Physics of Semiconductors (13)

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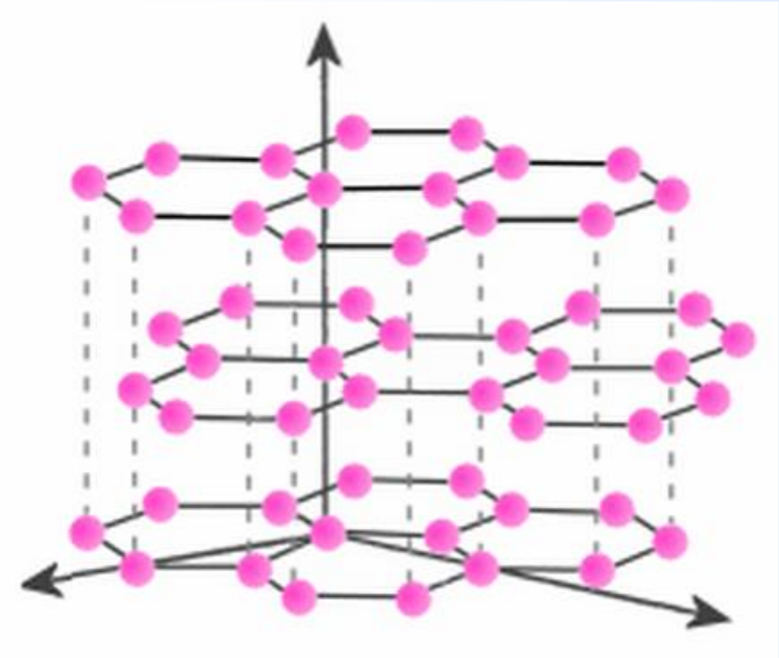
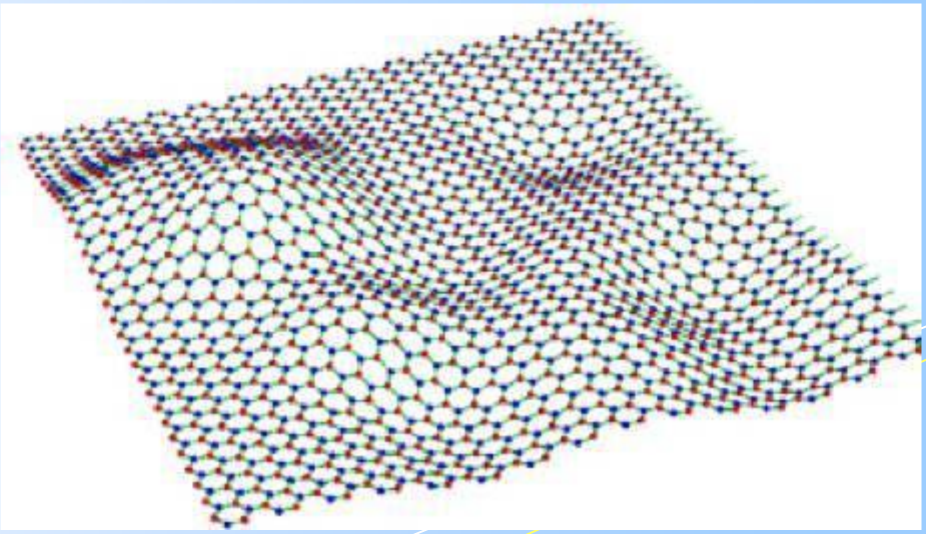
Graphite and Graphene

