Lecture on

Semiconductors / 半導体

(Physics of semiconductors)

2021.6.23 Lecture 11

10:25 – 11:55

Institute for Solid State Physics, University of Tokyo

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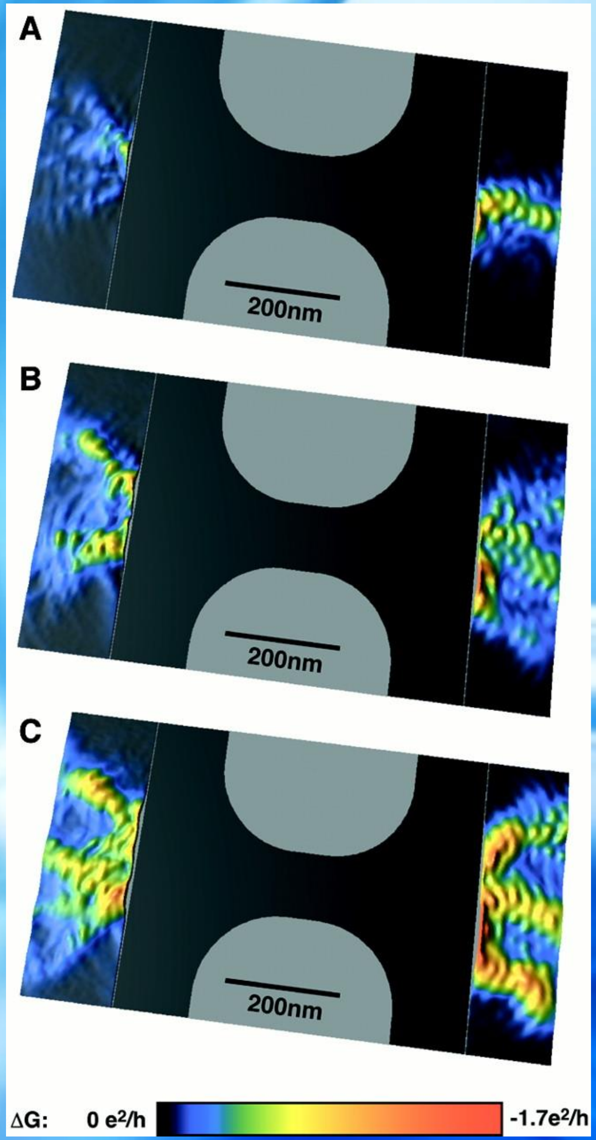
Review of last week

- Two-dimensional electrons at heterointerface
- Quantum point contacts, quantum wires
- Core-shell nanowires
- Two dimensional systems → quantum dots
- Self assembled quantum dots
- Colloidal quantum dots
- Optical devices with minority carrier confinement
  Solar cells, DH LEDs, Laser diodes

Chapter 8

Basics of Quantum Transport



M. A. Topinka et al. Science 2000;289:2323-2326

Quantum entanglement

$$|\psi\rangle = |A\rangle + |B\rangle \qquad |\varphi\rangle = |1\rangle + |2\rangle$$

$\lvert A\rangle$	$\lvert B\rangle$	
$\lvert A\rangle\lvert 1\rangle$		$\lvert 1\rangle$
	$\lvert B\rangle\lvert 2\rangle$	$\lvert 2\rangle$

Direct product $|\Psi\rangle = |\psi\rangle \otimes |\varphi\rangle = |A\rangle|1\rangle + |A\rangle|2\rangle + |B\rangle|1\rangle + |B\rangle|2\rangle$

Maximally entangled state $|\Phi\rangle = |A\rangle|1\rangle + |B\rangle|2\rangle$

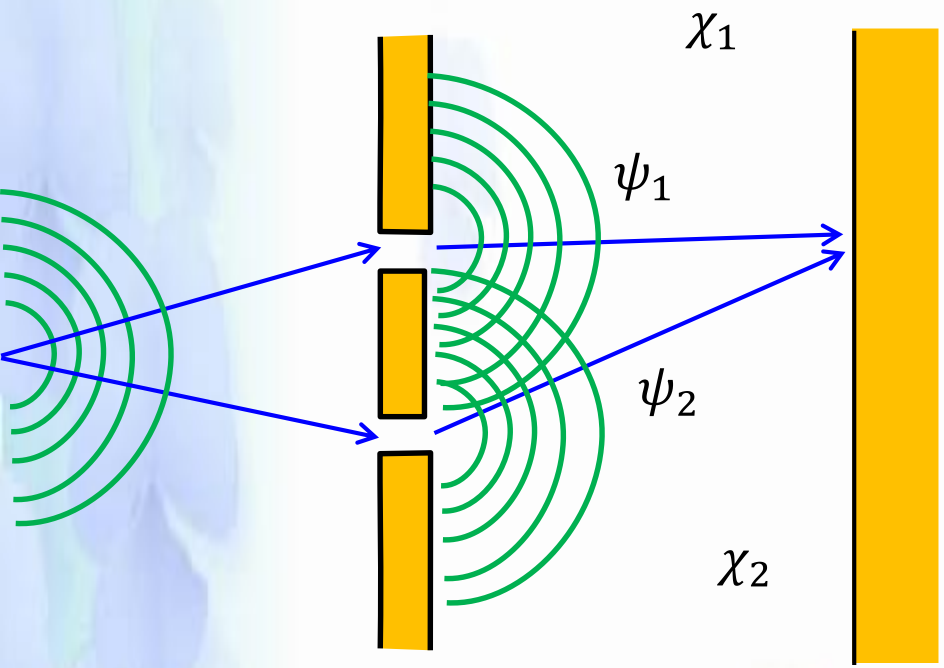
Quantification of Entanglement?

von Neumann entropy (entanglement entropy)

Density matrix $\rho = \sum |\psi\rangle\langle\psi|$

$$S = \mathrm{tr}(\rho \ln \rho)$$

Boundary between classical and quantum



$$|\psi|^2 = |\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + 2|\psi_1||\psi_2|\cos\theta$$

Environment wavefunction: χ

should associate with electron paths

$$\psi_1 \to \psi_1 \otimes \chi_1, \quad \psi_2 \to \psi_2 \otimes \chi_2$$

Then the interference term is $2|\psi_1||\psi_2|\cos\theta\langle\chi_1|\chi_2\rangle$

$\langle\chi_1|\chi_2\rangle = 1$: Full interference

$\langle\chi_1|\chi_2\rangle = 0$: No interference Particle-Environment maximally entangled

Electron transport:

Electron – Phonon inelastic scattering
Electron – Electron inelastic scattering
Electron – Localized spin scattering

Length limit quantum coherence (Coherence length)

Monochromaticity: Thermal length

Energy width: $\Delta E = k_{\mathrm{B}} T$

Diffusion length: $l = \sqrt{D\tau}$

Phase width: $2\pi \Delta f \tau = 2\pi \frac{\Delta E \tau}{h} = 2\pi \frac{k_{\mathrm{B}} T \tau}{h}$ $\rightarrow 2\pi :$ $\tau_{\mathrm{c}} = \frac{h}{k_{\mathrm{B}} T}$

Thermal diffusion length $l_{\mathrm{th}} = \sqrt{\frac{hD}{k_{\mathrm{B}} T}}$

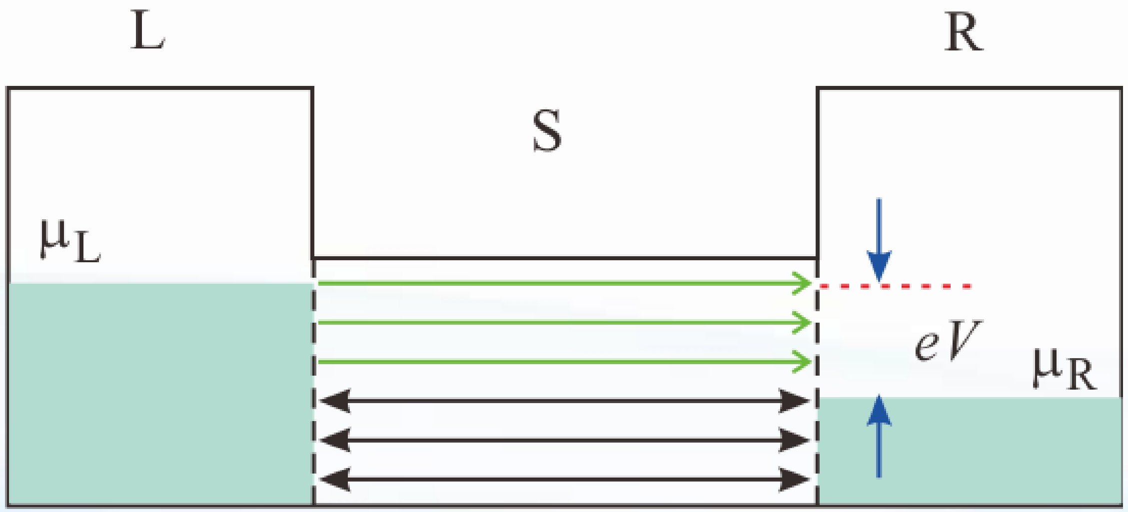
Ballistic thermal length $l_{\mathrm{th}} = \frac{h v_{\mathrm{F}}}{k_{\mathrm{B}} T}$

(Some) inelastic scattering time: τ_{inel}

Ballistic transport: $l_{\mathrm{inel}} = v_{\mathrm{F}} \tau_{\mathrm{inel}}$

Diffusive transport: $l_{\mathrm{inel}} = \sqrt{D \tau_{\mathrm{inel}}}$

Conductance quantum



L, R : Particle reservoirs

Thermal equilibrium:
well defined chemical potentials

Instantaneous thermalization:
particles loose quantum coherence

$$j(k) = \frac{e}{L} v_{\mathrm{g}} = \frac{e}{\hbar L} \frac{dE(k)}{dk}$$

L: wavefunction normalization length

$$J = \int_{k_{\mathrm{R}}}^{k_{\mathrm{L}}} j(k) \frac{L}{2\pi} dk = \frac{e}{h} \int_{\mu_{\mathrm{R}}}^{\mu_{\mathrm{L}}} dE = \frac{e}{h} (\mu_{\mathrm{L}} - \mu_{\mathrm{R}}) = \frac{e^2}{h} V$$

$$G = \frac{J}{V} = \frac{e^2}{h} \equiv G_{\mathrm{q}} \quad \text{Conductance quantum} \quad \left(\frac{2e^2}{h} \equiv G_{\mathrm{q}} \ \ \text{spin freedom} \right)$$

Conductance quantum as uncertainty relation

Space coordinate-wavenumber

Wave packet: $\Delta k \to \Delta x = \dfrac{2\pi}{\Delta k}, \quad v_{\rm g} = \dfrac{\Delta E}{\hbar \Delta k}$

