

Physics of Semiconductors (2)

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3 Charge carriers in semiconductors

Charge carriers in semiconductors play a central role in so called “structural sensitivity”. Here we have a look at very general properties of such carriers.

3.1 Effective mass

When a Bloch type electron wavefunction $\psi_{n\mathbf{k}}(\mathbf{r})$ has a dispersion relation $E_n(\mathbf{k})$, the group velocity is written as

$$\mathbf{v}_n(\mathbf{k}) = \hbar^{-1} \nabla_{\mathbf{k}} E_n(\mathbf{k}). \quad (2.1)$$

Hence, the acceleration is given as

$$\frac{d\mathbf{v}_n}{dt} = \frac{d\mathbf{k}}{\hbar dt} \cdot \nabla_{\mathbf{k}} (\nabla_{\mathbf{k}} E_n(\mathbf{k})) = \frac{\nabla_{\mathbf{k}}}{\hbar^2} \sum_{j=x,y,z} \frac{\partial E_n(\mathbf{k})}{\partial k_j} F_j. \quad (2.2)$$

Here, $\mathbf{F} = d\mathbf{p}/dt = \hbar d\mathbf{k}/dt$ is a vector of a “force”. Now we define the **inverse effective mass tensor** $1/m^*$, which is the inverse matrix of the **effective mass tensor** with

$$\left(\frac{1}{m^*} \right)_{ij} \equiv \frac{1}{\hbar^2} \frac{\partial^2 E(\mathbf{k})}{\partial k_i \partial k_j}. \quad (2.3)$$

Then (2.2) can be re-written as

$$\frac{dv_i(\mathbf{k})}{dt} = \sum_j \left(\frac{1}{m^*} \right)_{ij} F_j, \quad (2.4)$$

which is equivalent to

$$F_i = \sum_j m_{ij}^* \frac{dv_j(\mathbf{k})}{dt}. \quad (2.5)$$

For simplicity, we consider an energy band with an isotropic energy dispersion $E(k) = ak^2$. m^* , in general is a tensor, becomes a scalar $\hbar^2/(\partial^2 E(k)/\partial k^2) = \hbar^2/2a$. Let it be more specific. Consider the case of eq. (1.9), where a gap opens up in the nearly free electron approximation (NFEA). Around $\Delta k \sim 0$,

$$E_{\pm} \approx \epsilon_z \pm V_0 \left[1 + \frac{\epsilon_z}{2V_0} \left(\frac{\Delta k}{k_g} \right)^2 \right], \quad k_g \equiv \frac{\sqrt{2m_0 V_0}}{\hbar}, \quad (2.6)$$

which reads to the effective mass of

$$m^* = \pm \frac{\hbar^2}{2} \frac{2V_0}{\epsilon_z} \frac{2m_0}{\hbar^2} = \pm \frac{2V_0}{\epsilon_z} m_0 = \pm \frac{\epsilon_g}{\epsilon_z} m_0. \quad (2.7)$$

Here ϵ_z is the band width, ϵ_g the band gap. In this naive approximation, the ratio between the band width and the band gap determines the effective mass, namely, the wider the energy gap in comparison with the band width, the heavier the effective mass. This is a kind of “toy model” but can predict at least some trend in the effective masses in the same type of energy bands, *i.e.*, with the same symmetry at the same point in the reciprocal lattices. For example we can see such tendency in the effective masses at Γ point of conduction band in GaAs, InP, InAs.

In (2.7), we have a negative effective mass for E_- , which means, in this band, an electron is accelerated to the opposite direction of the force. In particular at the top of the band, the velocity of the electron is zero. So if you apply, say, an

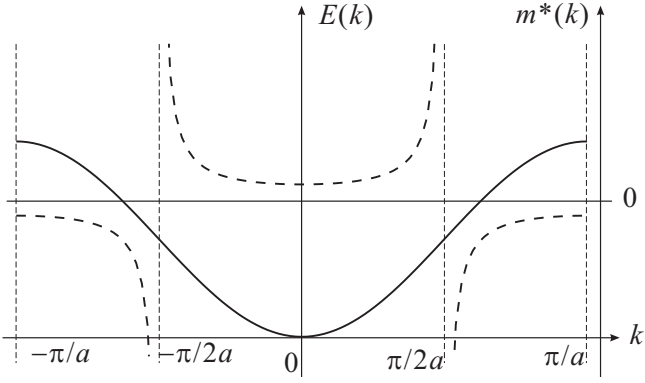


Figure 2.1: Effective mass calculated from (2.7) in the cosine band shown in Fig.1.4. It diverges and its sign is reversed at $k = \pm\pi/2a$.