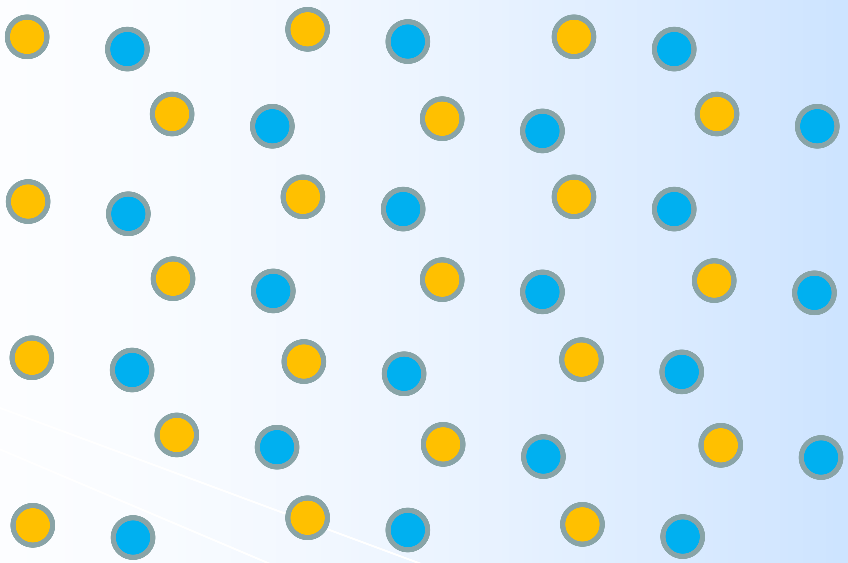


Graphite

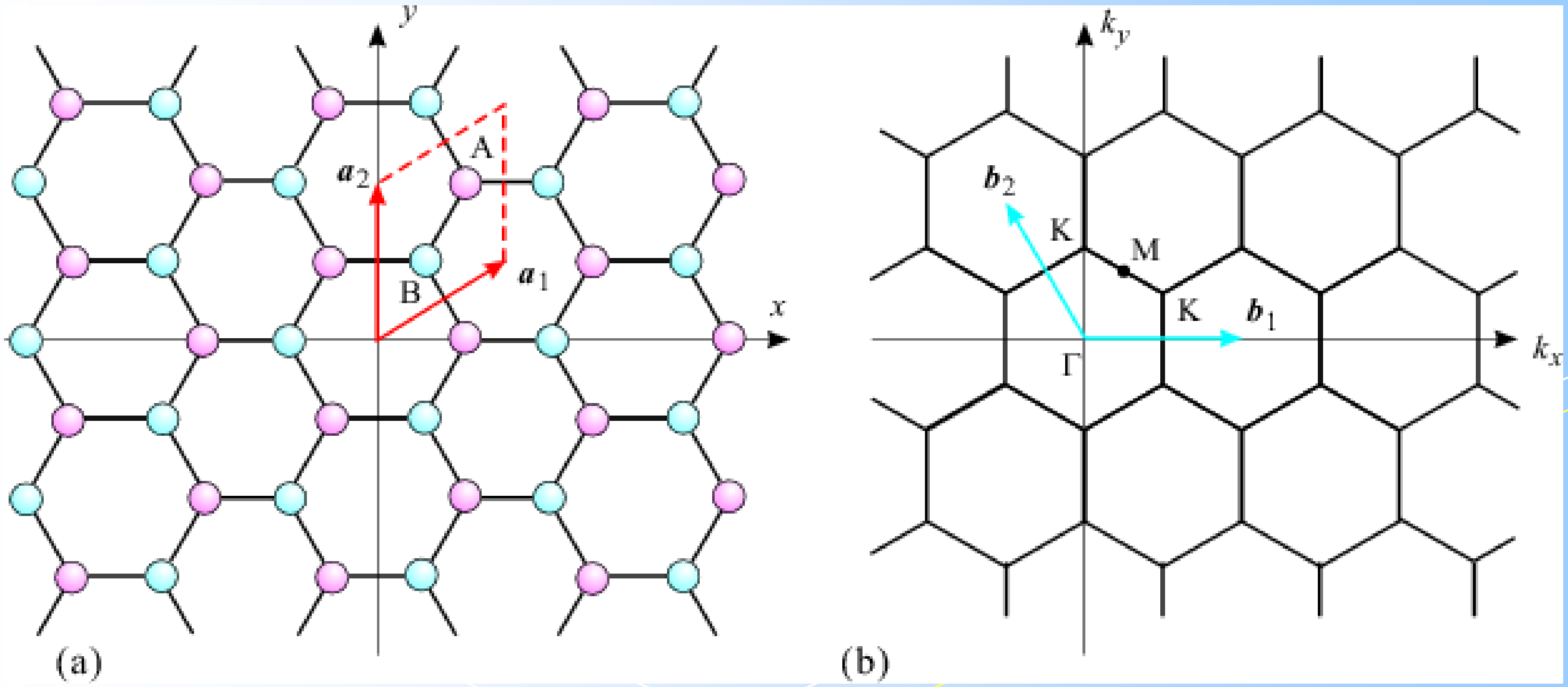


Graphene

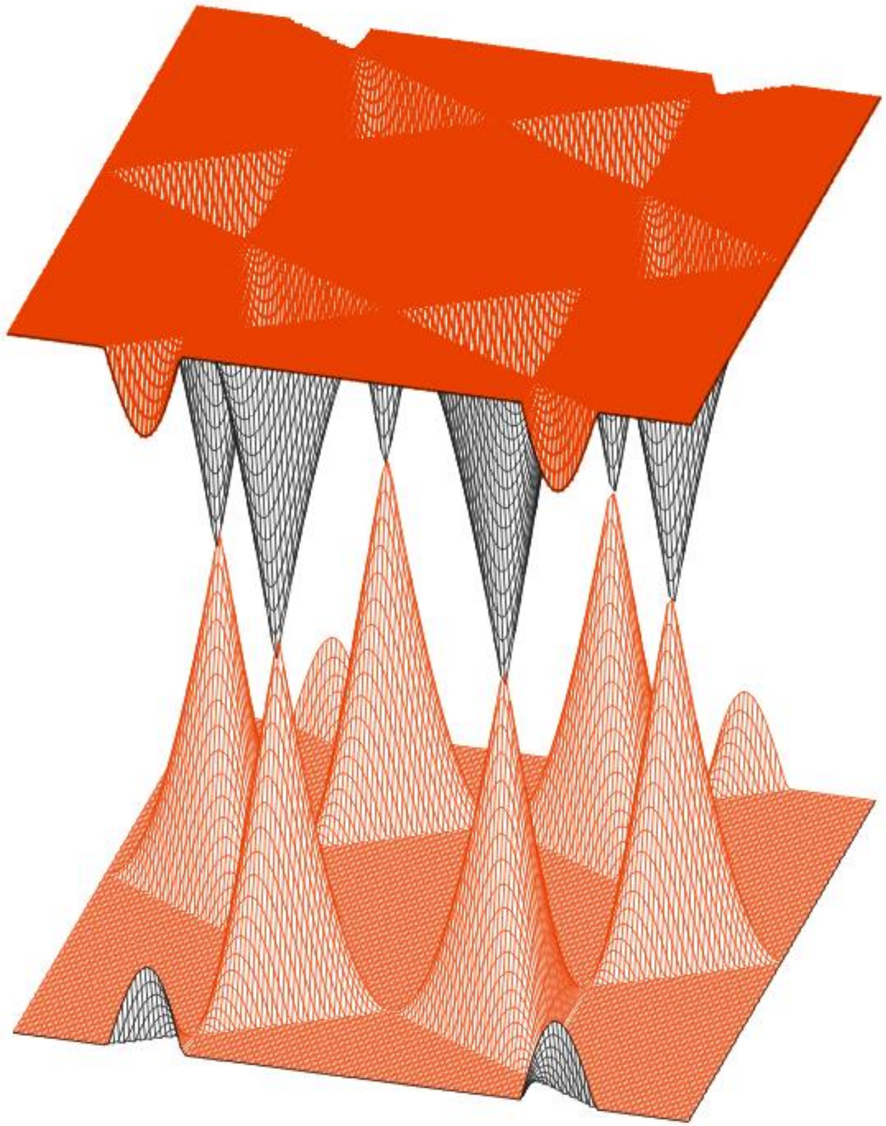
Graphene lattice and reciprocal lattice

