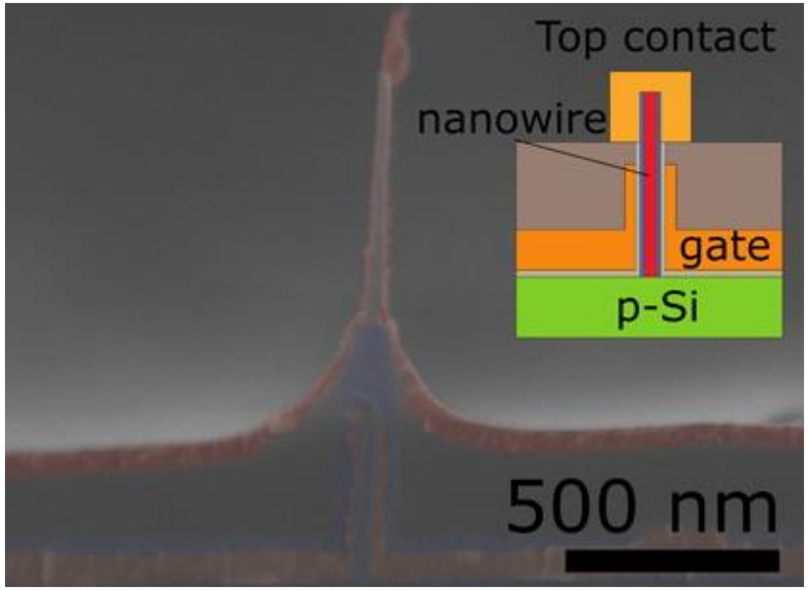


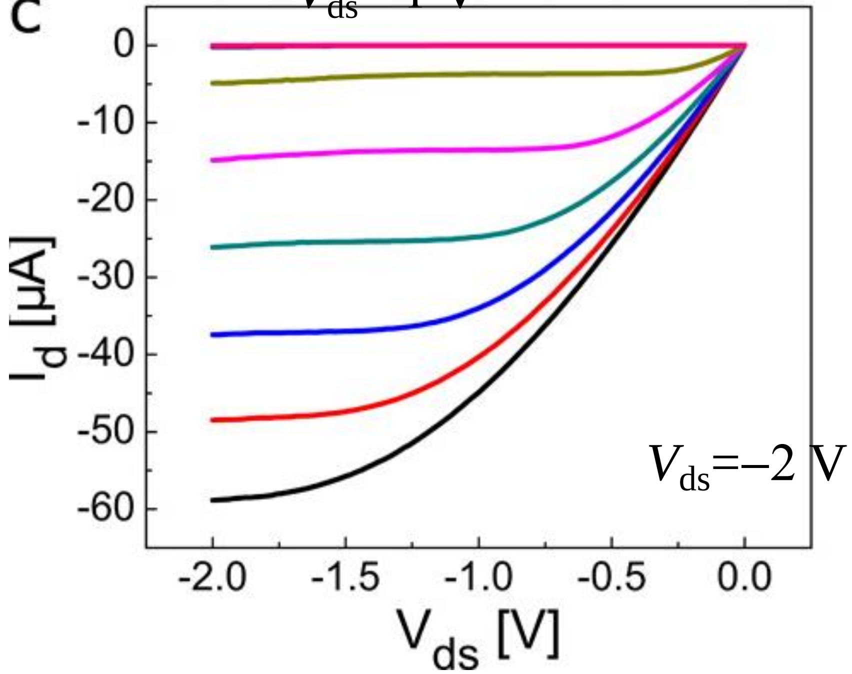


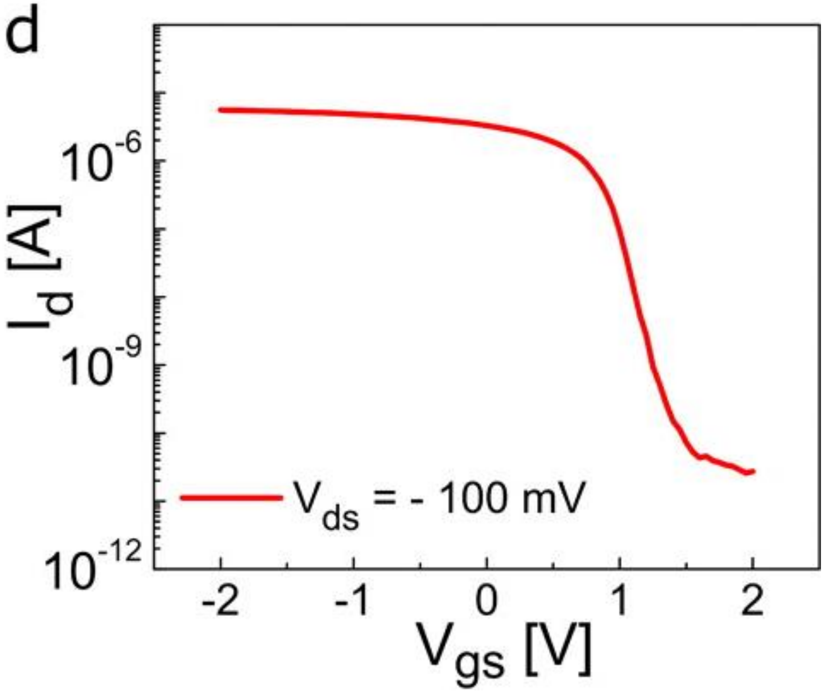
http://iemn.univ-lille1.fr/sites_perso/vignaud/english/35_nanowires.htm

