Lecture on

Semiconductors / 半導体

(Physics of semiconductors)

2021.7.14 Lecture 14

10:25 – 11:55

Institute for Solid State Physics, University of Tokyo

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What we have seen

Semiconductor basics

- Band structure
- Effective mass approximation
- Carrier statistics
- Electron-photon couplings
- Thermodynamics
- Semi-classical transport (Boltzmann equation)

Spatial modulation basics

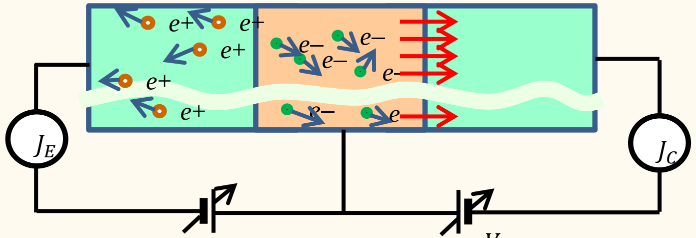
- Modulation doping: pn-junctions
- Schottky junctions, MOS junctions
- Hetero-junctions
- Quantum confinement
- Quantum wells, wires and dots
- Minority carrier confinement

Quantum physics in semiconductors

- Fermion transport: Landauer (-Büttiker) formalism
- T-matrix, S-matrix
- Boson transport, Bose-Einstein condensation
- Quantum dots: Single electron effect, quantum confinement
- Quantum Hall: Edge mode, topological number

