Lecture on

Semiconductors / 半導体

(Physics of semiconductors)

2021.6.30 Lecture 12

10:25 – 11:55

Institute for Solid State Physics, University of Tokyo

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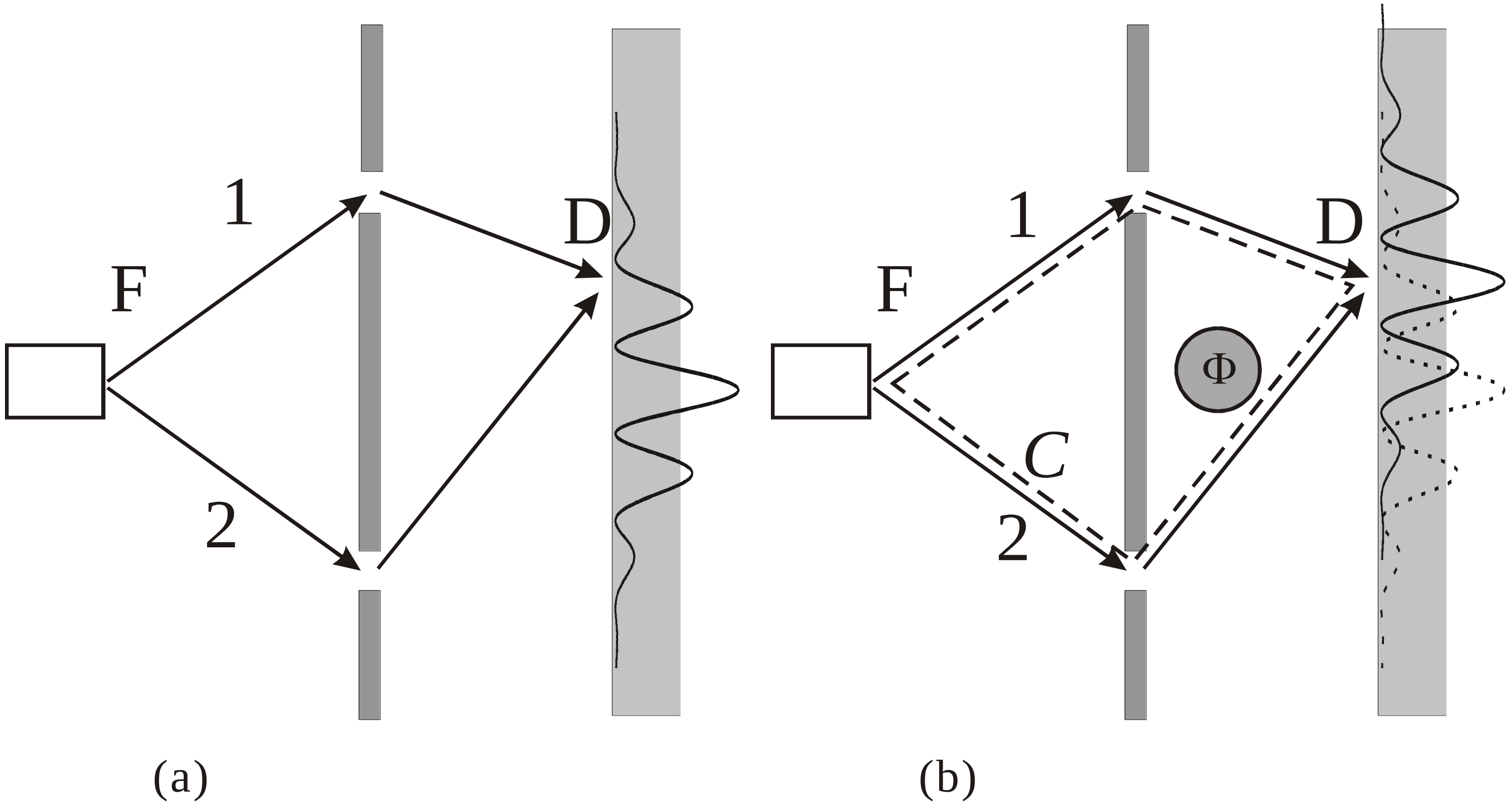
Chapter 8 Basics of Quantum Transport

- Boundary between classical and quantum (coherence length)
- Conductance quantum
- Quantum point contact
- Landauer formula for two-terminal conductance
- Scattering matrix (S-matrix)
- Onsager reciprocity
- Landauer-Büttker formula for multi-terminal conductance

Aharonov-Bohm effect

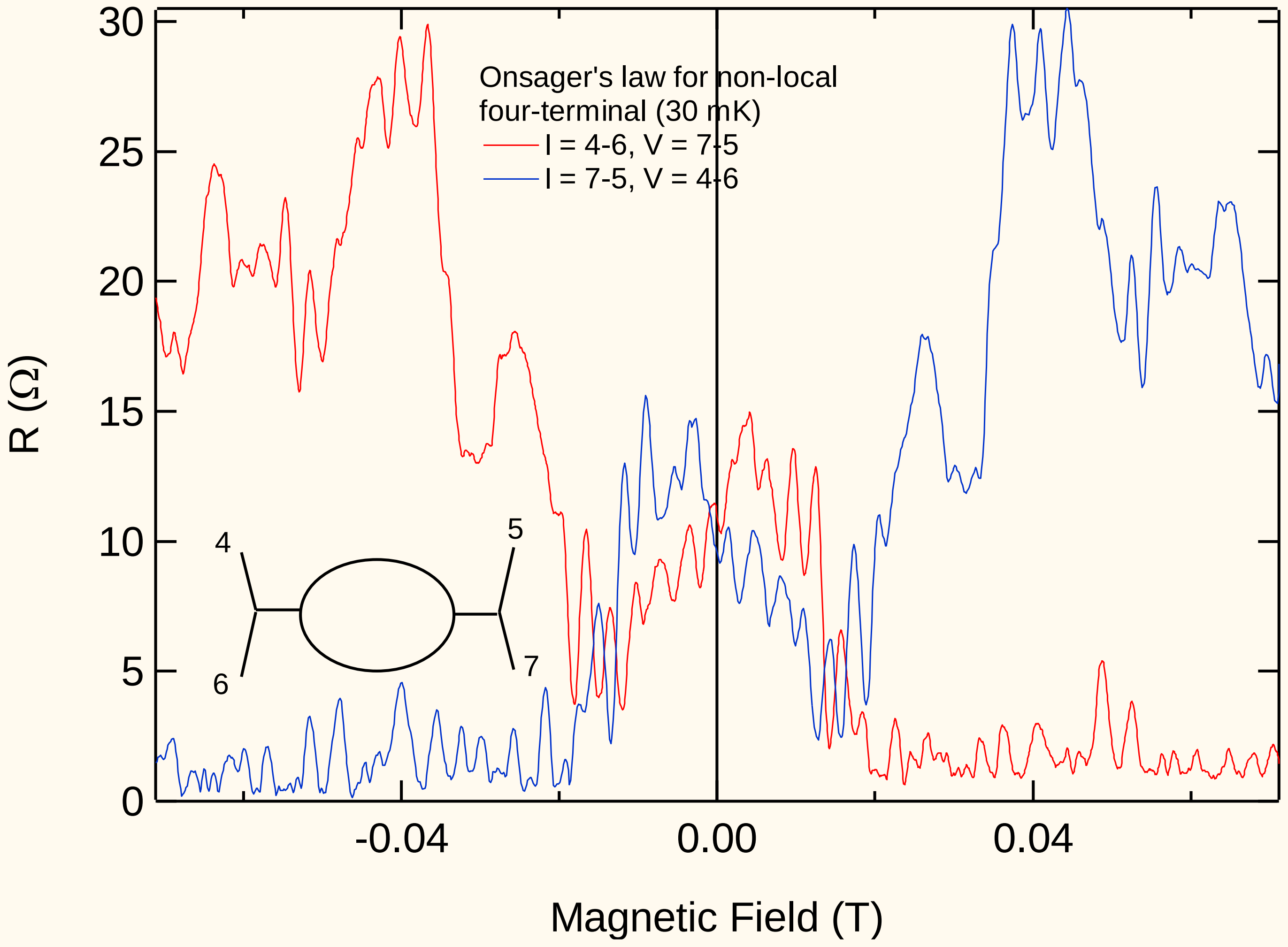


$$\boldsymbol{p} = m\boldsymbol{v} + e\boldsymbol{A} \qquad p = \hbar k = \frac{h}{\lambda}$$

$$\Delta\theta = \frac{e}{\hbar}\oint_C \Delta\boldsymbol{A}\cdot \mathrm{d}\boldsymbol{s} = \frac{e}{\hbar}\int_S \boldsymbol{B}\cdot\mathrm{d}\boldsymbol{n} = 2\pi\frac{\Phi}{\Phi_0} \qquad \Phi_0 \equiv \frac{h}{e}$$

Onsager reciprocity in AB ring

$$\mathcal{R}_{ij,kl}(B) = \mathcal{R}_{kl,ij}(-B)$$



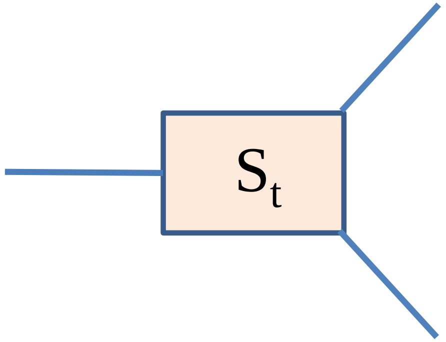
In the case of two-terminal measurement

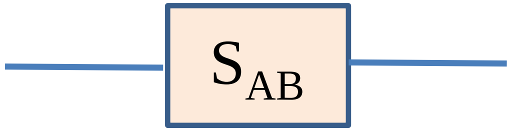
$$R(B) = R(-B)$$

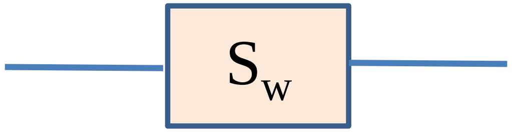
Magnetoresistance: Universal conductance fluctuation including AB oscillation

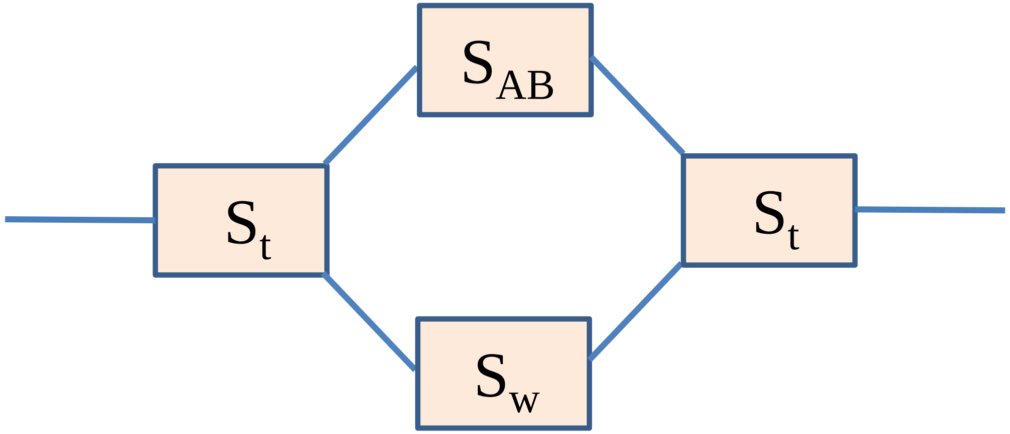
S-matrix: Application to Aharonov-Bohm ring