Core-shell nanowire transistor



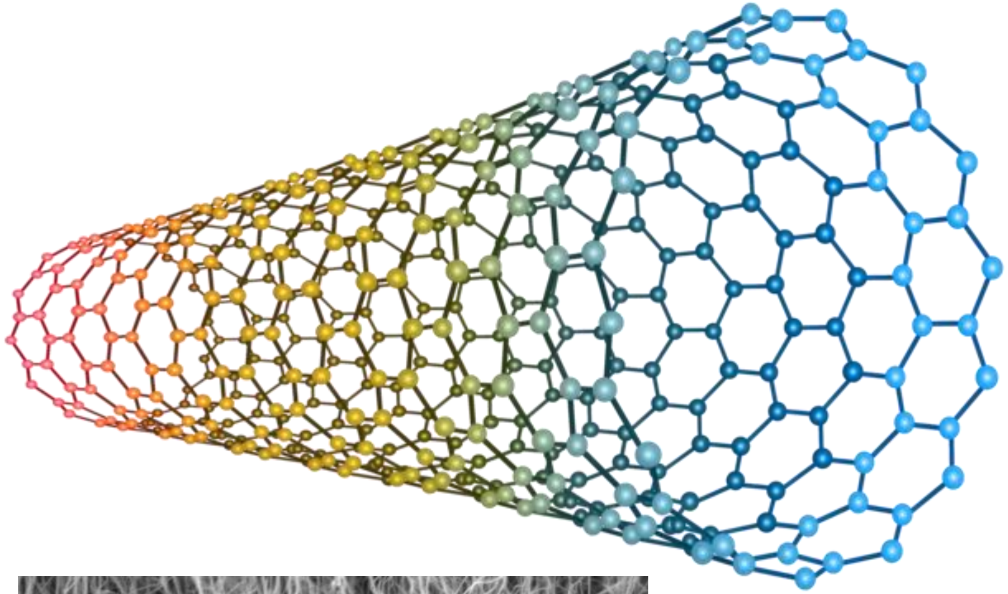


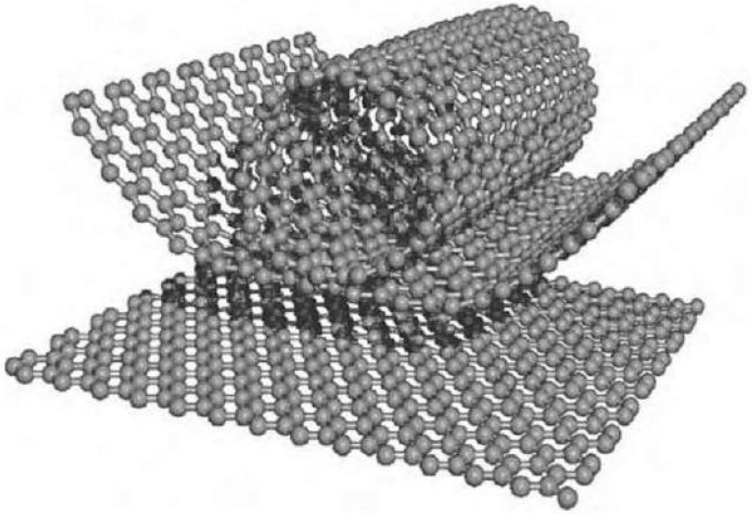


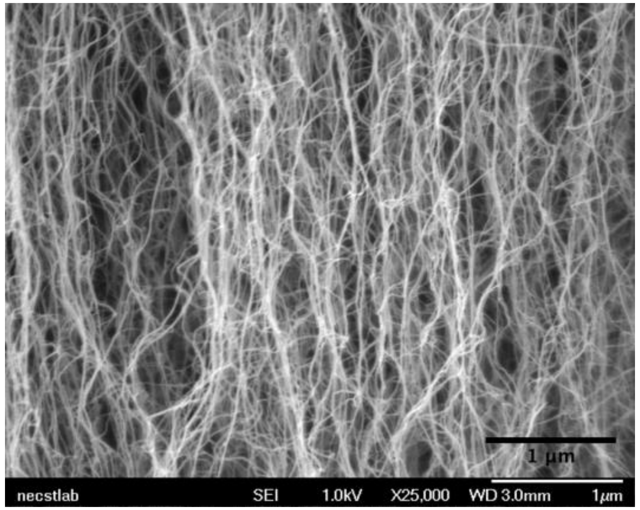


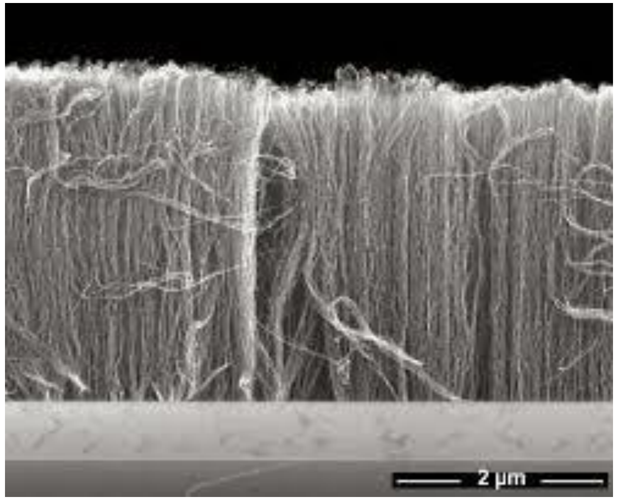
L. Chen *et al.*,
Nano Letters **16**, 420 (2016).