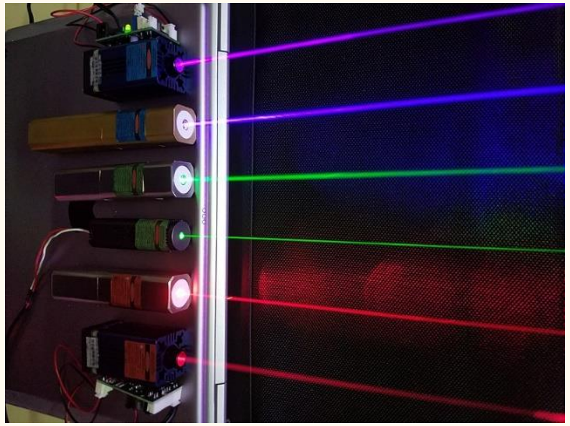
Part of topics

Charge (kinetic) freedom

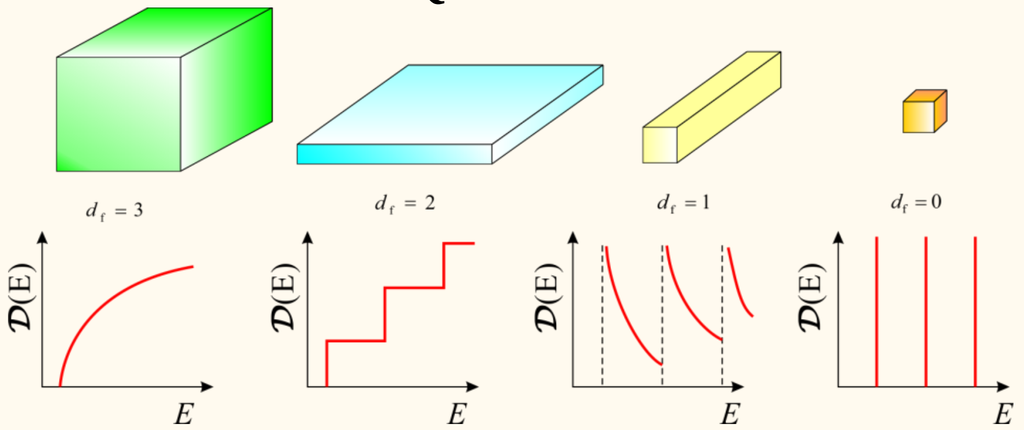


Semiclassical transport

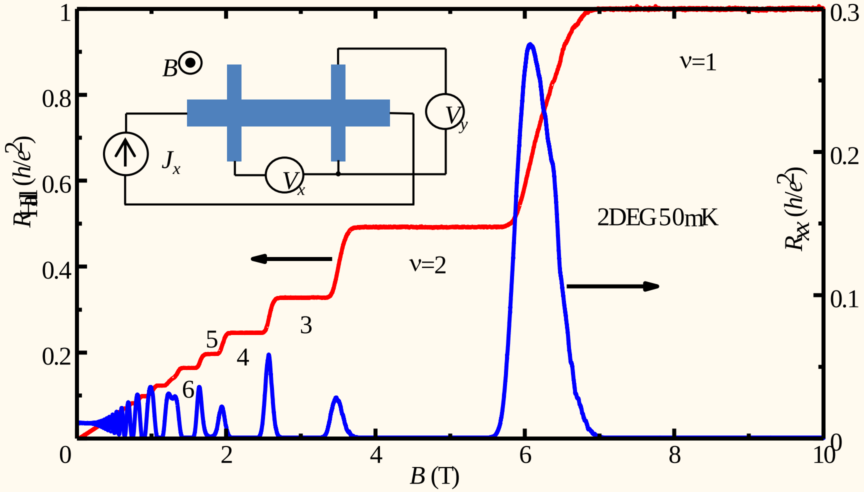
Laser diode



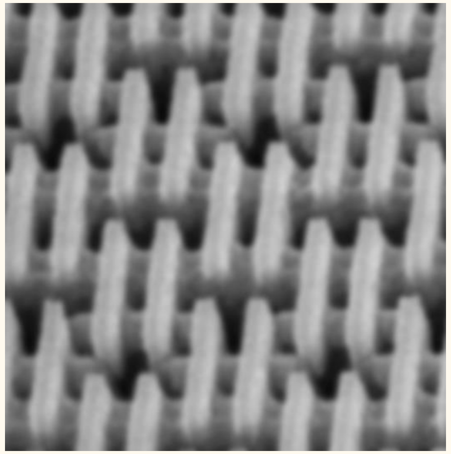
Quantum confinement



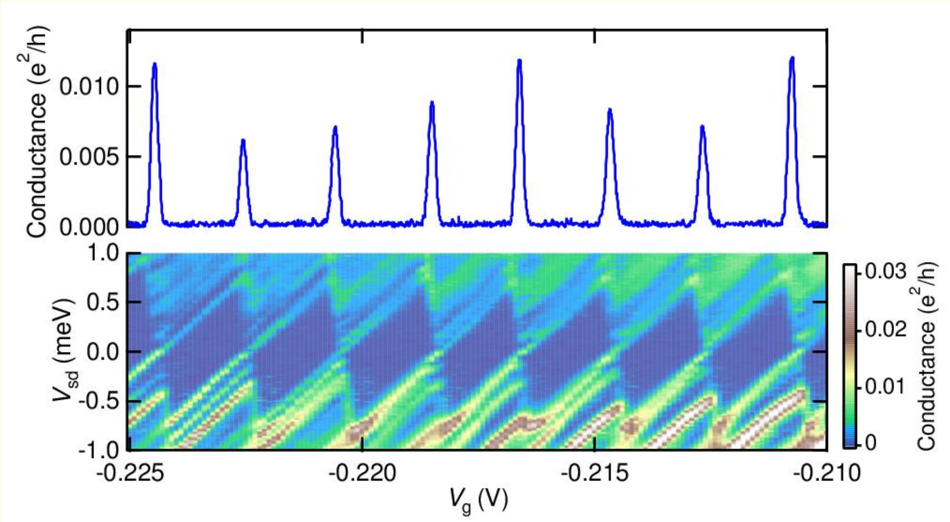
Quantum wells, wires, dots



Quantum Hall and topology in solid state physics



Si technology: FinFET



Quantum dot: single electron, quantum confinement

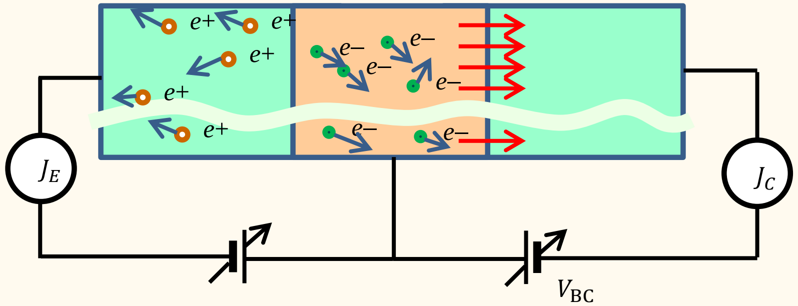
Chapter 10a Spintronics I

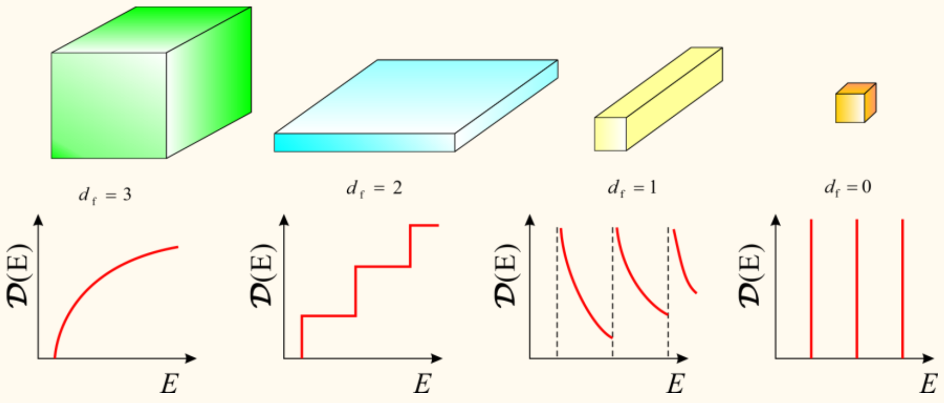
Two current model

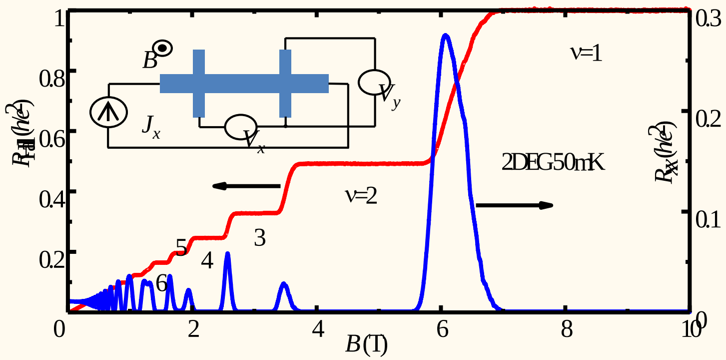
Spin injection

Spin degree of freedom: A new paradigm

Charge (kinetic) freedom







Spin degree of freedom

Giant magnetoresistance
spin valve

Spin injection

Spin-manipulation of
quantum information

Topological insulators

Nobel laureates

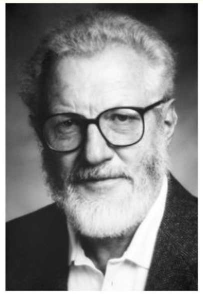

Photo from the Nobel Foundation archive.
William Bradford Shockley


John Bardeen
Prize share: 1/3


Photo from the Nobel Foundation archive.
Walter Houser Brattain

1956


Photo from the Nobel Foundation archive.
Zhores I. Alferov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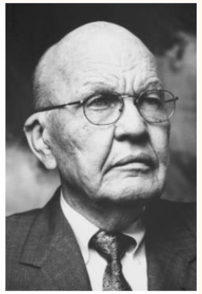
Herbert Kroemer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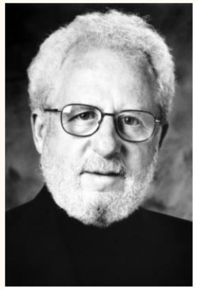
Jack S. Kilby

2000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Alan J. Heeger


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Hideki Shirakawa

2000 (Chemistry)


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Leo Esaki

1973


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Klaus von Klitzing

1985


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2010


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2016


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1998


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Shuji Nakamura

2014

Charge (kinetic) freedom

Spin degree of freedom


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Albert Fert


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Peter Grünberg

2007

The two current model

Nevill Mott
1905-1996

Divide a current to the one with ↑ spin and the one with ↓ spin.

$$\sigma = \sigma_\uparrow + \sigma_\downarrow, \quad \frac{1}{\rho} = \frac{1}{\rho_\uparrow} + \frac{1}{\rho_\downarrow}$$

Drude: $$\sigma_s = \frac{e^2 n_s \tau_s}{m_s^*} \quad (s = \uparrow, \downarrow)$$

Condition: spin diffusion length $\lambda_s \gg l$ mean free path (or other lengths)

Spin polarized current: $\boldsymbol{j}_{\rm p\uparrow} = \boldsymbol{j}_\uparrow - \boldsymbol{j}_\downarrow$

$$P_{\rm c} = \frac{|\boldsymbol{j}_\uparrow - \boldsymbol{j}_\downarrow|}{|\boldsymbol{j}_\uparrow + \boldsymbol{j}_\downarrow|} = \frac{j_{\rm p\uparrow(\downarrow)}}{j_{\rm c}}$$

$$\boldsymbol{j}_{\rm ps} = \boldsymbol{\sigma}_s \boldsymbol{E} - eD_s(-\nabla\delta n_s)$$

drift diffusion

Einstein relation for metals: $$\sigma_s = e^2 N_s(E_{\rm F}) D_s \quad (cf.\ \sigma = e^2 (n/k_{\rm B}T) D)$$

ϵ_s: local Fermi energy, $\delta\epsilon_s$: Shift from thermal equilibrium

$$\boldsymbol{j}_s = -\frac{\sigma_s}{e}\left[e\nabla\phi - \frac{D_s \nabla \delta n_s}{\sigma_s}\right] = \frac{\sigma_s}{e}[-e\nabla\phi + \nabla\delta\epsilon_s]$$

$\mu_s \equiv -e\phi + \epsilon_s$ Spin-dependent chemical potential $\boldsymbol{j}_s = -\dfrac{\sigma_s}{-e}\nabla\mu_s$

Spin current

Remember Boltzmann equation $$\frac{\partial f}{\partial t} + \frac{\boldsymbol{p}}{m^*} \cdot \frac{\partial f}{\partial \boldsymbol{r}} + \boldsymbol{F} \cdot \frac{\partial f}{\partial \boldsymbol{p}} = -\left(\frac{\partial f}{\partial t}\right)_c$$

Because spin carriers are dipoles it is difficult to apply forces (needs magnetic field gradient)→Diffusion current only

Spin current (simplest) definition $$\boldsymbol{j}^s(\boldsymbol{r}, t) = \frac{\hbar}{2(-e)}(\boldsymbol{j}_\uparrow - \boldsymbol{j}_\downarrow)$$

Angular momentum conservation $$\frac{\partial s_z}{\partial t} + \mathrm{div}\, \boldsymbol{j}^s = 0$$

With spin relaxation $$\frac{\partial s_z}{\partial t} + \mathrm{div}\, \boldsymbol{j}^s = \frac{\partial s_z}{\partial t} + \frac{\hbar}{2(-e)} \nabla \cdot (\boldsymbol{j}_\uparrow - \boldsymbol{j}_\downarrow) = \frac{\hbar}{2}\left(\frac{\delta n_\uparrow}{\tau_\uparrow} - \frac{\delta n_\downarrow}{\tau_\downarrow}\right)$$

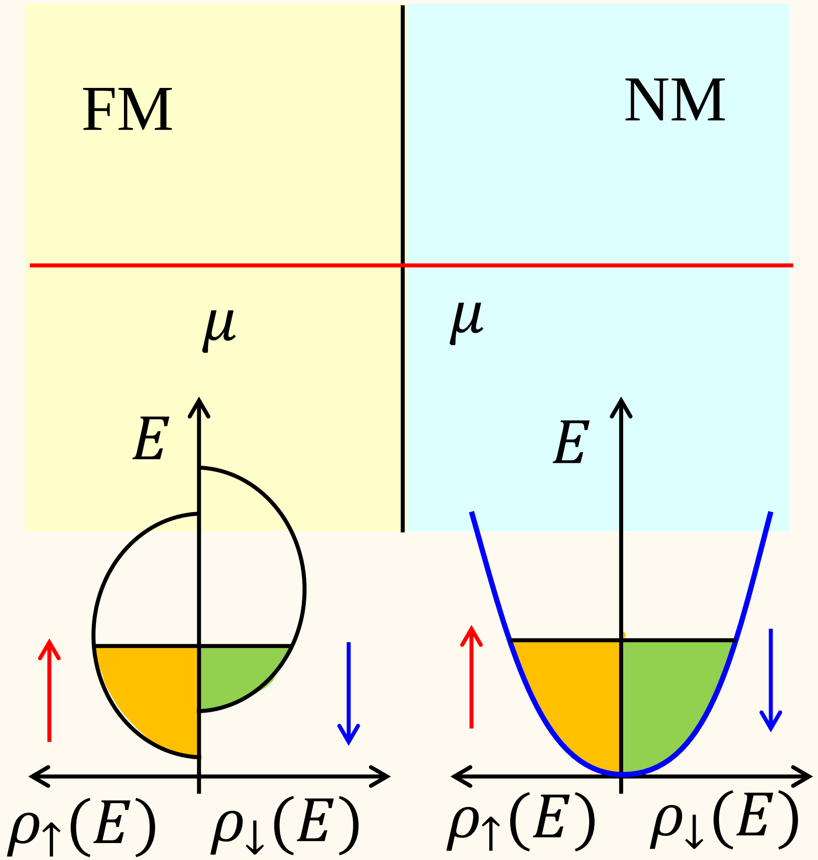
cf. Charge conservation $$\frac{\partial \rho}{\partial t} + \mathrm{div} \boldsymbol{j} = \frac{\partial \rho}{\partial t} + \nabla \cdot (\boldsymbol{j}_\uparrow + \boldsymbol{j}_\downarrow) = 0$$