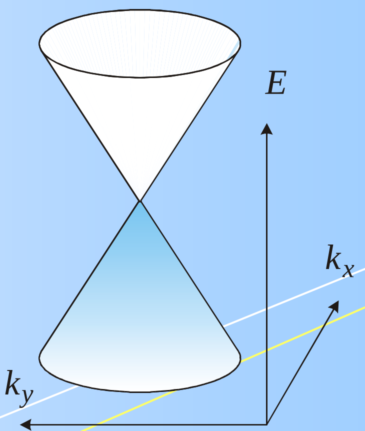


Graphene lattice and reciprocal lattice



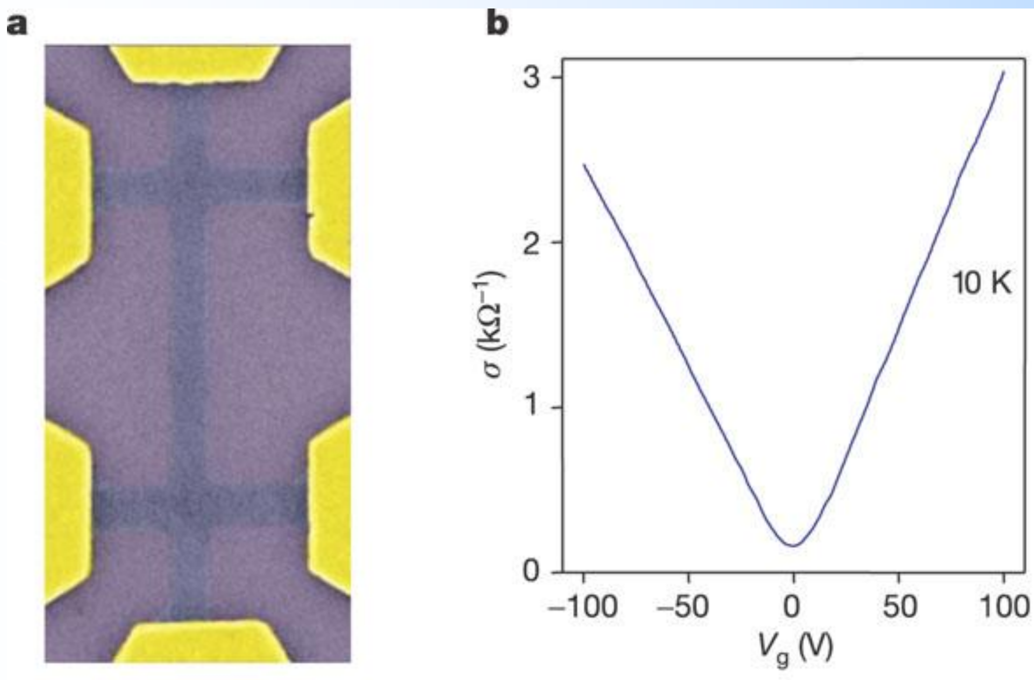
Dirac point, Dirac corn

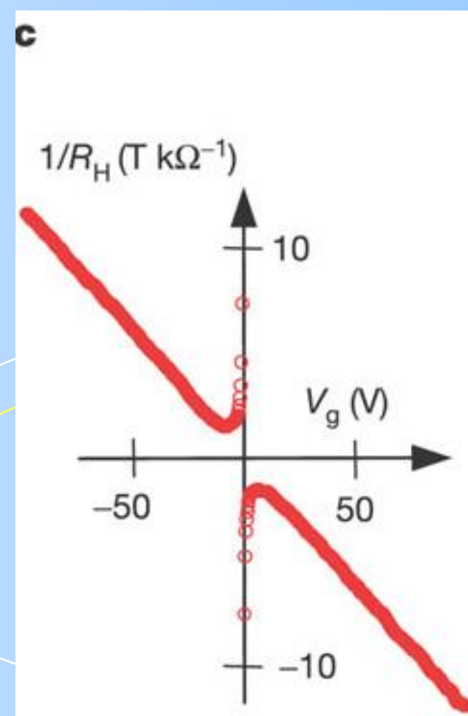




Transport in graphene

Andre Geim





Landau quantization in graphene

canonical momentum $\boldsymbol{p}_c = \boldsymbol{\pi} - e\boldsymbol{A} = \hbar\boldsymbol{k} - e\boldsymbol{A}$ cyclotron radius $r_c = \dfrac{\hbar k}{eB}$