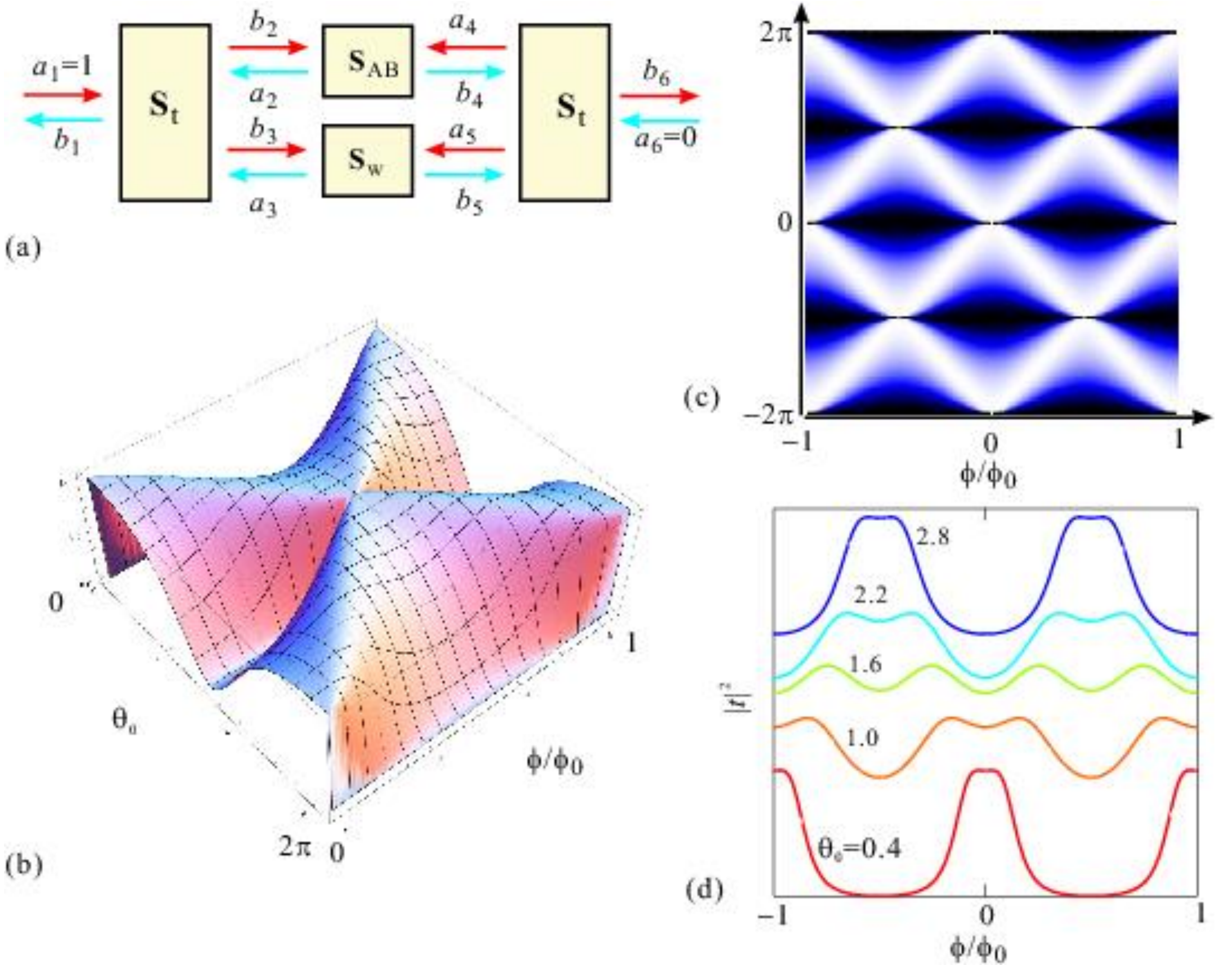
$$S_t = \begin{pmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{pmatrix} \qquad S_{\mathrm{AB}} = \begin{pmatrix} 0 & e^{i\theta_{\mathrm{AB}}} \\ e^{-i\theta_{\mathrm{AB}}} & 0 \end{pmatrix}, \qquad \theta \equiv 2\pi\frac{\phi}{\phi_0} = \frac{e}{\hbar}\phi \qquad S_{\mathrm{w}} = \begin{pmatrix} 0 & e^{i\theta_0} \\ e^{i\theta 0} & 0 \end{pmatrix}$$





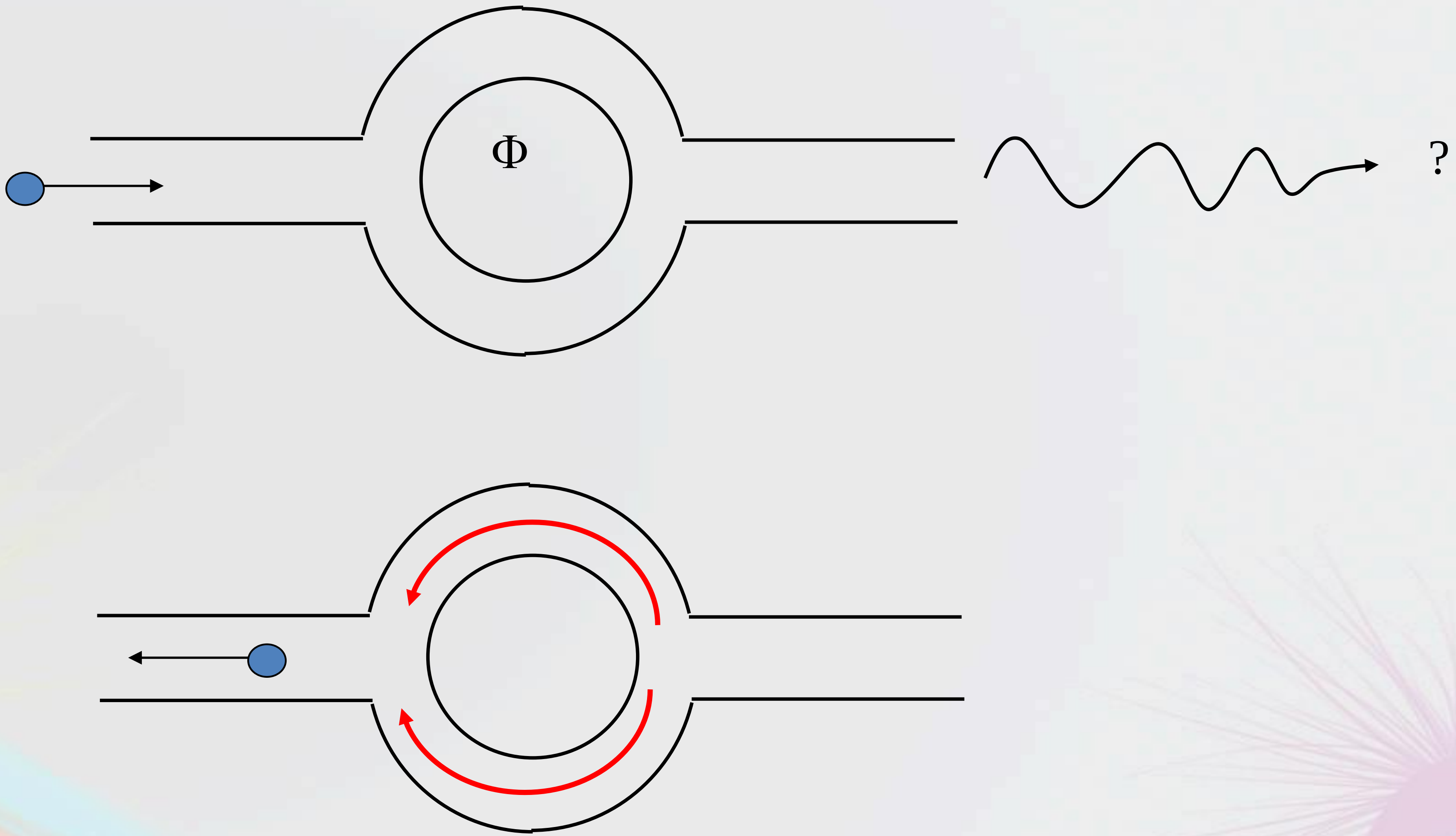






$$t = \frac{4\sin\theta_0}{1 + e^{i\theta_{\mathrm{AB}}}\left(e^{i\theta_{\mathrm{AB}}} + e^{i\theta_0} - 3e^{-i\theta_0}\right)}$$

Disappearance of electrons?



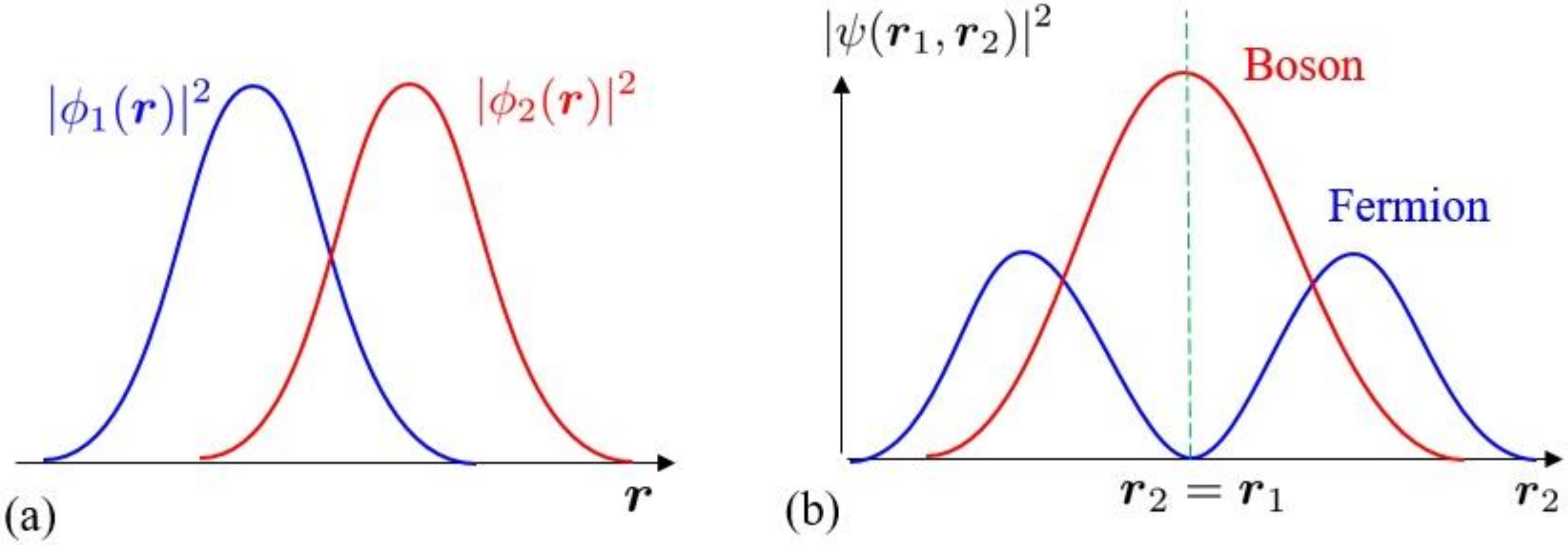
Bunching and anti-bunching of particles

Two-particle wavefunction: $$\psi(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{1}{\sqrt{2}}[\phi_1(\boldsymbol{r}_1)\phi_2(\boldsymbol{r}_2) \pm \phi_1(\boldsymbol{r}_2)\phi_2(\boldsymbol{r}_1)]$$ (+: boson, –: fermion)

Probability of finding two-particles at the same position $$|\psi(\boldsymbol{r}_1, \boldsymbol{r}_1)|^2 = \begin{cases} 2|\phi_1(\boldsymbol{r}_1)|^2|\phi_2\boldsymbol{r}_1|^2 & \text{(boson)}, \\ 0 & \text{(fermion)} \end{cases}$$

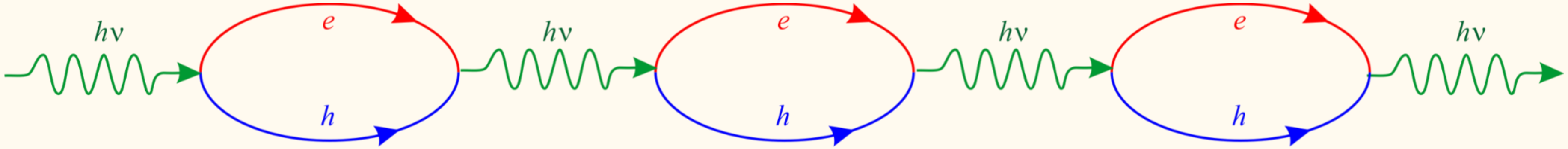
Boson: bunching, bosonic stimulation → laser oscillation, Bose-Einstein Condensation

Fermion: anti-bunching, conductance quantization, shot noise reduction



Waveguide for exciton-polariton

exciton-polariton



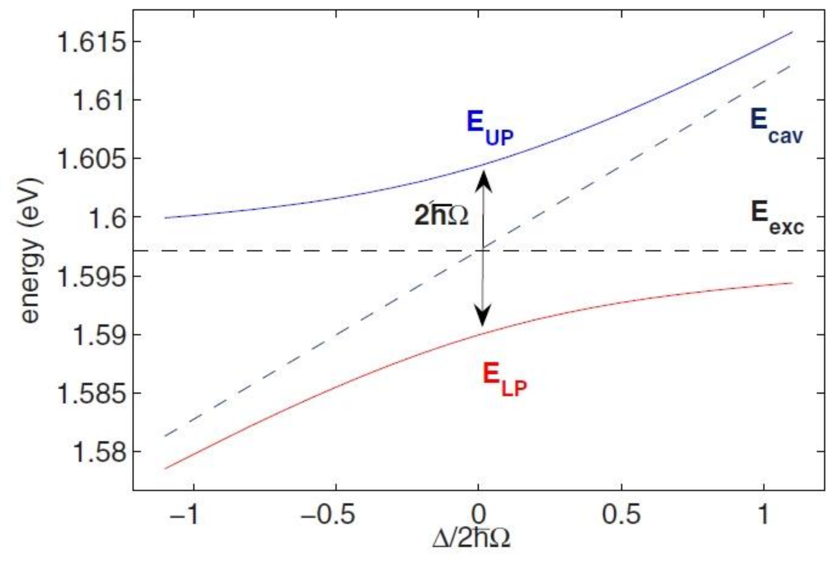
Chain of photon-exciton (photon-dressed exciton)