electric field to an electron at the top of this band, the electron is accelerated to climb up the electrostatic potential. This is not a mystery, of course. An electron at the top of E_- band sits still in average, or in the meaning of standing wave. So acceleration to one of the components is nothing but deceleration to the other. And from the band top, the amplitude of the accelerated component diminishes with any shift. Hence, the “potential climbing up electrons” are consuming their inner kinetic energy to gain potential energy.

There is another comment on the effective mass. Figure in the right is a partial reproduction of Fig.1.2(b), where an energy gap opens at a zone edge in NFEA. Here we shift the origin of the energy to ϵ_0 and approximate the energy dispersions of the first and the second bands ($n = 0, 1$) with linear ones. After the approximation we shift the origin of the wavenumber to the zone edge π/a .

$$E_p(k) = c\hbar k, \quad E_q(k) = -c\hbar k.$$

These are dispersions for massless particles such as photons or phonons with a velocity c . Now we add the potential $V(x) = 2E_0 \cos(k_w x)$, then the dispersions change to

$$E_{\pm} = \pm \sqrt{(c\hbar k)^2 + E_0^2} \approx \pm E_0 \left[1 + \frac{1}{2} \left(\frac{c\hbar}{E_0} \right)^2 k^2 \right],$$

around $k \approx 0$. From (2.3), the effective masses are readily obtained as

$$m^* = \pm E_0/c^2 \quad \therefore E_0 = \pm m^* c^2. \quad (2.8)$$

The double sign corresponds to particle/anti-particle. This result tells us that the mechanism how a Bloch electron gains an effective mass at zone edges is very general, common with masses of materials in that it is given from an interaction between the particles which are flying in opposite directions.

3.2 Bloch oscillation

We write the dispersion relation in the tight-binding approximation (1.15) as $E(k) = E_0(1 - \cos ka)$ and the effective mass given in (2.7) is $m^*(k) = \hbar^2(E_0 L^2 \cos ka)^{-1}$. As shown in Fig.2.1, the mass diverges and changes its sign at $k = \pm\pi/2a$. If we put $k = 0$ at $t = 0$, the acceleration equation gives $k = Ft/\hbar$. Accordingly the expectation value of the electron position in real space coordinate $\langle x \rangle$ oscillates as

$$\langle x \rangle = \frac{E_0}{F} \left(1 - \cos \frac{Fa}{\hbar} t \right) = \frac{E_0}{F} (1 - \cos \omega_B t), \quad \omega_B \equiv \frac{Fa}{\hbar}. \quad (2.9)$$

The oscillation is called “Bloch oscillation”.

A Bloch oscillating state has an oscillation in the kinetic energy, hence is not an energy eigenstate. Here we treat the system as a potential problem. The external force can be a consequence of a slanted potential $\phi = -Fx$ added to the lattice potential, where the force F is given by $F = -d\phi/dx$. Let us apply the tight-binding approximation, in which an energy band is formed through the tunneling between neighboring localized states. Now the potential ϕ gives an energy difference $\Delta E = Fa$ to neighboring sites and an extended energy eigenstate cannot be formed. Instead, even with tunneling, each site has a localized energy eigen state. The system has a modulated translational symmetry, in that the potential structure remains the same but a constant shift after a spatial translation. Hence the energy eigenvalues of the localized states are in series with an equidistance in energy. Such localized states are called Stark ladder state. From