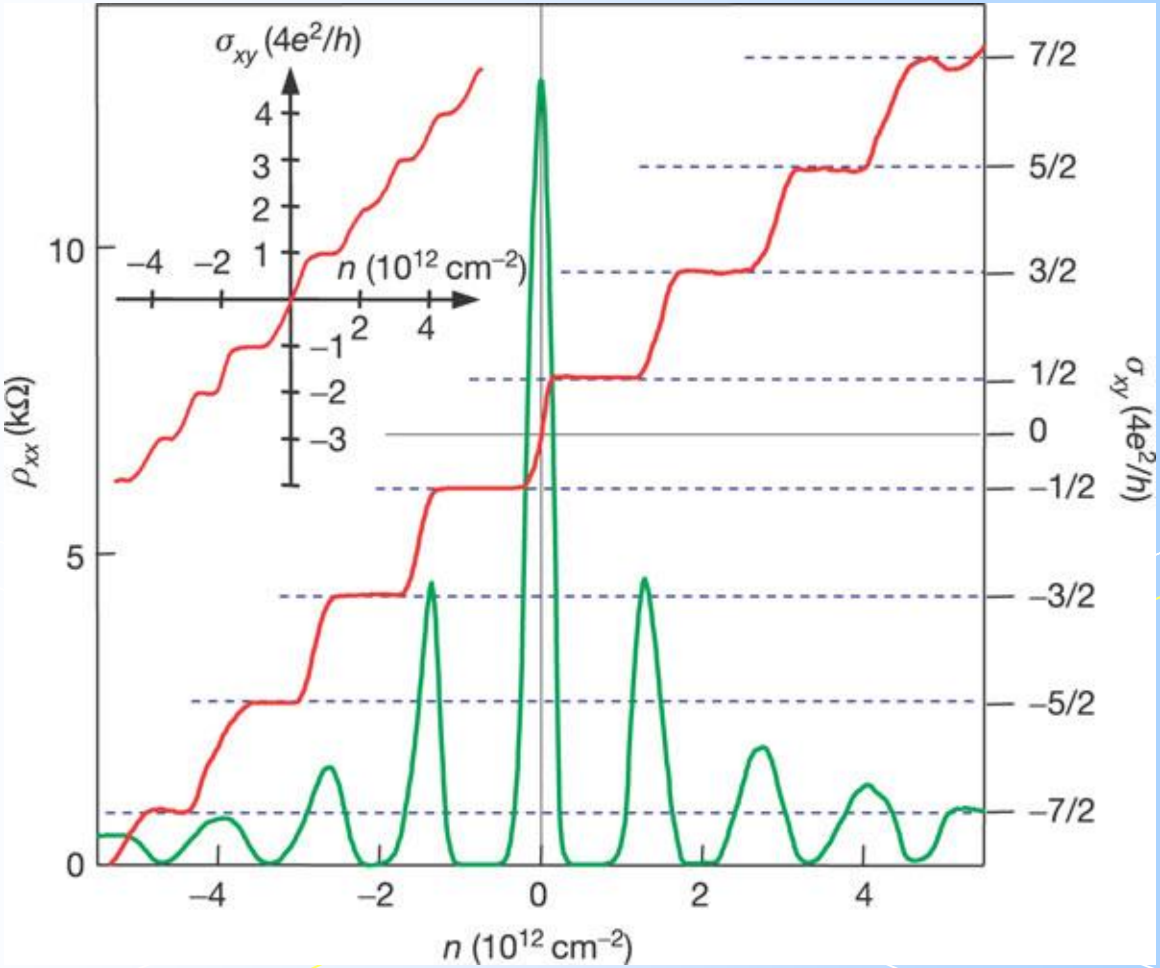
phase acquired along the circumference $2\pi r_c k + \theta_{\mathrm{AB}} = 4\pi\dfrac{\Phi}{\Phi_0} - 2\pi\dfrac{\Phi}{\Phi_0} = 2\pi\dfrac{\Phi}{\Phi_0}$

single-valuedness condition: $\Phi(= \pi r_c^2 B) = n\Phi_0 = \dfrac{nh}{e} \quad \therefore\ r_c = \sqrt{\dfrac{2n\hbar}{eB}}$