Fermion statistics: electron charge concentration $= \dfrac{e}{\Delta x} = \dfrac{e\Delta k}{2\pi}$

$$J = e n_{\rm electron} v_{\rm g} = \frac{e}{\Delta x}\frac{\Delta E}{\hbar \Delta k} = \frac{e^2}{h}V$$

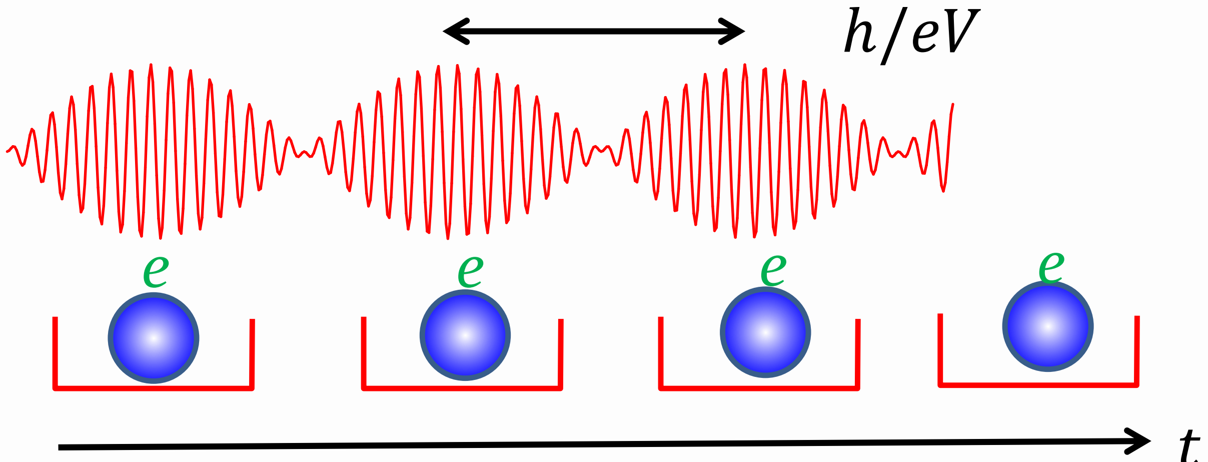
Energy-time

Energy width: $\Delta E = eV$

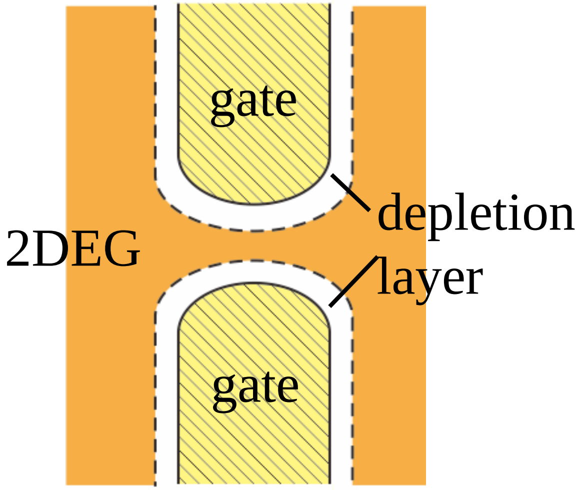
Wave packet width in time: $\Delta t = \dfrac{h}{\Delta E} = \dfrac{h}{eV}$

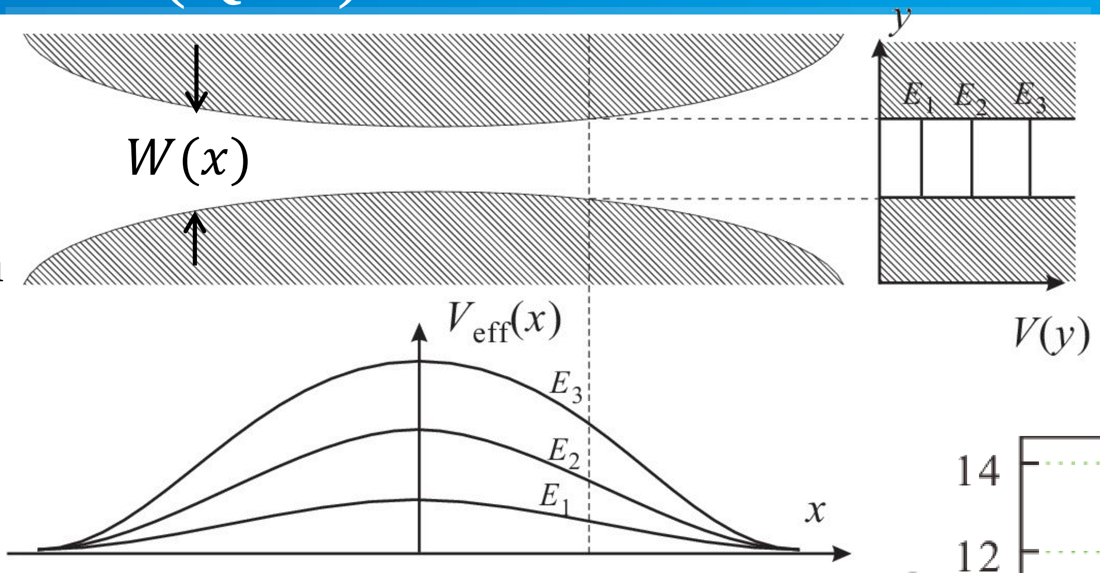
Fermion anti-bunching effect: $J = \dfrac{e}{\Delta t} = \dfrac{e^2}{h}V$

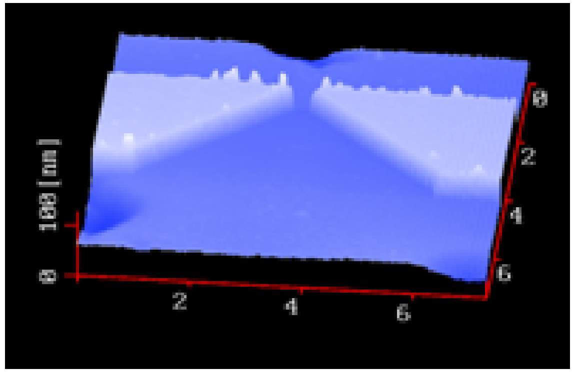
Conductance quantum comes from fermion statistics of electrons



Quantum point contact (QPC)



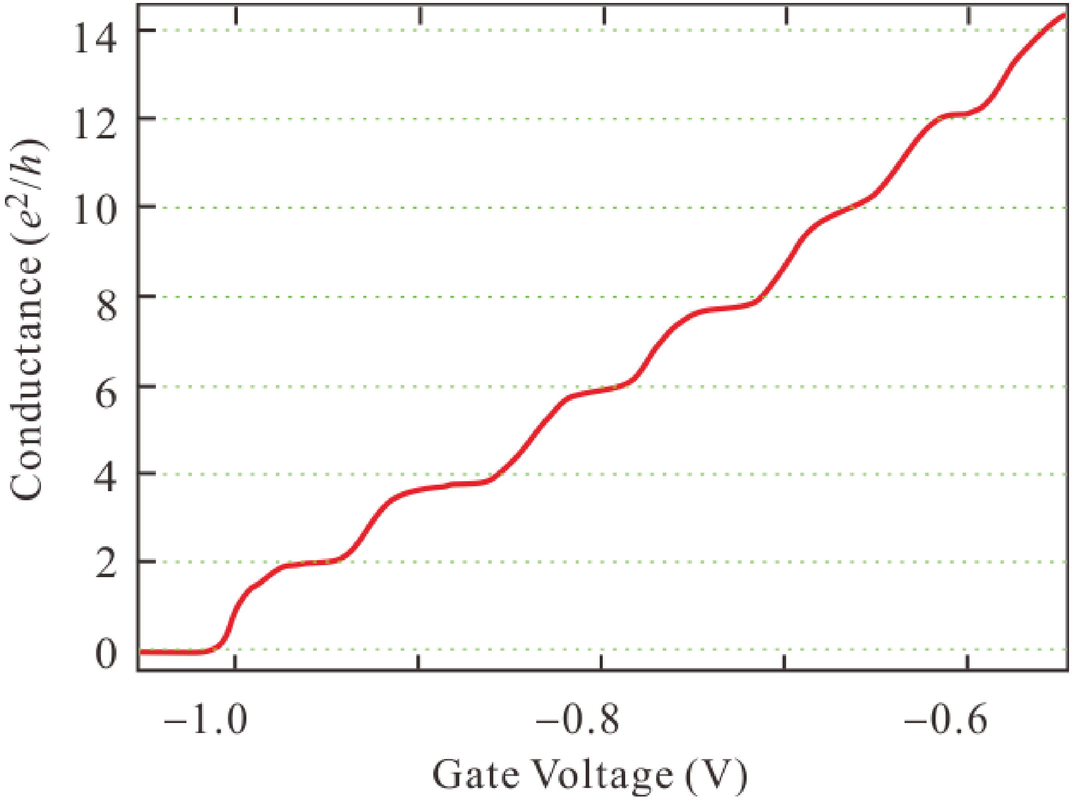




$$H\psi(x,y) = \frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\varphi_n(y)\phi(x)$$

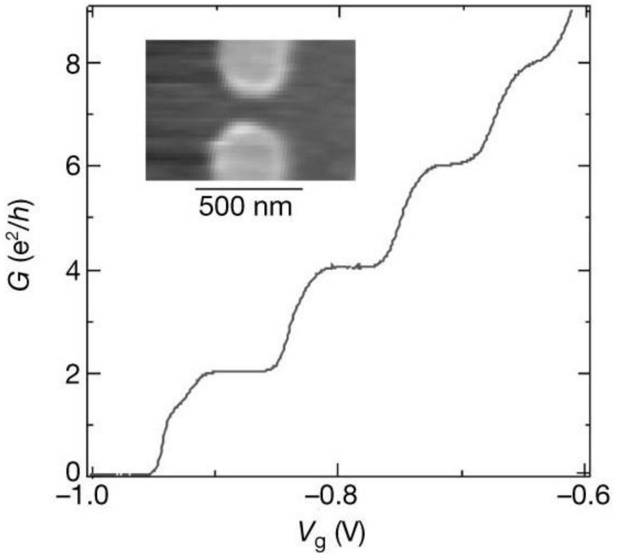
$$= \varphi_n(y)\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \left(\frac{n\pi}{2W}\right)^2\right)\phi(x) = E\varphi_n(y)\phi(x)$$