Carbon nanotube



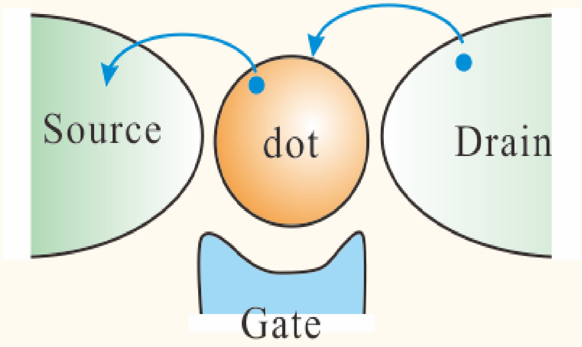


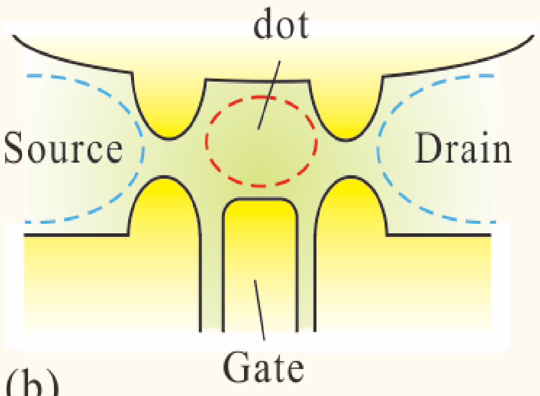


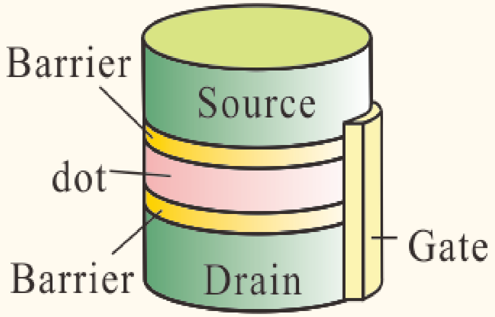


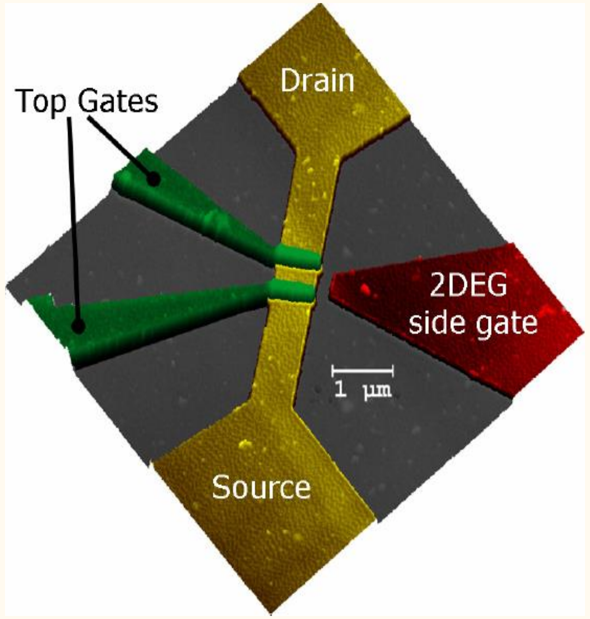
Quantum dots: zero-dimensional system

Quantum dots with nano-fabrication techniques

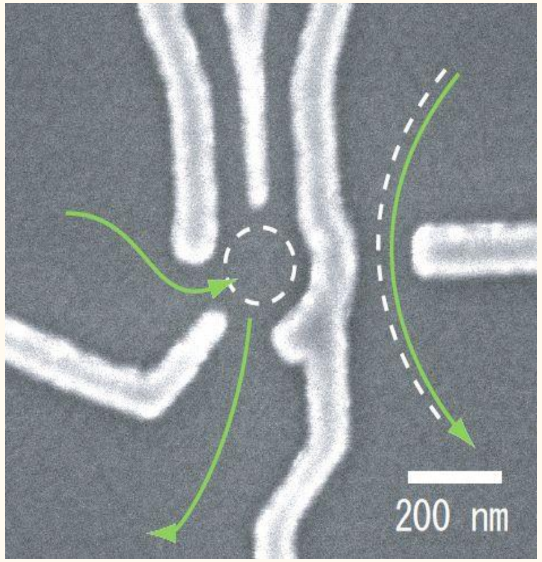






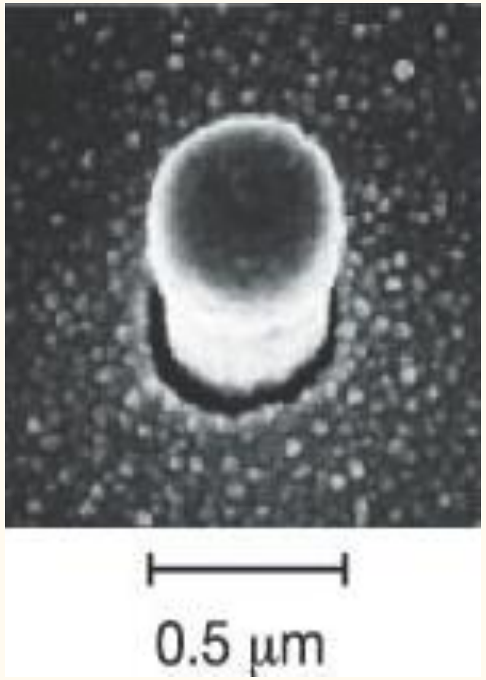


wrap gate



split gate

with charge detector

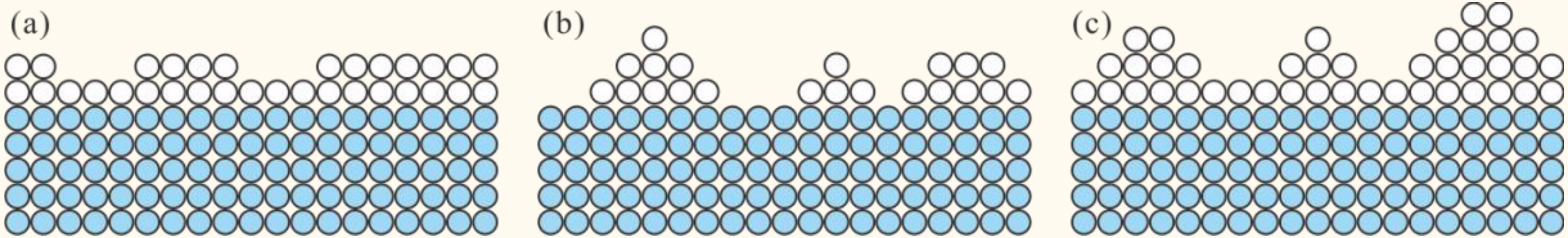


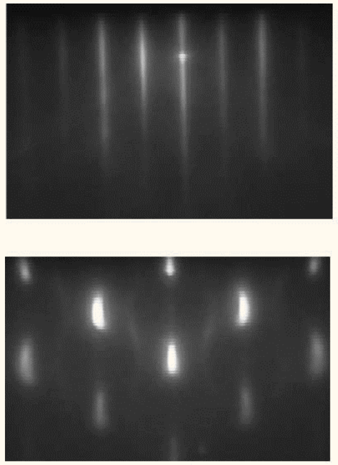
vertical type

Formation of quantum dots: self assemble

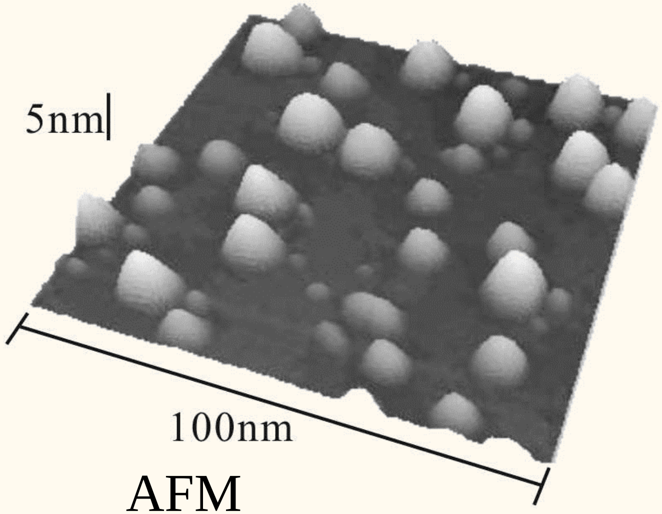
MBE growth modes

Frank-van der Merve   Volmer-Weber   Stranski-Krastanow

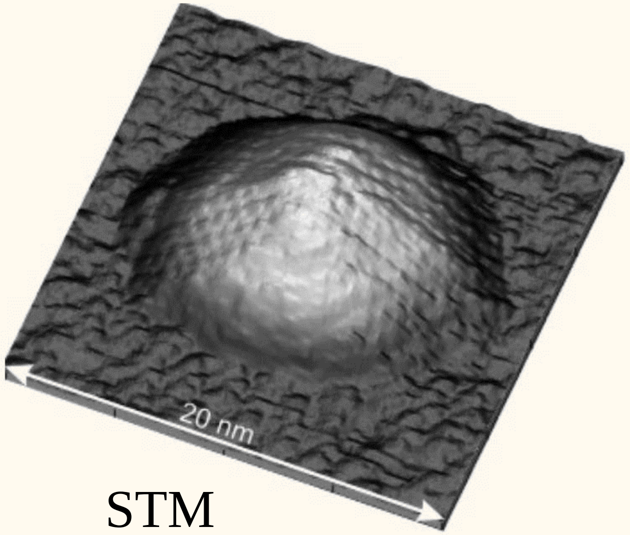




RHEED

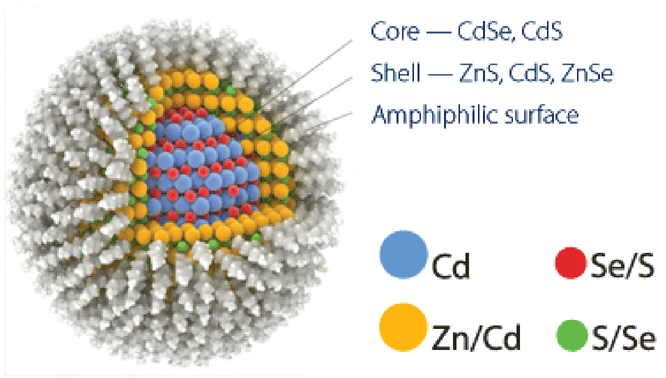


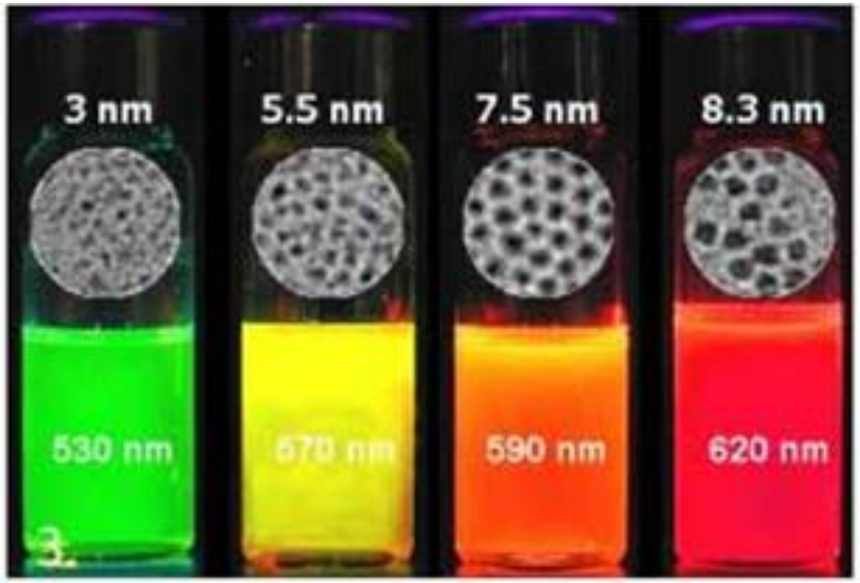
AFM

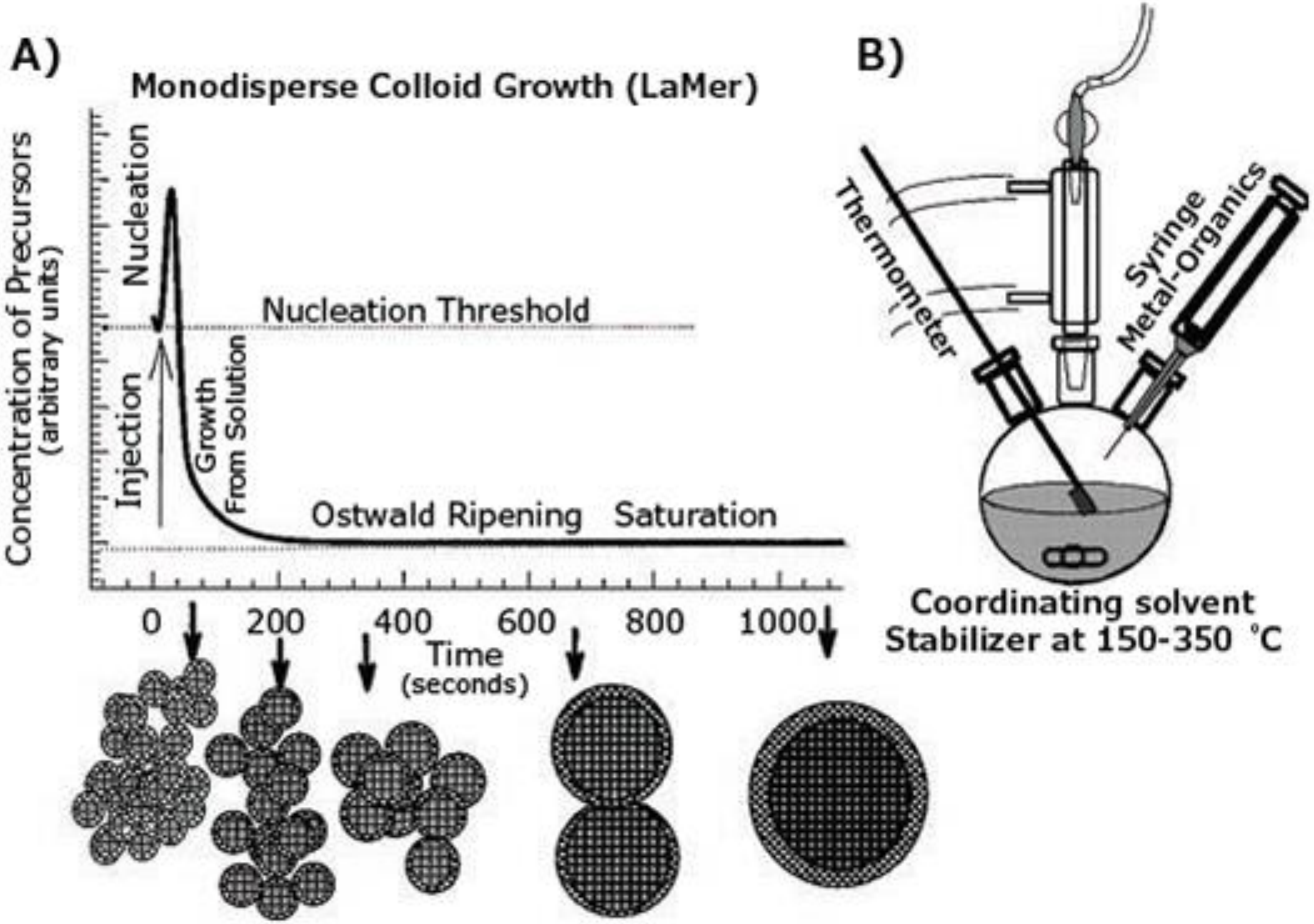


STM

Formation of quantum dots: Colloidal nano-crystals

