Physics of Semiconductors (4)

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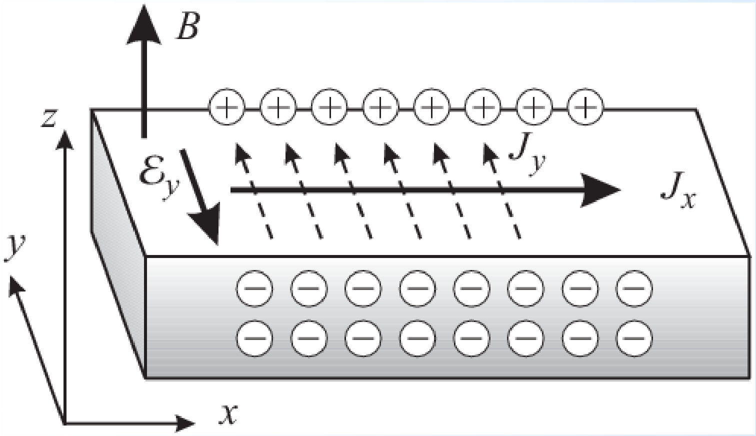
Hall effect



Hall coefficient : $$R_H = \frac{\mathcal{E}_y}{J_x B_z}$$

$$R_H = -\frac{1}{ne}\frac{\langle\tau^2\rangle_E}{\langle\tau\rangle_E^2} = \frac{1}{n(-e)}\frac{\Gamma(2s+5/2)\Gamma(5/2)}{(\Gamma(s+5/2))^2} = \frac{r_H}{n(-e)}$$

Thermal transport and electric transport

Thermal flux

$$j_{qx} = \langle n v_x (E-\mu) \rangle = \int_0^\infty v_x (E-\mu) f(E) \mathscr{D}(E) dE.$$

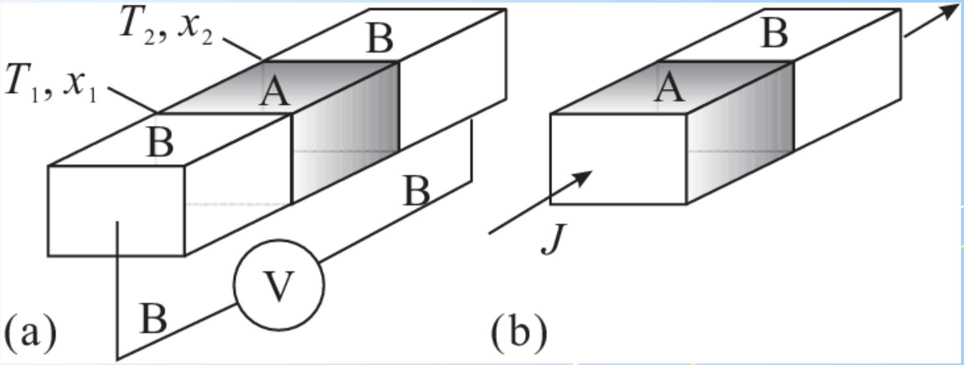
Thermal conductivity

$$\kappa_n = -\frac{j_{qx}}{\partial T / \partial x}.$$

vector format

$$\boldsymbol{j}_q = -\hat{\kappa} \nabla T.$$

Thermoelectric effect



Seebeck effect

$$S_{\mathrm{AB}} = \frac{V_{\mathrm{AB}}}{\Delta T}$$

Peltier effect

$$\Pi_{\mathrm{AB}} = \frac{Q_{\mathrm{AB}}}{J}$$

Thomson effect

$$\tau = \frac{\partial Q / \partial x}{J (\partial T / \partial x)}$$

Kelvin relation

$$\Pi_{\mathrm{AB}} = S_{\mathrm{AB}} T, \quad \tau_{\mathrm{A}} - \tau_{\mathrm{B}} = T \frac{d S_{\mathrm{AB}}}{dT},$$

Boltzmann equation and thermoelectric coefficients

Material specific Seebeck coefficient

$$S_A(T) \equiv \int_0^T \frac{\tau_A(T')}{T'} dT'$$

$$S_{AB} = S_A - S_B$$

Boltzmann equation for steady states

$$\boldsymbol{v} \cdot \nabla f + \frac{\boldsymbol{F}}{m} \nabla_v f = -\frac{f - f_0}{\tau(E)}.$$

Replace f in LHS with f_0.

$$\nabla f_0 = \nabla T \frac{\partial f_0}{\partial T}.$$

$$a = -\frac{E - E_F}{k_B T}$$

$$\frac{\partial f_0}{\partial T} = \frac{\partial f_0}{\partial E} \frac{\partial E}{\partial a} \frac{\partial a}{\partial T} = \frac{\partial f_0}{\partial E} (-k_B T) \frac{E - E_F}{k_B T^2} = \frac{\partial f_0}{\partial E} \frac{E_F - E}{T}$$

$$\nabla f_0 = \nabla T \frac{E_F - E}{T} \frac{\partial f_0}{\partial E}.$$

$$\nabla_v f_0 = \nabla_v E \frac{\partial f_0}{\partial E} = m\boldsymbol{v} \frac{\partial f_0}{\partial E}.$$

$$f = f_0 - \tau(E) \boldsymbol{v} \cdot \left[-e\mathbf{E} + \frac{E_F - E}{T} \nabla T \right] \frac{\partial f_0}{\partial E}.$$