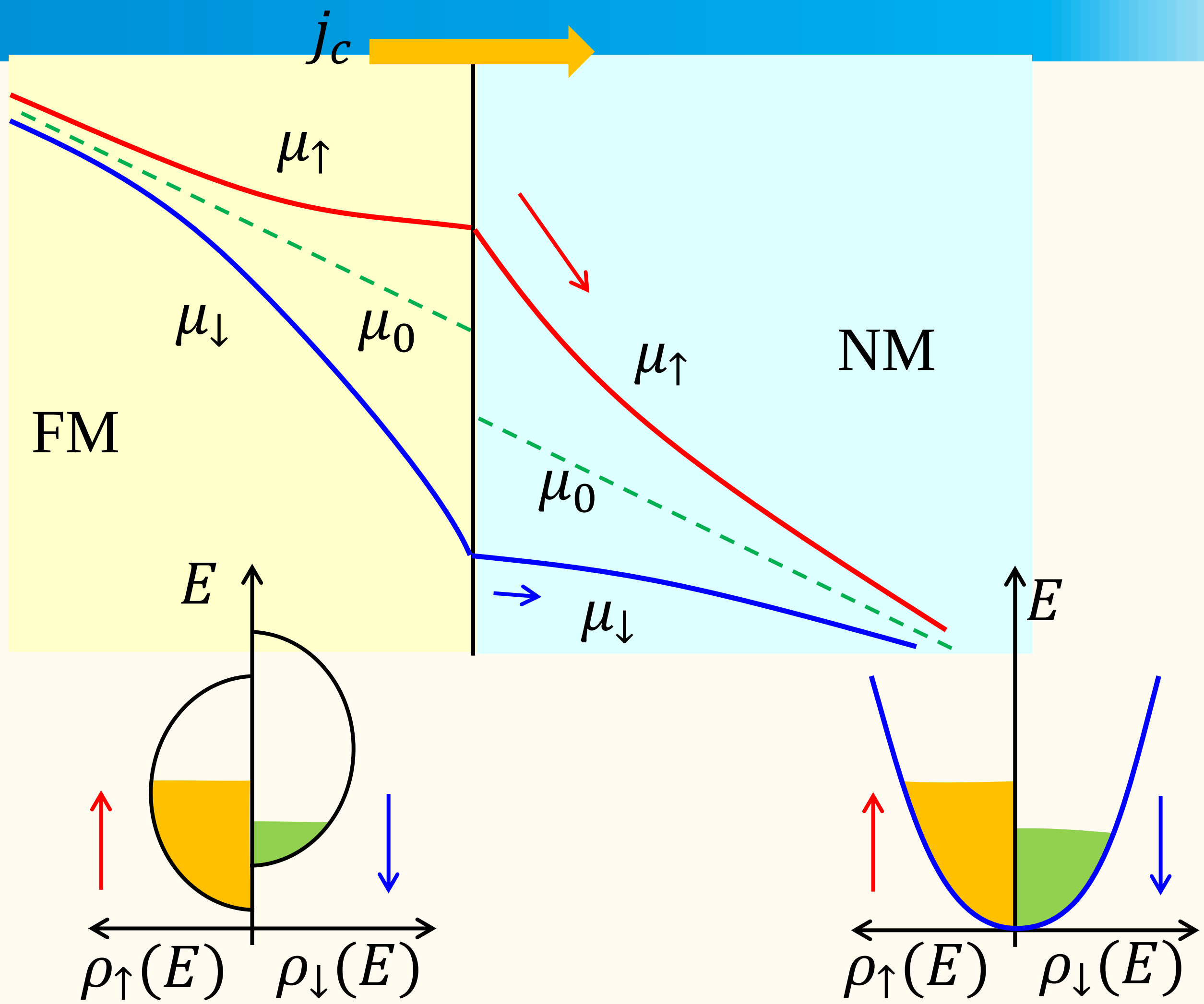
Steady state $$N_\uparrow \tau_\downarrow = N_\downarrow \tau_\uparrow$$

spin diffusion equation $$\nabla^2(\sigma_\uparrow \mu_\uparrow + \sigma_\downarrow \mu_\downarrow) = 0,\ \nabla^2(\mu_\uparrow - \mu_\downarrow) = \frac{1}{(\lambda_{\mathrm{sf}}^{\mathrm{F}})^2}(\mu_\uparrow - \mu_\downarrow)$$

spin diffusion length $$\left(\frac{1}{(\lambda_{\mathrm{sf}}^{\mathrm{F}})^2} = \frac{1}{(\lambda_\uparrow^{\mathrm{F}})^2} + \frac{1}{(\lambda_\downarrow^{\mathrm{F}})^2}\right)$$

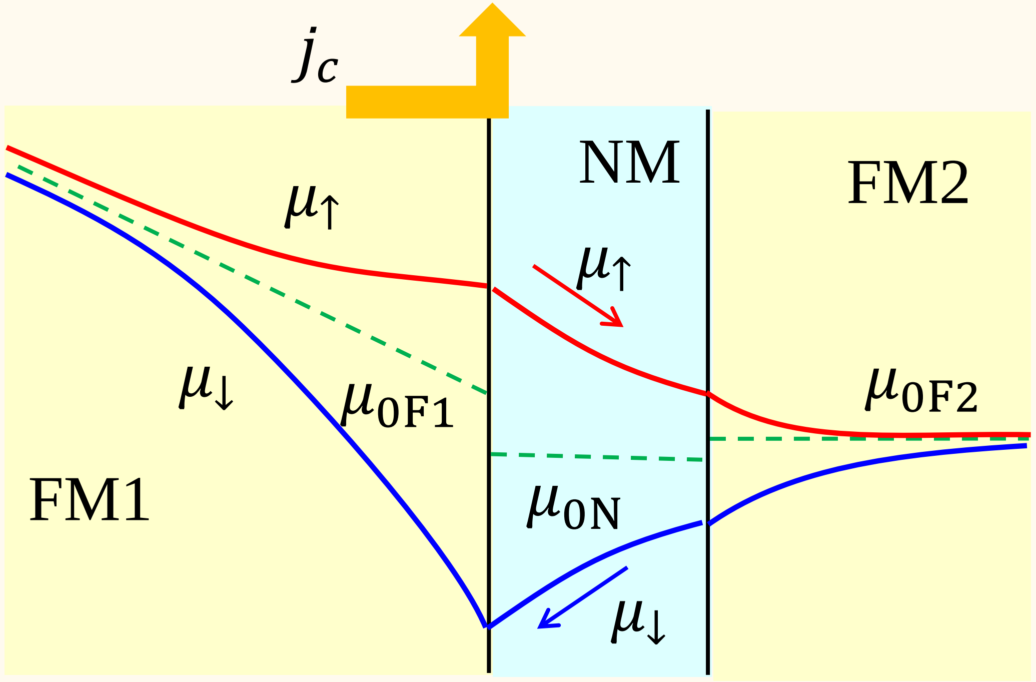
Spin injection

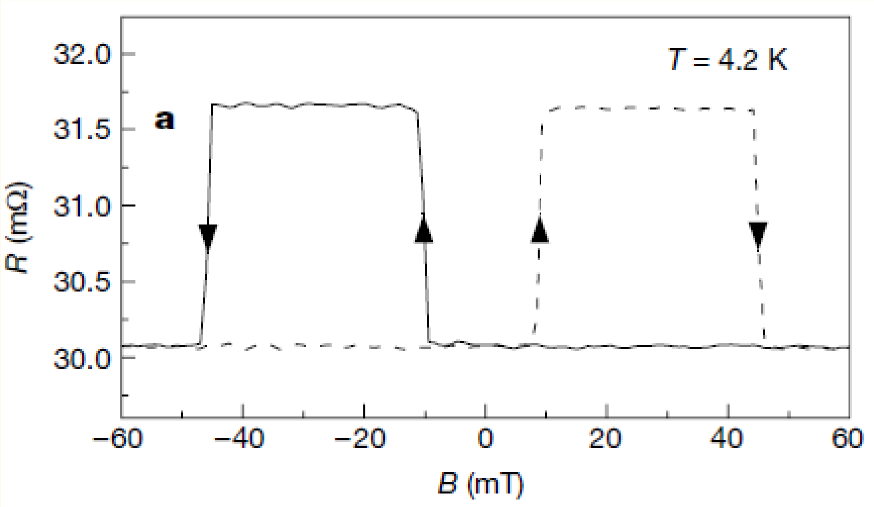


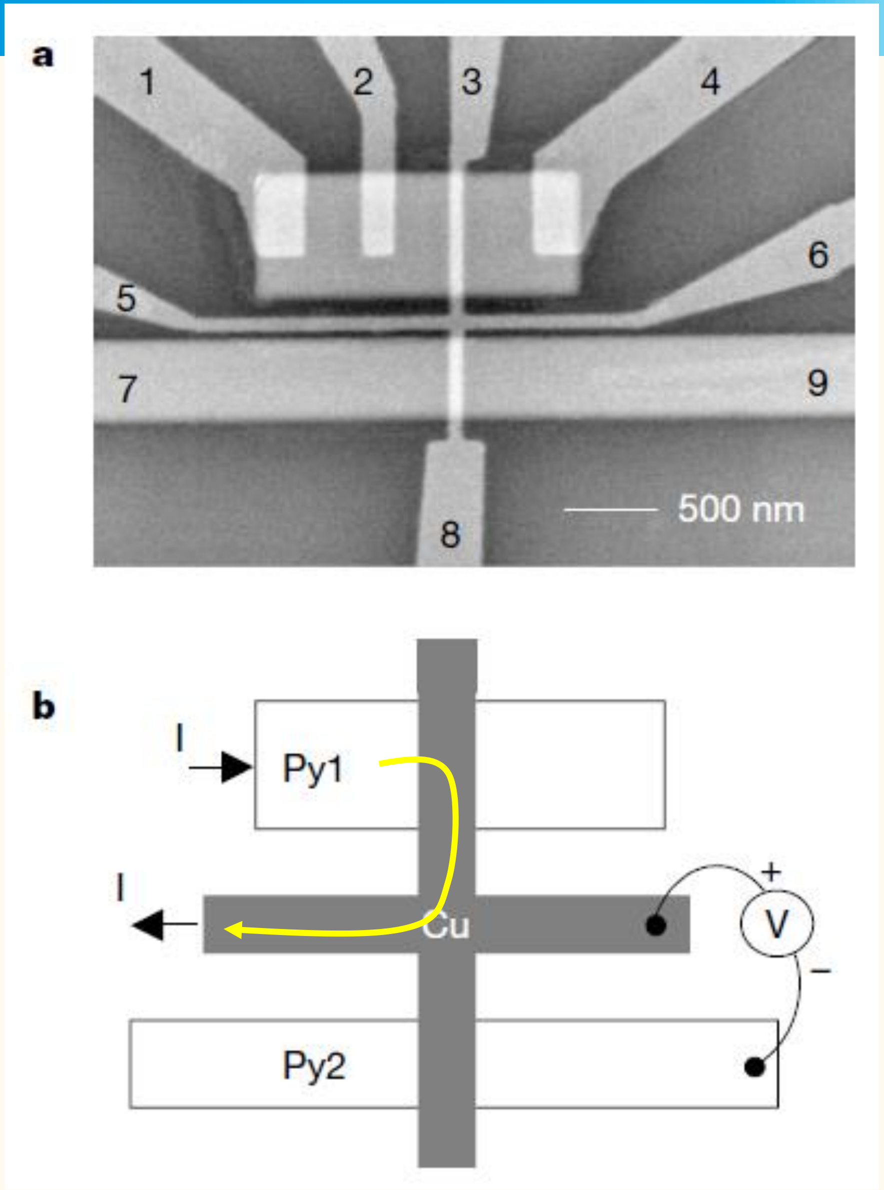


$$\mu_s^{\mathrm{M}} = a^{\mathrm{M}} + b^{\mathrm{M}} x \pm \frac{c^{\mathrm{M}}}{\sigma_s^{\mathrm{M}}} \exp\left(\frac{x}{\lambda_{\mathrm{sf}}^{\mathrm{M}}}\right) \pm \frac{d^{\mathrm{M}}}{\sigma_s^{\mathrm{M}}} \exp\left(-\frac{x}{\lambda_{\mathrm{sf}}^{\mathrm{M}}}\right) \qquad \mathrm{M} = \mathrm{F, N}$$

Spin injection and detection







Jedema et al. Nature **410**, 345 (2001).