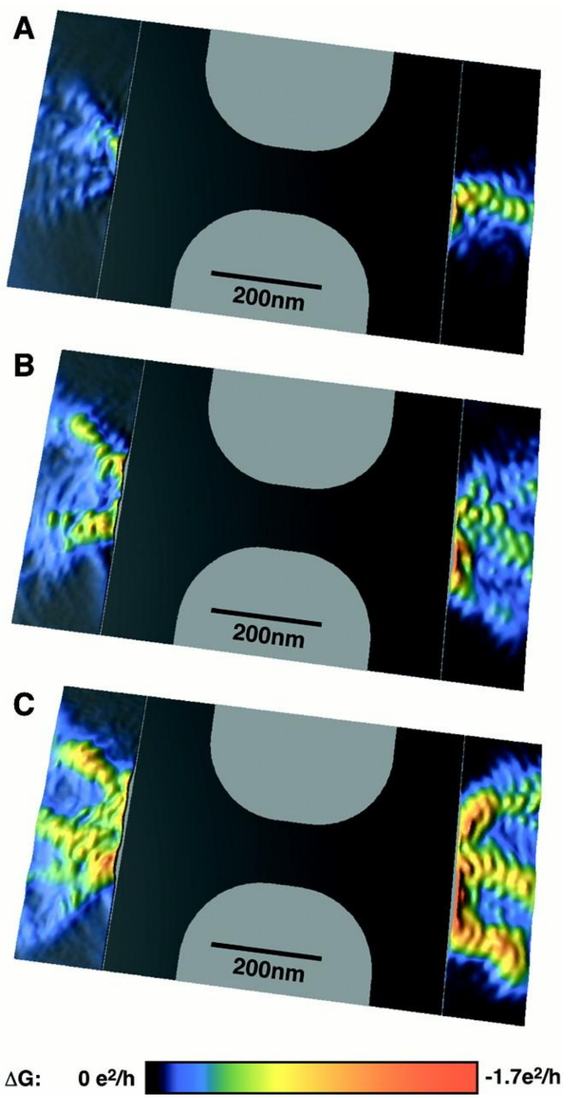
$$V_{\text{eff}}(n,x) = \frac{\hbar^2}{2m}\left(\frac{n\pi}{2W(x)}\right)^2$$

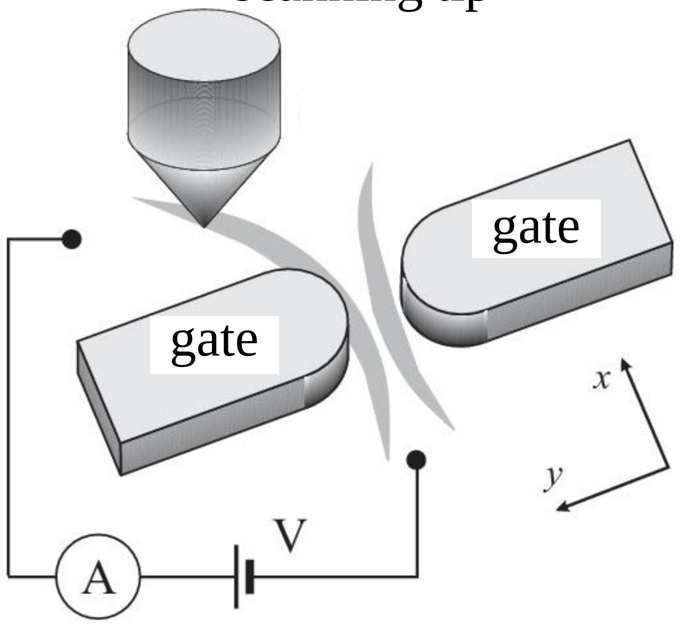


Transmissible one-dimensional system: Conductance Channel

Scanning tip conductance measurement







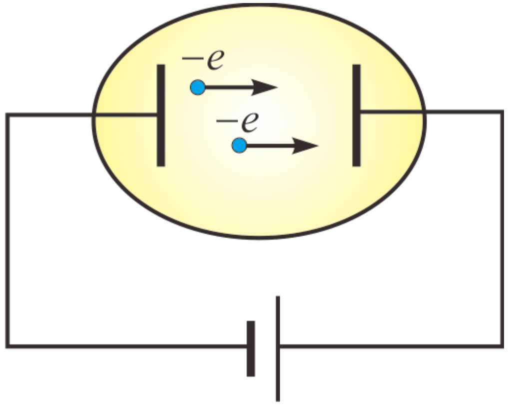
Tip image potential scatters electrons

→ conductance shifts from quantized value

Scattering amplitude ∝ $|\psi|^2$

M. A. Topinka et al.,
Nature **410**, 183 (2001)

Shot noise reduction on the conductance plateaus



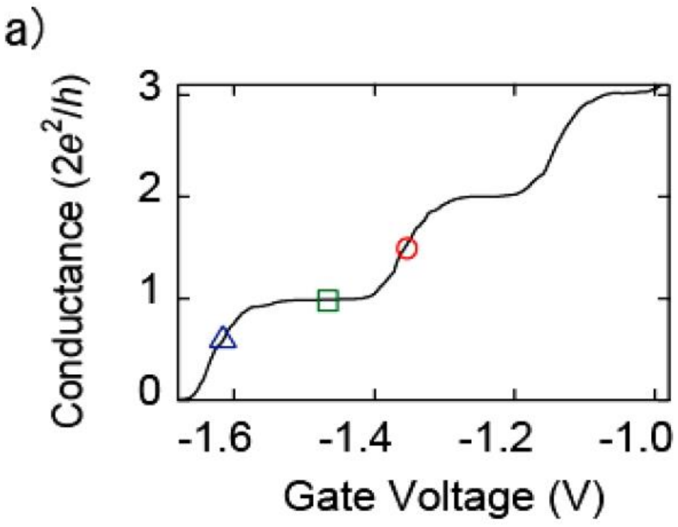
Flow of single electron: Time domain: δ-function approximation

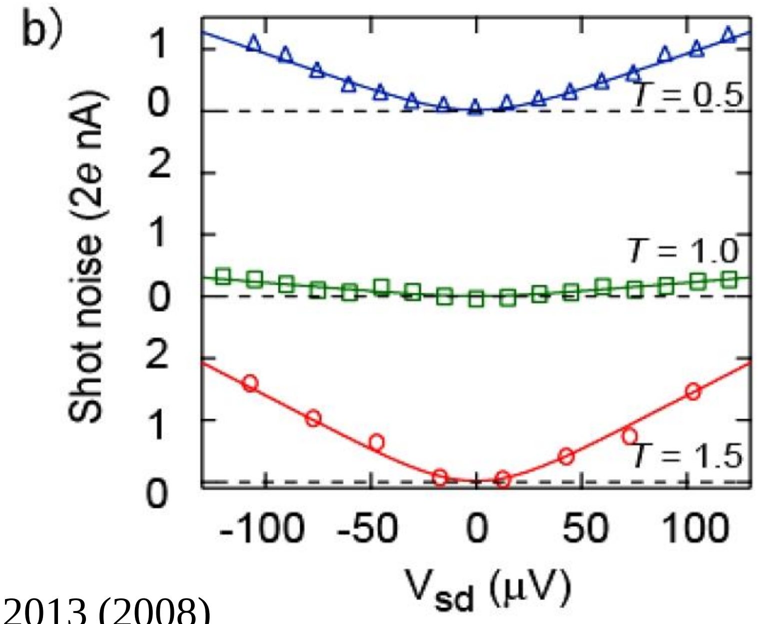
$$J_e(t) = e\delta(t-t_0) = e\int_{-\infty}^{\infty} e^{2\pi i f(t-t_0)}df = 2e\int_0^{\infty}\cos\left[2\pi f(t-t_0)\right]df$$

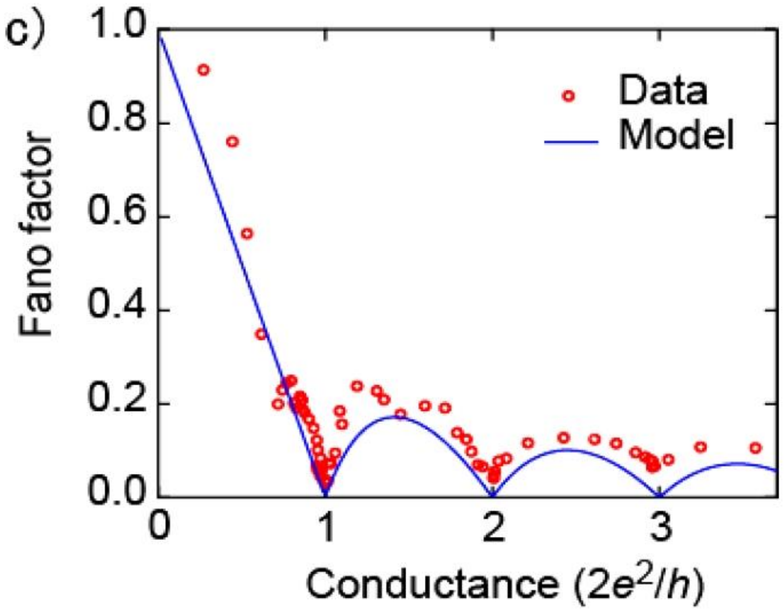
Current fluctuation density for infinitesimal band df $\quad \delta J = d\sqrt{\langle J_e^2\rangle} = \frac{2e}{\sqrt{2}}df = \sqrt{2}e df$

$$\overline{\langle \delta j^2\rangle} = (j_p + j_q e^{i\phi})(j_p + j_q e^{-i\phi}) = j_p^2 + j_q^2 + 2j_p j_q\cos\phi = j_p^2 + j_q^2 = 2\times(\sqrt{2}e)^2 df$$

Flow of *N*-electrons $\quad \overline{\langle \delta J^2\rangle} = N\times 2e^2 df = 2e\bar{J}df \quad (\bar{J} = eN) \quad$: Poissonian noise