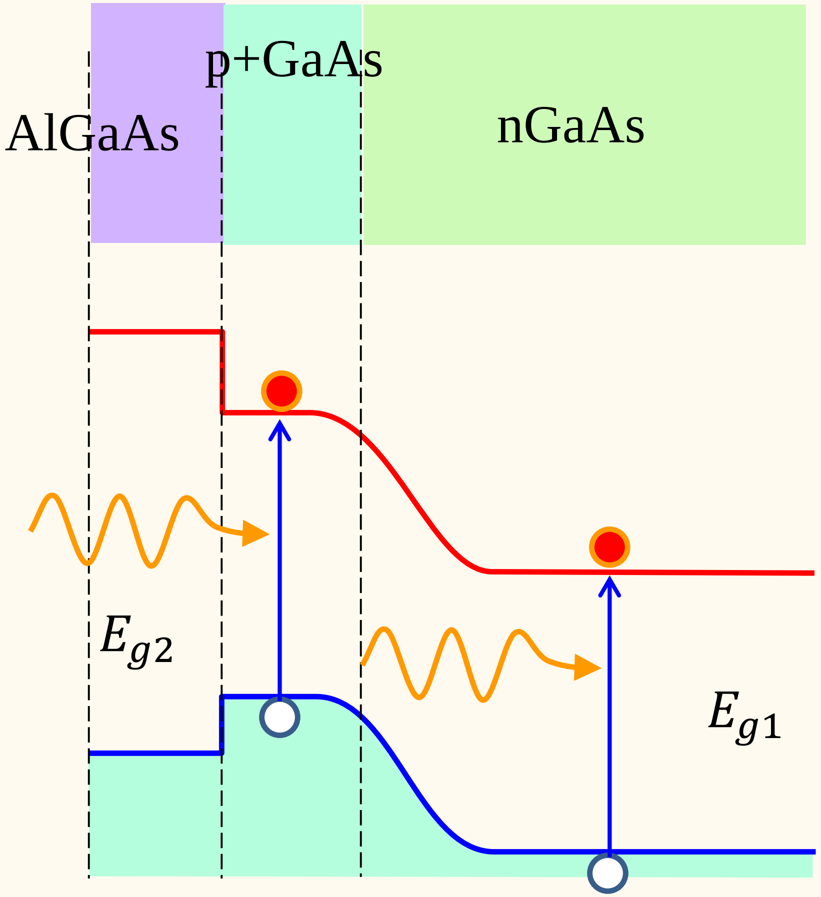






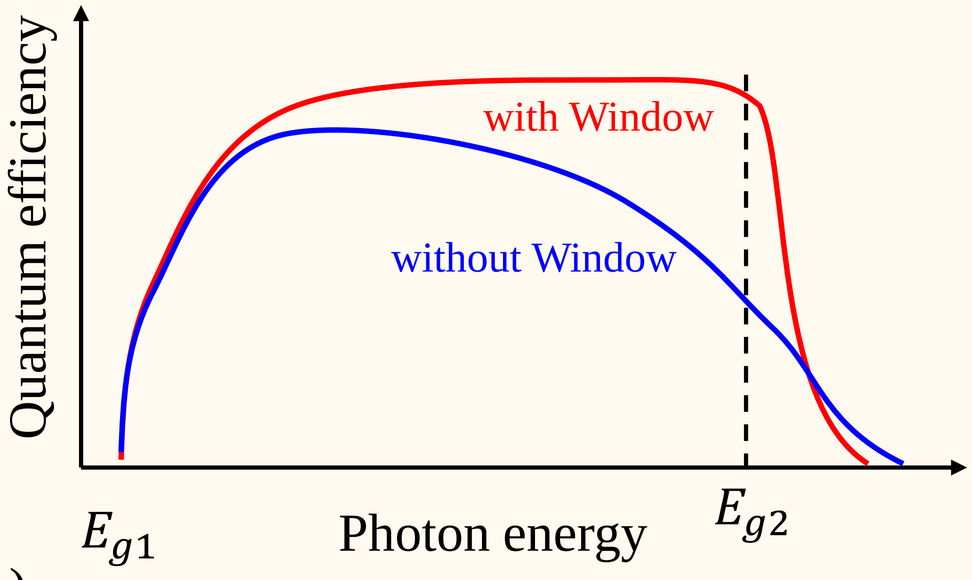
Optical devices with minority carrier confinement

Window layer in solar cells



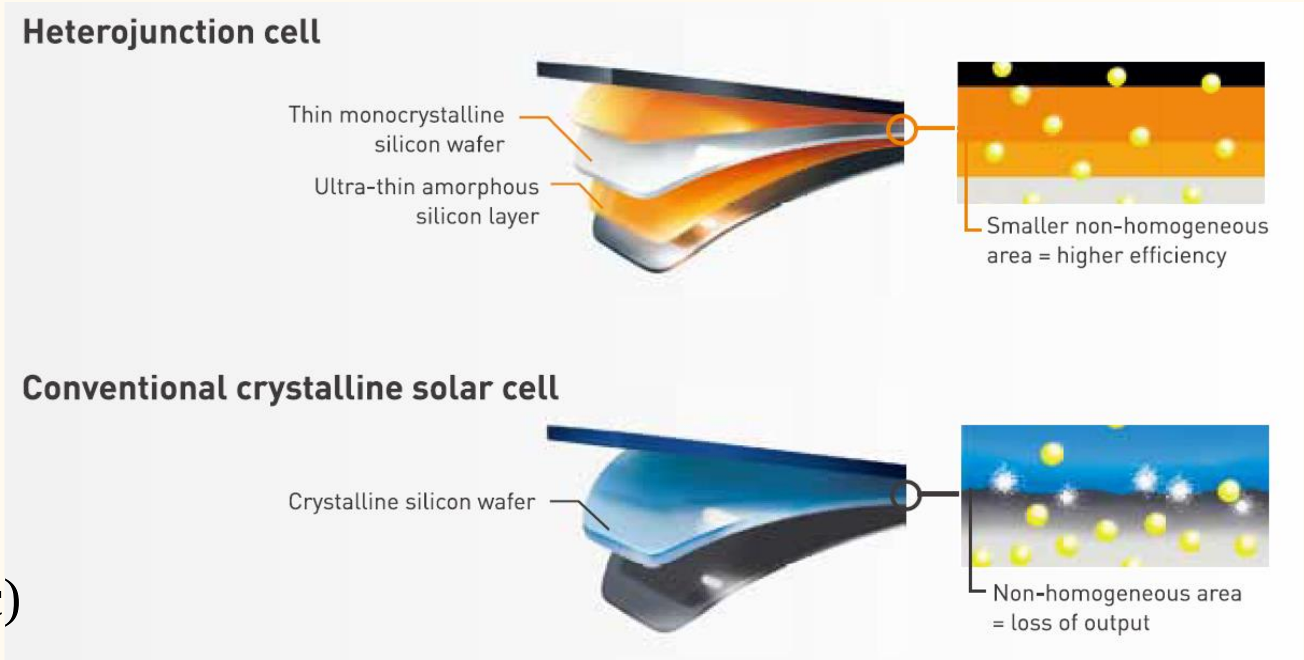
$E_{g2} > E_{g1}$

(a)



(b)

Panasonic HIT



(c)

Light emitting diodes (LEDs)

Emission spectrum $$I(\nu) \propto \nu^2 (h\nu - E_{\rm g})^{1/2} \exp\left[\frac{-(h\nu - E_{\rm g})}{k_{\rm B}T}\right]$$

Quantum efficiency $$\eta_{\rm q} \equiv \frac{R_{\rm r}}{R} = \frac{\tau_{\rm nr}}{\tau_{\rm nr} + \tau_{\rm r}} = \frac{\tau_{\rm tot}}{\tau_{\rm r}}, \qquad \frac{1}{\tau_{\rm tot}} \equiv \frac{1}{\tau_{\rm nr}} + \frac{1}{\tau_{\rm r}}$$

non-radiative ($\tau_{\rm nr}$) radiative ($\tau_{\rm r}$)

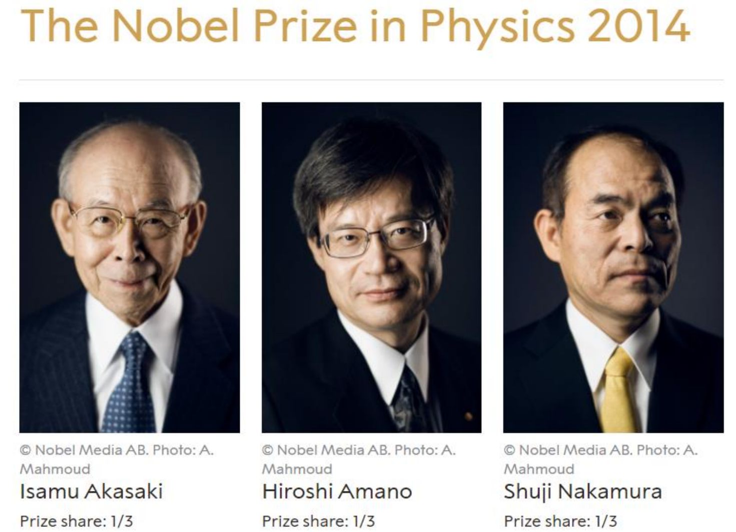
Minority carrier diffusion $$j_e + j_h = e\left[\frac{D_e n_{p0}}{L_e} + \frac{D_h p_{n0}}{L_h}\right]\left[\exp\left(\frac{eV}{k_{\rm B}T}\right) - 1\right]$$