Spin precession

Zeeman Hamiltonian $\mathscr{H} = \frac{e\hbar}{2m_0} g B_0 \hat{s}_z = g\mu_B B_0 \hat{s}_z \qquad [\hat{s}_j, \hat{s}_k] = i\hat{s}_l/2$

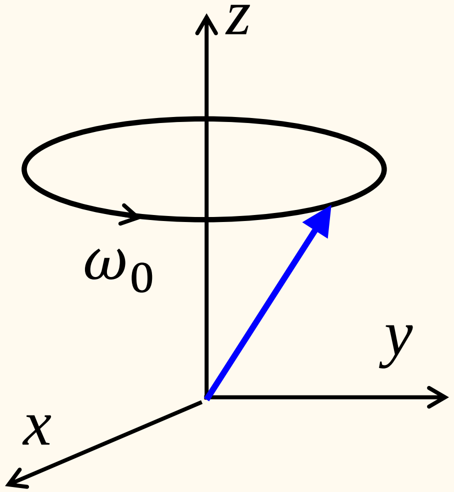
$$[\mathscr{H}, \hat{s}_x] = ig\mu_B B_0 \hat{s}_y, \quad [\mathscr{H}, \hat{s}_y] = -ig\mu_B B_0 \hat{s}_x, \quad [\mathscr{H}, \hat{s}_z] = 0$$

From Heisenberg equation: $\frac{\partial \langle s_x \rangle}{\partial t} = -\frac{g\mu_B}{\hbar} B_0 \langle s_y \rangle, \quad \frac{\partial \langle s_y \rangle}{\partial t} = \frac{g\mu_B}{\hbar} B_0 \langle s_x \rangle, \quad \frac{\partial \langle s_z \rangle}{\partial t} = 0$

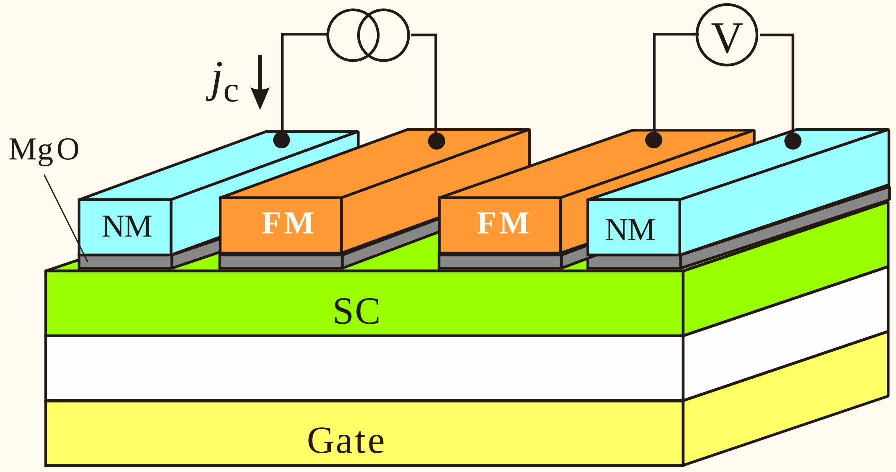
Solution $\langle s_x \rangle = A\cos\omega_0 t, \quad \langle s_y \rangle = A\sin\omega_0 t, \quad \langle s_z \rangle = C$

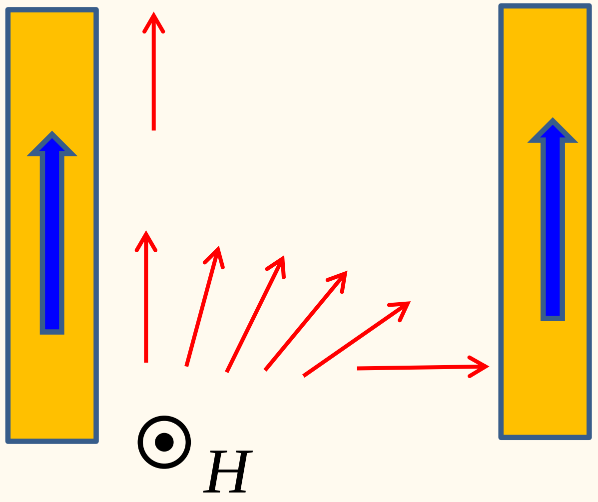
$$A^2 + C^2 = s^2, \quad \omega_0 = \frac{eg}{2m_0} B_0$$