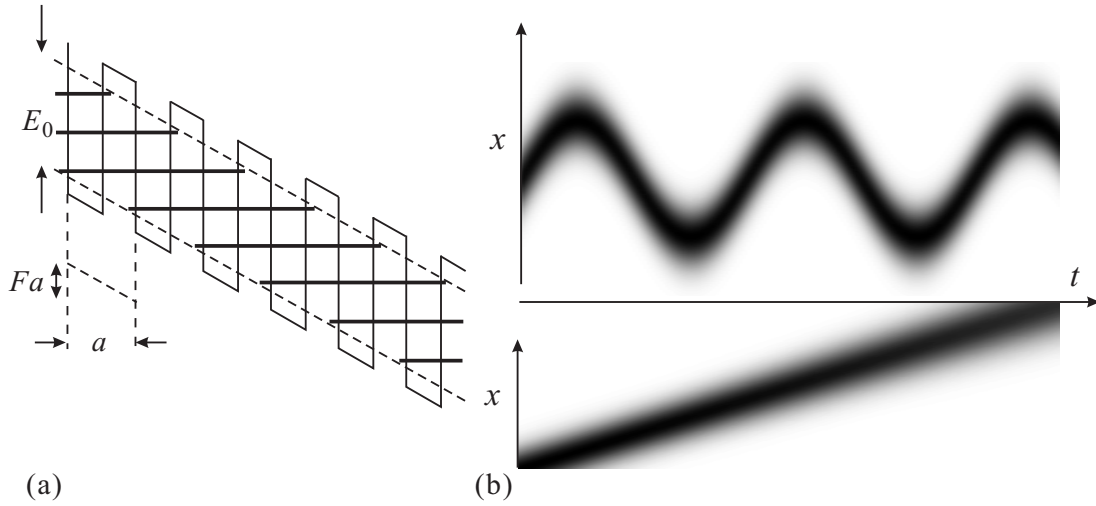


Figure 2.2: (a) Schematic of Stark ladder states, which appear in superlattices under electric fields and are localized along the direction perpendicular to the ultra-thin films. Horizontal thick solid lines indicate the energy levels of Stark ladder states. And the length of the lines indicate the regions where the states exist. (b) Time evolution of a wave packet under Bloch oscillation, which is obtained through the motions of plane waves forming the packet. The absolute value of wavefunction $|\psi|$ is plotted in gray scale (the darker the larger in amplitude). In the upper panel a wave packet in a cosine band is under an electric field and showing a Bloch oscillation. The lower panel shows an ordinary time evolution of a wave packet with a parabolic energy dispersion. In the lower the packet spreads with time due to the different phase velocities in different wavenumber while in the upper, all the components in the packet experiences wavenumbers in the first Brillouin zone and the packet re-focuses. In the actual oscillation the electrons interacts with electromagnetic environment and such focusing does not occur.

eq.(2.9) the spatial size of the localized states is about E_0/F . In the energy diagram in Fig.2.2(a), we see that the size corresponds to the region in which a state with a constant energy can exist (the length of the solid horizontal bars in the figure).

The frequency of Bloch oscillation is given as $\omega_B = \Delta E/\hbar$, from which we can see that the Bloch oscillation is a quantum coherence oscillation between neighboring Stark ladder states. Even for $F = 0$, if we take a localized state as an initial state, we observe a coherence oscillation as we see in Appendix A for a double well potential. The frequency is $\omega_t = |T|/\hbar$ where T is the tunneling matrix element. Hence the total coherence oscillation has the frequency $\omega = (1/2)(\omega_B \pm \sqrt{\omega_B^2 + \omega_t^2})$.

Fig.2.2(b) simulates a motion of a wave packet under a constant electric field with assumption that each plane wave component of the packet has its own Bloch oscillation in a cosine band. Interestingly, in this approximation with no interaction with electromagnetic field other than the static force, the wave packet does not show dispersion because every plane wave component experiences the whole wavenumber within a period of Bloch oscillation.

3.3 Experiment on Bloch oscillation

As seen above, the concepts of Bloch oscillation, Stark ladder state stem from a very naive band picture though the experimental realization had many big difficulties. Usually the widths of energy bands in semiconductors exceed 1eV, which is about 10^4 K in thermal energy, accelerated electrons thus get scattering from phonons or other perturbations and relax in energy far before they reach the band top. Coherent acceleration hence seemed to be hopeless. On the other hand, there is a group of materials, which have narrow bands though the effective masses are very heavy and the acceleration is very difficult.