Boltzmann equation and thermoelectric coefficients

$$j_x = -e\langle n v_x \rangle = -e \int_0^\infty v_x f(E)\mathscr{D}(E)dE = e \int_0^\infty v_x^2 \tau \left[-e\mathcal{E}_x + \frac{E_F - E}{T}\frac{\partial T}{\partial x} \right] \frac{\partial f_0}{\partial E} \mathscr{D}(E)dE.$$

$$j_x = 0,$$

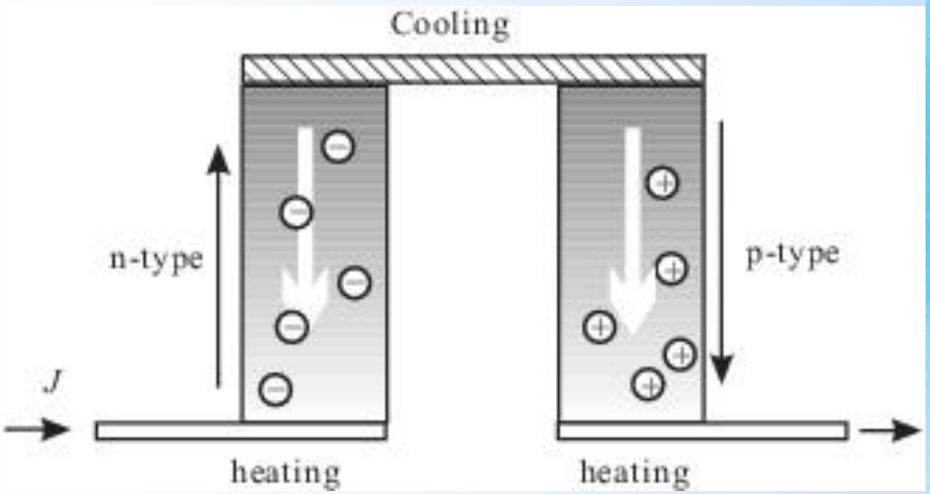
$$S = \frac{\mathcal{E}_x}{\partial T/\partial x} = \int_0^\infty \frac{E_F - E}{eT}\frac{\partial f_0}{\partial E}\mathscr{D}(E)dE \Big/ \int_0^\infty v_x^2 \tau \frac{\partial f_0}{\partial E}\mathscr{D}(E)dE$$
$$= \frac{1}{eT}\left[E_F - \int_0^\infty \tau E^2 \frac{\partial f_0}{\partial E}\mathscr{D}(E)dE \Big/ \int_0^\infty \tau E \frac{\partial f_0}{\partial E}\mathscr{D}(E)dE \right].$$

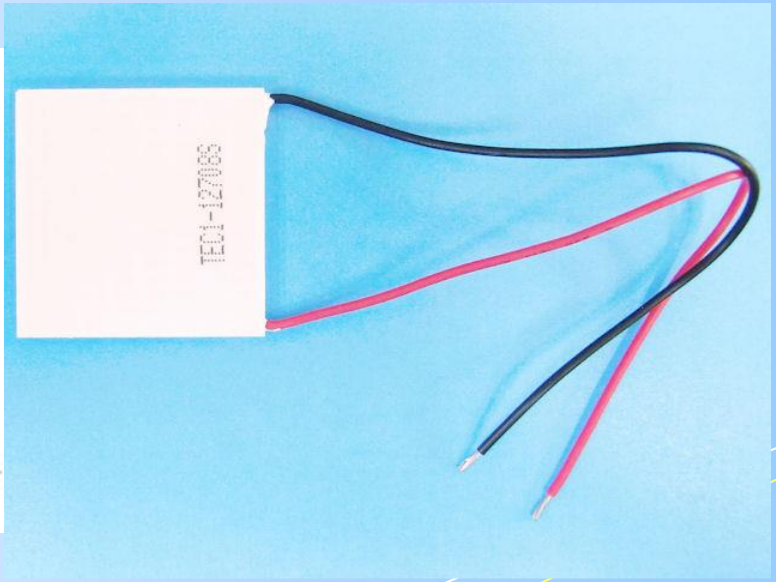
Maxwellian approximation $\partial f_0/\partial E = -f_0/k_B T$,