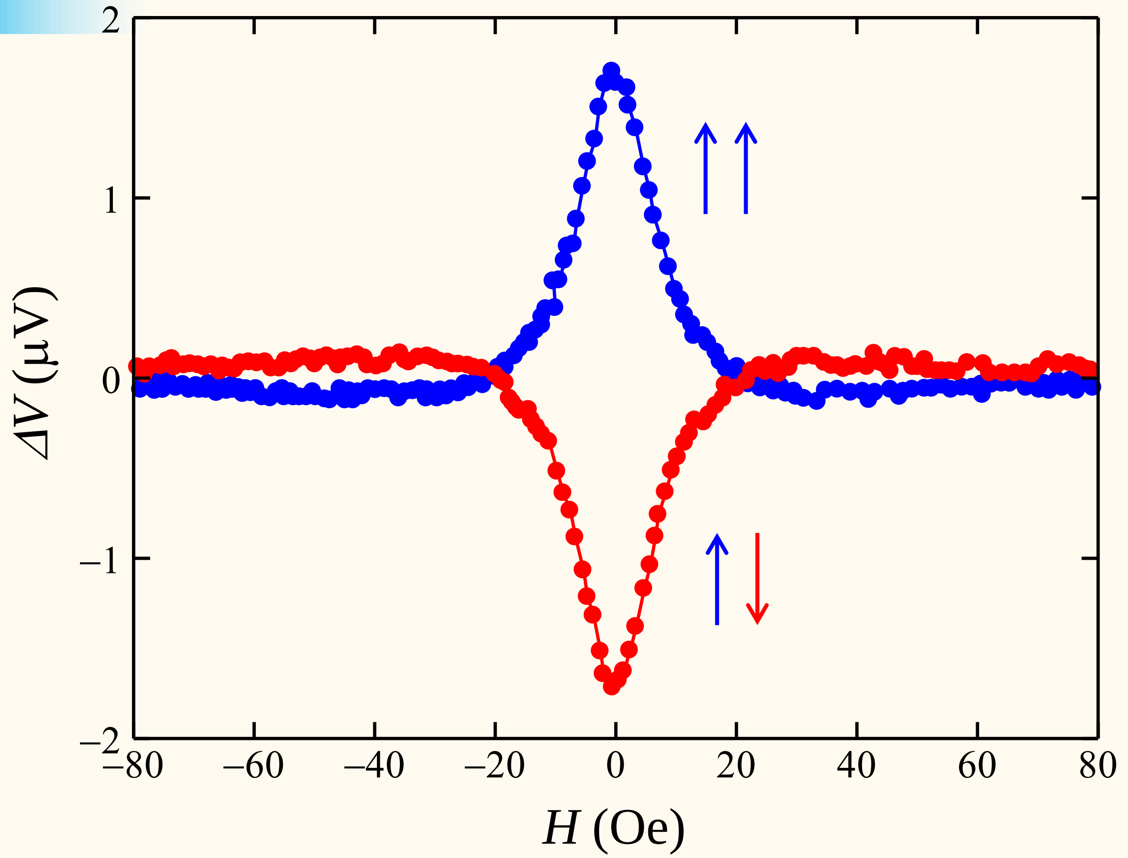
Larmor frequency



Spin precession experiment



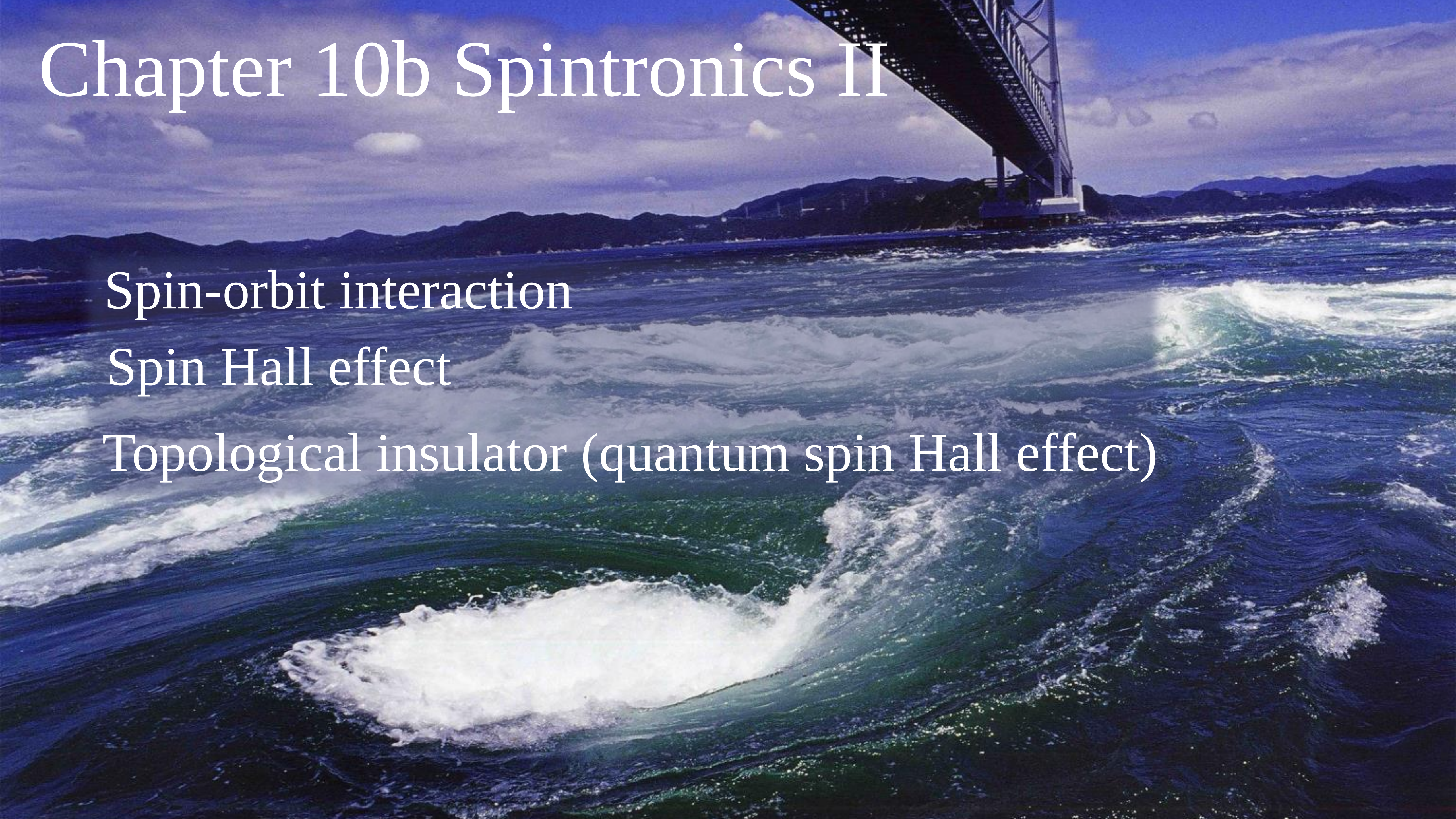




$$\Delta V = \pm \frac{j_c P_j^2}{e^2 N_{SC}} \int_0^\infty dt \varphi(t) \cos \omega t,$$

$$\varphi(t) = \frac{1}{\sqrt{4\pi D t}} \exp\left(-\frac{d^2}{4Dt}\right) \exp\left(-\frac{t}{\tau_{sf}}\right)$$

Chapter 10b Spintronics II



Spin-orbit interaction

Spin Hall effect

Topological insulator (quantum spin Hall effect)

Spin-orbit interaction (in electron motion)

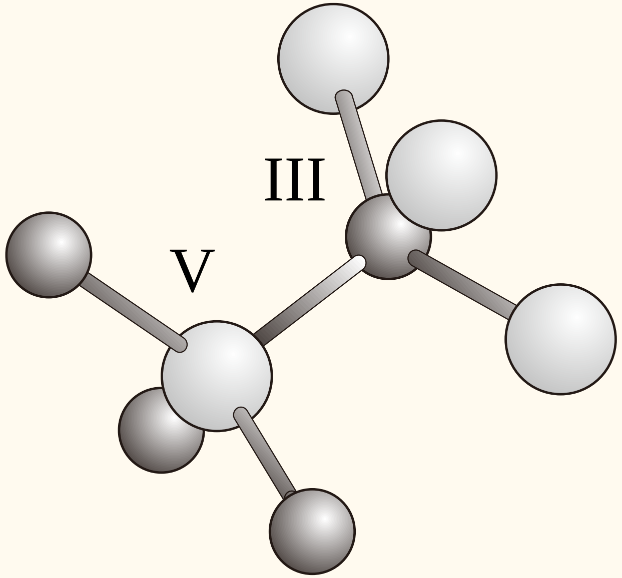
Pauli approximation of Dirac equation:

$$\frac{|P|^2}{3}\left\{\left(\frac{2}{E_{\mathrm{g}}}+\frac{1}{E_{\mathrm{g}}+\Delta}\right)k^2+V-\left(\frac{1}{E_{\mathrm{g}}}-\frac{1}{E_{\mathrm{g}}+\Delta}\right)\frac{e\boldsymbol{\sigma}\cdot\boldsymbol{B}}{\hbar}\right.$$
$$+\left[\frac{1}{E_{\mathrm{g}}^2}-\frac{1}{(E_{\mathrm{g}}+\Delta)^2}\right]e\boldsymbol{\sigma}\cdot(\boldsymbol{k}\times\boldsymbol{\mathcal{E}})$$
$$\left.-\left[\frac{2}{E_{\mathrm{g}}^2}+\frac{1}{(E_{\mathrm{g}}+\Delta)^2}\right]\frac{e\nabla\cdot\boldsymbol{\mathcal{E}}}{2}\right\}\psi_c=E'\psi_c$$

: Spin-orbit interaction

$\boldsymbol{\mathcal{E}}$: electric field

Finite $\boldsymbol{\mathcal{E}}$: requires inversion asymmetry.



BIA: Bulk inversion asymmetry

SIA: Structure inversion asymmetry

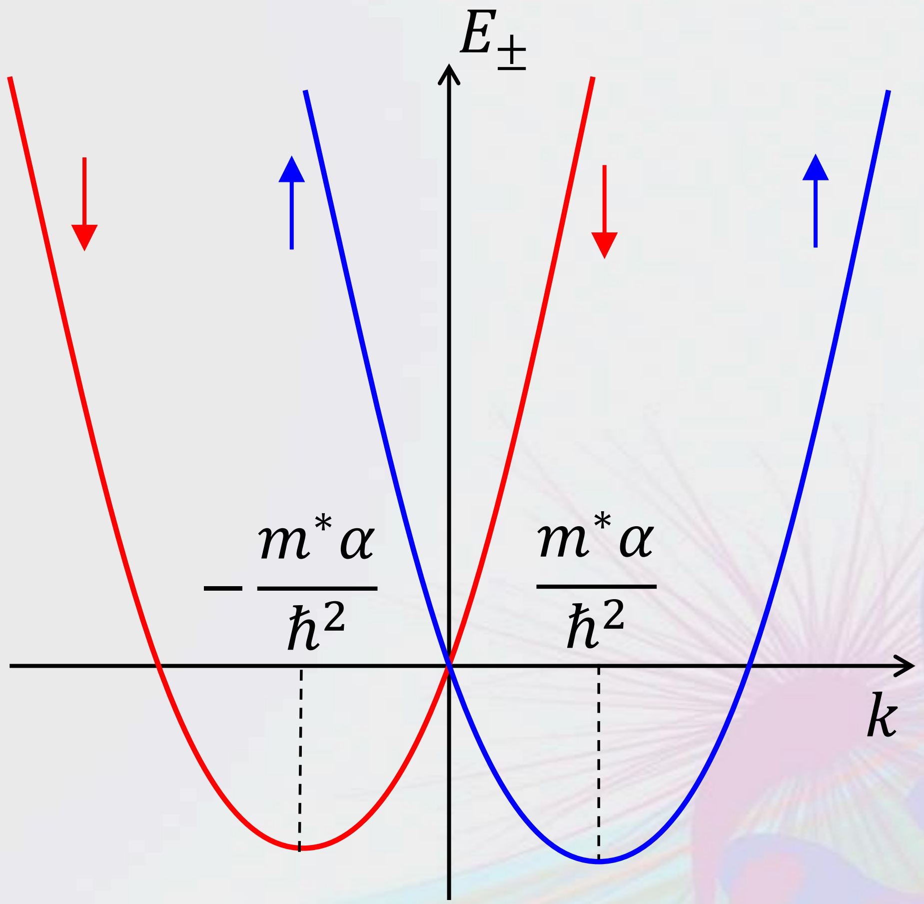
SIA-SOI Rashba-type SOI

BIA SOI $\mathscr{H}_{\rm DSO}^{\rm 2d} = \gamma\hbar^2[k_x(k_y^2 - \langle k_z^2\rangle)\sigma_x + k_y(\langle k_z^2\rangle - k_x^2)\sigma_y] = \beta(k_y\sigma_y - k_x\sigma_x) + \gamma\hbar^2(k_x k_y^2\sigma_x - k_y k_x^2\sigma_y)$

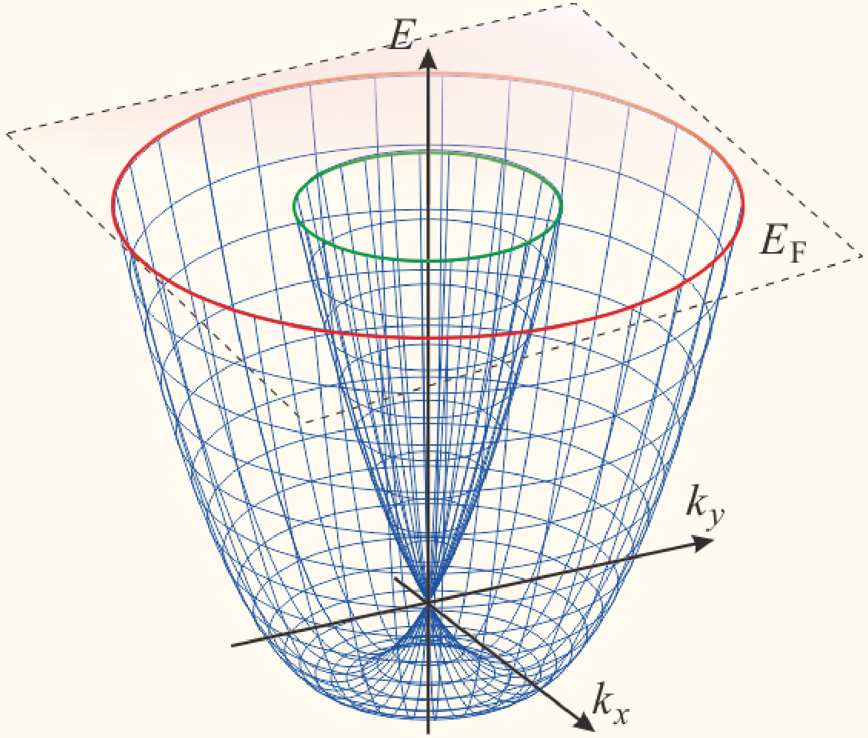
$\boldsymbol{\mathcal{E}} = (0, 0, \mathcal{E})$ on a 2DEG (x-y) (Actually through the valence band)