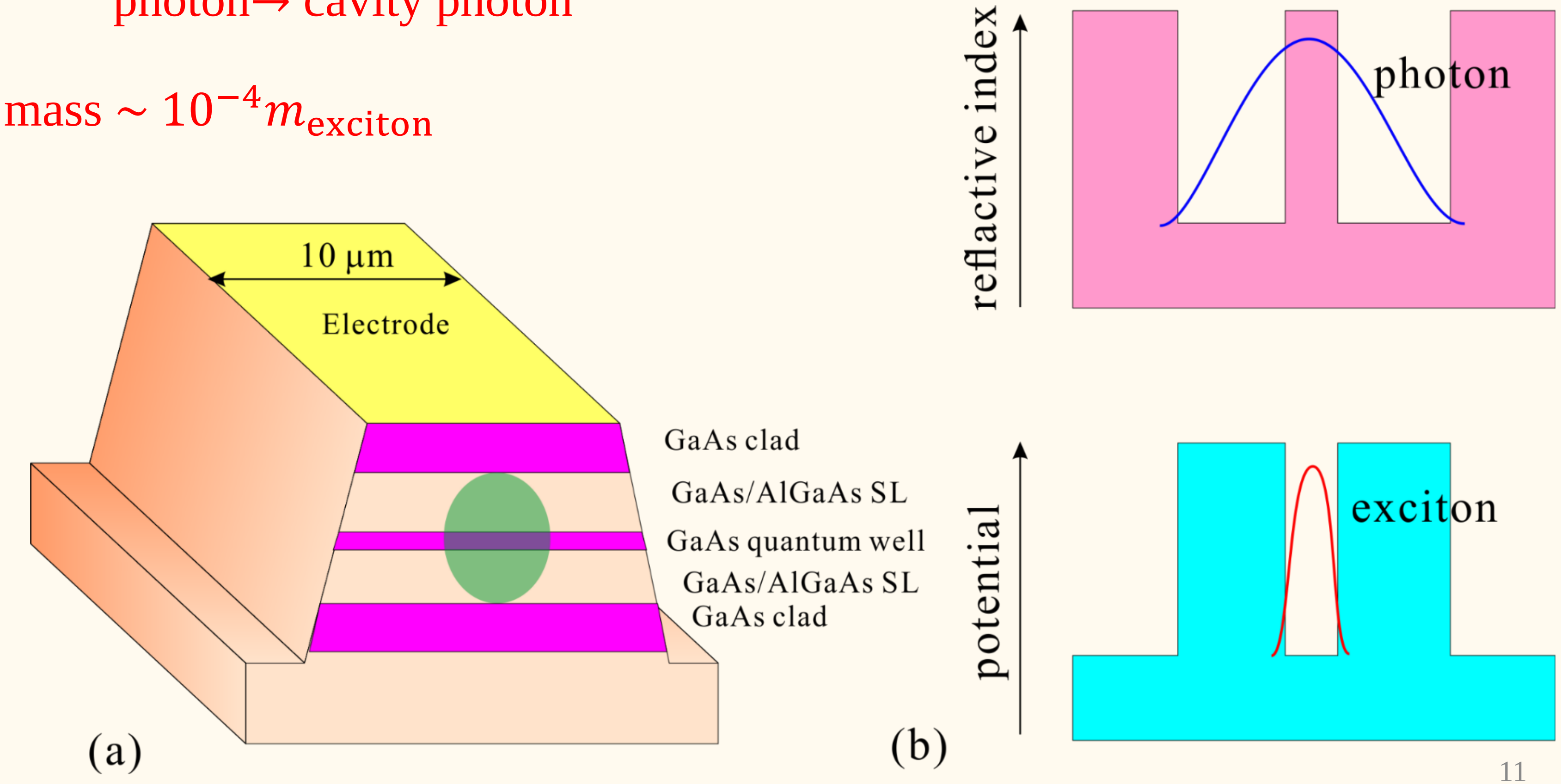
1 cycle ∼ few fs

coherent propagation in solids

photon→ cavity photon

dispersion relation: light effective mass ∼ $10^{-4} m_{\mathrm{exciton}}$



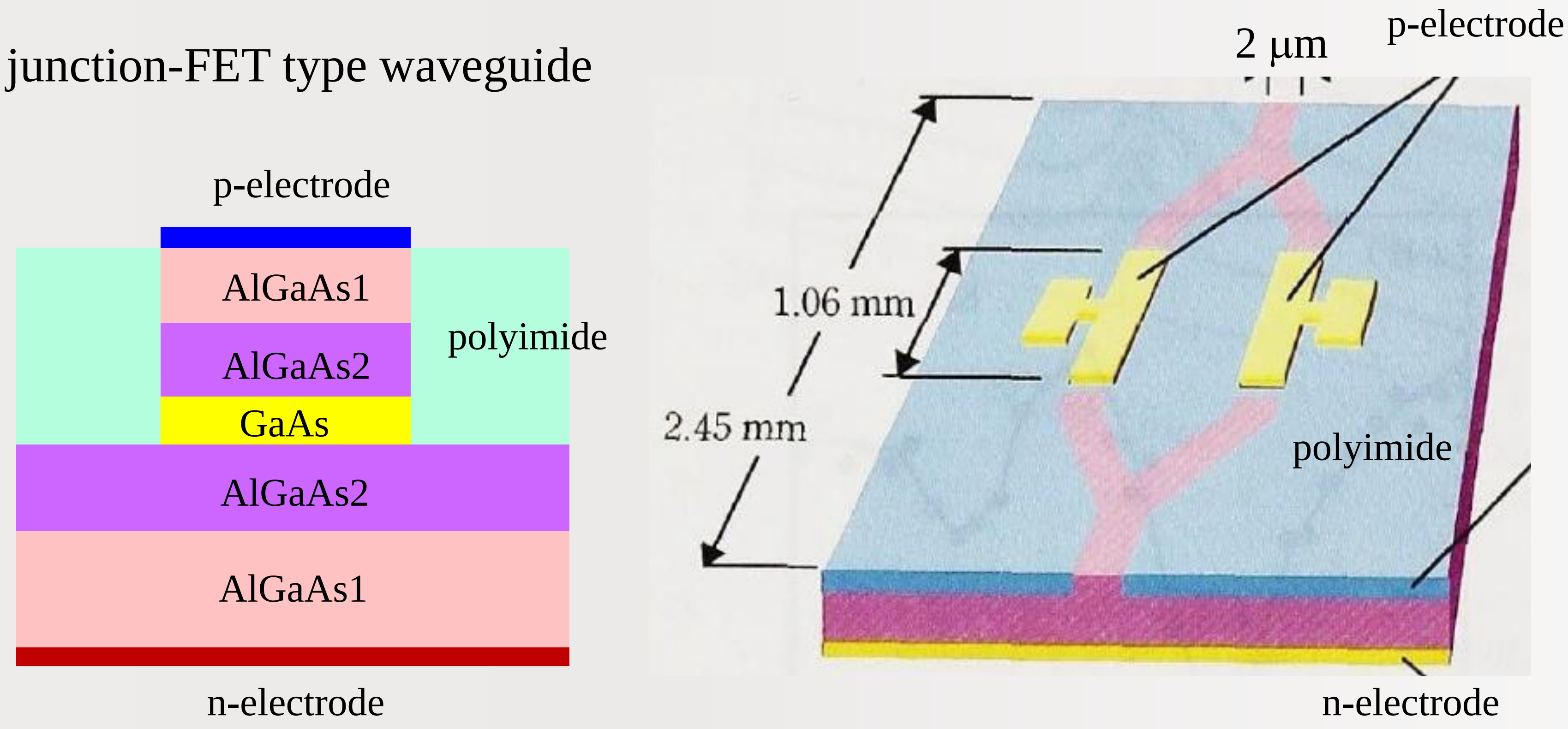


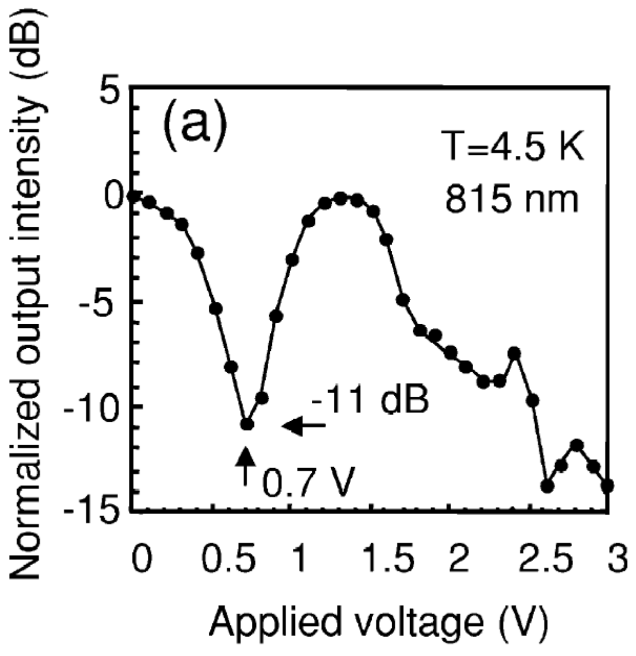
Mach-Zehnder interferometer (voltage-type)

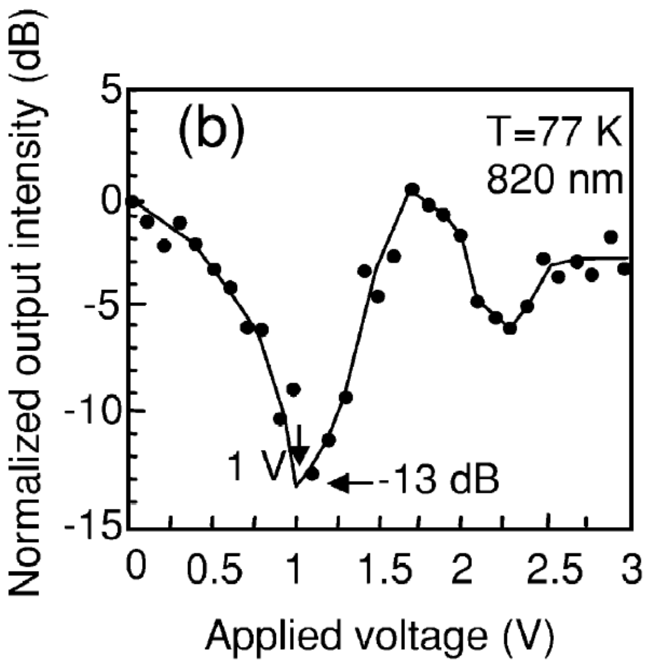
Kinetic phase shift with electric field: $\Delta\varphi = L\left[\frac{\sqrt{2mE_k}}{\hbar} - \frac{\sqrt{2m(E_k - \delta E)}}{\hbar}\right]$

δE: energy shift due to the depletion of quantum well

junction-FET type waveguide







Katsuyama, Hosomi, Micro. Eng. **63**, 23 (2002).

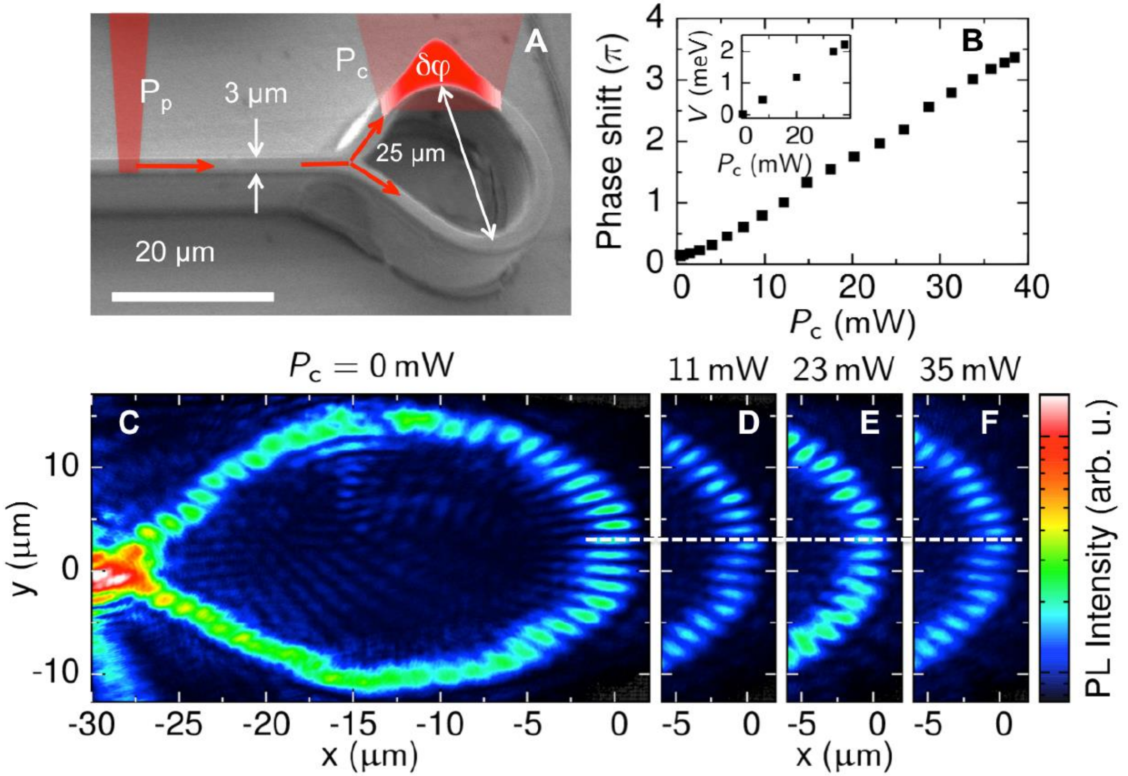
Voltage control of optical output through interference