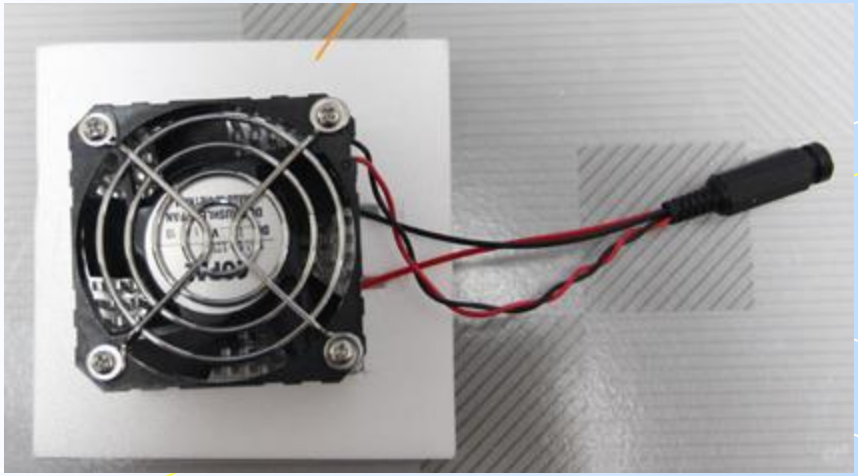
$$\tau \propto E^s,$$

$$S = -\frac{1}{eT}\left[\frac{\langle \tau E \rangle_E}{\langle \tau \rangle_E} - E_F \right] = -\frac{1}{eT}\left[\left(\frac{5}{2} + s \right) k_B T - E_F \right]$$