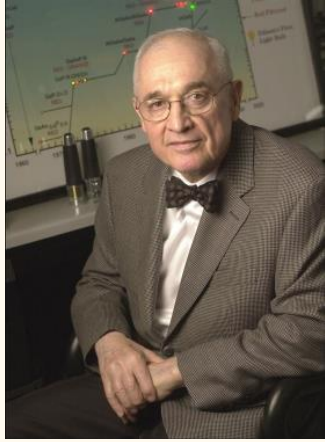
Carrier recombination in depletion layer $$j_{\rm R} = \frac{e n_i w_d}{2\tau_0}\left[\exp\left(\frac{eV}{2k_{\rm B}T}\right) - 1\right]$$

injection efficiency $$\gamma = \frac{j_e}{j_e + j_h + j_{\rm R}}$$

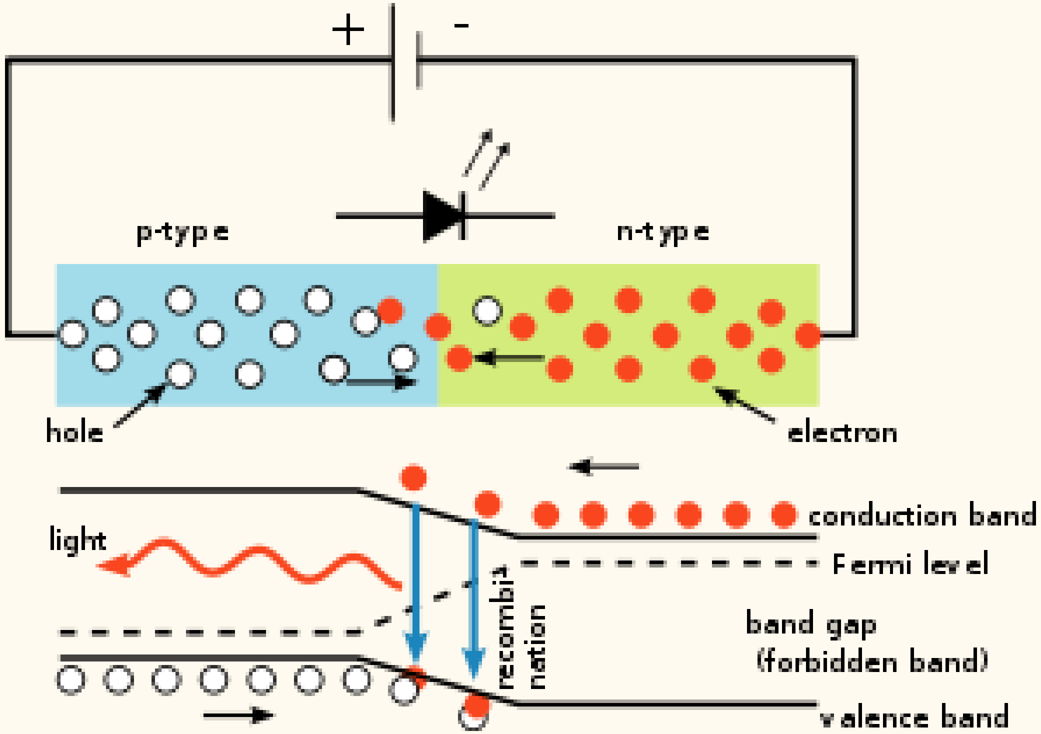
Internal quantum efficiency

$$\eta_{\rm iq} = \gamma \eta_{\rm q}$$

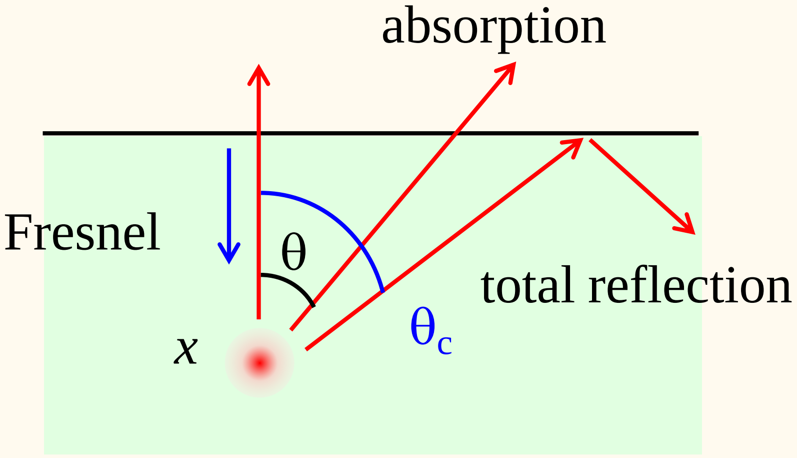




Nick Holonyak Jr.



External quantum efficiency



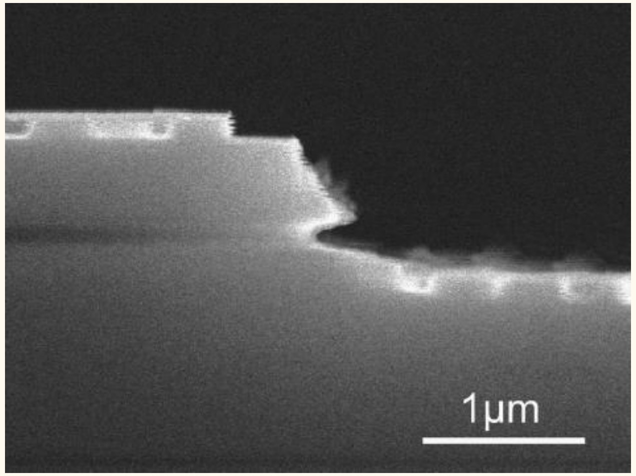
Optical losses

- Absorption loss $\zeta_{\mathrm{abs}} = 1 - \exp\left(-\frac{\alpha x}{\cos\theta}\right)$
- Fresnel loss $\Gamma = \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_1 + \bar{n}_2}\right)^2$
- Total reflection loss $\theta > \theta_{\mathrm{c}} = \sin^{-1}\frac{\bar{n}_1}{\bar{n}_2}$

Optical efficiency: η_{opt}

External quantum efficiency $\eta_{\mathrm{exq}} = \eta_{\mathrm{opp}}\eta_{\mathrm{iq}}$

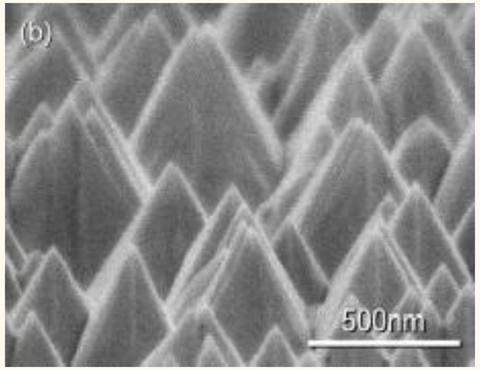
Device example



Windisch *et al*., APL **74**, 2256 (1999).