The realization was accomplished with **superlattice**, comprehension of which requires those of epitaxial crystal growth and heterojunctions. Here we take a shortcut and introduce the very essence of them. Then introduce an experiment, which utilized many modern sophisticated experimental techniques. In the lecture I use PowerPoint.

3.3.1 Photon emission and Bloch oscillation

The results of experiment are shown in Fig.2.3. In Fig.2.3(b), they applied static electric fields to a GaAs/Al_{0.3}Ga_{0.7}As and irradiated a light of energy $h\nu$ and measured the photocurrent I_{ph} through the specimen. In GaAs/Al_{0.3}Ga_{0.7}As superlattices the energy barrier in the valence side is low and the Stark ladder in the hole side can be ignored. When $h\nu$

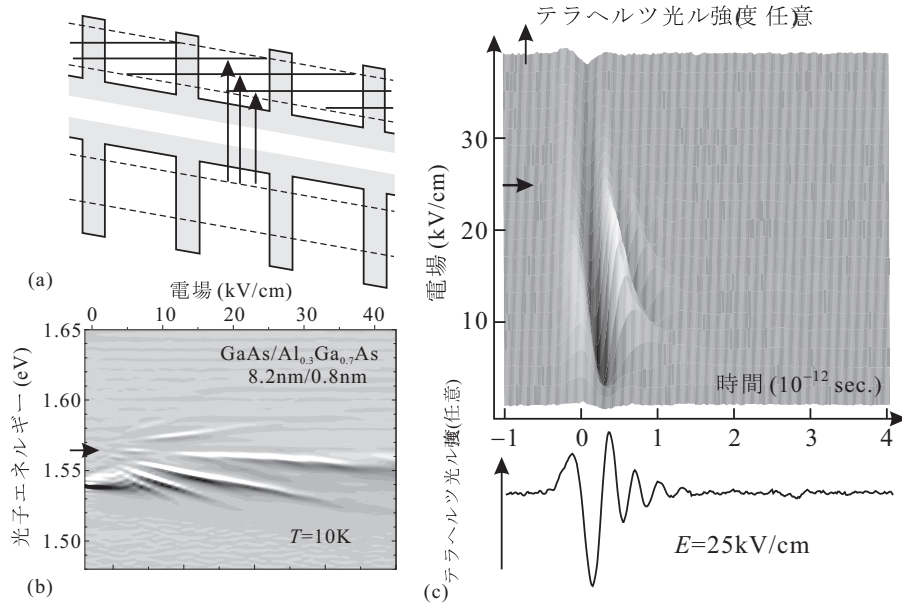


Figure 2.3: (a) Schematic picture of a superlattice potential with an electric field. The arrows indicate excitation processes to Stark ladder states in a quantum well. (b) Photocurrent I_{ph} through a GaAs/Al_{0.3}Ga_{0.7}As superlattice under electric fields. In order to emphasize the peak structures, the derivative of I_{ph} to the energy of incident light $h\nu$, $dI_{ph}/h\nu$ is plotted in gray scale. With increasing the electric field, the peak positions spread like a fan from the position indicated by an arrow. (c) Amplitude of electric field in THz electromagnetic wave emitted from the superlattice is 3D gray scale plotted against the plane of time and the strength of applied static electric field. From $t = 0$ clear damping oscillations appear, the frequency of which increases with applied electric field while the amplitude diminishes. The lower panel shows the data for 25kV/cm. Courtesy of Prof. Kaz Hirakawa, Institute of Industrial Science, University of Tokyo (Phys. Rev. Lett. **90**, 046806 ('03), *ibid.* **94**, 057408 ('05)).

coincides with the energy between the bottom of the hole miniband and the Stark ladder states in the conduction side, the photon is absorbed, creating an electron-hole pair and contributes to I_{ph} as illustrated in Fig.2.3(a). Hence I_{ph} forms peak structures versus $h\nu$ under electric field E_m at

$$h\nu_n = E_g^* + n\hbar\omega_{BO} = E_g^* + nedE_m \quad (n = 0, \pm 1, \pm 2, \dots). \quad (2.10)$$

The intrinsic band gap and the hole miniband are represented by E_g^* . Equation(2.10) predicts that the peak positions spread as a fan versus the electric field and we see the behavior in Fig.2.3(b).