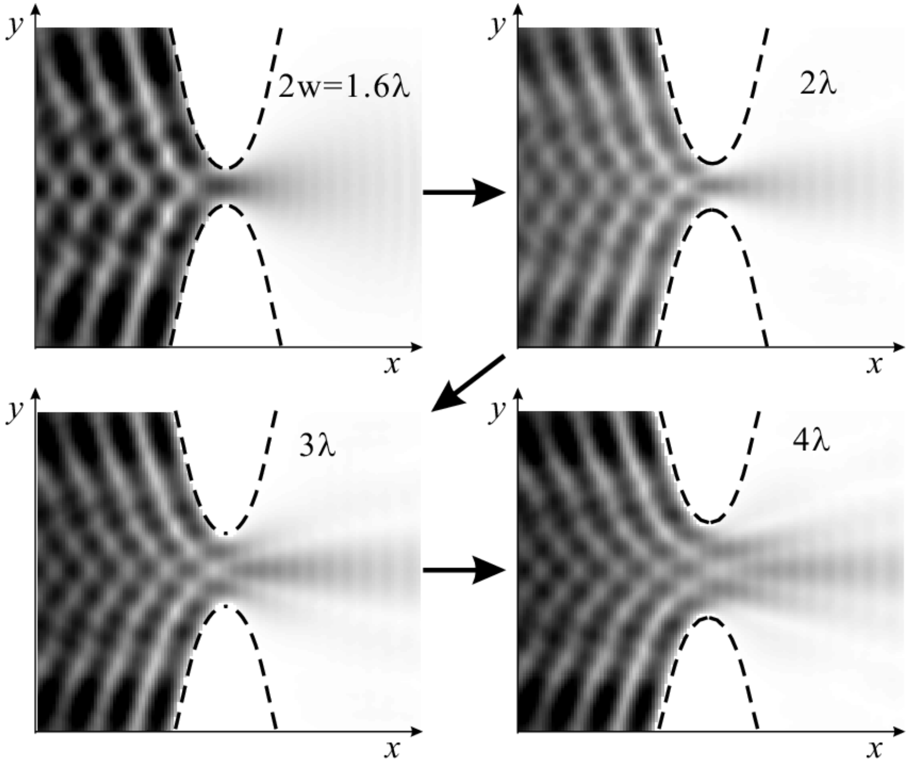




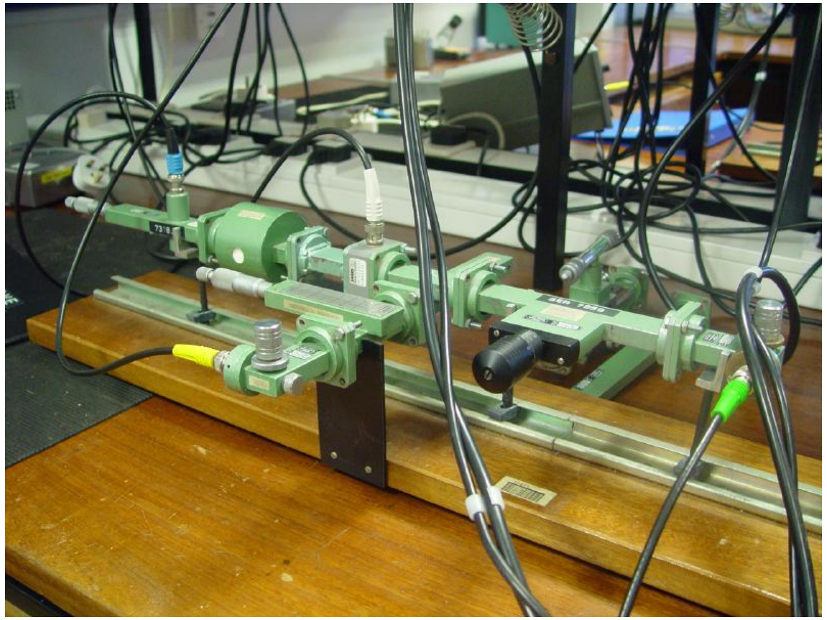
M. Hashisaka, *et al.*, J. Phys.: Conf. Ser. **109**, 012013 (2008)

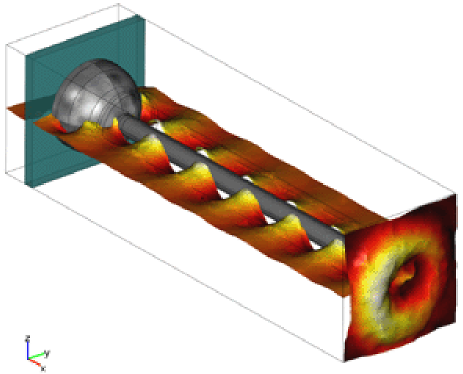
Microwave and electron waveguides

Quantum point contact



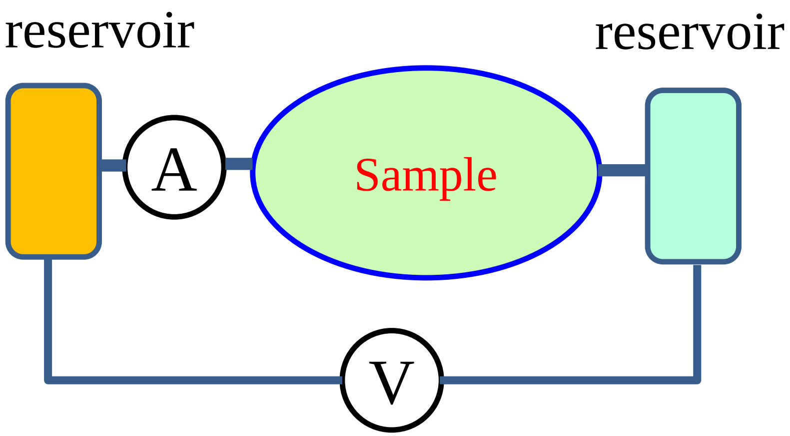
Microwave waveguide

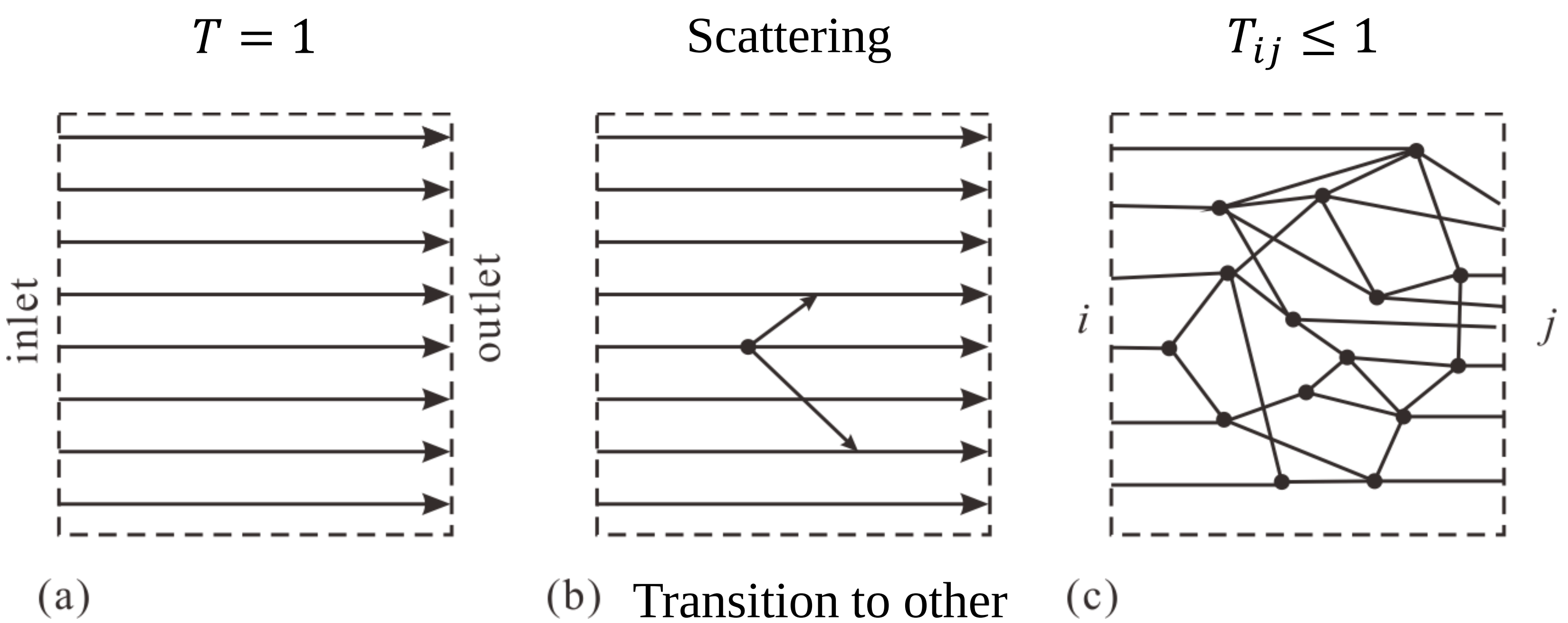




Quantum point contacts or quantum wires can be viewed as "electron waveguides."

Landauer formula for two-terminal conductance





Transition to other channels

$$G = 2\frac{e^2}{h}\sum_{i,j} T_{ij}$$

Electron spin

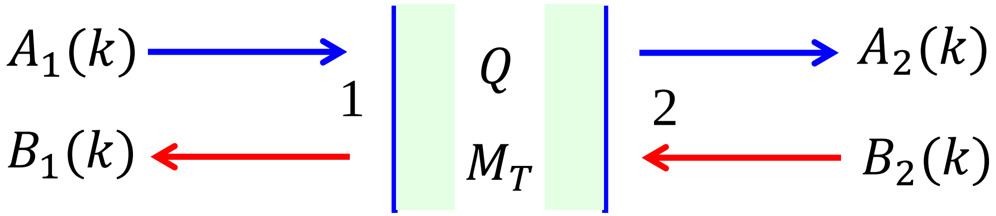
Fermion antibunching

Waveguide connection

Rolf Landauer
1927 - 1999

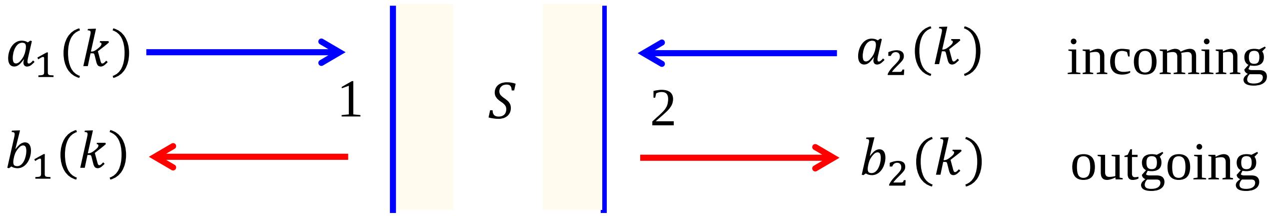
Scattering matrix (S-matrix)

T-matrix



Transfer matrix: M_T $$\begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} \equiv M_T \begin{pmatrix} A_1 \\ B_1 \end{pmatrix}$$