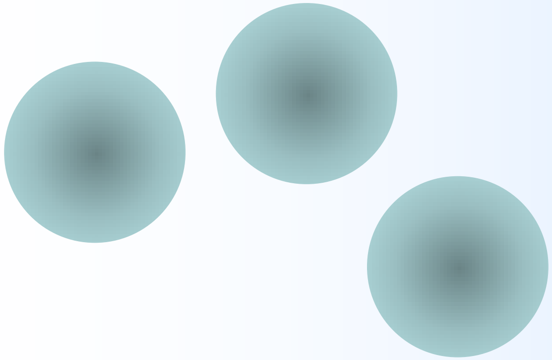
Peltier device



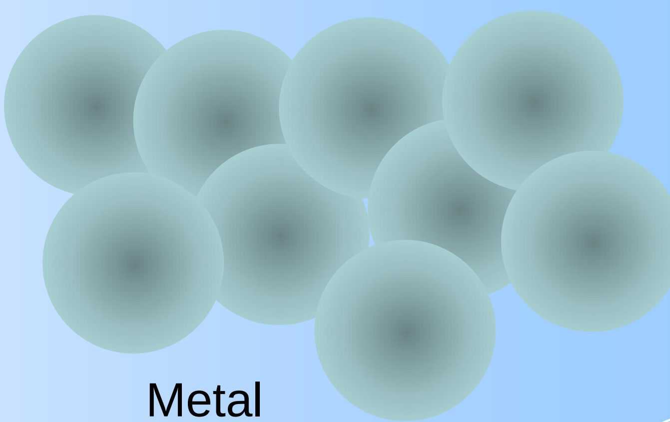




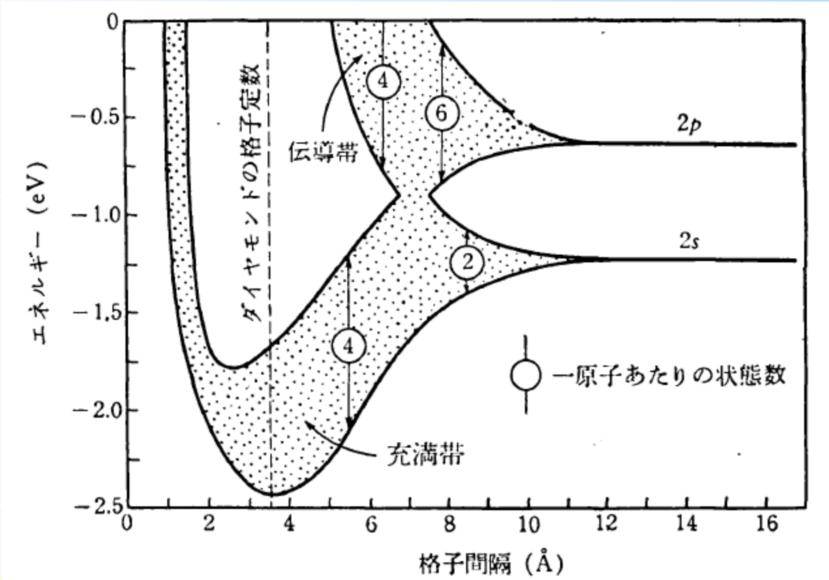
Metal-insulator transition



Insulator