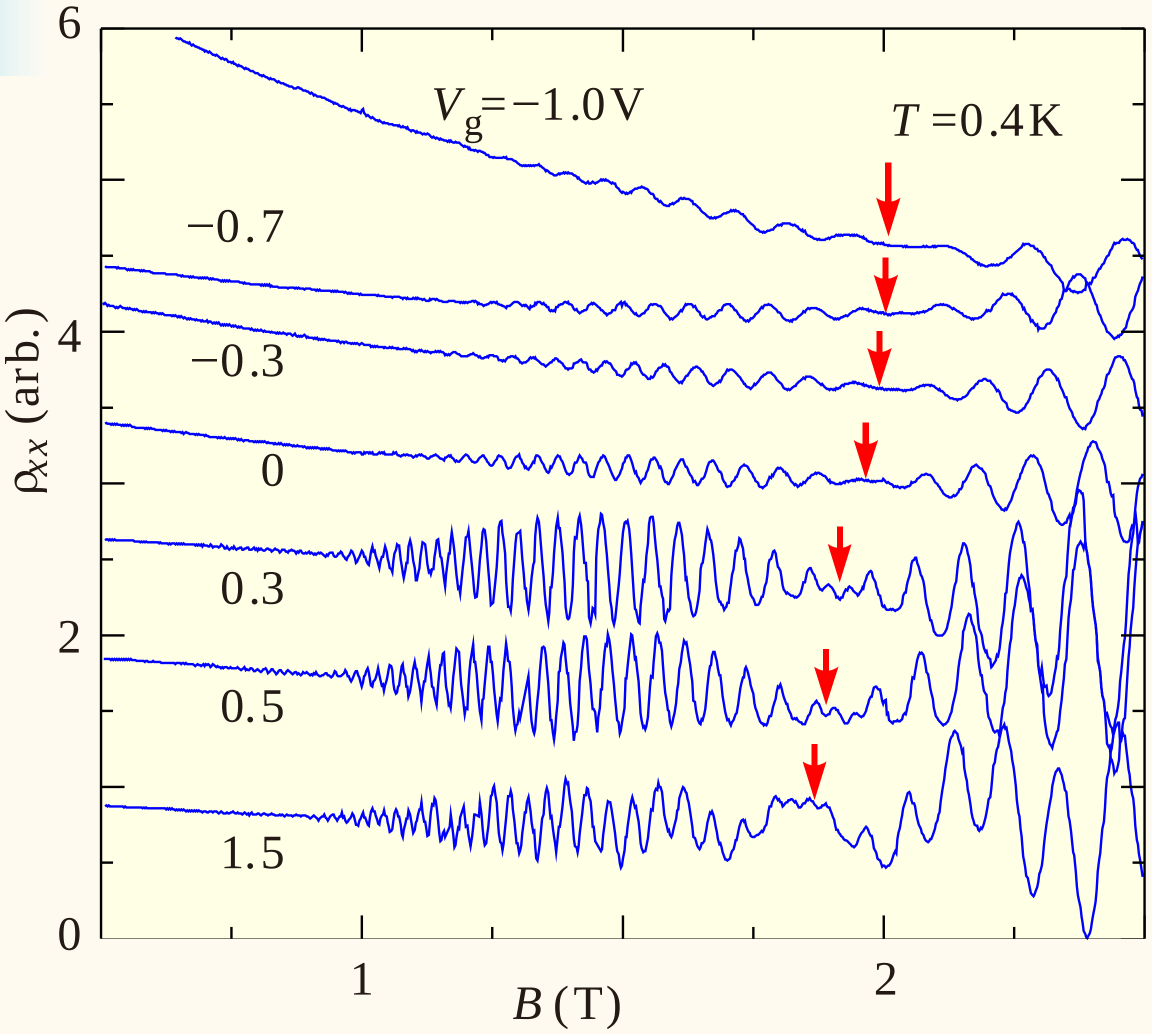
$$\mathscr{H}_{\rm RSO} = \alpha\boldsymbol{\sigma}\cdot(\boldsymbol{k}\times\boldsymbol{e}_z) = \alpha(k_y\sigma_x - k_x\sigma_y)$$

$$E_\pm = \frac{\hbar^2 k^2}{2m^*} \mp \alpha k = \frac{\hbar^2}{2m^*}\left(k \mp \frac{m^*\alpha}{\hbar^2}\right)^2 - \frac{m^*}{2\hbar^2}\alpha^2$$



SOI and SdH oscillation



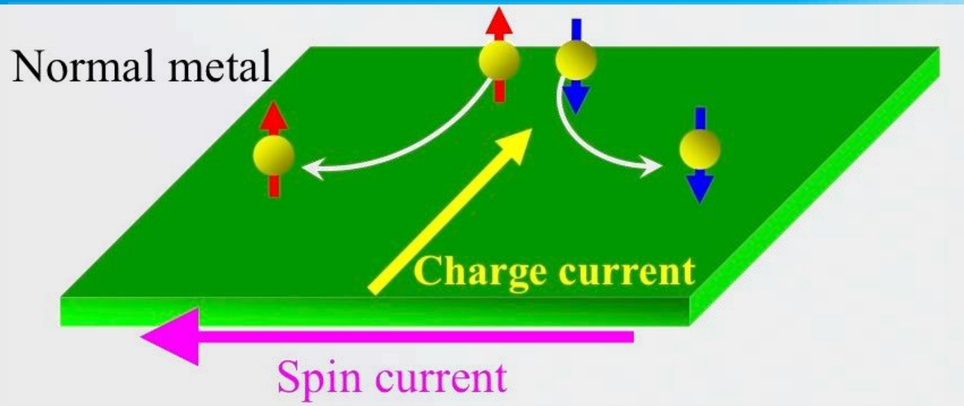


Nitta *et al*., Phys. Rev. Lett. **78**, 1335 (1997).