Now an off-peak light with high intensity excites a linear combination of two Stark ladder states and a Bloch oscillation begins. A Bloch oscillation of an electron is that of a charged particle hence emits an electromagnetic wave. By using a high-tech laser technique, they measured such emission in THz frequency region. The amplitude of the emission is plotted as a function of duration in Fig.2.3(c). A clear damping oscillation appears directly indicating the excitation and damping of a Bloch oscillation. Increase of the oscillation frequency according to (??) with increasing applied electric field is also observed.

3.4 Concept of holes

A semiconductor is not a metal then the Fermi level exists in a bandgap not in an energy band. We call the energy band just below the Fermi level the **valence band** while the one just above the Fermi level, the **conduction band**. In the ground state, the valence band is full while the conduction band is empty.

A wavefunction which describes a many-electron state should be anti-symmetric, that is, it should change its sign with an interexchange of two electrons. Ignoring the mutual Coulomb interaction between the electrons, the eigenstate of many-electron state can be written as a Slater determinant

$$\psi(x_1, x_2, \dots, x_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \phi_1(x_1) & \cdots & \phi_1(x_N) \\ \vdots & \ddots & \vdots \\ \phi_N(x_1) & \cdots & \phi_N(x_N) \end{vmatrix}, \quad (2.11)$$

where ϕ_i are one electron energy eigenstates of the lattice potential, N the total number of electrons. The effect of Coulomb interaction can be taken into account within the Hartree-Fock (mean field) approximation with replacing the

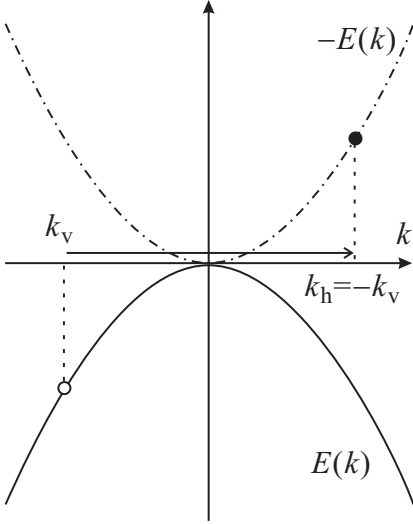


Figure 2.4: Schematic diagram of dispersion relation in a valence band. The origin of energy is taken at the top of the band. The open circle indicates an emptied state at $k = k_v$ in the valence band. In this “hole state” the total energy has decreased from the band-full state by $E(k_v)$ (hence the energy is $-E(k)$), and the total momentum is $k_h = -k_v$ because the band-full state has zero momentum in total ($\sum \mathbf{k} = 0$) due to the symmetry of valence band. Hence we should plot the “hole state” in this diagram at the position of the black bullet though we often describe “hole” with the open white circle particularly in optical absorption or emission. We should thus be careful in the treatment of “holes” in such k -space energy diagrams.

single-electron wavefunction ϕ_i in (2.11) with the solution of equation for mean field approximation (Single electron approximation of Coulomb interaction).

We further assume that this single-electron wavefunctions can be written in the Bloch form of (1.5) and they do not change with the total electron number N .¹ Then we can identify a many electron states with a single electron band diagram like Fig.1.3 by indicating occupied and unoccupied states. We need to be careful, though, in labeling of electrons. For example there are many ways to create $N - 1$ - electron state from N -electron state and emptying a k state is just one of them. You may be need to describe an “emptied state” with a superposition of many such k -emptied states. This is just like we need to describe a wave packet with a superposition of many k -states.

Now we introduce the concept of **holes**. Again consider a ground state of a valence band filled with N electrons. We create an $N - 1$ state by putting out an electron from the ground state and call it a single hole state. Further extraction creates two, three, \dots hole states. Now consider a hole state that is created from extraction of an electron labeled with k_v . The energy is decreased by $E(k_v)$ from the ground state and the total momentum is $-k_v$ because in the ground state the total momentum is zero ($\sum \mathbf{k} = 0$) due to the symmetry of valence bands (otherwise the valence band has a flow in a symmetrical lattice). We call this state “a hole state with a wavenumber of ”, the energy of which is $-E(k_v) = -E(-k_h)$ (Fig.2.4).