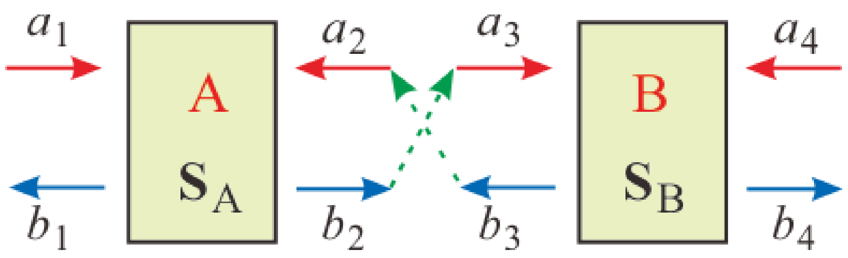
S-matrix



$$\begin{pmatrix} b_1(k) \\ b_2(k) \end{pmatrix} = S \begin{pmatrix} a_1(k) \\ a_2(k) \end{pmatrix} = \begin{pmatrix} r_{\mathrm{L}} & t_{\mathrm{R}} \\ t_{\mathrm{L}} & r_{\mathrm{R}} \end{pmatrix} \begin{pmatrix} a_1(k) \\ a_2(k) \end{pmatrix}$$

Complex probability density flux $$a_i(k) = \sqrt{v_{\mathrm{Fi}}}\psi_{ai}(k_{\mathrm{F}})$$

Series connection of S-matrix



$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = S_A \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} r_{\mathrm{L}}^{(\mathrm{A})} & t_{\mathrm{R}}^{(\mathrm{A})} \\ t_{\mathrm{L}}^{(\mathrm{A})} & r_{\mathrm{R}}^{(\mathrm{A})} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix},$$

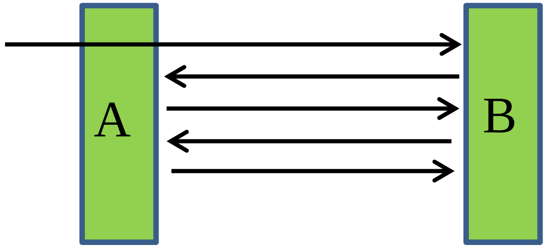
$$\begin{pmatrix} b_3 \\ b_4 \end{pmatrix} = S_B \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} r_{\mathrm{L}}^{(\mathrm{B})} & t_{\mathrm{R}}^{(\mathrm{B})} \\ t_{\mathrm{L}}^{(\mathrm{B})} & r_{\mathrm{R}}^{(\mathrm{B})} \end{pmatrix} \begin{pmatrix} a_3 \\ a_4 \end{pmatrix}$$

$$b_2 = a_3, \quad a_2 = b_3$$

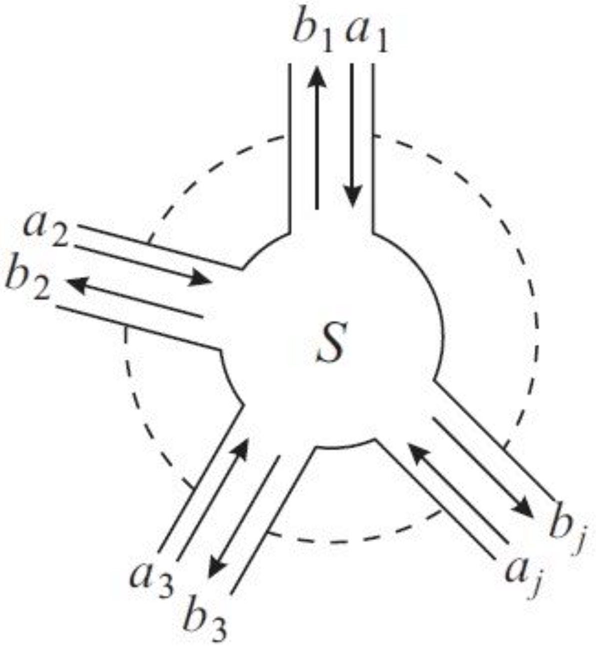
$$S_{\mathrm{AB}} = \begin{pmatrix} r_{\mathrm{L}}^{(\mathrm{A})} + t_{\mathrm{R}}^{(\mathrm{A})} r_{\mathrm{L}}^{(\mathrm{B})} \left(I - r_{\mathrm{R}}^{(\mathrm{A})} r_{\mathrm{L}}^{(\mathrm{B})}\right)^{-1} t_{\mathrm{L}}^{(\mathrm{A})} & t_{\mathrm{R}}^{(\mathrm{A})} \left(I - r_{\mathrm{L}}^{(\mathrm{B})} r_{\mathrm{R}}^{(\mathrm{A})}\right)^{-1} t_{\mathrm{R}}^{(\mathrm{B})} \\ t_{\mathrm{L}}^{(\mathrm{B})} \left(I - r_{\mathrm{R}}^{(\mathrm{A})} r_{\mathrm{L}}^{(\mathrm{B})}\right)^{-1} t_{\mathrm{L}}^{(\mathrm{A})} & r_{\mathrm{R}}^{(\mathrm{B})} + t_{\mathrm{L}}^{(\mathrm{B})} \left(I - r_{\mathrm{R}}^{(\mathrm{A})} r_{\mathrm{L}}^{(\mathrm{B})}\right)^{-1} r_{\mathrm{R}}^{(\mathrm{A})} t_{\mathrm{R}}^{(\mathrm{B})} \end{pmatrix}$$

S-matrix

$$\left(I - r_{\mathrm{R}}^{(\mathrm{A})} r_{\mathrm{L}}^{(\mathrm{B})}\right)^{-1} = I + r_{\mathrm{R}}^{(\mathrm{A})} r_{\mathrm{L}}^{(\mathrm{B})} + (r_{\mathrm{R}}^{(\mathrm{A})} r_{\mathrm{L}}^{(\mathrm{B})})^2 + (r_{\mathrm{R}}^{(\mathrm{A})} r_{\mathrm{L}}^{(\mathrm{B})})^3 + \cdots$$



Multi-channel



$$\boldsymbol{b} = \begin{pmatrix} b_1 \\ \vdots \\ b_i \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} S_{11} & \cdots & S_{1i} & \cdots & S_{1n} \\ \vdots & \ddots & & & \vdots \\ S_{i1} & & S_{ii} & & S_{in} \\ \vdots & & & \ddots & \vdots \\ S_{n1} & \cdots & S_{ni} & \cdots & S_{nn} \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{pmatrix} = \mathrm{S}\boldsymbol{a}$$

Reciprocity $S_{ij} = S_{ji}$
(time-reversal symmetry)

Unitarity $\sum_j S_{ji} S_{jk}^* = \delta_{ik}$

Onsager reciprocity

$$\left[\frac{(i\hbar\nabla + e\boldsymbol{A})^2}{2m} + V\right]\psi = E\psi$$

Complex conjugate and $\boldsymbol{A} \rightarrow -\boldsymbol{A}$

Lars Onsager
1903-1976

$$\left[\frac{(i\hbar\nabla + e\boldsymbol{A})^2}{2m} + V\right]\psi^* = E\psi^*$$

$$\{\psi^*(-B)\} = \{\psi(B)\}$$

Scattering solution: Sc$\{a \rightarrow b\}$ $\mathrm{Sc}\{\boldsymbol{a}(B) \rightarrow \boldsymbol{b}(B)\} \in \{\psi(B)\}, \quad i.e., \quad \boldsymbol{b}(B) = S(B)\boldsymbol{a}(B)$

$$\boldsymbol{b}^*(B) = S^*(B)\boldsymbol{a}^*(B)$$

$$\mathrm{Sc}\{\boldsymbol{b}^*(-B) \rightarrow \boldsymbol{a}^*(-B)\} \in \{\psi^*(-B)\} = \{\psi(B)\} \quad i.e. \quad \boldsymbol{a}^*(-B) = S(B)\boldsymbol{b}^*(-B)$$

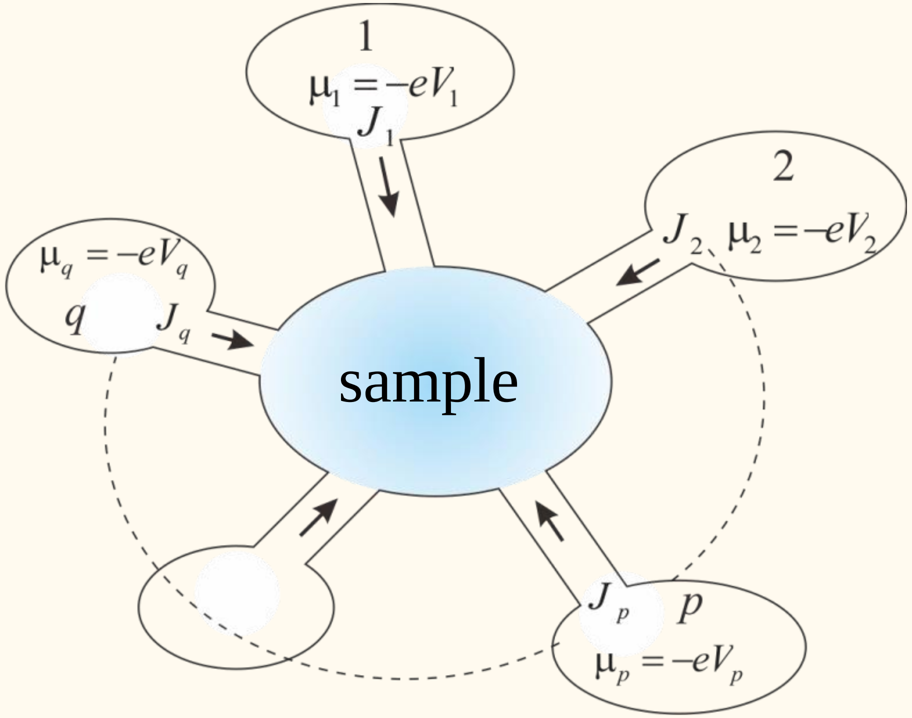
$$\boldsymbol{b}^*(B) = S^{-1}(-B)\boldsymbol{a}^*(B)$$

$$S^*(B) = S^{-1}(-B) = S^\dagger(-B) \quad (\quad \text{unitarity} \quad SS^\dagger = S^\dagger S = I)$$

$$S(B) = {}^tS(-B) \qquad S_{ij}(B) = S_{ji}(-B)$$

Landauer-Büttiker formula

Makus Büttiker
1950-2013



$$J_p = -\frac{2e}{h}\sum_q [T_{q\leftarrow p}\mu_p - T_{p\leftarrow q}\mu_q]$$

$$\mathscr{T}_{pq} \equiv T_{p\leftarrow q} \quad (p \neq q), \qquad \mathscr{T}_{pp} \equiv -\sum_{q\neq p} T_{q\leftarrow p}$$