Metal



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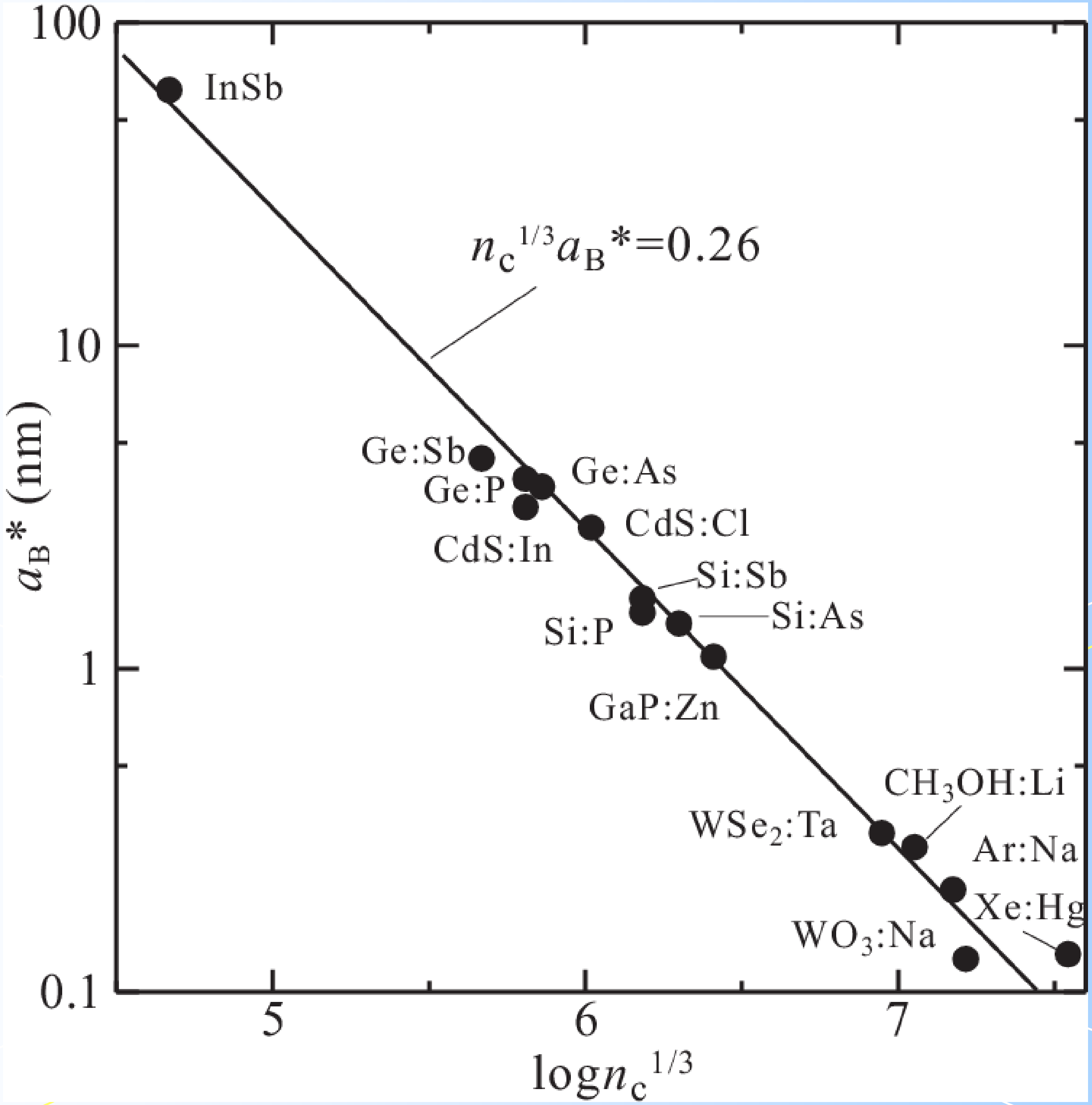
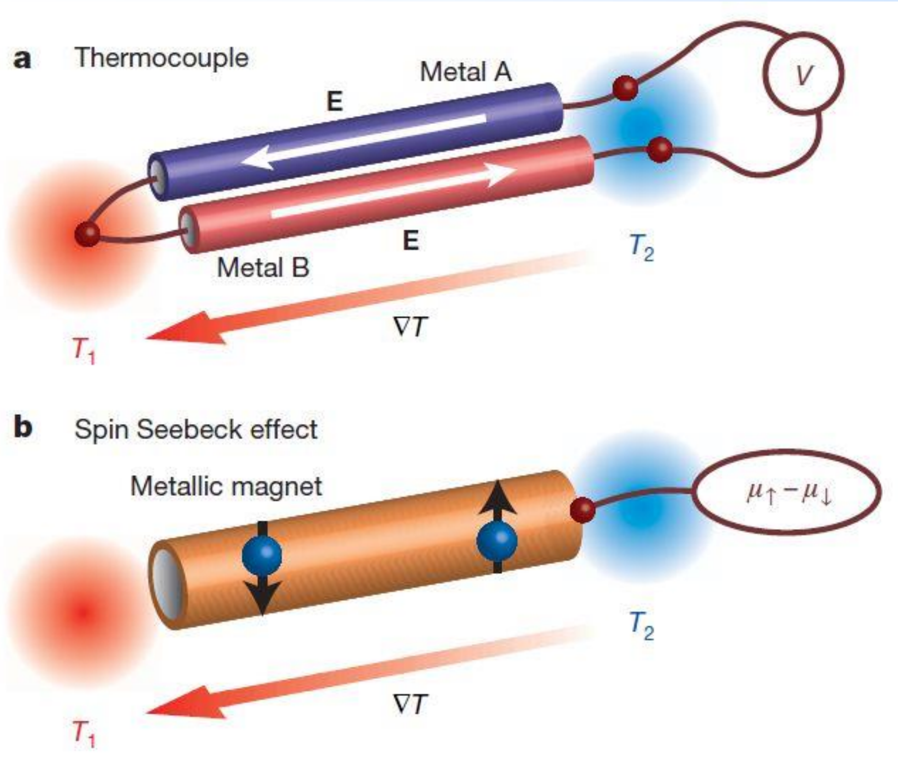
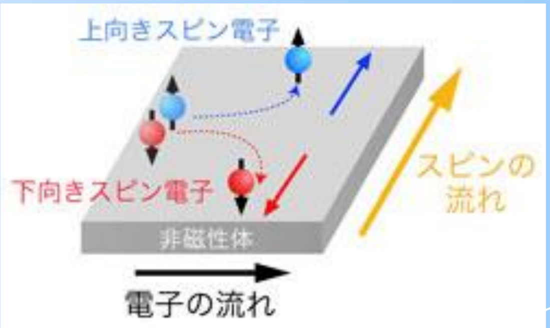