We test the concept of holes by applying an electric field \mathbf{E} to a hole state with a wavenumber $k_h = -k_v$. The acceleration equation

$$-e\mathbf{E} = \mathbf{F} = \frac{d\mathbf{p}}{dt} = \hbar \frac{d\mathbf{k}}{dt}, \quad (2.12)$$

is applicable to the electrons filling the k -states in the valence band other than k_v . The electrons shift in the k -space in parallel with a (k -space) velocity of Therefore the emptied state is reserved as an emptied state in the k -space and shift also with a velocity of $-eE/\hbar$. We write down the situation as $dk_v/dt = -eE/\hbar$. Now $dk_h/dt = -dk_v/dt$, so the acceleration equation for a hole is described as if the hole has a positive charge of $+e$. It may be a bit confusing but the acceleration equation for the emptied state is still for a particle with a negative charge of $-e$.

Next we form a wave packet of a hole with superposing many k_v states and consider the group velocity v_v . The motion of this hole wave packet in real space should be the same as that of emptied electrons. If we adopt the effective mass in NFEA (2.7), $m^* = -(\epsilon_g/\epsilon_z)m_0$,

$$\frac{dv_v}{dt} = -\frac{\epsilon_z}{\epsilon_g} \frac{-eE}{m_0} = \frac{\epsilon_z}{\epsilon_g} \frac{eE}{m_0} = \frac{dv_h}{dt}. \quad (2.13)$$

Now the wavepacket of emptied states is in real space and at the position of the packet the charge increased by $+e$ from the valence band filled ground state. Hence it is the hole wavepacket itself.

The above discussion tells that a hole state has $+e$ charge, and the effective mass is $m_h^* = -m_e^*$. Because m_e^* is negative at the top of valence band, a hole behaves as a particle with a positive charge and a positive effective mass. As we saw a hole is a quasi-particle description of many electron states in a valence band.

¹This is a kind of **rigid band model** in that the single-electron band does not change with the electron-electron interaction. Though this seems to be a very coarse approximation, it has been proven to be good in many cases and they often widely expand this to *e. g.*, the case in which an electric field is applied.

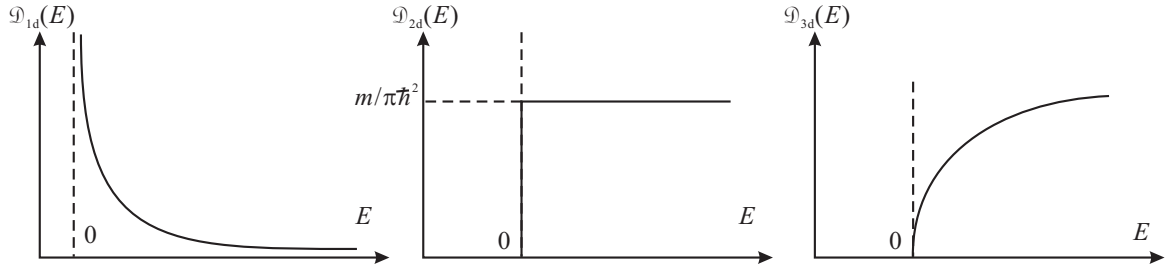


Figure 2.5: Schematic diagrams of density of states for 1 to 3 dimensional systems in eq.(2.14).

3.5 Intrinsic semiconductors

When a semiconductor has a perfect, defect-free crystal structure and no impurity, we call it an intrinsic semiconductor. An intrinsic semiconductor is, of course, an approximation, which holds when we can ignore the effect of imperfections. The concept of intrinsic semiconductor has two important roles; 1) the approximation actually holds at finite temperatures when the thermal energy exceeds characteristic energy scales for electron scattering by imperfections; 2) an intrinsic semiconductor can be a starting point of approximations for impure, non-uniform semiconductors.