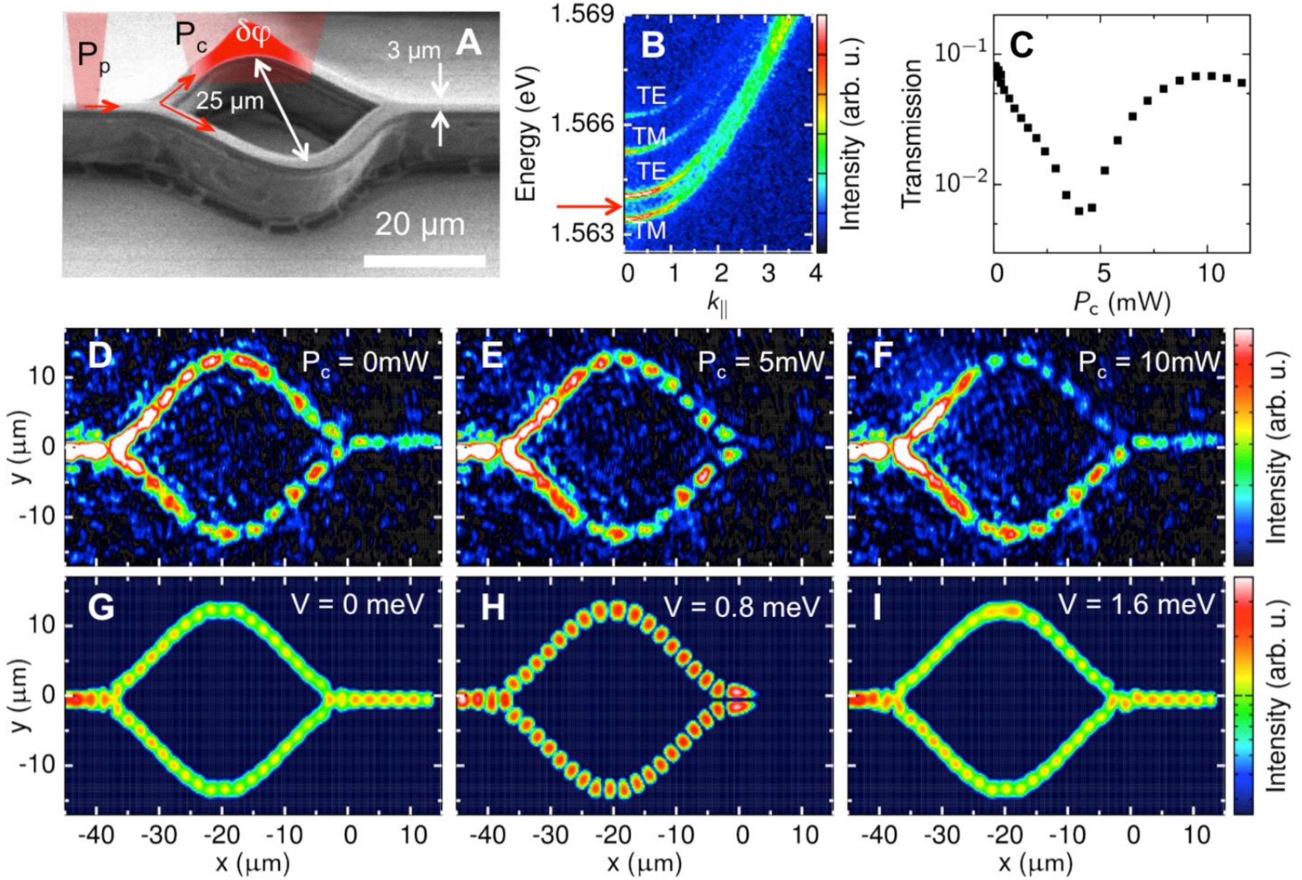
Mach-Zehnder interferometer 2 (optical control)

Kinetic phase shift with electric field: $\Delta\varphi = L\left[\frac{\sqrt{2mE_k}}{\hbar} - \frac{\sqrt{2m(E_k-\delta E)}}{\hbar}\right]$

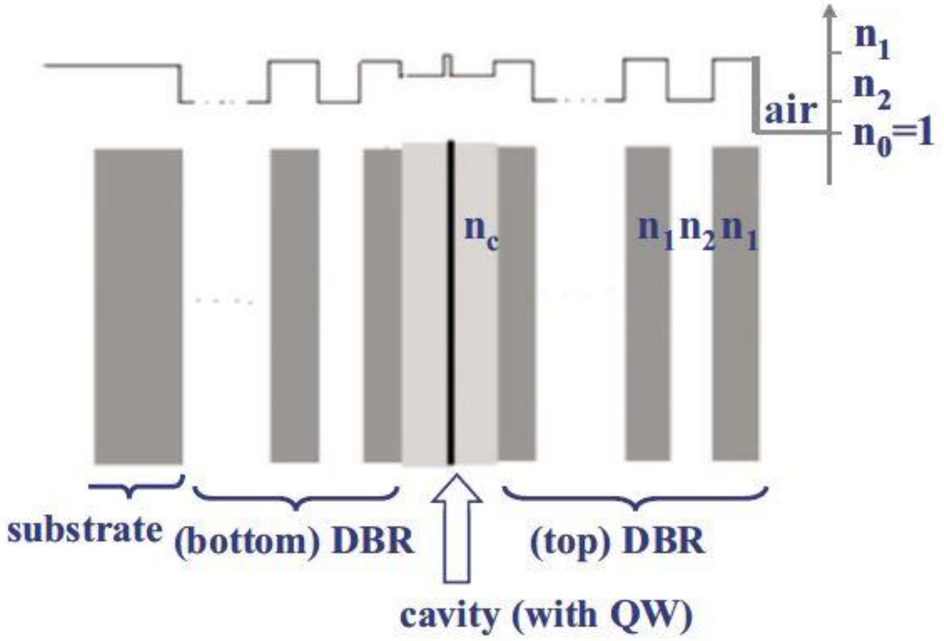
δE: energy shift due to the barrier by optically excited carriers (quasi-Fermi levels)

Sturm *et al.*, Nature Comm. **5**, 3278 (2014)

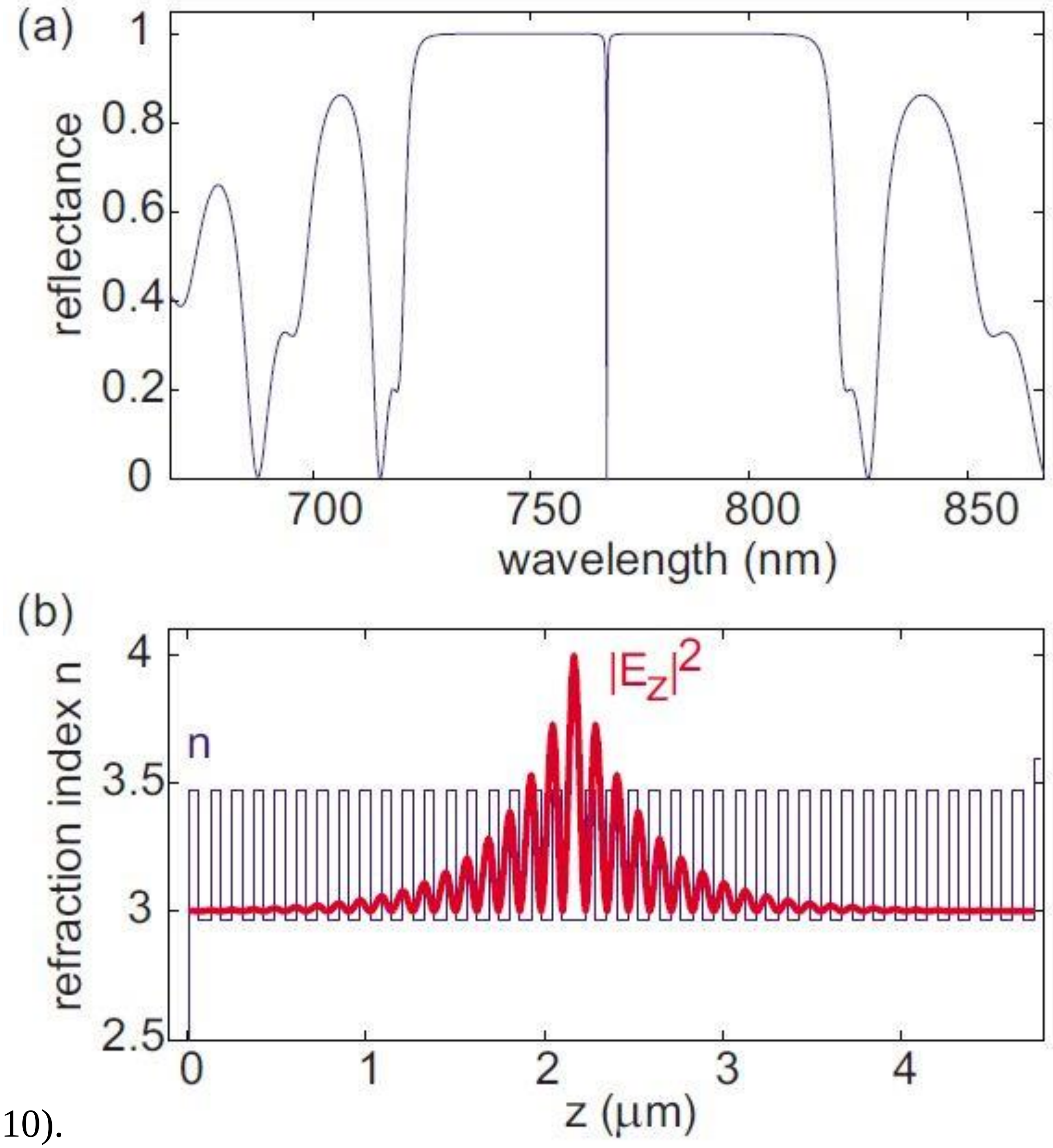




Exciton-polariton condensation

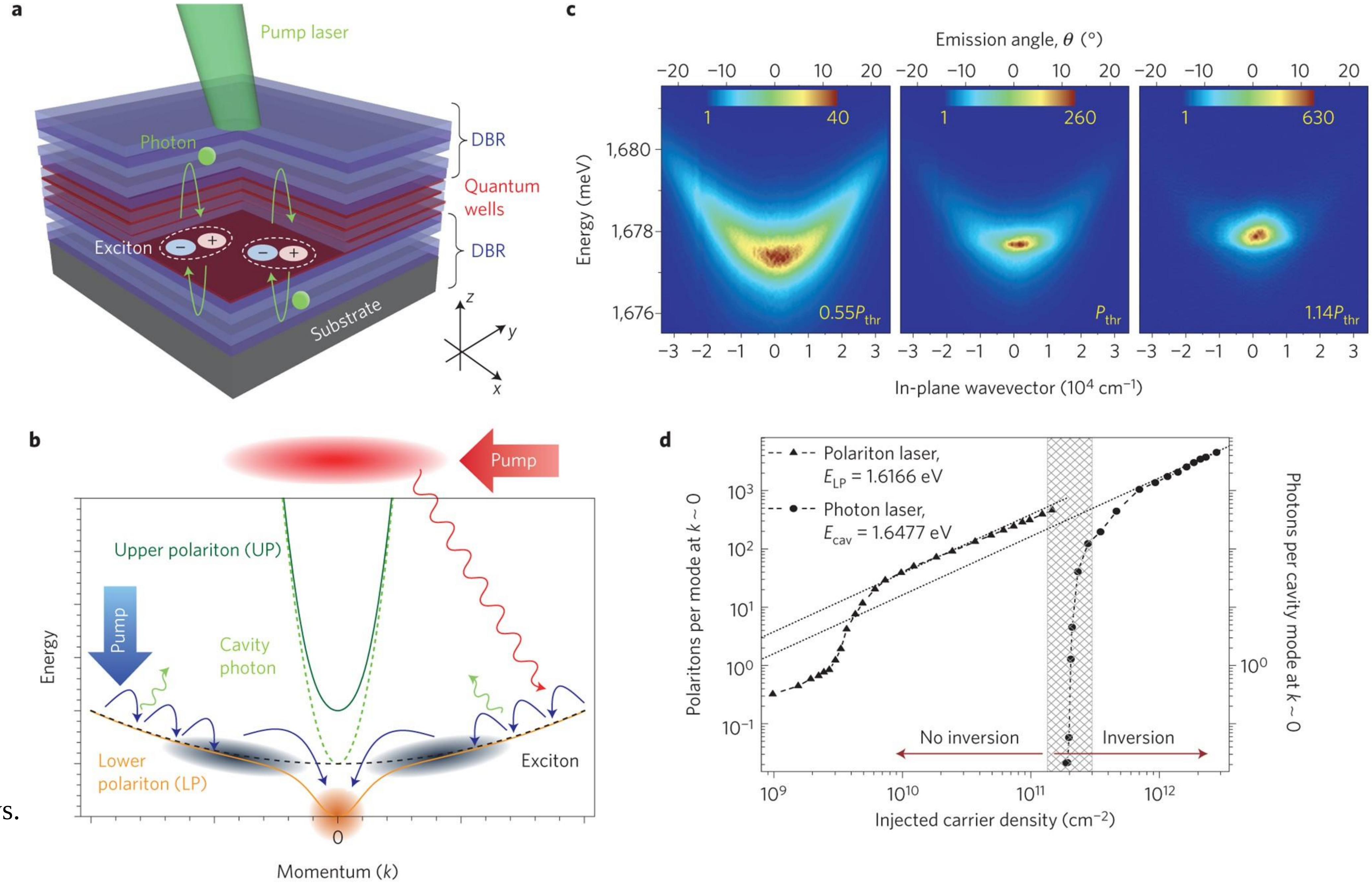


$$T = \frac{(1-R_1)(1-R_2)}{[1-\sqrt{R_1 R_2}]^2 + 4\sqrt{R_1 R_2}\sin^2(\phi/2)}$$



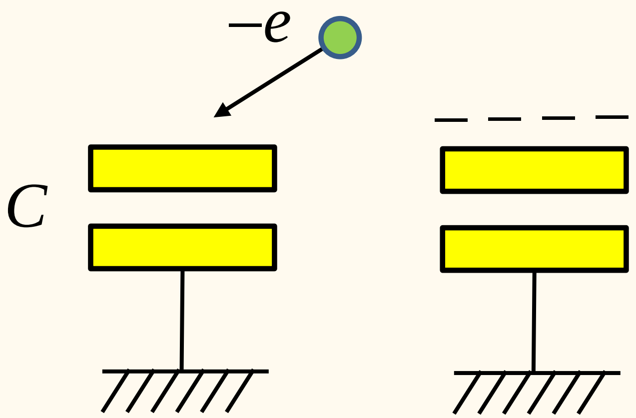
Deng, Haug, Yamamoto, Rev. Mod. Phys. **82**, 1489 (2010).