Spin Seebeck effect

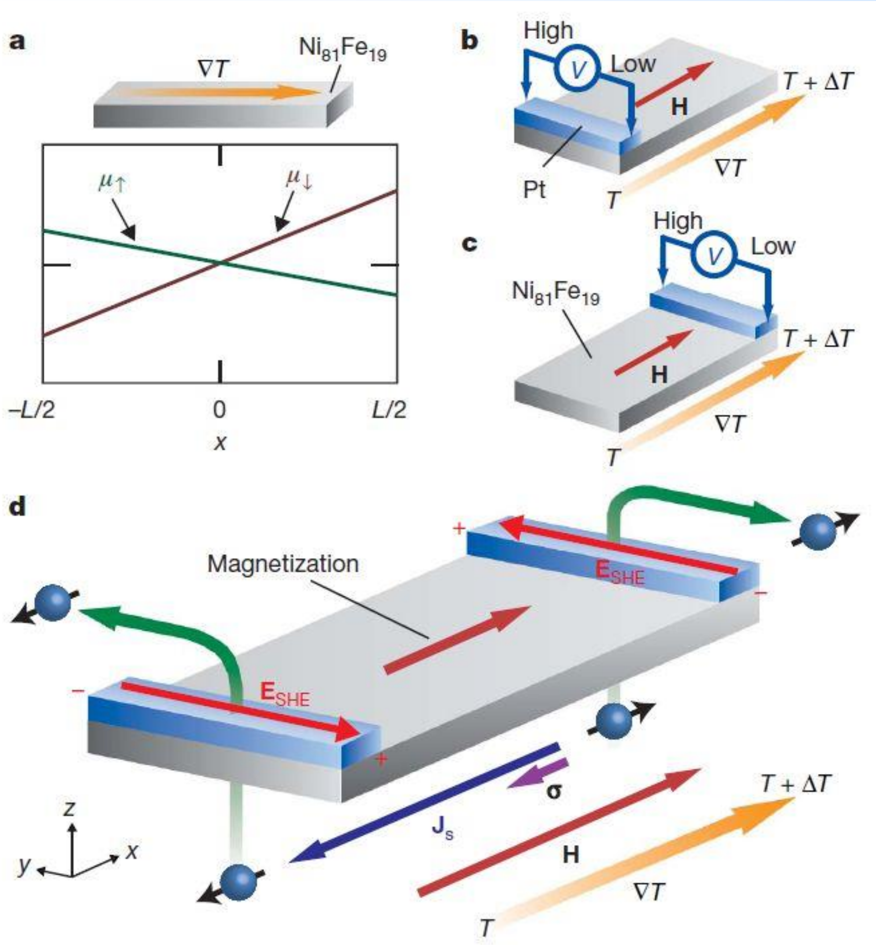


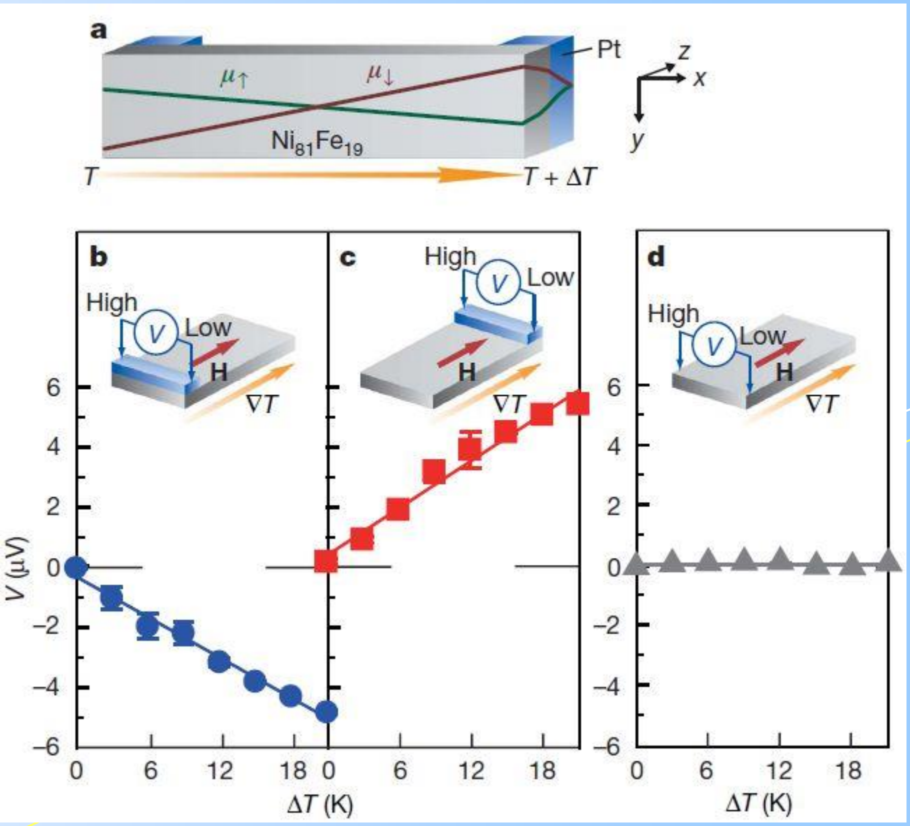
$$E_{\mathrm{SHE}} = D_{\mathrm{ISHE}} \boldsymbol{J}_s \times \boldsymbol{\sigma}_s$$



K. Uchida, S. Takahashi, K. Harii, J. Ieda, W. Koshibae, K. Ando, S. Maekawa and E. Saitoh, Nature **455**, 778 (2008).

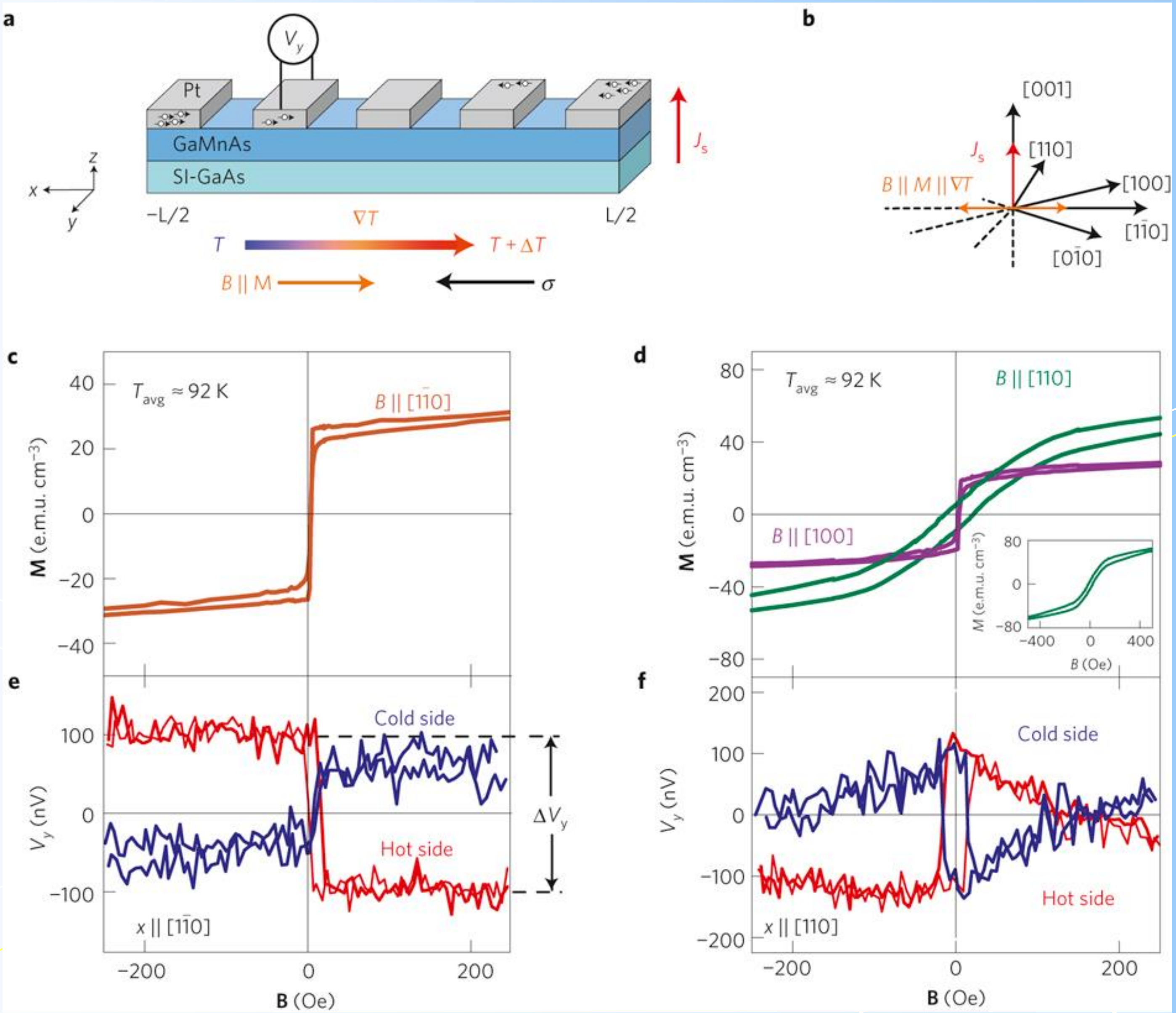
Spin Seebeck effect





Spin Seebeck effect (?) in a ferromagnetic semiconductor

Jarowski *et al.*
Nature Materials
9, 898 (2010)