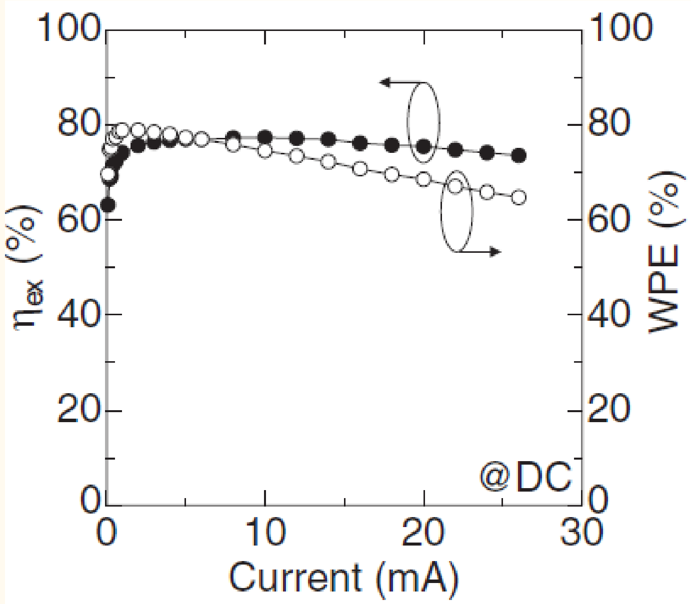
Textured surface AlGaAs/GaAs

η_{exq} >30 %



GaN/InGaN/GaN

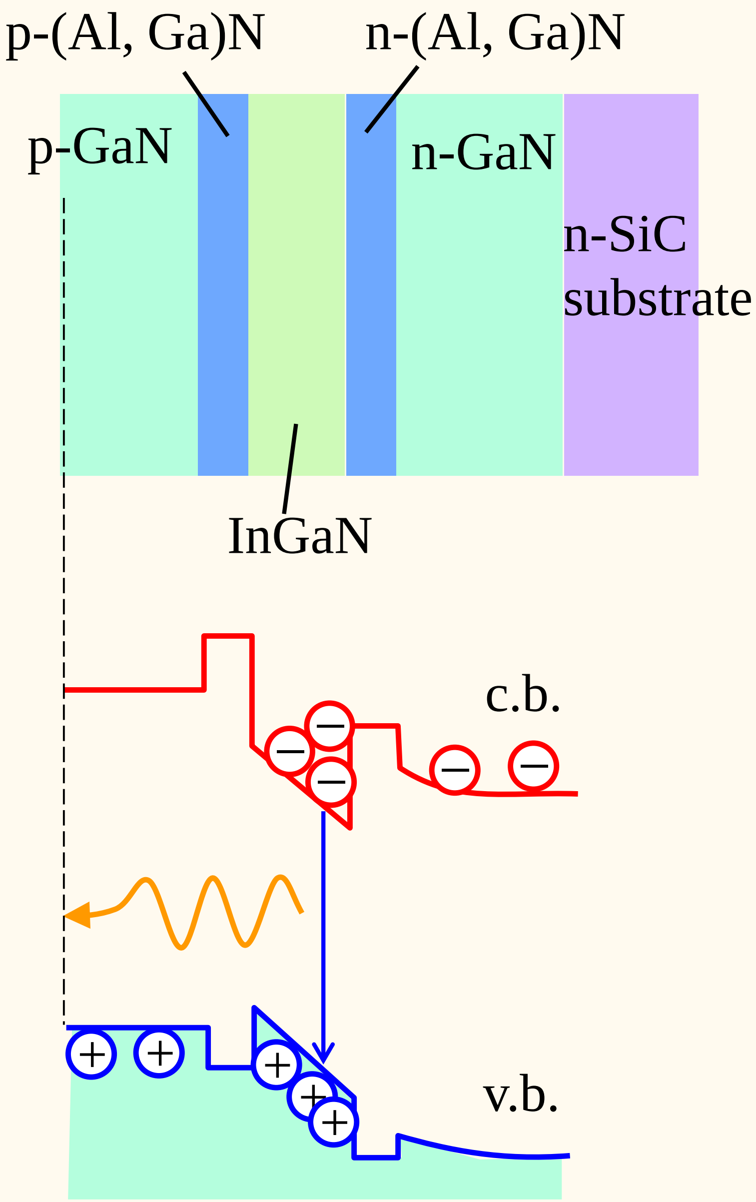
Fujii *et al*., APL **84**, 855 (2004).



InGaN-based YAG

Narukawa *et al*., JJAP **41**, L1431 (2007).

Double heterojunction (DH) LED



Advantages of DH LED

High internal quantum efficiency

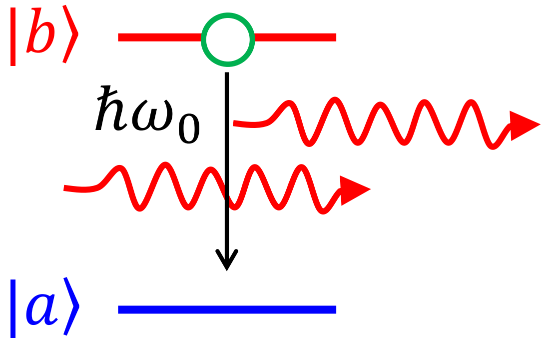
- Narrow active region → high *np* product
- No need for doping in active layer → less concentration of non-radiative recombination center
- Diffusion of minority carrier to surface, recombination centers is reduced.

Low absorption loss

- Energy of emitted photons is lower than the band gaps of the top and the bottom layers.

Laser diode

LASER: Light Amplification with Stimulated Emission of Radiation



(c) stimulated emission

Coherent state: Classical oscillating electromagnetic field

$$\boldsymbol{p} = m\omega_\lambda \boldsymbol{r}_0 \cos(\omega_0 t)$$

$$\vec{e}\cdot\boldsymbol{p} = \frac{\omega_\lambda m}{e}\vec{e}\cdot\boldsymbol{\mu}$$

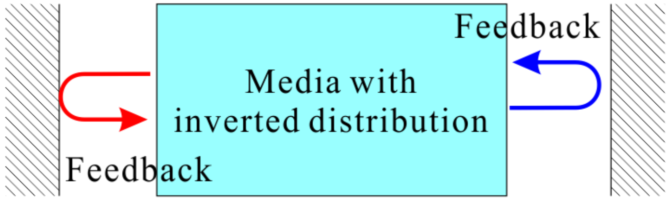
Probability of stimulated emission $$P_{ba}(t) = \frac{\omega_\lambda}{\epsilon\epsilon_0\hbar V}|\langle a|\vec{e}\cdot\boldsymbol{\mu}|b\rangle|^2 n_\lambda \frac{t^2}{2}$$

Energy absorption of media from the light (coherent state): $$\mathcal{E} = (N_a - N_b)P_{ba}(\tau)\hbar\omega_\lambda$$

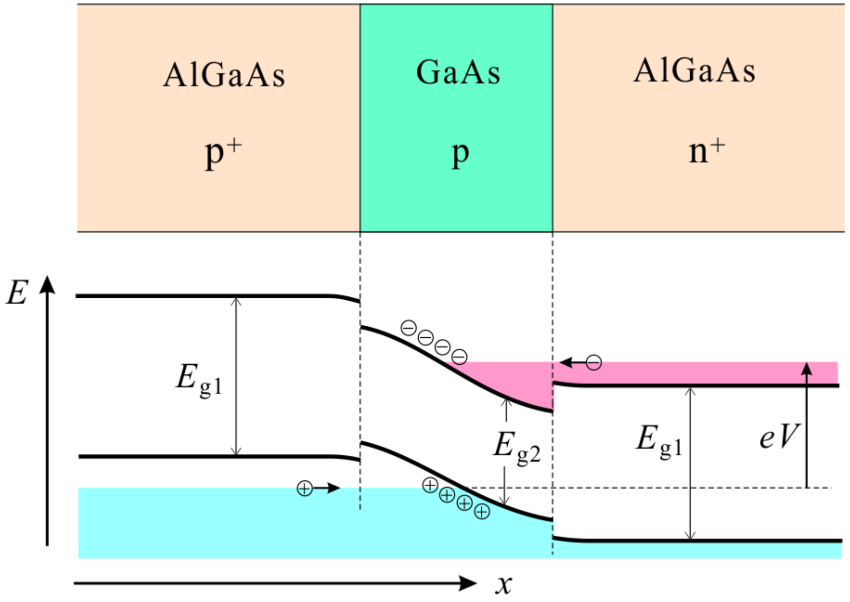
This means if a state with $N_b > N_a$ is realized, $\varepsilon < 0$, *i.e.*, the energy is absorbed from the media to light

Increase of amplitude of the coherent state: Bosonic stimulation

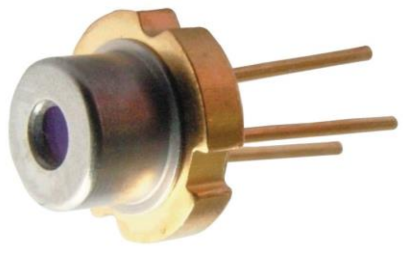
Laser diode



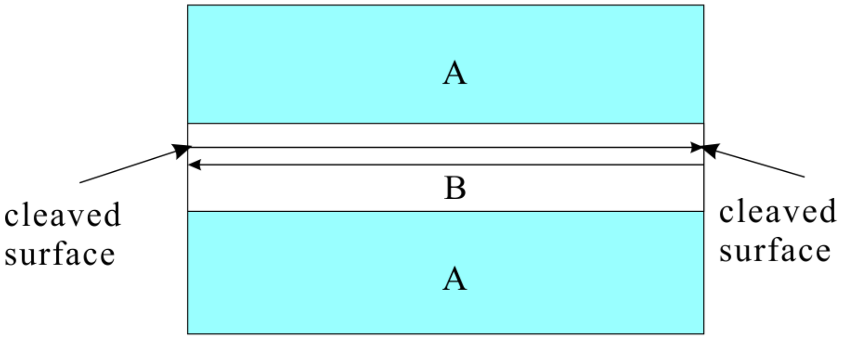
(a)



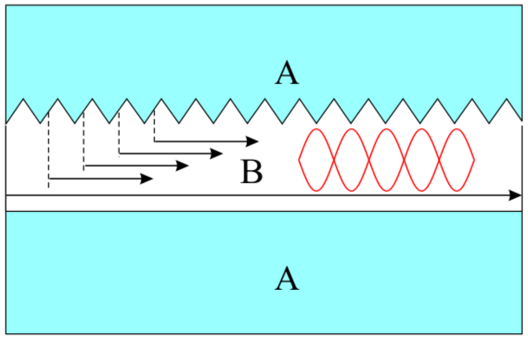
(b)