Exciton-polariton condensation2



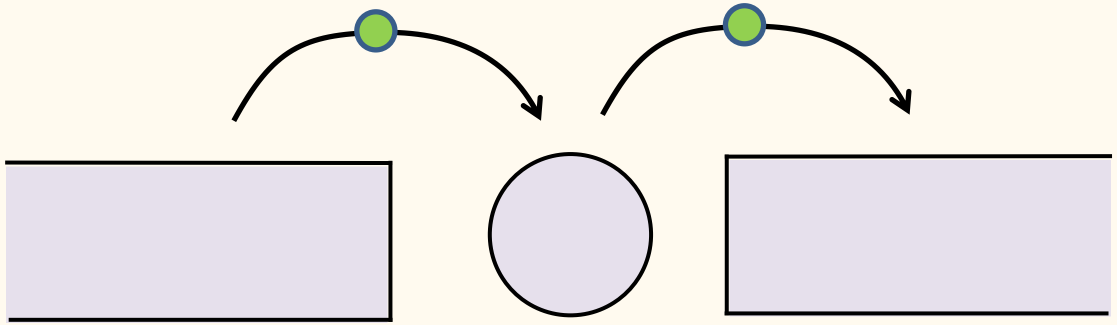
Byrnes, Kim, Yamamoto, Nat. Phys. **10**, 803 (2014),

Single electron effect

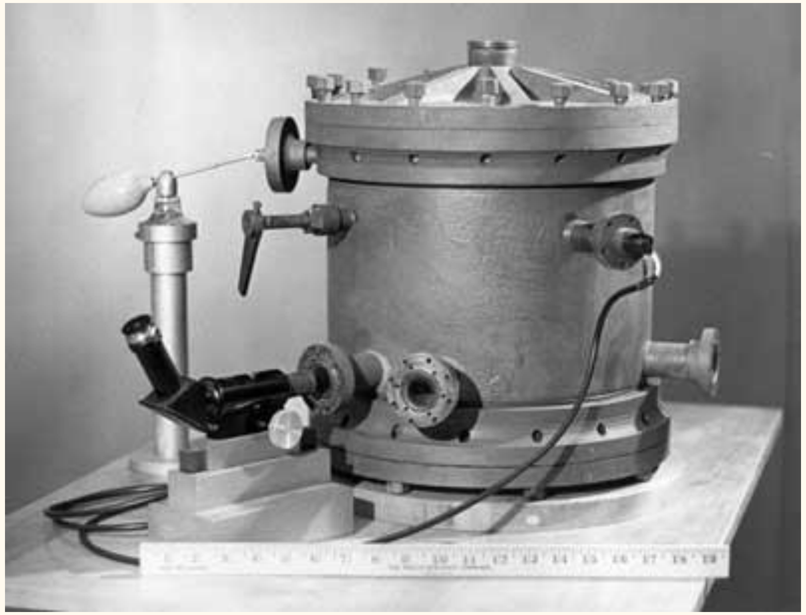


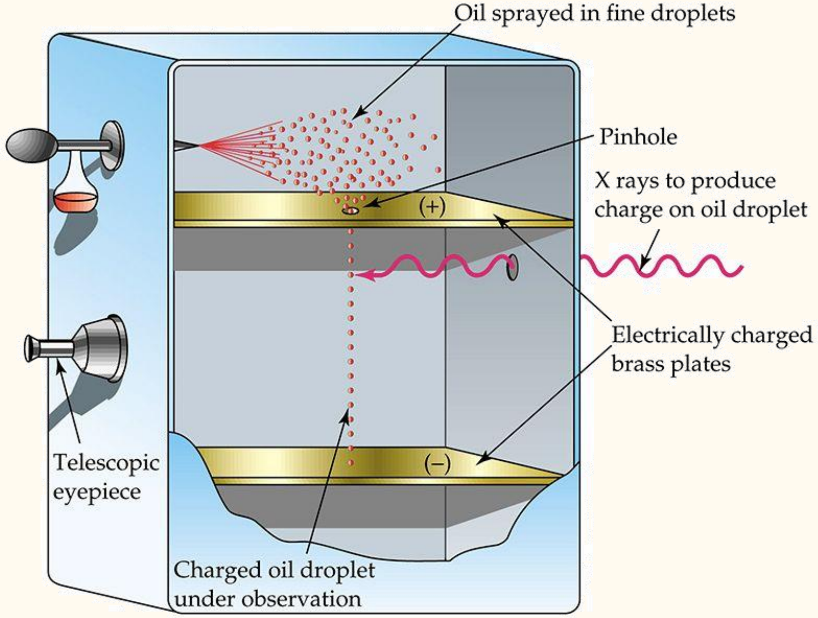
$$E_c = \frac{e^2}{2C} > k_B T$$

Coulomb blockade

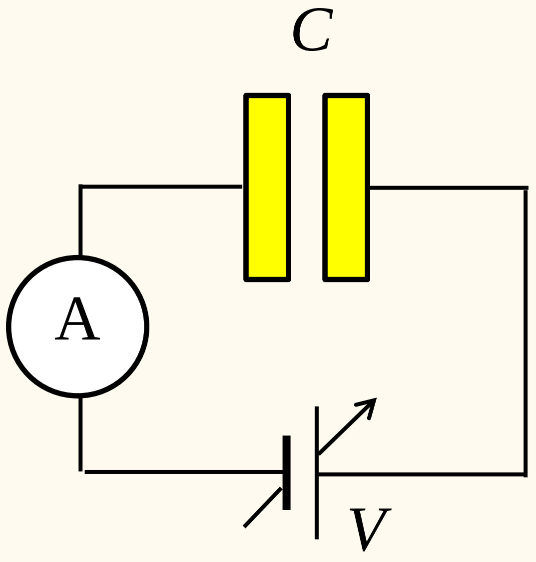


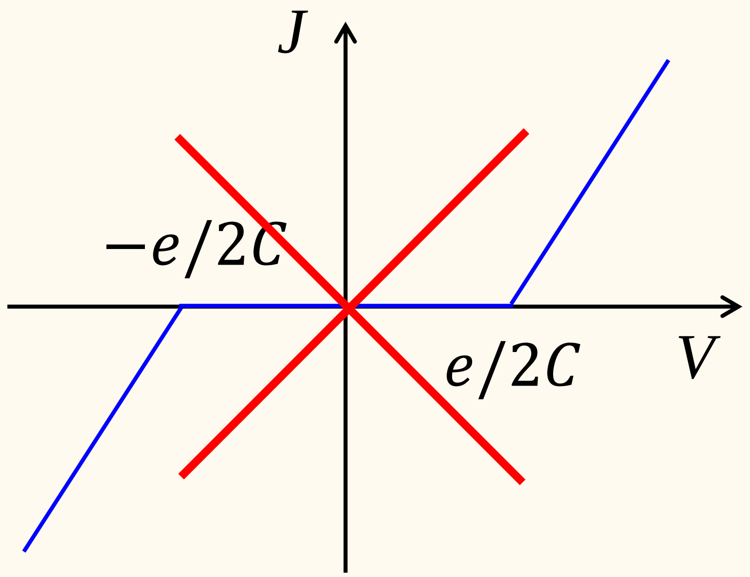
Millikan's oil droplet experiment





Role of power sources





Power sources: Automatically supply energy.

Energy → Enthalpy $H = U - PV$

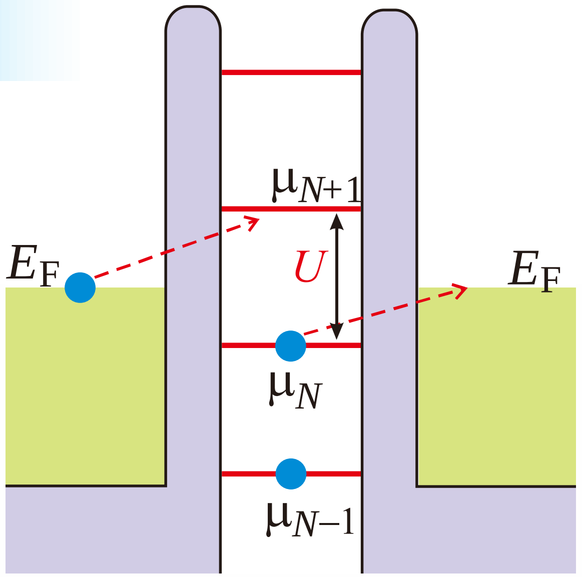
Constant interaction model, capacitor model

Constant interaction: U

Electron number: N

Interaction energy
$$E_{cN} = {}_N C_2 U = \frac{N(N-1)U}{2} = \frac{U(N-1/2)^2}{2} - \frac{U}{8}$$

Chemical potential
$$\Delta E_+(N) = (N-1)U$$

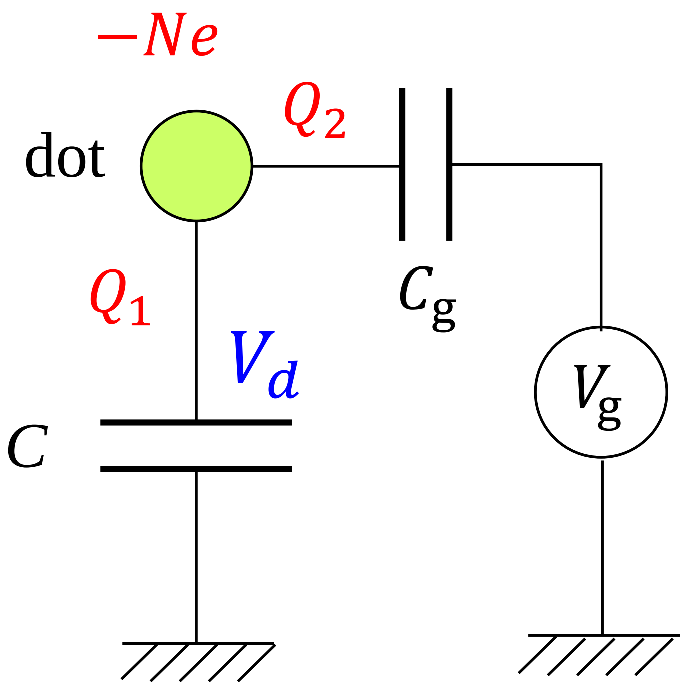


Charge relations
$$Q_1 + Q_2 = -eN, \quad Q_1 = CV_\mathrm{d},$$
$$Q_2 = C_\mathrm{g}(V_\mathrm{d} - V_\mathrm{g})$$