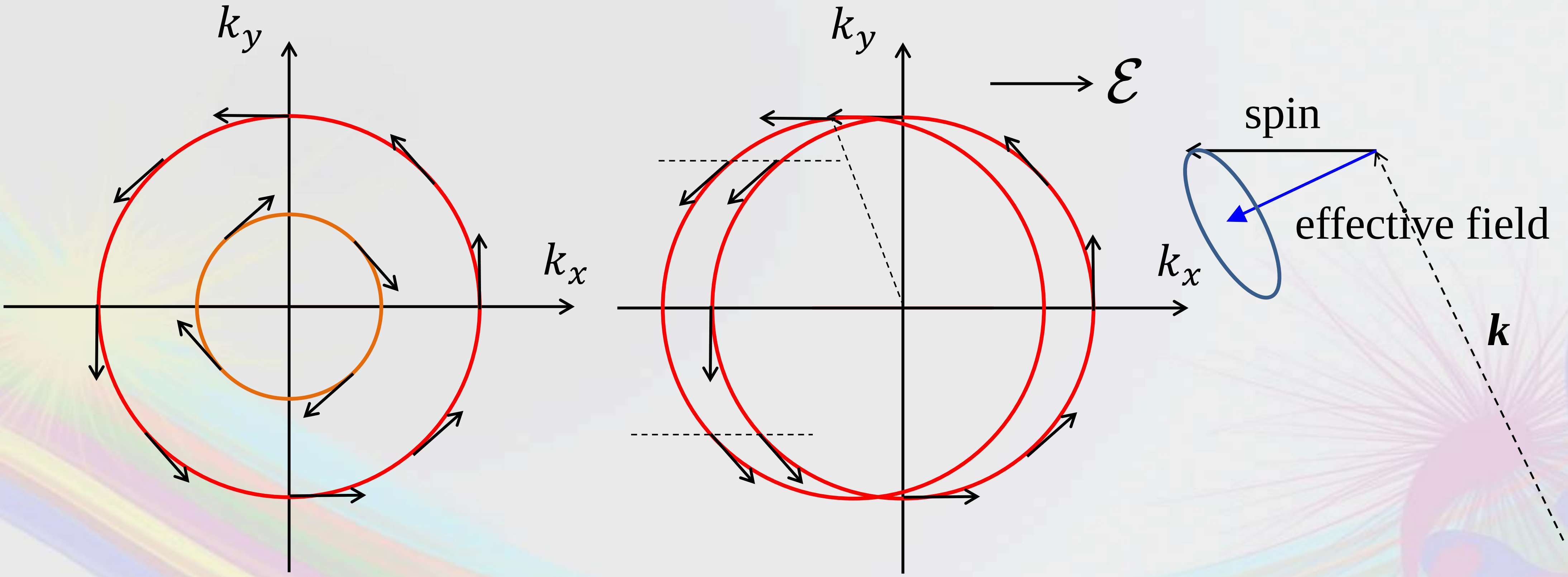
Spin Hall effect

$$J_{ij} = \sigma_s \sum_k \epsilon_{ijk} E_k$$

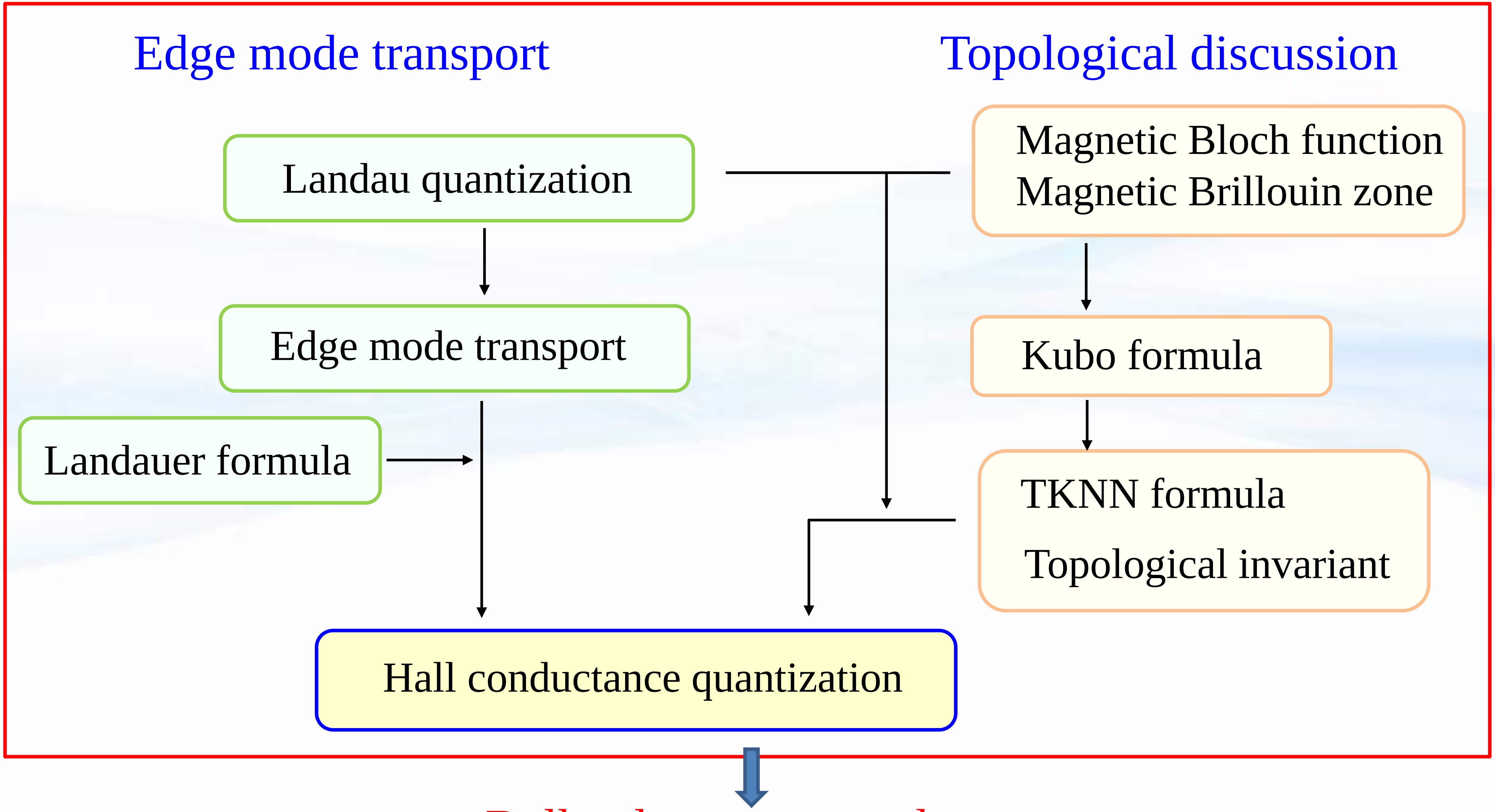
$$\mathscr{H}_{\mathrm{RSO}} = \alpha \boldsymbol{\sigma} \cdot (\boldsymbol{k} \times \boldsymbol{e}_z)$$



Effective magnetic field



How we understand the quantum Hall effect?



Bulk-edge correspondence

Spin Hall effect in an insulator

Remember k·p approximation $\mathscr{H}_{\boldsymbol{k}} u_{n\boldsymbol{k}}(\boldsymbol{r}) = E_{n\boldsymbol{k}} u_{n\boldsymbol{k}}(\boldsymbol{r}) \qquad |n\boldsymbol{k}\rangle$

$$\boldsymbol{A}_n(\boldsymbol{k}) = i\left\langle n\boldsymbol{k}\left|\frac{\partial}{\partial\boldsymbol{k}}\right|n\boldsymbol{k}\right\rangle, \quad \boldsymbol{B}_n(\boldsymbol{k}) = i\left\langle\frac{\partial(n\boldsymbol{k})}{\partial\boldsymbol{k}}\right|\times\left|\frac{\partial(n\boldsymbol{k})}{\partial\boldsymbol{k}}\right\rangle$$

Consider the case these are not zero. Then the discussion is in parallel with the TKNN formula.

$$\langle\boldsymbol{k}|\hat{\boldsymbol{r}}|\boldsymbol{k}'\rangle = (i\nabla_{\boldsymbol{k}} + \boldsymbol{A})\,\delta(\boldsymbol{k}-\boldsymbol{k}')$$

$$\langle\boldsymbol{k}|[\hat{x},\hat{y}]|\boldsymbol{k}'\rangle = (i\nabla_{\boldsymbol{k}}\times\boldsymbol{A})_z\,\delta(\boldsymbol{k}-\boldsymbol{k}') = iB_z\delta(\boldsymbol{k}-\boldsymbol{k}')$$

$$\left\langle\boldsymbol{k}\left|\frac{d\hat{x}}{dt}\right|\boldsymbol{k}'\right\rangle = \left[\frac{\partial E}{\partial k_x} - (\boldsymbol{F}\times\boldsymbol{B})_x\right]\frac{\delta(\boldsymbol{k}-\boldsymbol{k}')}{\hbar},$$

$$\left\langle\boldsymbol{k}\left|\frac{d\hat{k}_x}{dt}\right|\boldsymbol{k}\right\rangle = F_x\frac{\delta(\boldsymbol{k}-\boldsymbol{k}')}{\hbar}$$

Anomalous velocity and quantum spin Hall effect

Wave packet: $f = \sum_{\boldsymbol{k}} a_{\boldsymbol{k}} |\boldsymbol{k}\rangle$ Bloch wave expansion

$$\boldsymbol{F} = -e\boldsymbol{\mathcal{E}}$$

$$\frac{d\boldsymbol{r}_0}{dt} = \boldsymbol{v} = \left\langle f \middle| \frac{d\hat{\boldsymbol{r}}}{dt} \middle| f \right\rangle = \sum_{\boldsymbol{k}} \frac{\langle f|\boldsymbol{k}\rangle}{\hbar} (\nabla_{\boldsymbol{k}} E - \boldsymbol{F} \times \boldsymbol{B}) \langle \boldsymbol{k}|f\rangle$$

$$\approx \frac{1}{\hbar} (\nabla_{\boldsymbol{k}} E - \underline{\boldsymbol{F} \times \boldsymbol{B}})|_{\boldsymbol{k}=\boldsymbol{k}_0}$$

Anomalous velocity

$$\frac{d\boldsymbol{k}_0}{dt} = \frac{\boldsymbol{F}}{\hbar}$$

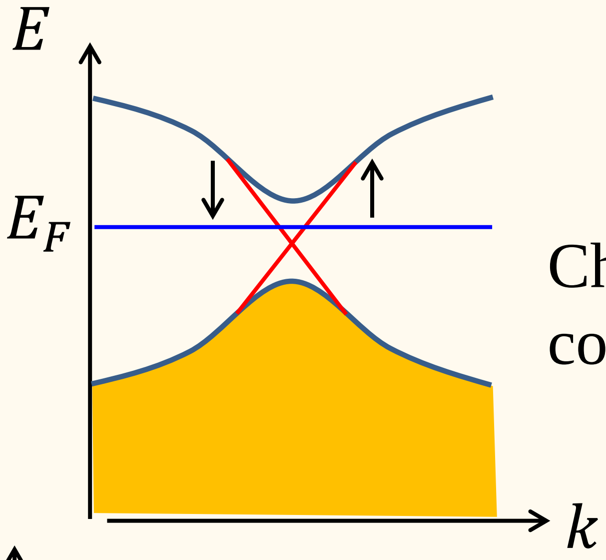
TKNN

$$\sigma_{xy}^s = \frac{\hbar}{-2e}(\sigma_{xy}^{\uparrow} - \sigma_{xy}^{\downarrow}) = \frac{-e}{4\pi}(\nu^{\uparrow} - \nu^{\downarrow}) = \frac{-e}{4\pi}\underline{\nu_{\mathrm{s}}}$$

Spin-subband Chern number

Spin Chern number

Topological insulator: helical edge state



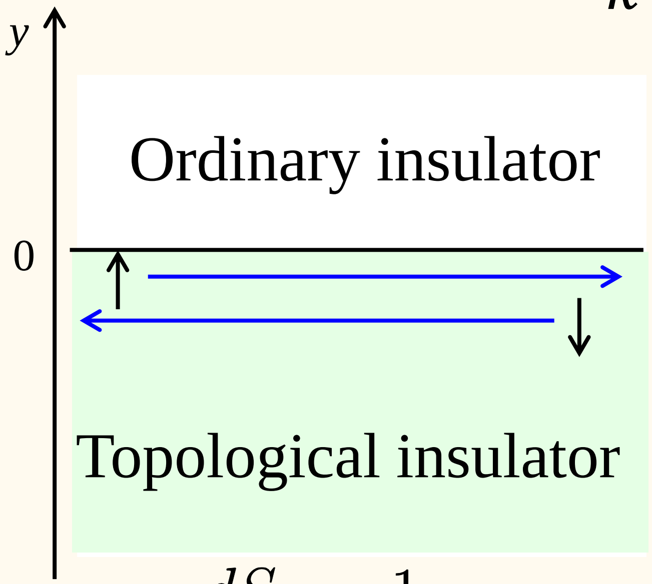
$$j_x^\chi = \Theta(y)\sigma_{xy}^\chi E_y, \quad j_y^\chi = -\Theta(y)\sigma_{xy}^\chi E_x \quad \chi = \uparrow, \downarrow$$