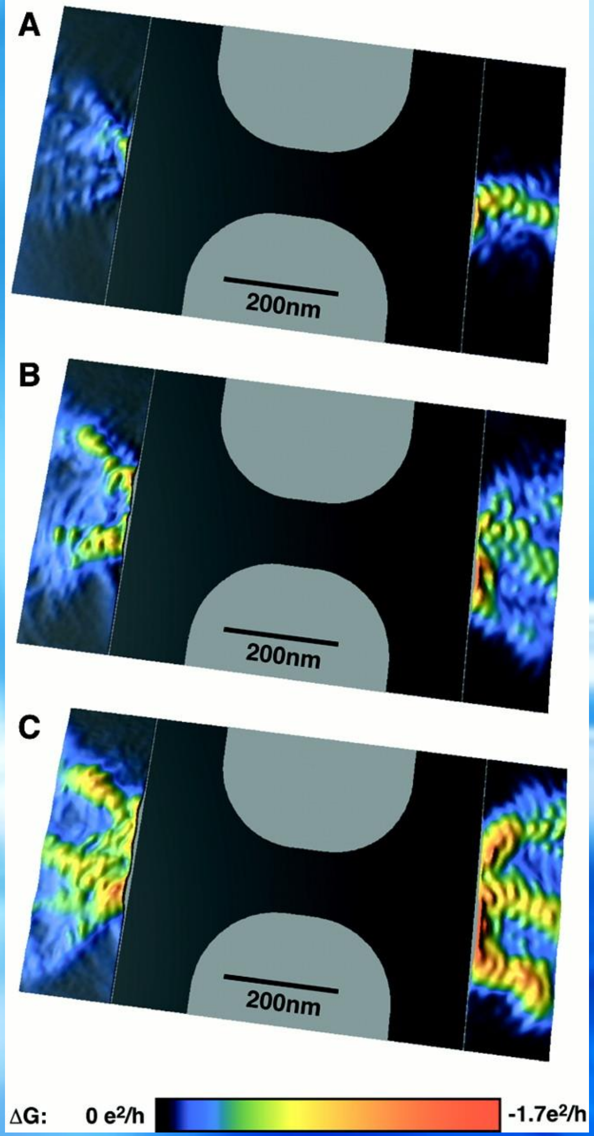
ADL-65074TL-1 - Laser Diode 655nm 7mW



(c) Fabry-Perot type



(d) Distributed Feedback type



M. A. Topinka et al. Science 2000;289:2323-2326

Chapter 8

Basics of Quantum Transport

Science
AAAS

Quantum entanglement

$|\psi\rangle = |A\rangle + |B\rangle$ $\qquad$ $|\varphi\rangle = |1\rangle + |2\rangle$

$\lvert A\rangle$	$\lvert B\rangle$	
$\lvert A\rangle\lvert 1\rangle$		$\lvert 1\rangle$
	$\lvert B\rangle\lvert 2\rangle$	$\lvert 2\rangle$

Direct product $|\Psi\rangle = |\psi\rangle \otimes |\varphi\rangle = |A\rangle|1\rangle + |A\rangle|2\rangle + |B\rangle|1\rangle + |B\rangle|2\rangle$

Maximally entangled state $|\Phi\rangle = |A\rangle|1\rangle + |B\rangle|2\rangle$

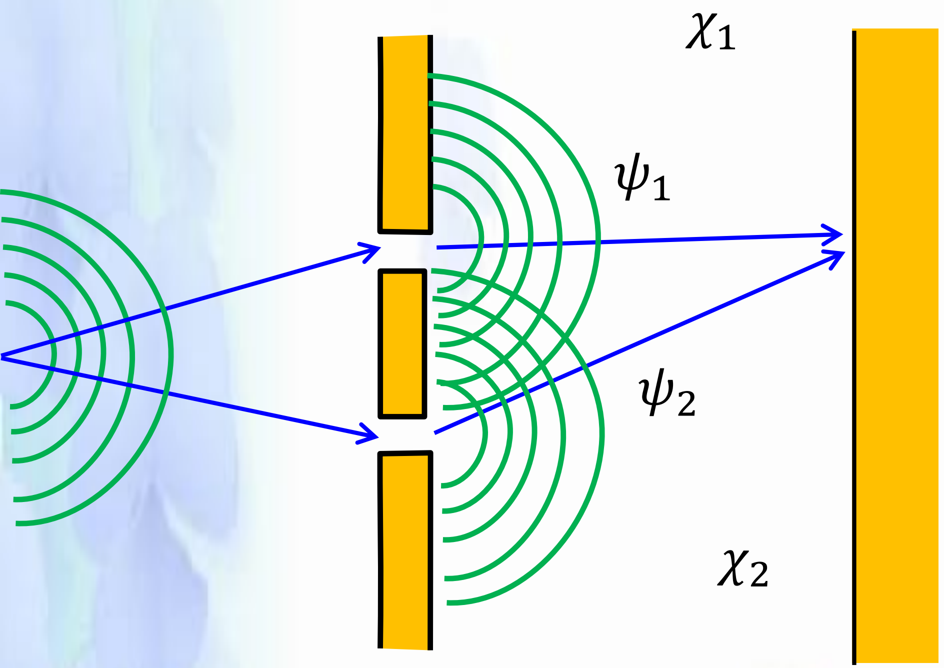
Quantification of Entanglement?

von Neumann entropy (entanglement entropy)

Density matrix $\rho = \sum |\psi\rangle\langle\psi|$

$$S = \mathrm{tr}(\rho \ln \rho)$$

Boundary between classical and quantum



$$|\psi|^2 = |\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + 2|\psi_1||\psi_2|\cos\theta$$

Environment wavefunction: χ

should associate with electron paths

$$\psi_1 \to \psi_1 \otimes \chi_1, \quad \psi_2 \to \psi_2 \otimes \chi_2$$

Then the interference term is $2|\psi_1||\psi_2|\cos\theta\langle\chi_1|\chi_2\rangle$

$\langle\chi_1|\chi_2\rangle = 1$: Full interference

$\langle\chi_1|\chi_2\rangle = 0$: No interference Particle-Environment maximally entangled

Electron transport: Electron – Phonon inelastic scattering
Electron – Electron inelastic scattering
Electron – Localized spin scattering

Length limit quantum coherence (Coherence length)

Monochromaticity: Thermal length

Energy width: $\Delta E = k_{\mathrm{B}}T$

Diffusion length: $l = \sqrt{D\tau}$

Phase width: $$2\pi\Delta f\tau = 2\pi\frac{\Delta E\tau}{h} = 2\pi\frac{k_{\mathrm{B}}T\tau}{h} \quad \to 2\pi : \quad \tau_{\mathrm{c}} = \frac{h}{k_{\mathrm{B}}T}$$

Thermal diffusion length $$l_{\mathrm{th}} = \sqrt{\frac{hD}{k_{\mathrm{B}}T}}$$

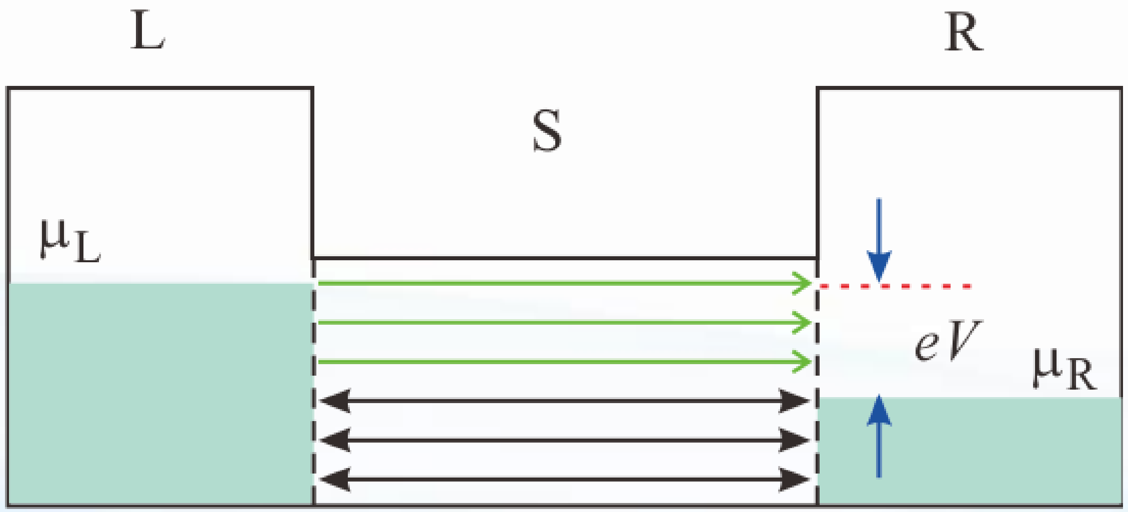
Ballistic thermal length $$l_{\mathrm{th}} = \frac{hv_{\mathrm{F}}}{k_{\mathrm{B}}T}$$