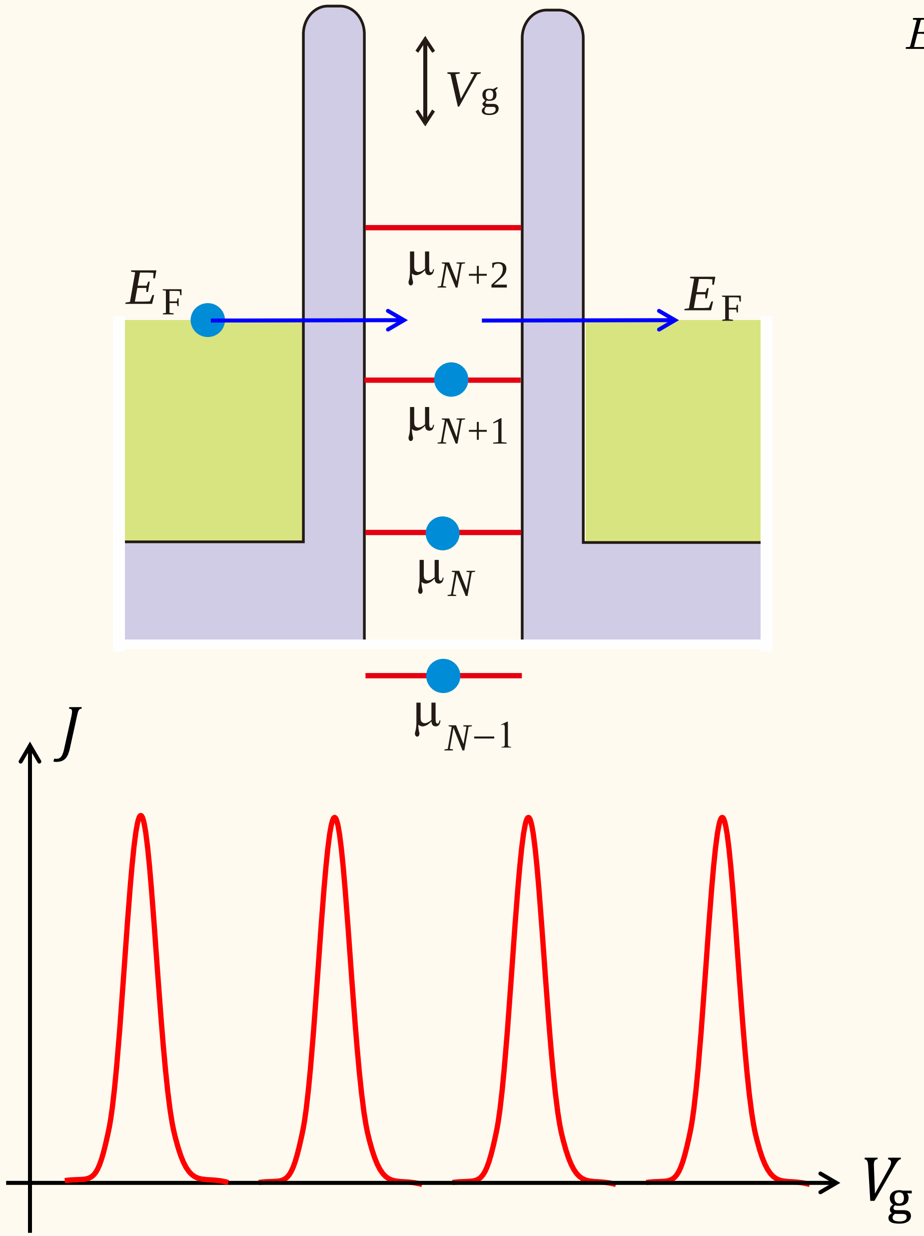
Electrostatic energy
$$E = \frac{1}{2}CV_\mathrm{d}^2 + \frac{1}{2}C_\mathrm{g}(V_\mathrm{d} - V_\mathrm{g})^2$$

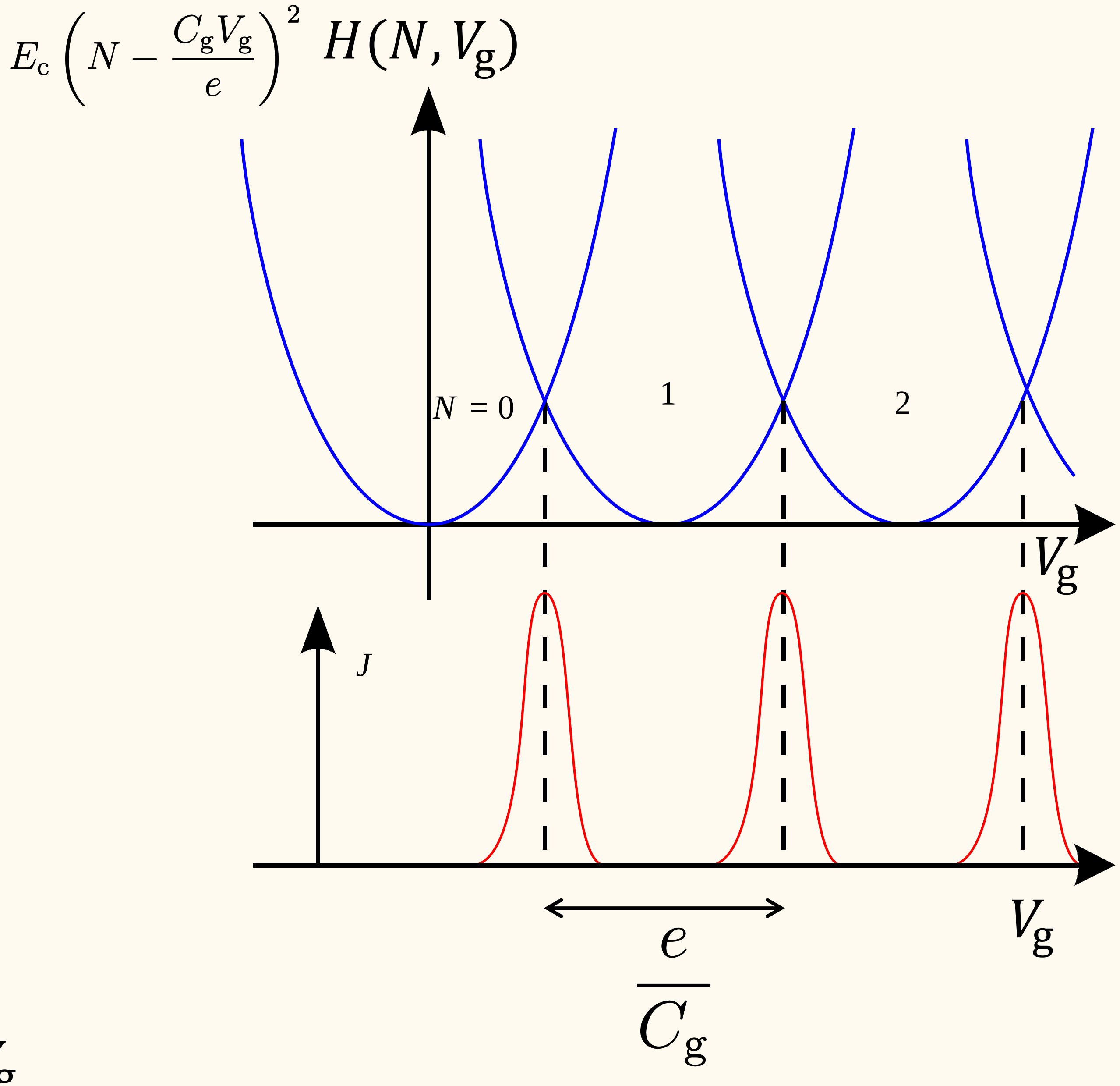
Enthalpy
$$H(N, V_\mathrm{g}) = \frac{(Ne - C_\mathrm{g}V_\mathrm{g})^2}{2(C + C_\mathrm{g})} \equiv \frac{(Ne - C_\mathrm{g}V_\mathrm{g})^2}{2C_\mathrm{s}}$$

Chemical potential
$$\mu_N \approx \frac{dH}{dN} = \frac{e(Ne - C_\mathrm{g}V_\mathrm{g})}{C_\mathrm{s}} = 2E_\mathrm{c}\left(N - \frac{C_\mathrm{g}V_\mathrm{g}}{e}\right)$$