Energy dispersion: $E = c\hbar k$

then $E = c\hbar k = \sqrt{2e\hbar c^2 Bn}$

Integer quantum Hall effect in graphene



Integer quantum Hall effect in graphene

$$\sigma_{xy} = 4\left(n + \frac{1}{2}\right) G_q$$

Degeneracy from K and K', and A, B sublattice

$\boldsymbol{k} = \boldsymbol{K} + \boldsymbol{q}$ $\quad$ $k \cdot p$ perturbation $\quad$ ζ_A, ζ_B envelope functions

$$\frac{\hbar^2}{2m}\sqrt{3}\xi \begin{pmatrix} 0 & q_x - iq_y \\ q_x + iq_y & 0 \end{pmatrix} \begin{pmatrix} \zeta_\mathrm{A} \\ \zeta_\mathrm{B} \end{pmatrix} = E \begin{pmatrix} \zeta_\mathrm{A} \\ \zeta_\mathrm{B} \end{pmatrix}$$

Landau gauge $\quad \boldsymbol{A} = (0, Bx)$

$$\hbar \begin{pmatrix} 0 & -i\frac{\partial}{\partial x} - i\frac{x}{l^2} - \frac{\partial}{\partial y} \\ -i\frac{\partial}{\partial x} + i\frac{x}{l^2} + \frac{\partial}{\partial y} & 0 \end{pmatrix} \begin{pmatrix} \zeta_\mathrm{A} \\ \zeta_\mathrm{B} \end{pmatrix} = E \begin{pmatrix} \zeta_\mathrm{A} \\ \zeta_\mathrm{B} \end{pmatrix}$$

Integer quantum Hall effect in graphene

Variable separation

$$\zeta_{\mathrm{A}}(\boldsymbol{q},\boldsymbol{r}) = \exp(iq_y y)\eta_{\mathrm{A}}(q_x,x), \quad \zeta_{\mathrm{B}}(\boldsymbol{q},\boldsymbol{r}) = \exp(iq_y y)\eta_{\mathrm{B}}(q_x,x)$$

$$\begin{pmatrix} 0 & -i\left(\frac{\partial}{\partial x}+\frac{x}{l^2}\right)-iq_y \\ -i\left(\frac{\partial}{\partial x}-\frac{x}{l^2}\right)+iq_y & 0 \end{pmatrix}\begin{pmatrix}\eta_{\mathrm{A}}\\ \eta_{\mathrm{B}}\end{pmatrix} = E\begin{pmatrix}\eta_{\mathrm{A}}\\ \eta_{\mathrm{B}}\end{pmatrix}$$

$$\left[-\frac{\partial^2}{\partial x^2}+\left(q_y-\frac{x}{l^2}\right)^2\right]\eta_{\mathrm{A}} = \left(\epsilon^2-\frac{1}{l^2}\right)\eta_{\mathrm{A}},$$

$$\left[-\frac{\partial^2}{\partial x^2}+\left(q_y-\frac{x}{l^2}\right)^2\right]\eta_{\mathrm{B}} = \left(\epsilon^2+\frac{1}{l^2}\right)\eta_{\mathrm{B}}$$

$$E = \pm\sqrt{2e\hbar c^2 B(n_{\mathrm{A}}+1)},$$

$$E = \pm\sqrt{2e\hbar c^2 B n_{\mathrm{B}}}$$

Degeneracy lifted at $E = 0$

Fractional quantum Hall effect