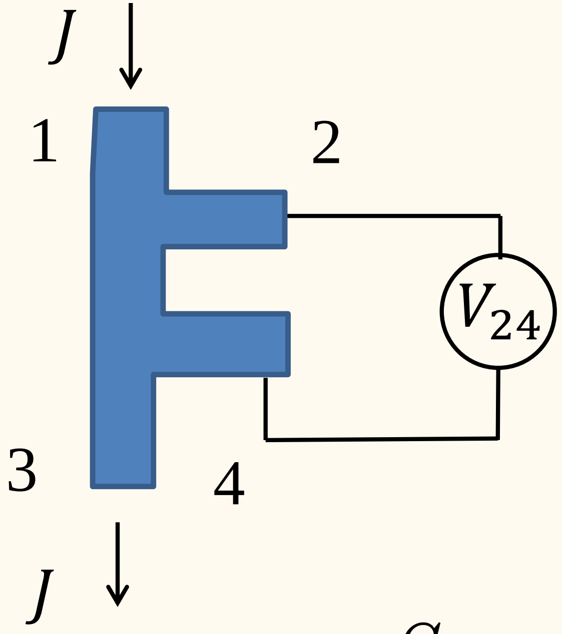
$$\boldsymbol{J} =^t (J_1, J_2, \cdots), \boldsymbol{\mu} =^t (\mu_1, \mu_2, \cdots)$$

$$\boldsymbol{J} = \frac{2e}{h}\mathscr{T}\boldsymbol{\mu}$$

$$V_q = \frac{\mu_q}{-e}, \qquad G_{pq} \equiv \frac{2e^2}{h}T_{p\leftarrow q} \ \text{ then } \ J_p = \sum_q [G_{qp}V_p - G_{pq}V_q]$$

$$\sum_q J_q = 0 \qquad \sum_q [G_{qp} - G_{pq}] = 0 \qquad G_{qp}(B) = G_{pq}(-B)$$

Landauer-Büttker formula: Application to 4-terminal measurement



$$\alpha_{11} = 2G_q[-\mathscr{T}_{11} - S^{-1}(\mathscr{T}_{14} + \mathscr{T}_{12})(\mathscr{T}_{41} + \mathscr{T}_{21})]$$

$$\alpha_{12} = 2G_q S^{-1}(\mathscr{T}_{12}\mathscr{T}_{34} - \mathscr{T}_{14}\mathscr{T}_{32})$$

$$\alpha_{21} = 2G_q S^{-1}(\mathscr{T}_{21}\mathscr{T}_{43} - \mathscr{T}_{23}\mathscr{T}_{41})$$

$$\alpha_{22} = 2G_q[-\mathscr{T}_{22} - S^{-1}(\mathscr{T}_{21} - \mathscr{T}_{23})(\mathscr{T}_{32} + \mathscr{T}_{12})]$$

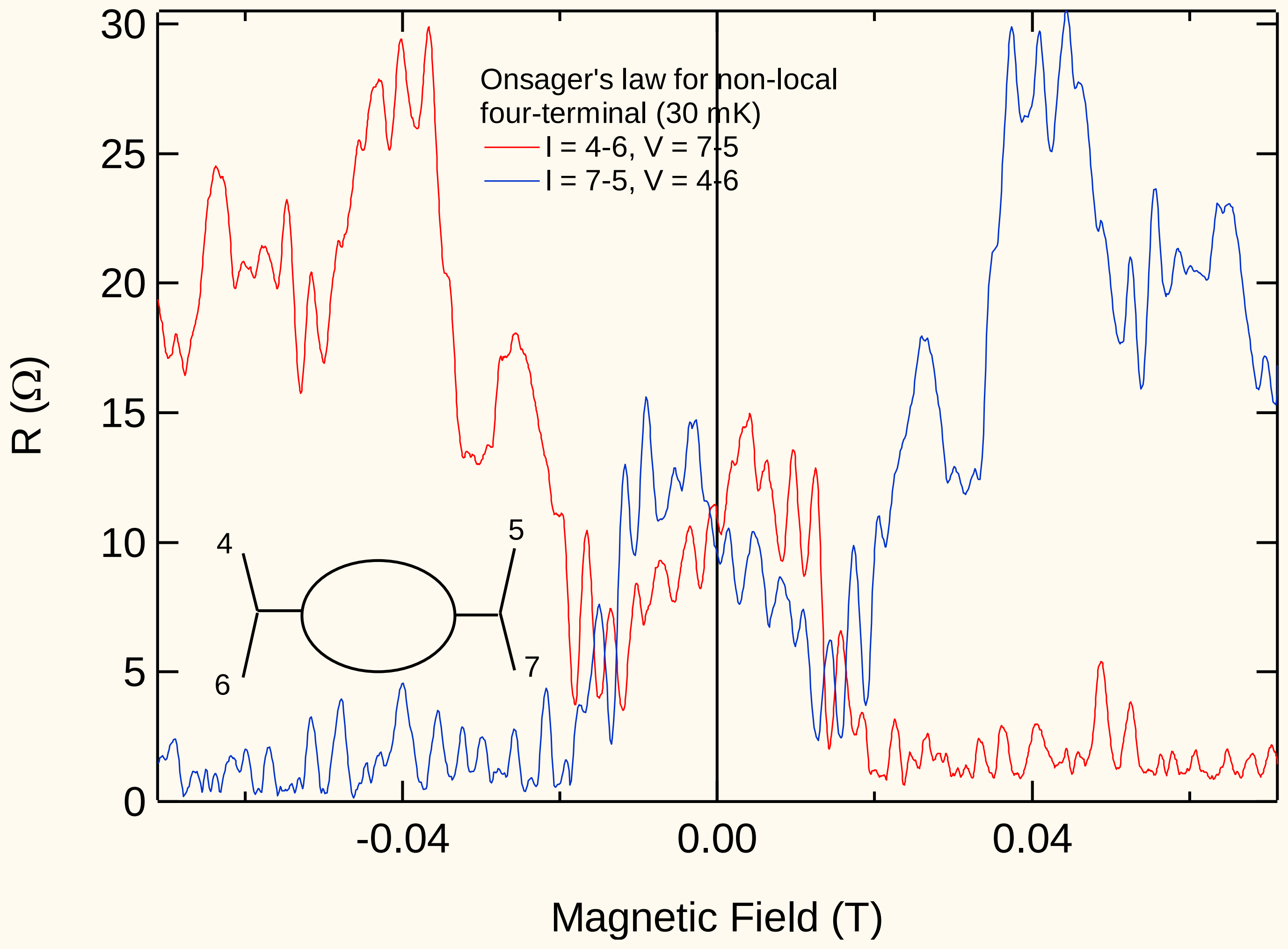
$$S = \mathscr{T}_{12} + \mathscr{T}_{14} + \mathscr{T}_{32} + \mathscr{T}_{34} = \mathscr{T}_{21} + \mathscr{T}_{41} + \mathscr{T}_{23} + \mathscr{T}_{43}$$

$$\mathcal{R}_{13,24} = \frac{V_2 - V_4}{J_1} = \frac{\alpha_{21}}{\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}}$$

$$\mathcal{R}_{mn,kl}(B) = -\mathcal{R}_{kl,mn}(-B)$$ Onsager reciprocity

Onsager reciprocity in AB ring

$$\mathcal{R}_{ij,kl}(B) = \mathcal{R}_{kl,ij}(-B)$$



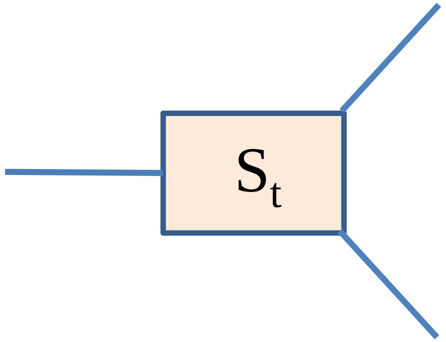
Magnetoresistance: Universal conductance fluctuation including AB oscillation

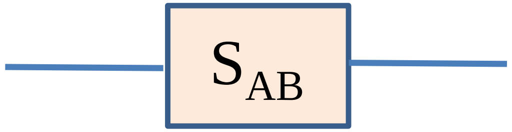
S-matrix: Application to Aharonov-Bohm ring

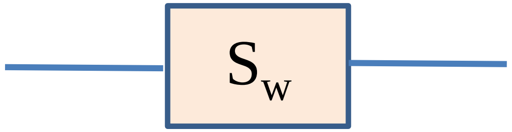
$$S_t = \begin{pmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{pmatrix}$$

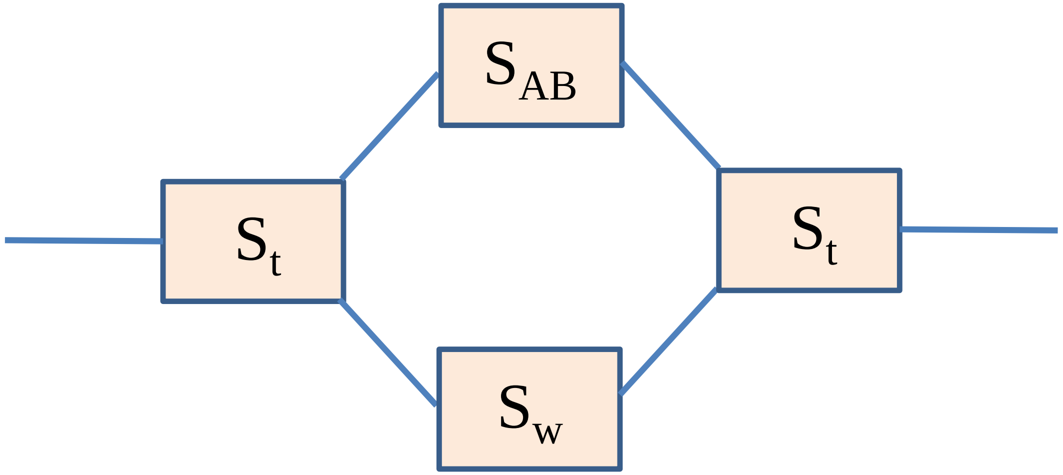
$$S_{\mathrm{AB}} = \begin{pmatrix} 0 & e^{i\theta_{\mathrm{AB}}} \\ e^{-i\theta_{\mathrm{AB}}} & 0 \end{pmatrix}, \qquad \theta \equiv 2\pi\frac{\phi}{\phi_0} = \frac{e}{\hbar}\phi$$