Charge conservation:

$$\frac{d\rho^\chi}{dt} + \nabla \cdot \boldsymbol{j}^\chi = \frac{d\rho^\chi}{dt} - \delta(y)\sigma_{xy}\chi E_x$$

$$= \frac{d\rho^\chi}{dt} - \delta(y)\nu^\chi \frac{e^2}{h} E_x = 0$$

$$\frac{d}{dt}(\rho^\uparrow - \rho^\downarrow) - \delta(y)\frac{e^2}{h}(\nu^\uparrow - \nu^\downarrow)E_x = 0$$



$$\frac{dS_z}{dt} = L\frac{-e}{2\pi}\nu_{\mathrm{s}} E_x \longrightarrow$$ Extra spin flow at the edge

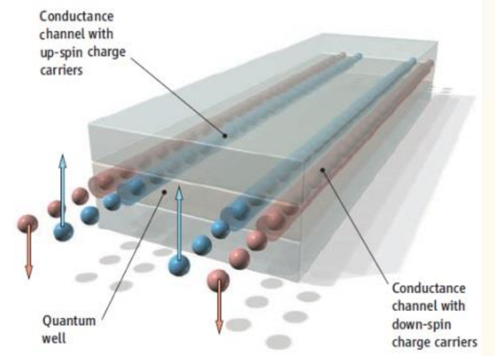
Helical edge mode:

$$E_k^{\uparrow\downarrow} = \pm v(\delta k_x - eE_x t) \quad \uparrow: +, \downarrow: -$$

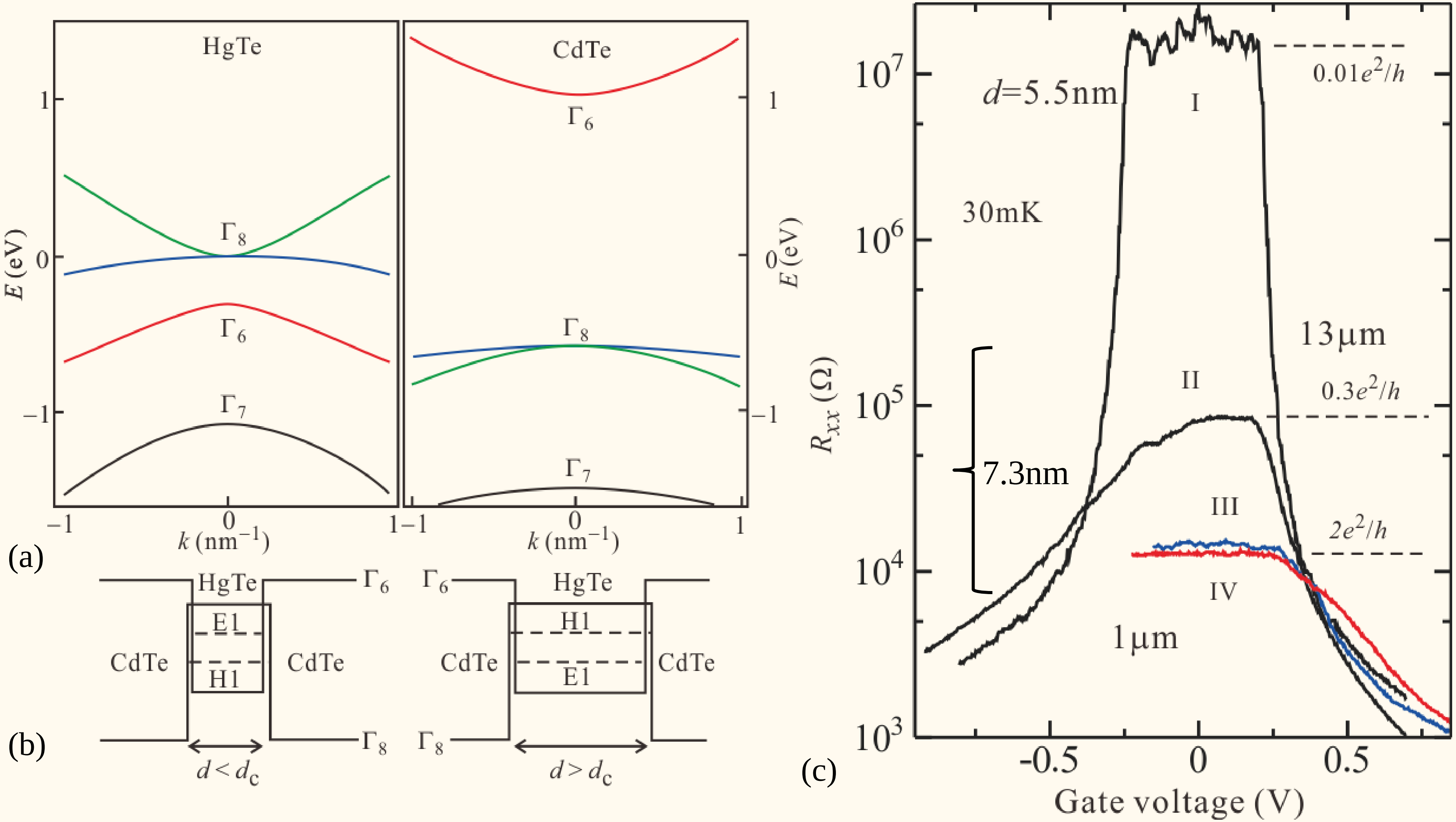
$$\frac{dS_z}{dt} = \frac{1}{2}(\delta N_\uparrow - \delta N_\downarrow) = L\frac{e}{2\pi}E_x$$

Edge mode number = Chern number

Topologically insulating quantum well

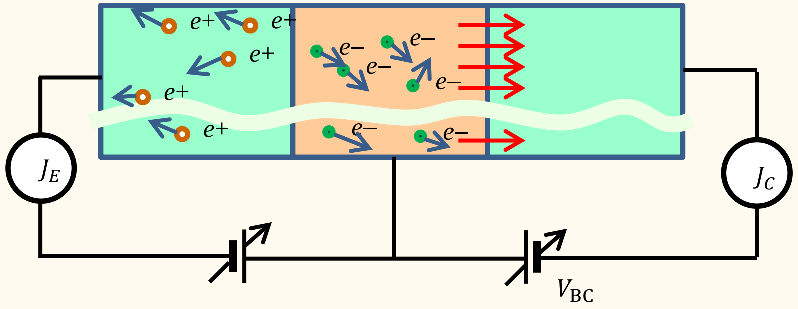


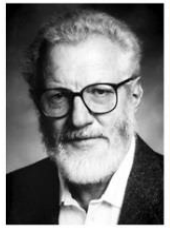
König *et al.*, Science **318**, 766 (2007).

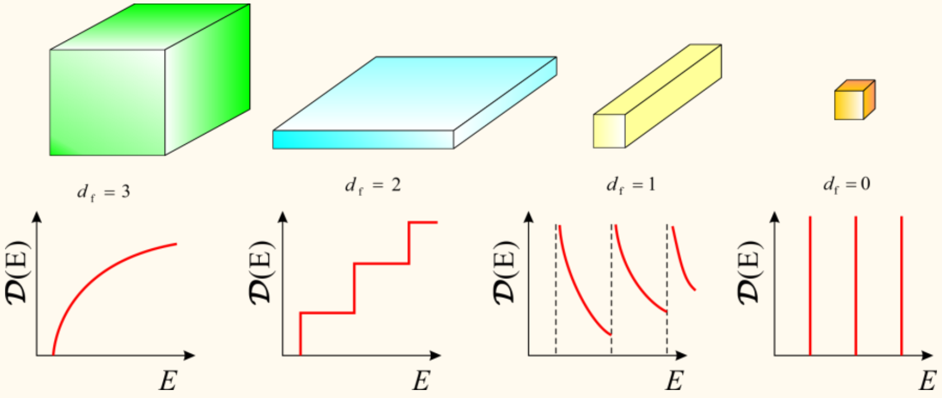


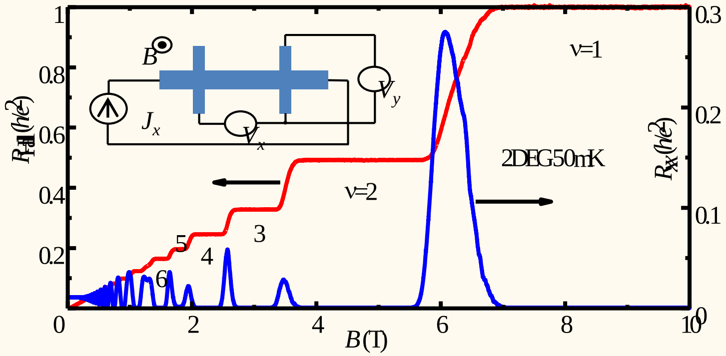
Summary

Charge (kinetic) freedom



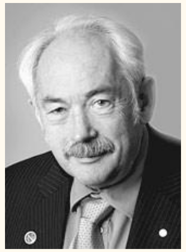




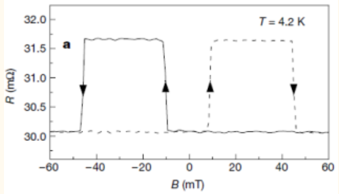


Spin degree of freedom

Giant magnetoresistance
spin valve



Spin injection



Spin-manipulation of
quantum information

Topological insulators