3.5.1 Density of states

We consider a simple lattice system which has a state per a unit cell with an edge length of a . We take the system size as $L = Na$ in one dimension. For an n -dimensional system, the volume $(2\pi/L)^n$ contains a single state in k -space. Given the kinetic energy as $E(k) = \hbar^2 k^2 / 2m$, the number of states per volume between E and $E + dE$ deviated by dE is

$$\mathcal{D}(E) = \frac{1}{L^d} \left(\frac{L}{2\pi} \right)^d \frac{dV_d(k)}{dE} = \frac{1}{(2\pi)^d} \frac{dV_d(k)}{dk} \frac{dk}{dE} = \frac{1}{(2\pi)^d} \frac{m}{\hbar^2} \frac{dV_d(k)}{k dk},$$

where $V_d(k)$ is the volume of d -dimensional sphere with the radius of k .

This $\mathcal{D}(E)$ is called **energy density of state**. Because $V_1 = 2k$, $V_2 = \pi k^2$, $V_3 = 4\pi k^3/3$,

$$d = 1 : \mathcal{D}_{1d}^{(0)} = \frac{1}{\pi\hbar} \sqrt{\frac{2m}{E}}, \quad d = 2 : \mathcal{D}_{2d}^{(0)} = \frac{m}{\pi\hbar^2}, \quad d = 3 : \mathcal{D}_{3d}^{(0)} = \frac{\sqrt{2m^3}}{\pi^2\hbar^3} \sqrt{E}. \quad (2.14)$$

Here we put the factor two from the freedom of spin.

In the case of electrons in crystals, the above expressions for density of states are applicable with replacing the mass with the effective mass where non-parabolicity of the band is ignorable, *e.g.*, around tops and bottoms of the bands. Where we cannot apply the parabolic approximation, we need to go back to the definition of the density of states. For a three dimensional system it is given from

$$\mathcal{D}(E) = \int_{E(\mathbf{k})=E} \frac{dS_{\mathbf{k}}}{(2\pi)^3} \frac{2}{\nabla_{\mathbf{k}} E(\mathbf{k})}. \quad (2.15)$$

The integral is over the equi-energy surface $E(\mathbf{k}) = E$ in k -space.

3.5.2 Carrier distribution in thermal equilibrium

Electrons are fermions, that is a single quantum state can be occupied only with a single electron, hence they obey the Fermi-Dirac distribution function

$$f_{\text{F}}(E) = \frac{1}{\exp((E - E_{\text{F}})/k_{\text{B}}T) + 1} \quad (2.16)$$

in a thermal equilibrium at temperature T . E_{F} is the Fermi energy, k_{B} the Boltzmann constant.

Let us see how electrons and holes distribute in energy space at a finite temperature obeying (2.16). The numbers of electrons and holes which exist in $E \sim E + dE$ are

$$g_e(E)dE = \mathcal{D}_e(E)f(E)dE, \quad (2.17a)$$

$$g_h(E)dE = \mathcal{D}_h(E)[1 - f(E)]dE \equiv \mathcal{D}_h(E)f_h(E)dE, \quad (2.17b)$$

respectively. Here we have introduced the distribution function for holes as (see Fig.2.6(c))

$$f_h(E) = 1 - f(E) = \frac{1}{1 + \exp(E_{\text{F}} - E)/k_{\text{B}}T)}. \quad (2.18)$$

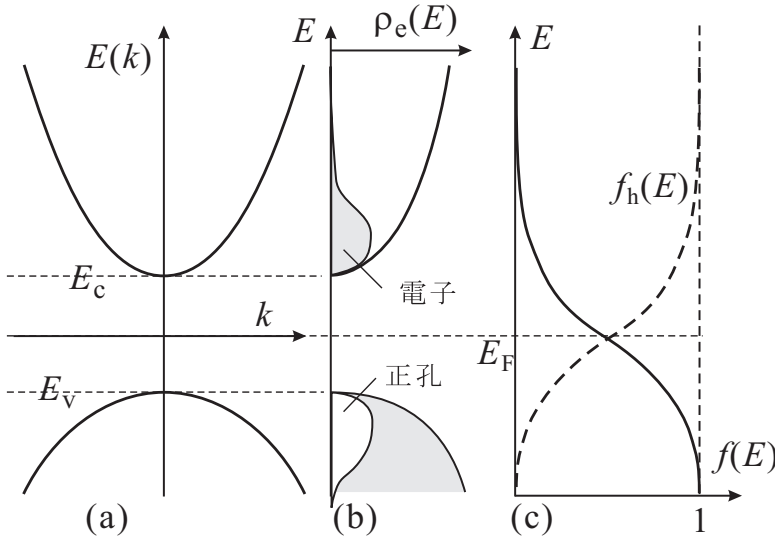


Figure 2.6: (a) Schematic chart of a conduction band and a valence band for an intrinsic semiconductor. (b) Density of states corresponding to the chart in (a). The electron distribution $n(E)$ is plotted in gray and the hole distribution $p(E)$ in white. (c) Distribution functions for electrons ($f(E)$, solid line), and for holes ($f_h(E)$, broken line).