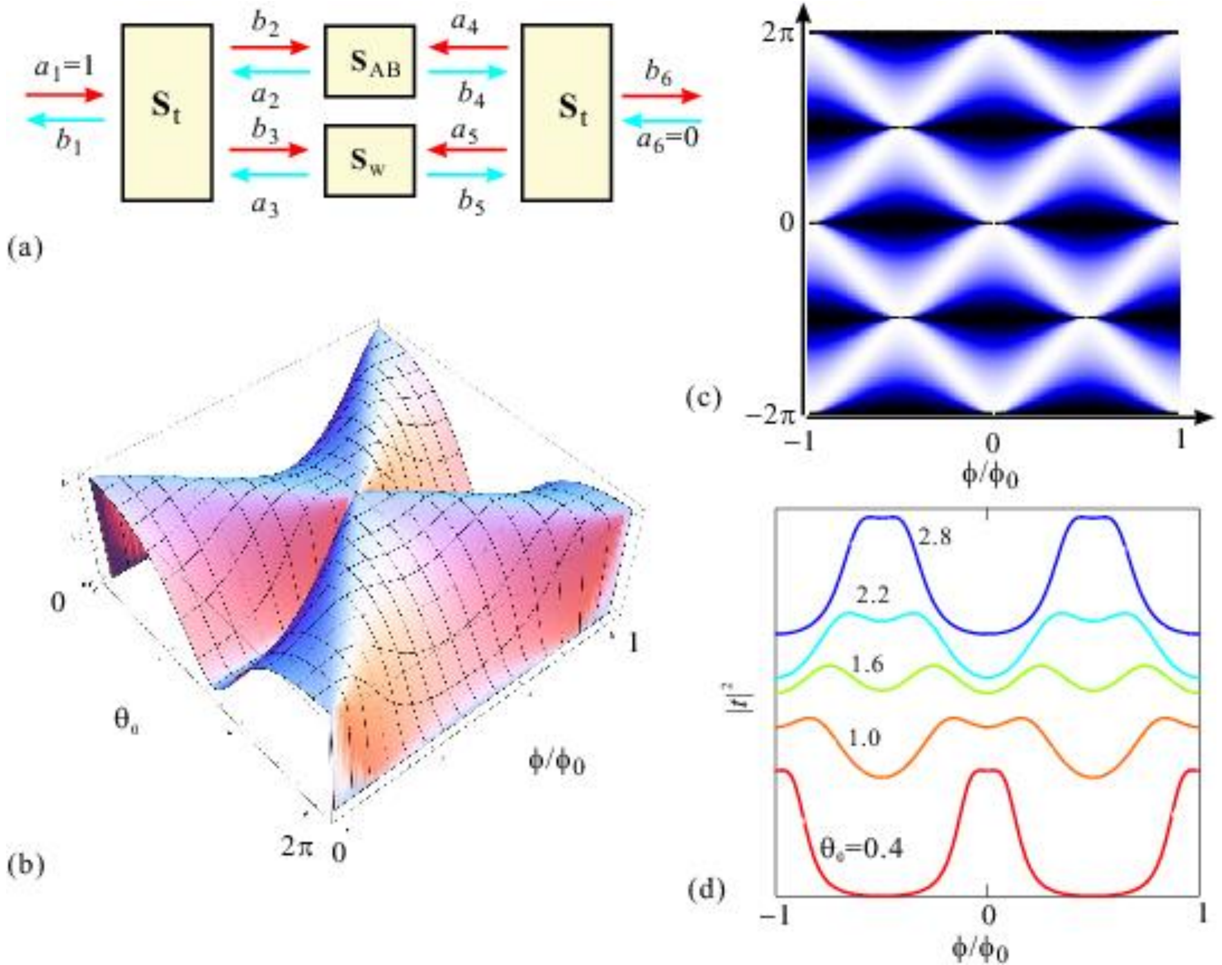
$$S_{\mathrm{w}} = \begin{pmatrix} 0 & e^{i\theta_0} \\ e^{i\theta 0} & 0 \end{pmatrix}$$











$$t = \frac{4\sin\theta_0}{1 + e^{i\theta_{\mathrm{AB}}}\left(e^{i\theta_{\mathrm{AB}}} + e^{i\theta_0} - 3e^{-i\theta_0}\right)}$$

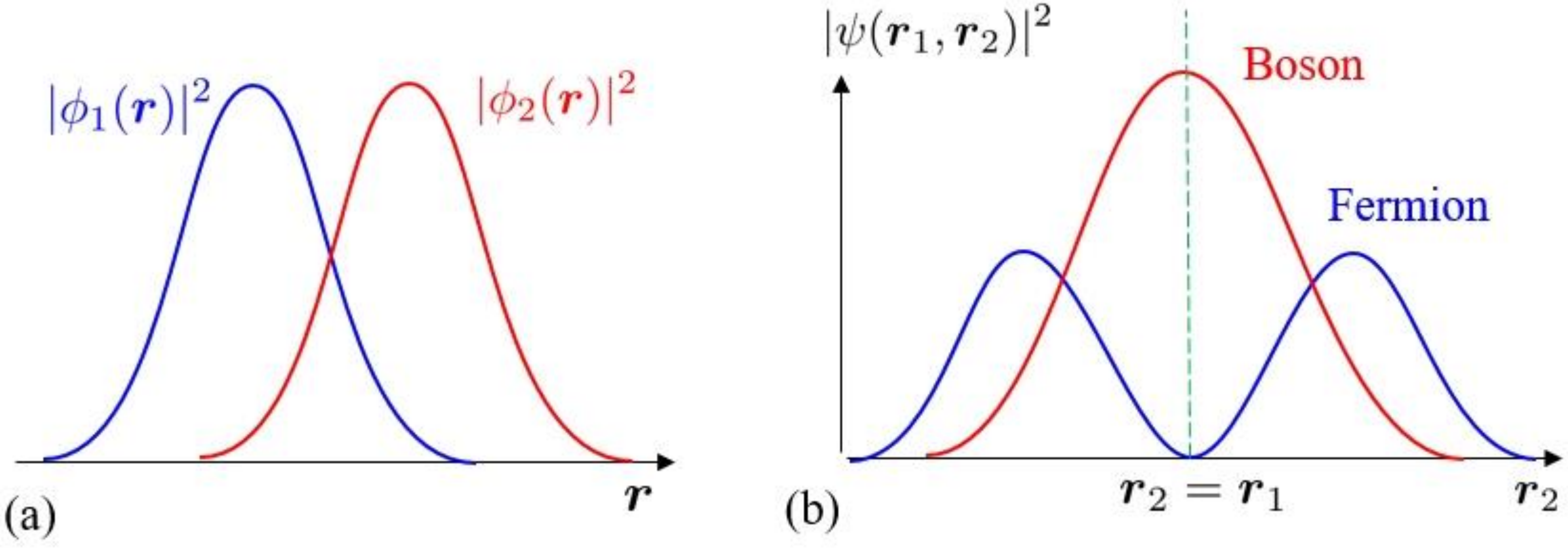
Bunching and anti-bunching of particles

Two-particle wavefunction: $\psi(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{1}{\sqrt{2}}[\phi_1(\boldsymbol{r}_1)\phi_2(\boldsymbol{r}_2) \pm \phi_1(\boldsymbol{r}_2)\phi_2(\boldsymbol{r}_1)]$ (+: boson, –: fermion)

Probability of finding two-particles at the same position

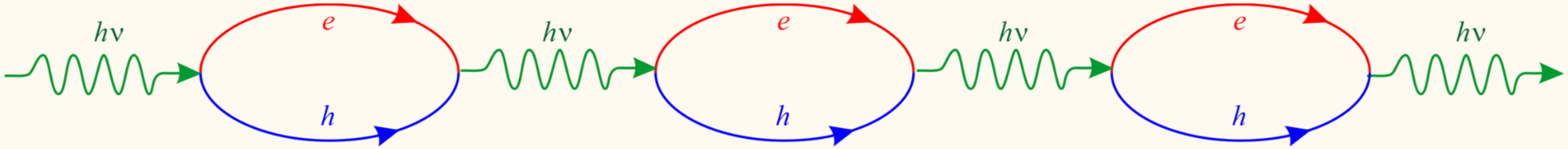
$$|\psi(\boldsymbol{r}_1, \boldsymbol{r}_1)|^2 = \begin{cases} 2|\phi_1(\boldsymbol{r}_1)|^2|\phi_2\boldsymbol{r}_1|^2 & (\text{boson}), \\ 0 & (\text{fermion}) \end{cases}$$

Boson: bunching, bosonic stimulation → laser oscillation, Bose-Einstein Condensation



Waveguide for exciton-polariton

exciton-polariton



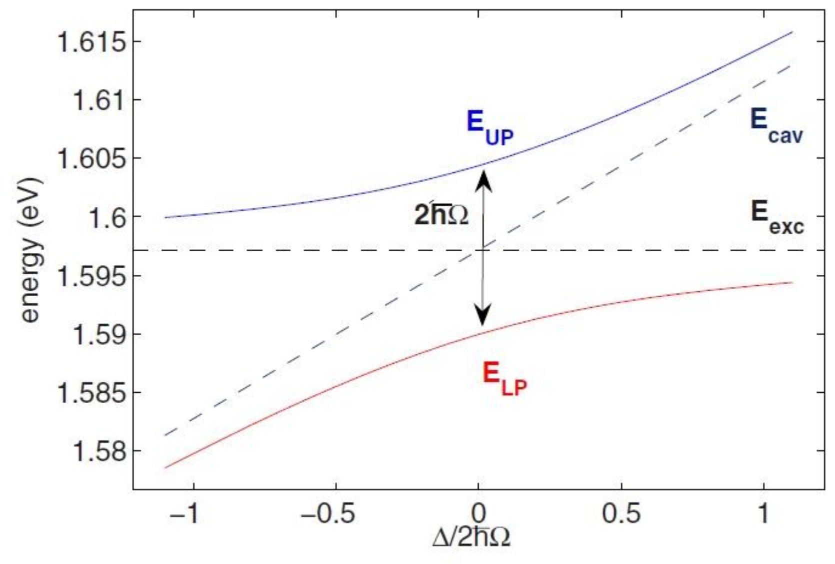
Chain of photon-exciton (photon-dressed exciton)