(Some) inelastic scattering time: τ_{inel}

Ballistic transport: $l_{\mathrm{inel}} = v_{\mathrm{F}}\tau_{\mathrm{inel}}$

Diffusive transport: $l_{\mathrm{inel}} = \sqrt{D\tau_{\mathrm{inel}}}$

Conductance quantum



L, R : Particle reservoirs

Thermal equilibrium:
well defined chemical potentials

Instantaneous thermalization:
particles loose quantum coherence

$$j(k) = \frac{e}{L} v_{\mathrm{g}} = \frac{e}{\hbar L}\frac{dE(k)}{dk}$$

L: wavefunction normalization length

$$J = \int_{k_{\mathrm{R}}}^{k_{\mathrm{L}}} j(k) \frac{L}{2\pi} dk = \frac{e}{h}\int_{\mu_{\mathrm{R}}}^{\mu_{\mathrm{L}}} dE = \frac{e}{h}(\mu_{\mathrm{L}} - \mu_{\mathrm{R}}) = \frac{e^2}{h} V$$

$$G = \frac{J}{V} = \frac{e^2}{h} \equiv G_{\mathrm{q}} \ \text{Conductance quantum} \ \left(\frac{2e^2}{h} \equiv G_{\mathrm{q}} \ \text{spin freedom}\right)$$

Conductance quantum as uncertainty relation

Wave packet: $\Delta k \to \Delta x = \dfrac{2\pi}{\Delta k}, \quad v_{\mathrm{g}} = \dfrac{\Delta E}{\hbar \Delta k}$

Fermion statistics: electron charge concentration $= \dfrac{e}{\Delta x} = \dfrac{e \Delta k}{2\pi}$

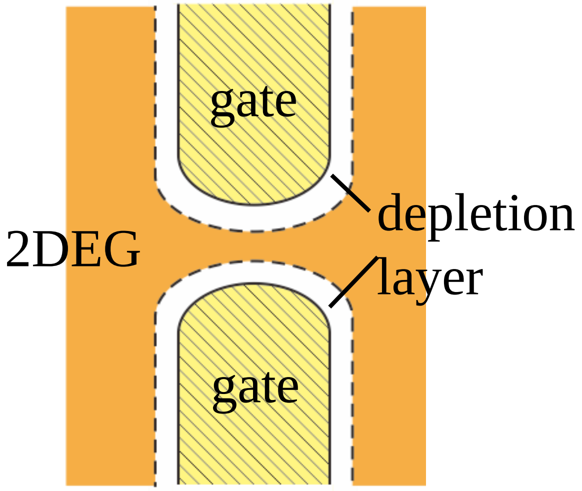
$$J = \frac{e}{\Delta x}\frac{\Delta E}{\hbar \Delta k} = \frac{e^2}{h}V$$

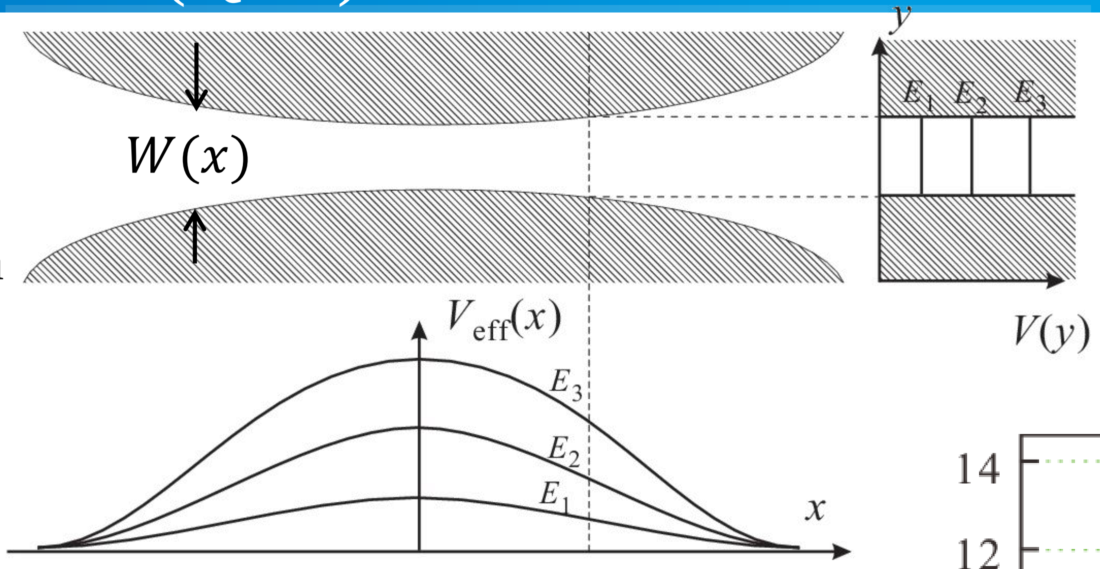
Energy width: $\Delta E = eV$ Wave packet width in time: $\Delta t = \dfrac{h}{\Delta E} = \dfrac{h}{eV}$

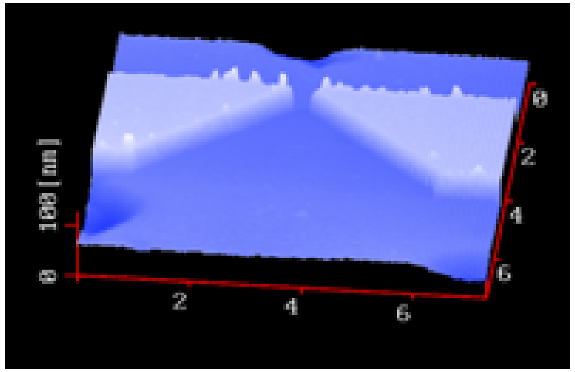
$$J = \frac{e}{\Delta t} = \frac{e^2}{h}V$$

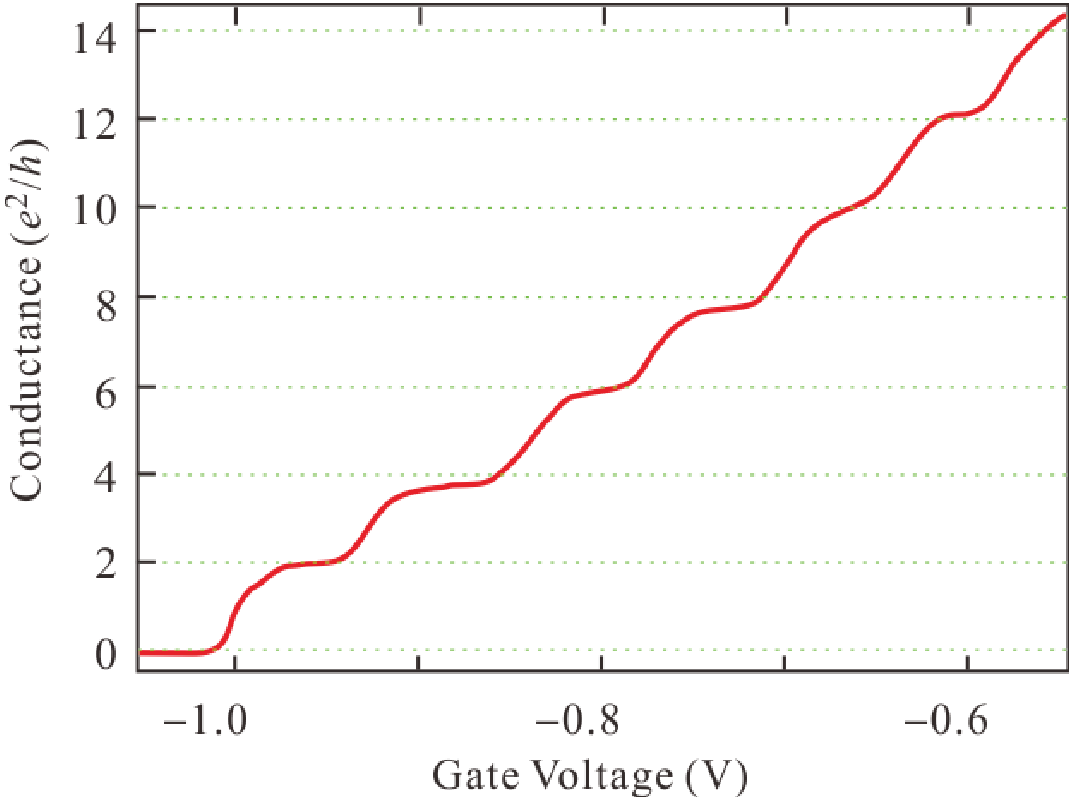
Conductance quantum comes from fermion statistics of electrons

Quantum point contact (QPC)









$$H\psi(x,y) = \frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\varphi_n(y)\phi(x)$$

$$= \varphi_n(y)\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \left(\frac{n\pi}{2W}\right)^2\right)\phi(x) = E\varphi_n(y)\phi(x)$$

$$V_{\text{eff}}(n,x) = \frac{\hbar^2}{2m}\left(\frac{n\pi}{2W(x)}\right)^2$$

Transmissible one-dimensional system: Conductance Channel