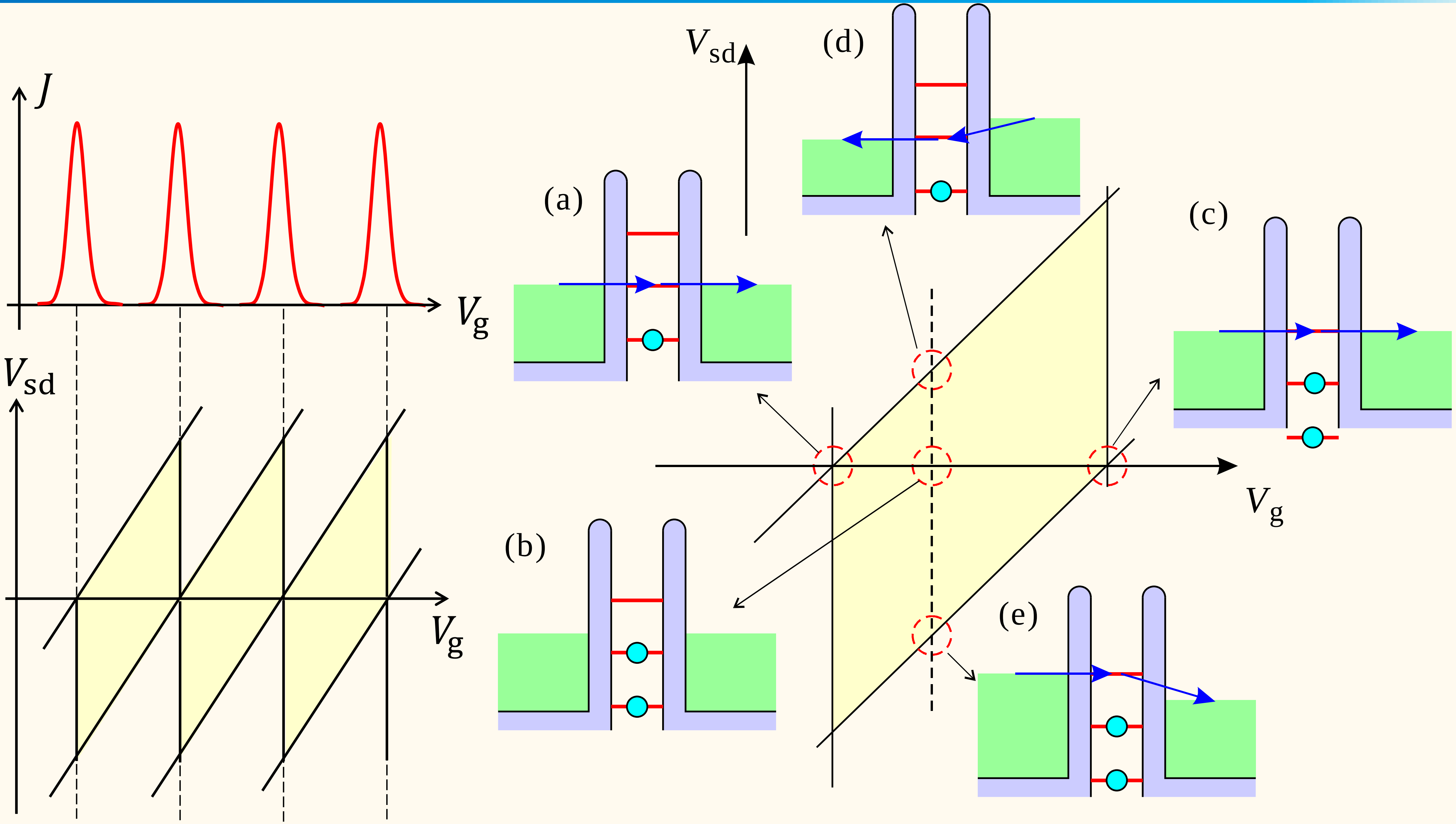


Coulomb oscillation

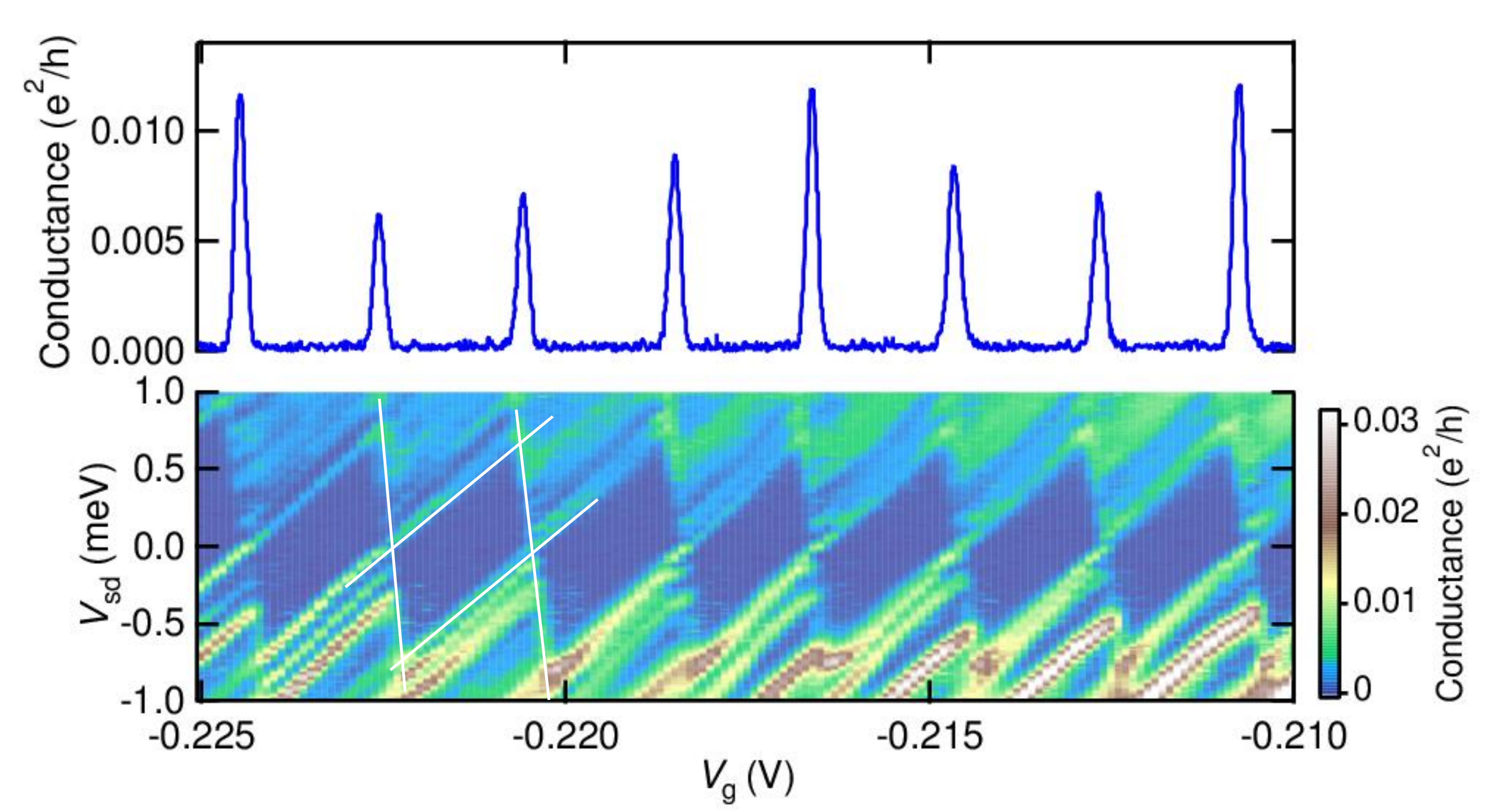




Coulomb diamond

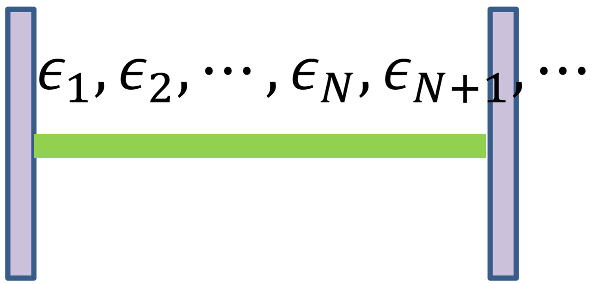


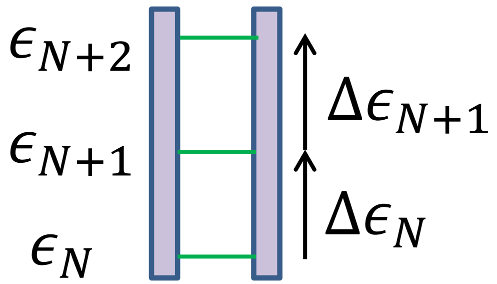
Coulomb oscillation and diamonds

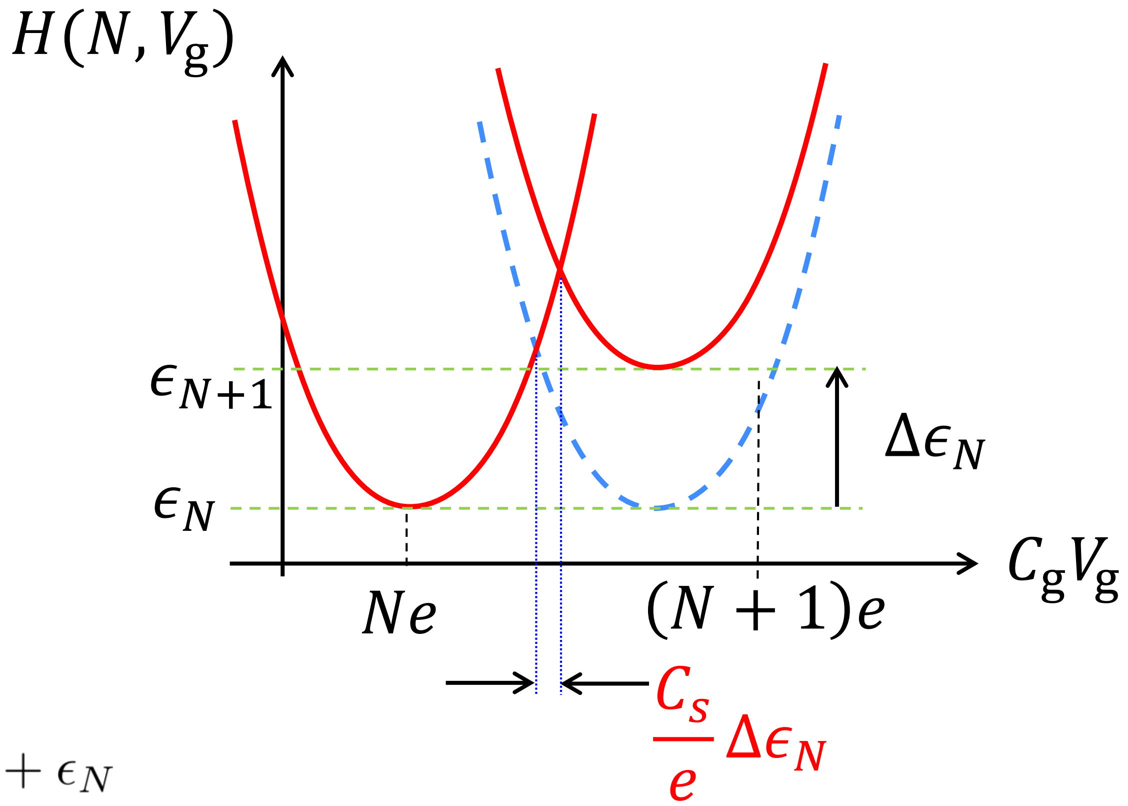


Quantum confinement

Zero-dimensional confinement to a quantum dot gives shifts in Coulomb peak positions.







Enthalpy shift by quantum confinement

$$H(N) = \frac{(Ne - C_g V_g)^2}{2C_s} + \epsilon_N$$

Chemical potential shift

$$\Delta H(N, N+1) = H(N+1) - H(N)$$

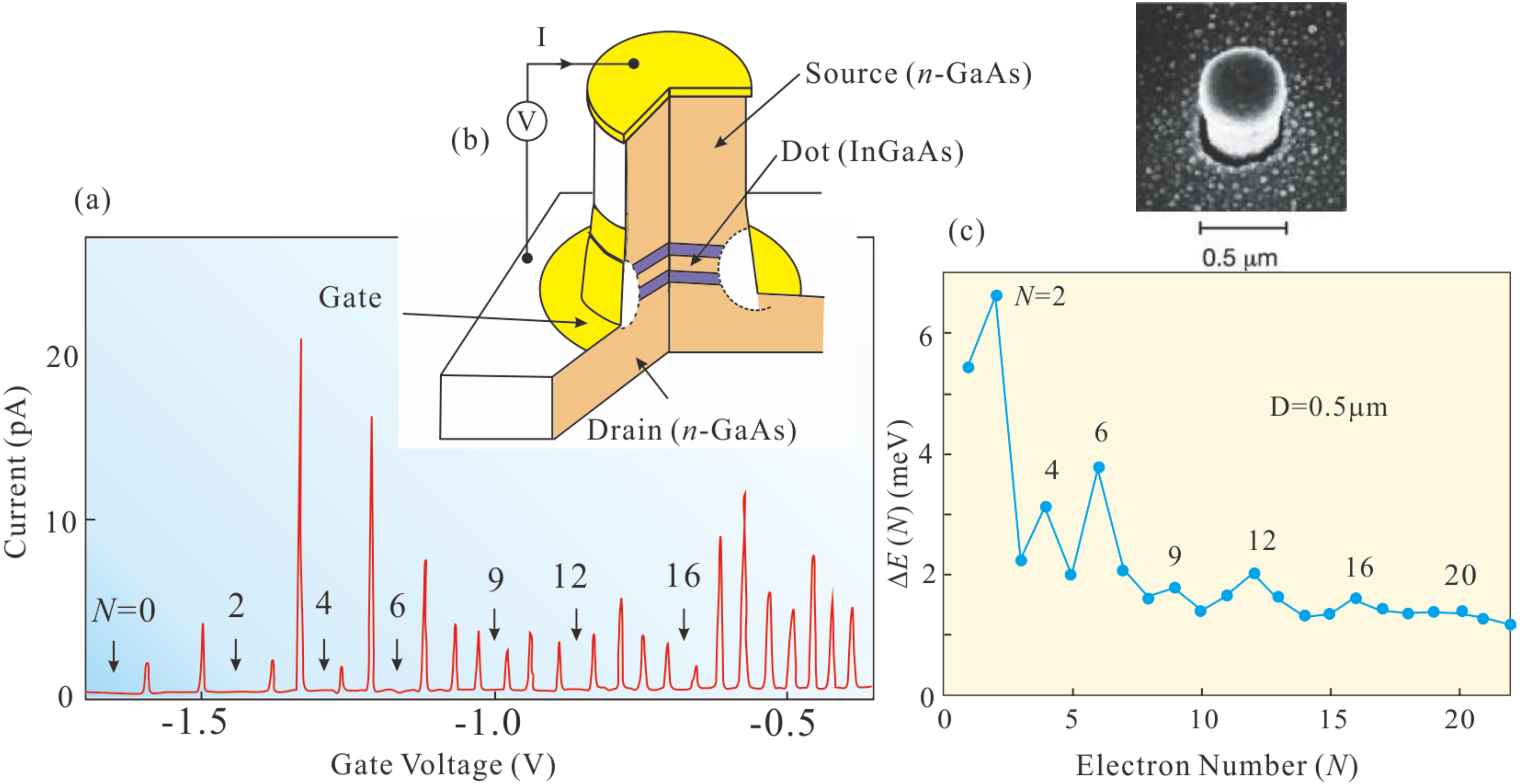
$$= \frac{e}{C_s}\left\{\left(N + \frac{1}{2}\right)e - C_g V_g\right\} + \Delta\epsilon_N$$

$$\Delta\epsilon_N \equiv \epsilon_{N+1} - \epsilon_N$$

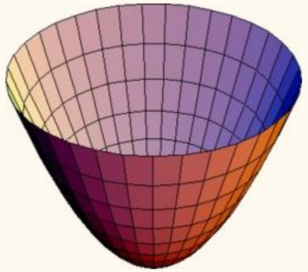
Shift in gate voltage

$$V_{gX}(N, N+1) = \frac{1}{C_g}\left\{\left(N + \frac{1}{2}\right)e + \frac{C_s}{e}\Delta\epsilon_N\right\}$$