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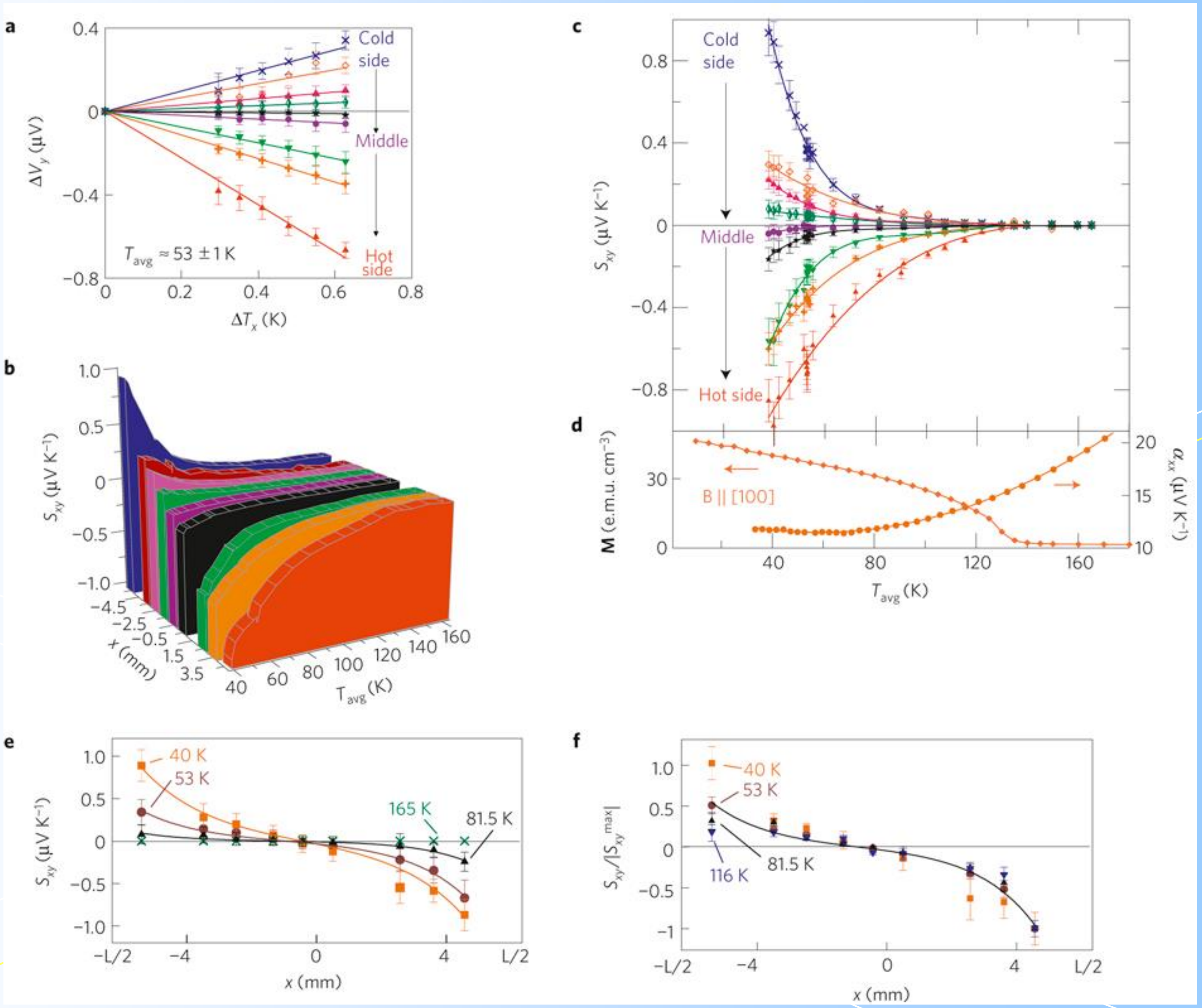