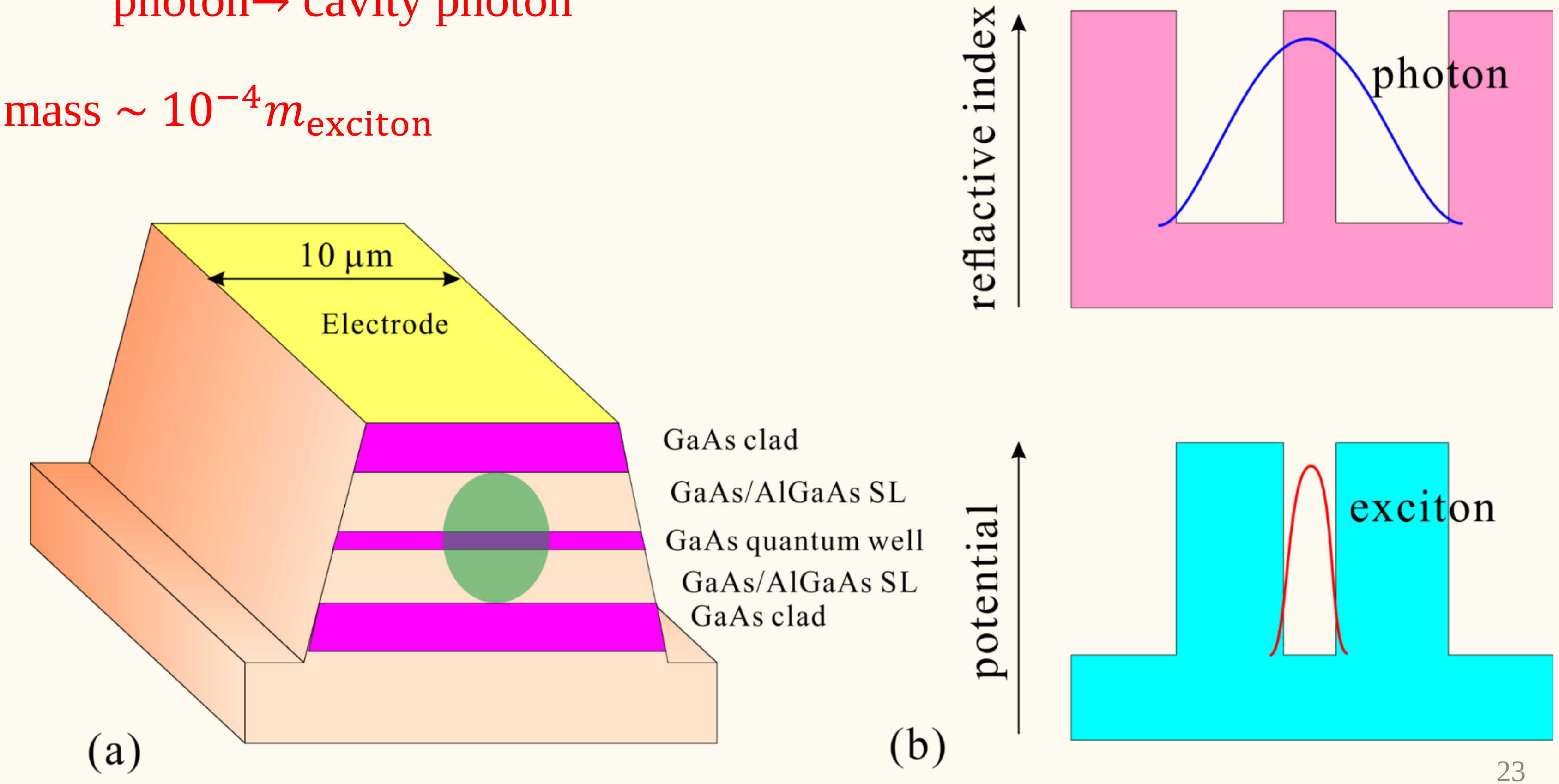
1 cycle ～ few fs

coherent propagation in solids

photon→ cavity photon

dispersion relation: light effective mass ~ $10^{-4} m_{\text{exciton}}$



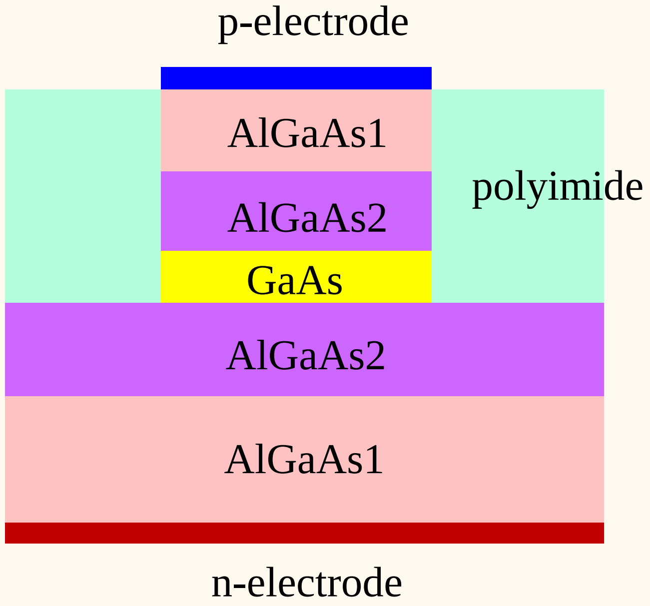


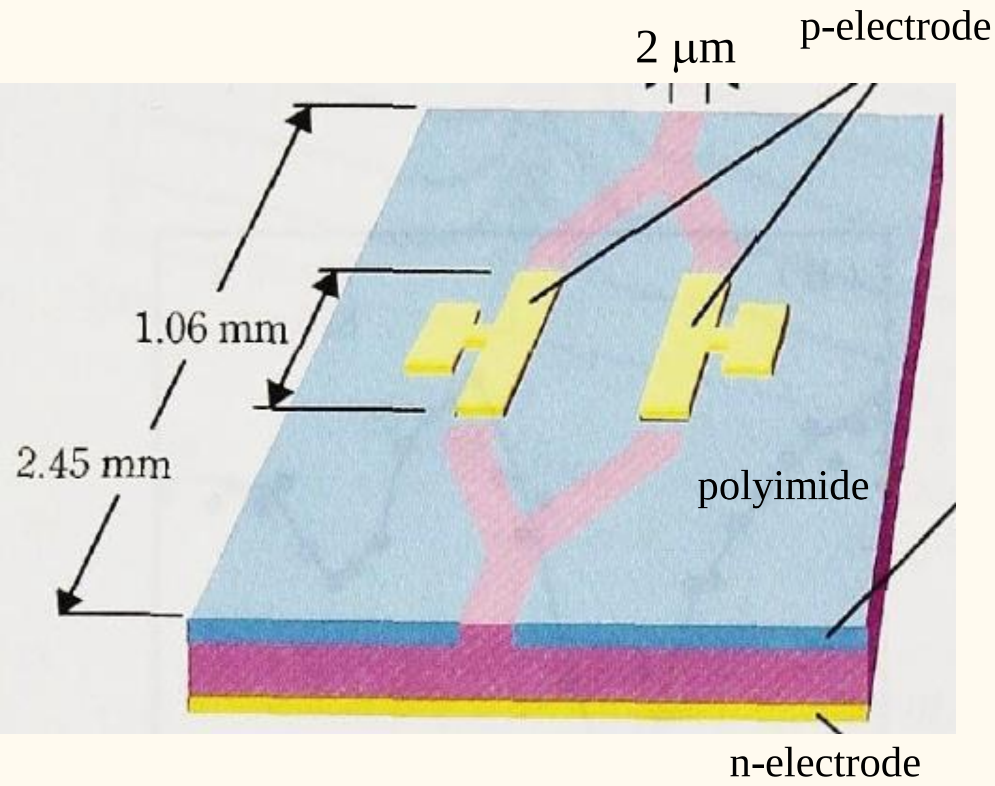
Mach-Zehnder interferometer (voltage-type)

Kinetic phase shift with electric field: $\Delta\varphi = L\left[\frac{\sqrt{2mE_k}}{\hbar} - \frac{\sqrt{2m(E_k-\delta E)}}{\hbar}\right]$

δE: energy shift due to the depletion of quantum well

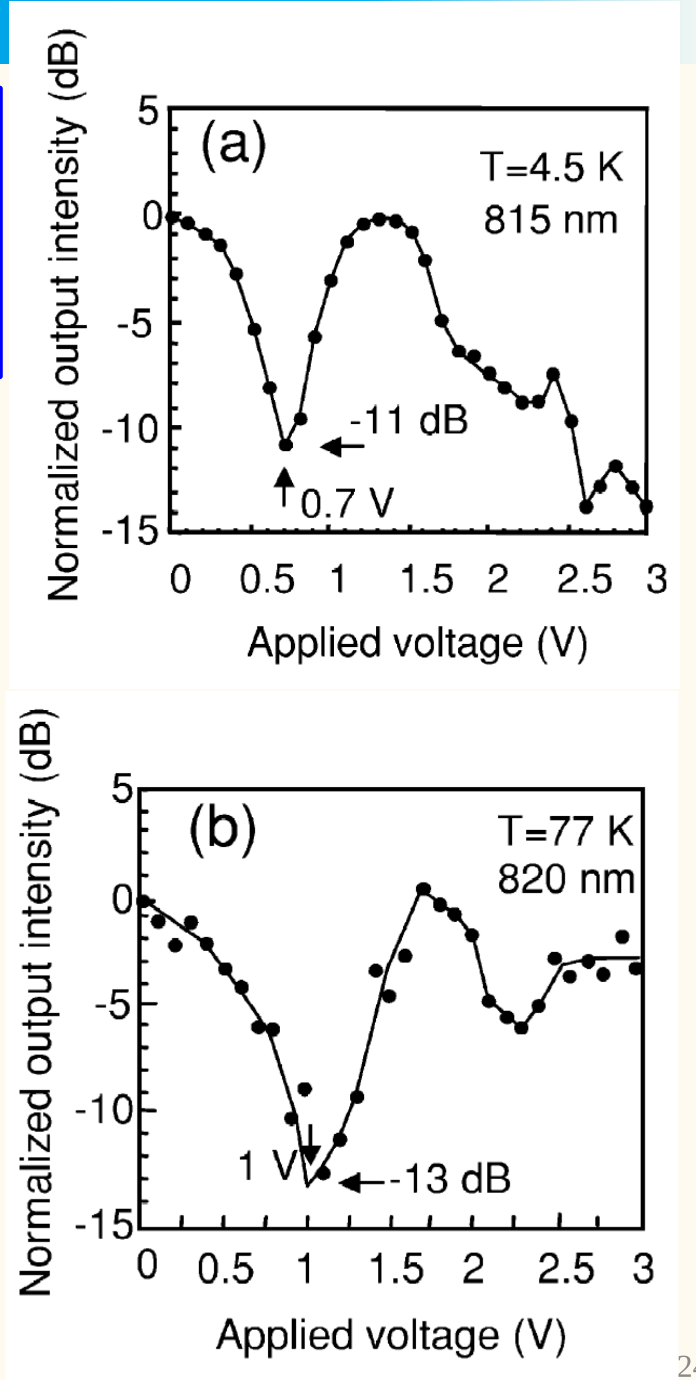
junction-FET type waveguide





Katsuyama, Hosomi, Micro. Eng. **63**, 23 (2002).

Voltage control of optical output through interference



Mach-Zehnder interferometer 2 (optical control)

Kinetic phase shift with electric field: $\Delta\varphi = L\left[\frac{\sqrt{2mE_k}}{\hbar} - \frac{\sqrt{2m(E_k - \delta E)}}{\hbar}\right]$

δE: energy shift due to the barrier by optically excited carriers (quasi-Fermi levels)

Sturm *et al.*, Nature Comm. **5**, 3278 (2014)