Quantum confinement effect in a vertical quantum dot



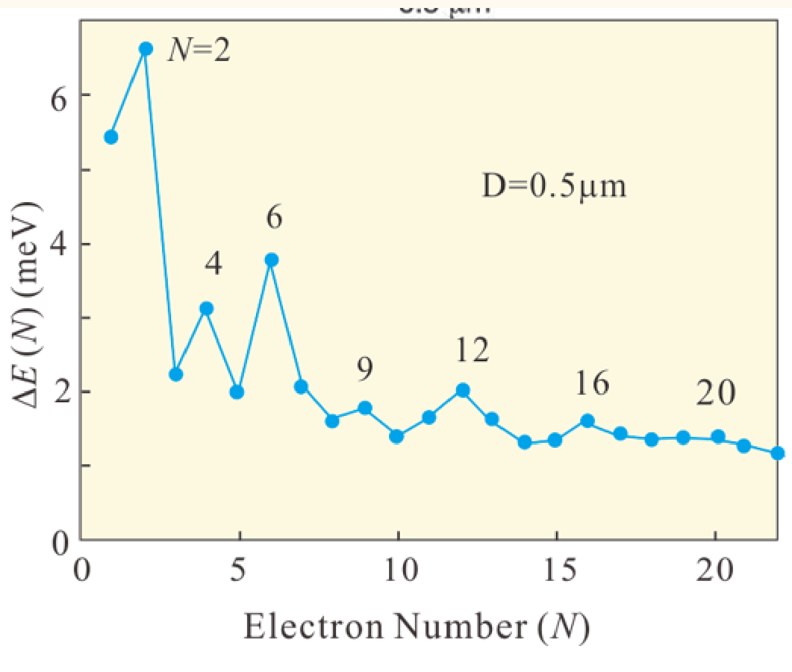
Two-dimensional harmonic potential

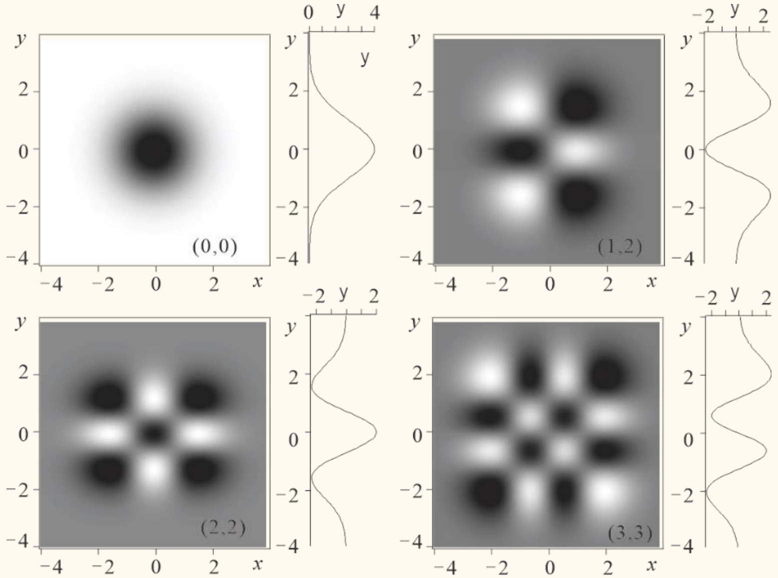


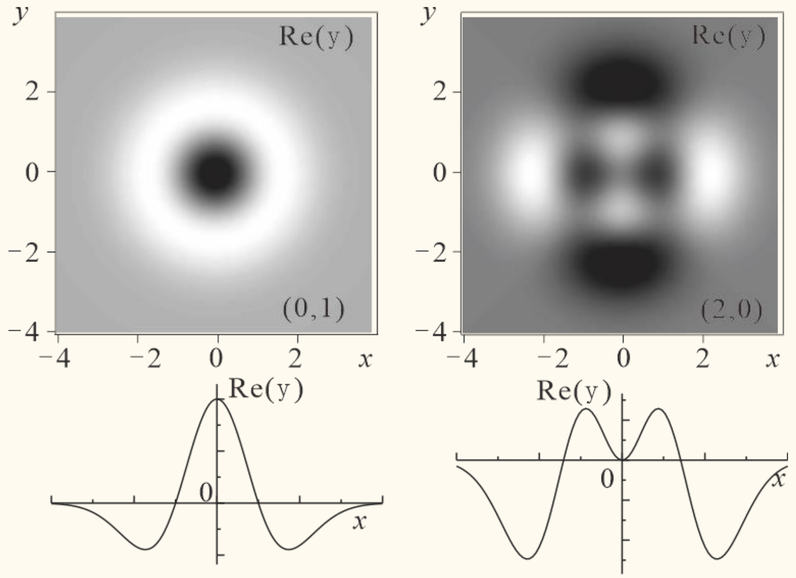
Potential shape: $V(x,y) = \frac{m\omega}{2}(x^2+y^2)$

Easy solutions from 1d harmonic potential $\psi_{n_x n_y} = A\exp\left[-\frac{m\omega(x^2+y^2)}{2\hbar}\right] H_{n_x}\left[\sqrt{\frac{m\omega}{\hbar}}x\right] H_{n_y}\left[\sqrt{\frac{m\omega}{\hbar}}y\right]$

Eigen energies: $E(n_x,n_y) = (n_x+n_y+1)\hbar\omega = (n_t+1)\hbar\omega \quad n_x+n_y \equiv n_t = 0,1,2,\cdots$







For fixed n_t $\quad n_x = 0,1,2,\cdots,n_t$ $\qquad n_t+1$ degeneracy

With spin degeneracy: 2, 4, 6, 8, ⋯

$N = 2, 6, 12, 20, \cdots, (n+1)(n+2), \cdots$

Quantum dot in magnetic field

Hamiltonian with $\boldsymbol{B} = (0,0,B)$

$$\mathscr{H} = \frac{(\boldsymbol{p} + e\boldsymbol{A})^2}{2m} + \frac{m}{2}\omega^2(x^2 + y^2) \quad \boldsymbol{A} = \left(-\frac{By}{2}, \frac{Bx}{2}, 0\right)$$

Expansion of the kinetic energy term

$$\frac{(\boldsymbol{p} + e\boldsymbol{A})^2}{2m} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \frac{ie\hbar B}{2m}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) + \frac{e^2B^2}{8m}(x^2 + y^2)$$

Definition of cyclotron frequency and composite harmonic confinement potential frequency

$$\omega_c = \frac{eB}{m} \quad \Omega \equiv \sqrt{\omega^2 + (\omega_c/2)^2}$$

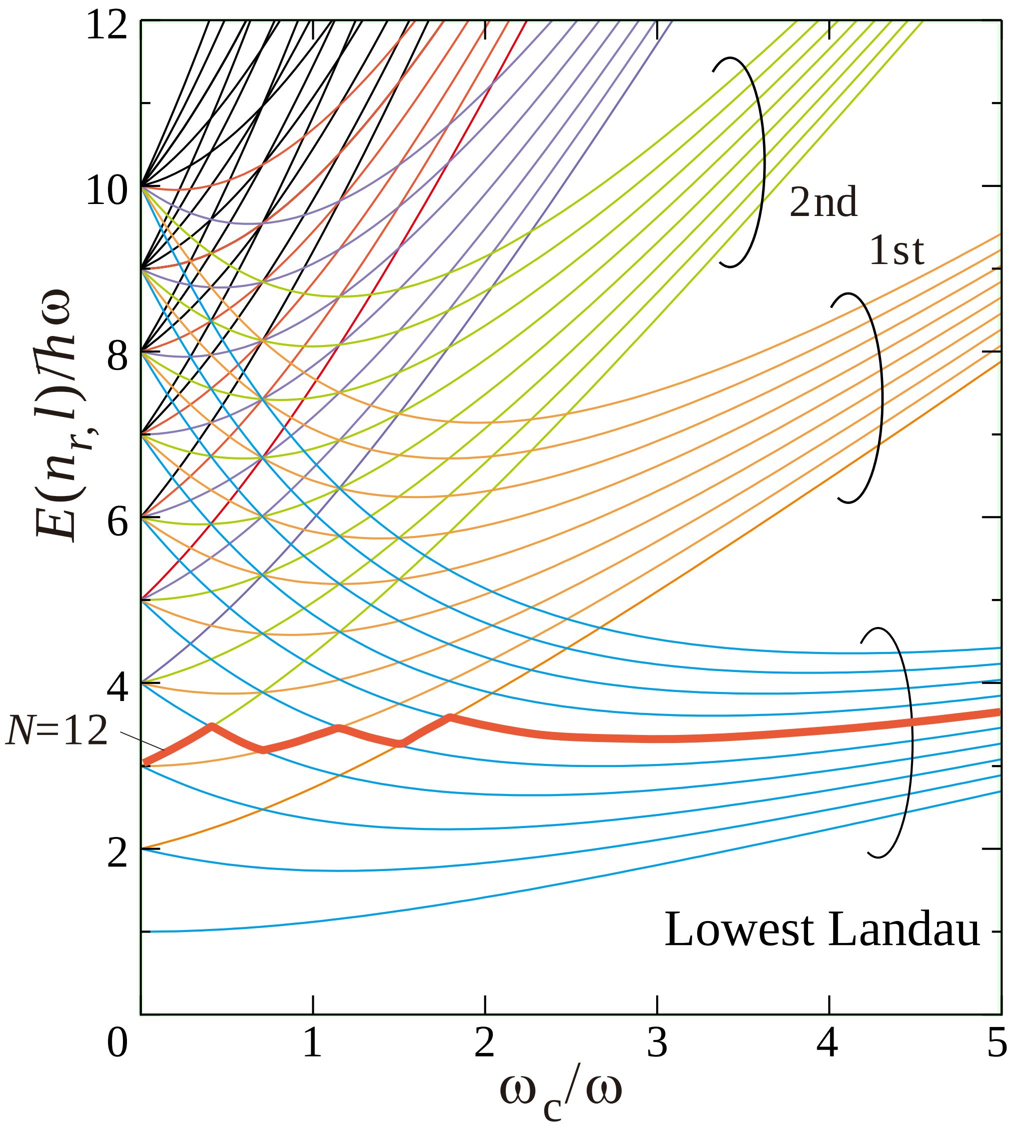
The Hamiltonian is rewritten as

$$\mathscr{H} = \frac{\hbar^2\nabla^2}{2m} + \frac{\Omega}{2m}(x^2 + y^2) + \frac{\omega_c \hat{L}_z}{2} = \mathscr{H}_\Omega + \frac{\omega_c \hat{L}_z}{2}$$

Fock-Darwin state eigen energies

$$E(n_r, l) = \hbar\Omega(2n_r + |l| + 1) + \hbar\omega_c l/2$$

Quantum dot in magnetic field



$$\mathscr{H} = \frac{(\boldsymbol{p} + e\boldsymbol{A})^2}{2m} + \frac{m}{2}\omega^2(x^2 + y^2) \quad \boldsymbol{A} = \left(-\frac{By}{2}, \frac{Bx}{2}, 0\right)$$

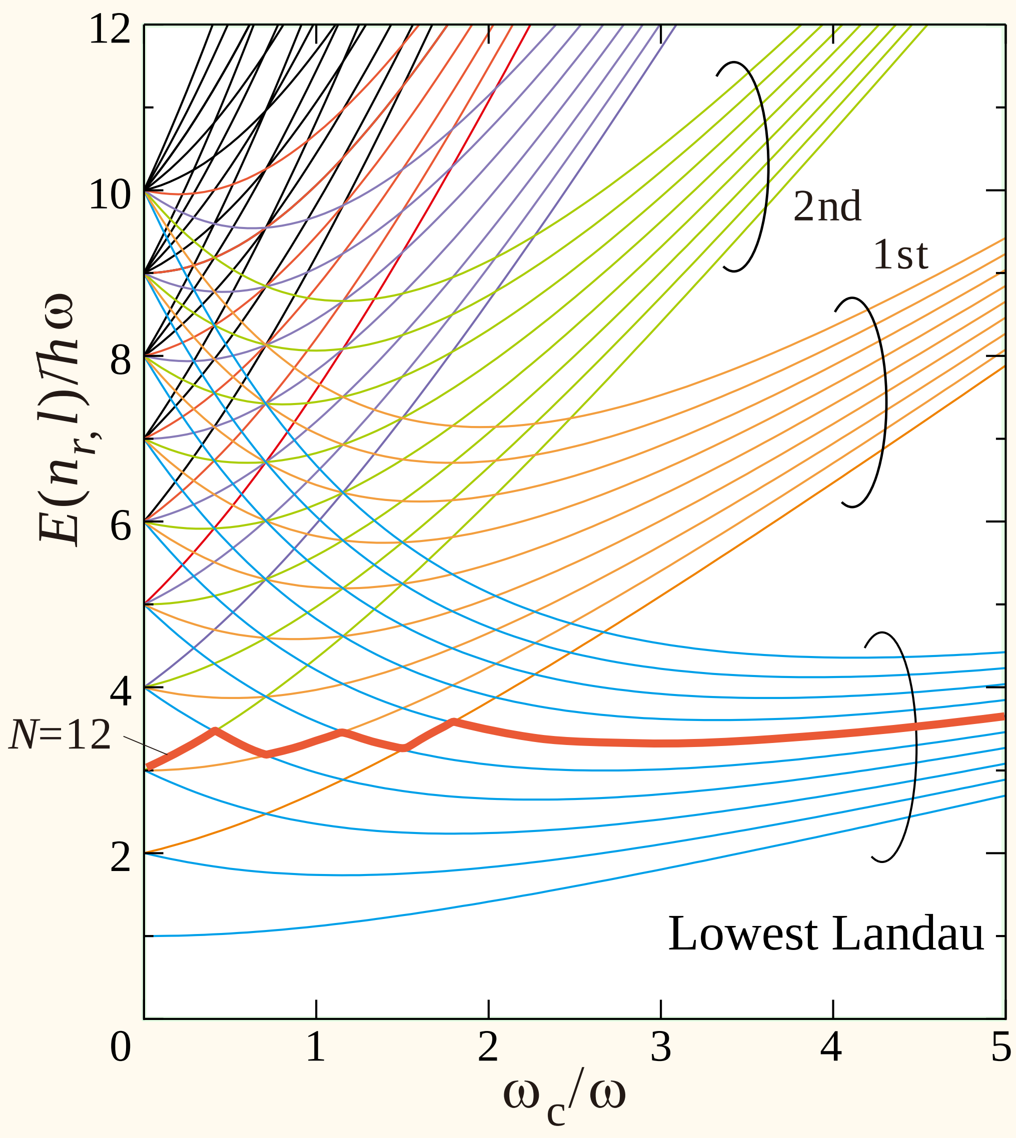
$$\frac{(\boldsymbol{p} + e\boldsymbol{A})^2}{2m} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \frac{ie\hbar B}{2m}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) + \frac{e^2B^2}{8m}(x^2 + y^2)$$

$$\omega_c = \frac{eB}{m} \quad \Omega \equiv \sqrt{\omega^2 + (\omega_c/2)^2}$$

$$\mathscr{H} = \frac{\hbar^2\nabla^2}{2m} + \frac{\Omega}{2m}(x^2 + y^2) + \frac{\omega_c \hat{L}_z}{2} = \mathscr{H}_\Omega + \frac{\omega_c \hat{L}_z}{2}$$

$$E(n_r, l) = \hbar\Omega(2n_r + |l| + 1) + \hbar\omega_c l/2$$

Fock-Darwin states



Level crossing points

$$\left(\frac{\omega_c}{\omega}\right)^2 = n_L - 2 + \frac{1}{n_L}$$

n_L: Landau index
= 1, 2, ….

• : Solutions