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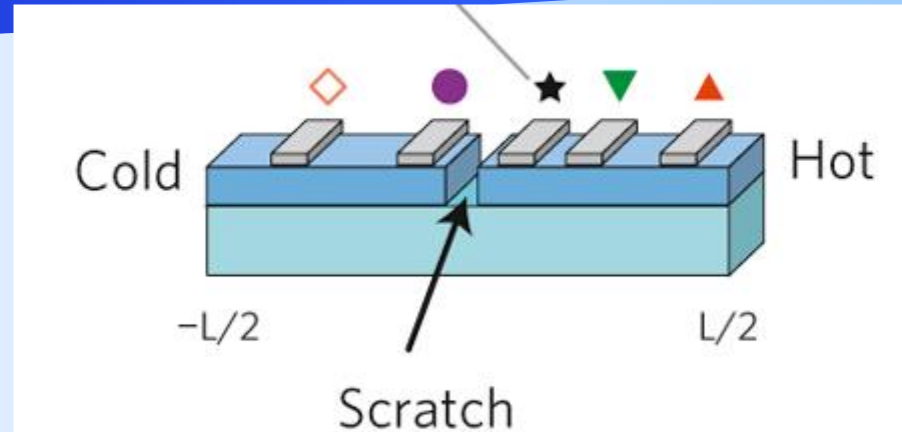

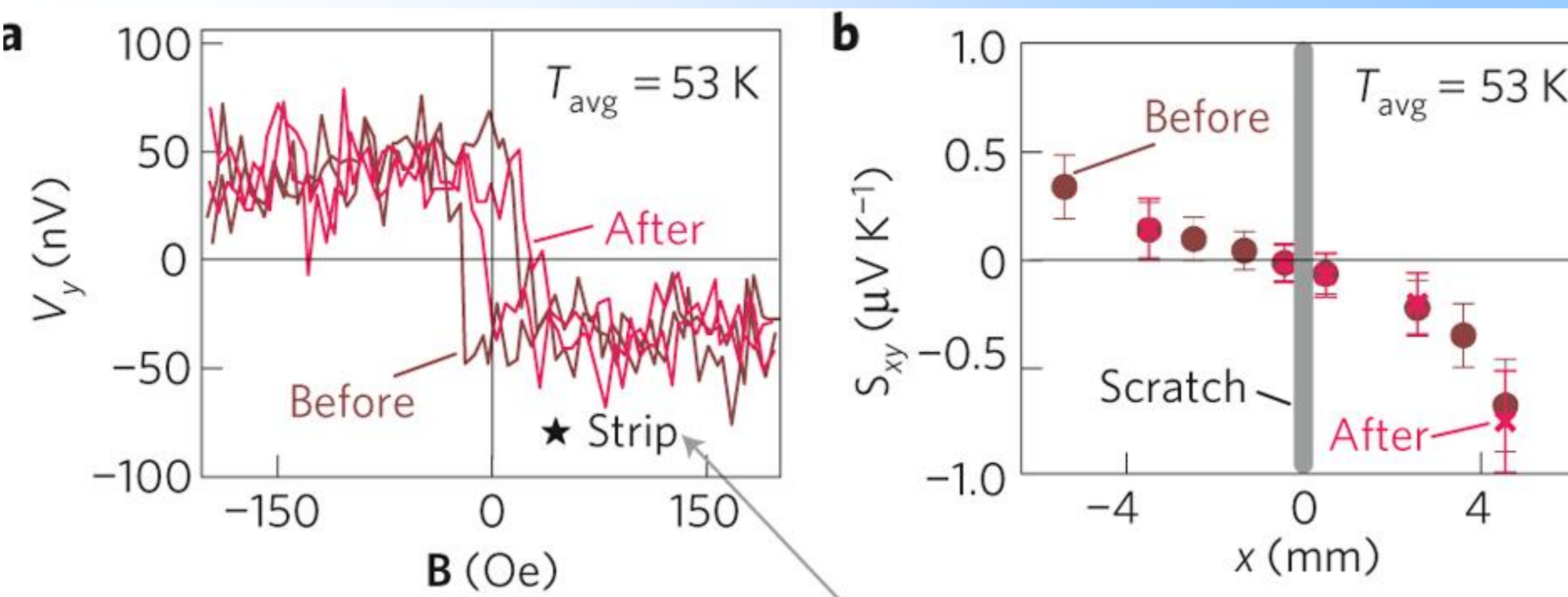

Exercise

1. p-Ge has Seebeck coefficient of 300μV/K and n-type Bi_2Te_3 –230μV/K. If one makes a thermocouple from these two materials, how high is the voltage caused by the temperature difference of 50K.

2. Obtain expressions for electron mobility and for diffusion constant in terms of the conductivity and the Hall coefficient.

3. Obtain the Mott's criterion $n_c{}^{1/3}a_B{}^*=0.26$ along the following guide.

Obtain the condition for the Schrodinger equation with screened Coulomb potential to have bound state with variational method.

$$\left[-\frac{\hbar^2}{2m^*}\nabla^2-\frac{e^2}{4\pi\varepsilon_0\varepsilon r}e^{-\frac{r}{\lambda}}\right]\psi(\boldsymbol{r})=E\psi(\boldsymbol{r}),\qquad \lambda=\sqrt{\frac{2\varepsilon\varepsilon_0E_\mathrm{F}}{3ne^2}}$$

$$E_F=\frac{\hbar^2}{2m^*}(3\pi^2n)^{2/3}$$