For the density of states, we here adopt parabolic approximation. From (2.14),

$$\mathcal{D}_e(E) = \frac{\sqrt{2m_e^*3}}{\pi^2\hbar^3} \sqrt{E - E_c} \quad (\text{conduction band}), \quad (2.19a)$$

$$\mathcal{D}_h(E) = \frac{\sqrt{2m_h^*3}}{\pi^2\hbar^3} \sqrt{E_v - E} \quad (\text{valence band}). \quad (2.19b)$$

Here E_c , E_v are the bottom of the conduction band and the top of the valence band as indicated in Fig.2.6(a).

Now we can obtain the thermal distribution of electrons and holes as shown in Fig.2.6(b). The total densities of electrons n and holes p are given as

$$n = \int_{E_c}^{\infty} g_e(E) dE = \frac{\sqrt{2m_e^*3}}{\pi^2\hbar^3} \int_{E_c}^{\infty} \frac{\sqrt{E - E_c} dE}{1 + \exp(E - E_F)/k_B T}, \quad (2.20a)$$

$$p = \int_{-\infty}^{E_v} g_h(E) dE = \frac{\sqrt{2m_h^*3}}{\pi^2\hbar^3} \int_{-\infty}^{E_v} \frac{\sqrt{E_v - E} dE}{1 + \exp(E_F - E)/k_B T}. \quad (2.20b)$$

In the case of $f_F(E) \ll 1 (E \geq E_c)$, $f_h(E) \ll 1 (E \leq E_v)$, we can apply the approximation with the Maxwell distribution as

$$f_F(E) \sim \exp(E_F - E)/k_B T, \quad f_h(E) \sim \exp(E - E_F)/k_B T, \quad (2.21)$$

putting $x = (E - E_F)/k_B T$ and use an identity of a definite integral

$$\int_0^{\infty} \sqrt{x} e^{-x} dx = \sqrt{\pi}/2,$$

to obtain

$$n = 2 \left(\frac{m_e^* k_B T}{2\pi\hbar} \right)^{3/2} \exp\left(\frac{E_F - E_c}{k_B T}\right) \equiv N_c \exp\left(\frac{E_F - E_c}{k_B T}\right), \quad (2.22a)$$

$$p = 2 \left(\frac{m_h^* k_B T}{2\pi\hbar} \right)^{3/2} \exp\left(\frac{E_v - E_F}{k_B T}\right) \equiv N_v \exp\left(\frac{E_v - E_F}{k_B T}\right). \quad (2.22b)$$

N_c , N_v are the kind of fake but convenient density of states, which gives the numbers n , p when one calculate them as if one has these numbers of state at E_c , E_v respectively. N_c , N_v are thus called effective density of states.

From (2.20a), (2.20b)

$$np = N_c N_v \exp\left(\frac{E_v - E_c}{k_B T}\right) = N_c N_v \exp\left(-\frac{E_g}{k_B T}\right). \quad (2.23)$$

$E_g \equiv E_c - E_v$ is the energy gap, which is one of the most important parameters in semiconductor physics. As we will see (2.23) also holds for doped semiconductors.

In an intrinsic semiconductor, only electrons and holes have electric charges (when the concept of holes is introduced, the positive charges at nuclei are included into holes) and from the electrical neutrality $n = p$. Then the position of the Fermi energy E_F is given from

$$E_F = \frac{E_c + E_v}{2} + \frac{k_B T}{2} \ln \frac{N_v}{N_c} = \frac{E_c + E_v}{2} + \frac{3k_B T}{4} \ln \frac{m_h}{m_e}. \quad (2.24)$$

At low temperatures the second term is small and E_F comes close to the center of the band gap.